

**Modeling of Tundish Steelmaking Process in Continuous Casting**  
**Prof. Pradeep K. Jha**  
**Department of Mechanical and Industrial Engineering**  
**Indian Institute of Technology, Roorkee**

**Lecture - 21**  
**Introduction to Turbulence in Fluid Flow**

Welcome to the lecture on Introduction to Turbulence in Fluid Flow. So, we had the discussion about the governing equations for fluid flow, and when we talk about fluid flow then in that case the flow may be laminar or may be turbulent and we need to know you know things about turbulence because most of the flows which we will deal they will be of the turbulent nature.

So, what are the essential terminologies? And what turbulence means? What needs to be modeled while we are solving for the you know fluid flow problems in the tundish or in any vessel where the flow is turbulent? So, we will have some introduction about these terms and we will know more about these turbulent flows modeling and their associated terms you know in our coming lectures.

So, as you know that when we talk about the flow then we basically decide by the Reynolds number whether the flow is laminar or turbulent.

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### Introduction

- ❖ At low Reynolds numbers, flows are laminar.
- ❖ At higher Reynolds numbers, flows are observed to become turbulent.
- ❖ The Reynolds number of a flow gives a measure of relative importance of inertia forces (associated with convective effects) and viscous forces.



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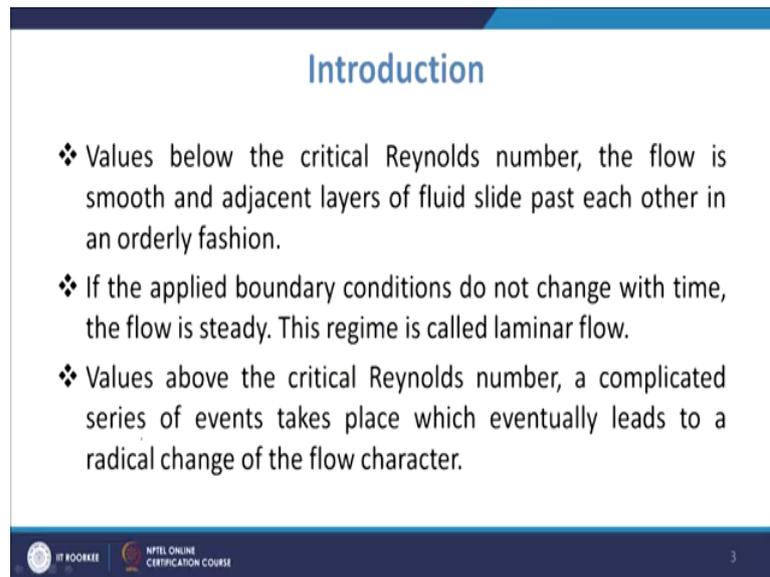
So, the Reynolds number value, so that will be less, in that case it will be laminar and if it is more, so at low Reynolds number flow will be laminar and at high Reynolds number flow is observed to be you know turbulent. So, your Reynolds number for that the expression will be re equal to  $\frac{u l}{\eta}$ . So, you will have these characteristic velocity characteristic length and then the kinematic viscosity of the fluid in the denominator. So, that way you get these value of the Reynolds number and then you differentiate whether the flow is laminar or the turbulent.

So, certainly, there will be also a situation when there will be transition from laminar to turbulence, so that will be you know another thing which will be there. Now, Reynolds number of flow gives a measure of relative importance of inertia forces and viscous forces. So, you know as we know that Reynold number is inertia force upon viscous force, so if it is of smaller value it means viscous forces are more dominant. And if the Reynolds number is very high it means inertia forces are dominating, so that way you know whenever you will have those reasons, so depending upon the Reynolds number we can say that which kind of forces are you know dominating which is more you know prominent.

Now, what happens that when we talk about the turbulence? So, basically you will have fluctuations. Now, the difference between the laminar and turbulence is that in case of turbulence what was happening that those length scales become quite large over which the mixing takes place. There will be fluctuations in the variable values like velocity, pressure or source, so they will be fluctuating over the mean values. So, in fact, the fluctuation in the velocities will give rise to another additional stress which we have seen that certainly that, in earlier lectures we have seen that you have stress terms and they are expressed in terms of you know velocity gradient term and all that.

Now, in this case your fluctuation terms which you are encountering because of the turbulence, now they lead to extra or additional stress terms. So, they are known Reynolds stress terms. So, they need to be modeled, they need to be taken into account. So, so these things are important when we talk about the Reynolds number of flow and as you see that you will have depending upon the number you see that which force is important, now you know that you have a critical Reynolds number. So, before below that it is laminar and above that it is you know turbulent.

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The slide is titled "Introduction" in blue text. It contains three bullet points, each starting with a blue diamond symbol. The first bullet point states that values below the critical Reynolds number result in smooth flow where adjacent layers slide past each other orderly. The second bullet point states that if boundary conditions are constant over time, the flow is steady and laminar. The third bullet point states that values above the critical Reynolds number lead to a radical change in flow character. At the bottom of the slide, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE, along with the number 3 in the bottom right corner.

- ❖ Values below the critical Reynolds number, the flow is smooth and adjacent layers of fluid slide past each other in an orderly fashion.
- ❖ If the applied boundary conditions do not change with time, the flow is steady. This regime is called laminar flow.
- ❖ Values above the critical Reynolds number, a complicated series of events takes place which eventually leads to a radical change of the flow character.

So, when the value of the critical Reynolds number, I mean value of the Reynolds number is below the critical Reynolds number then the flow is laminar and in that you have the adjacent you know when the flow is smooth and adjacent layers of fluid slide past each other. So, that is what the new traits of the laminar flow is that your flow will be smooth and your adjacent layers will be sliding one above other. So, they will not be mixing you know across you know in the cross devised manner.

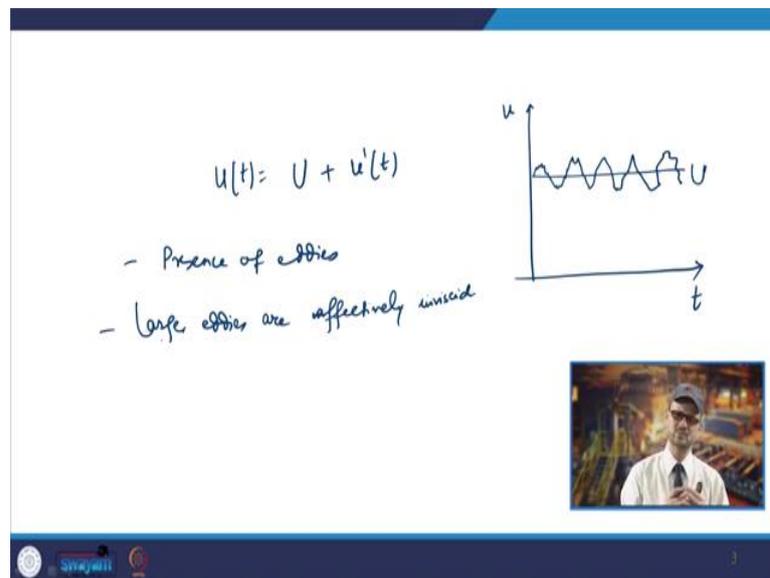
So, they will be going and then there may be some diffusion of the atoms will be taking place you know from the bottom layer to the top layer or top layer to the bottom layer like that. So, that is what is happening in the case of the laminar flow. And in that case if your applied boundary conditions are not changing with the time then we call it the flow is steady and the regime will be the laminar flow.

Now, there is another regime that is your turbulent regime and that is that you are encountering when you see that the Reynolds number value is going you know beyond a certain value that is critical value and in that basically there will be complicated series of events will be taking place. So, you that I mean there will be radical change of the flow. So, you will have a lot of you know events taking place in that and basically what happens that finally, the behavior of the fluid that becomes a random and chaotic.

In this it is in a very defined manner the flow is flowing one above other flow is there, but in that case it becomes very chaotic as well as random, there will be mixing, there will be

eddies, smaller eddies will be there, larger eddies will be there, so all the diffusion which is taking place they will be at higher scales. So, because there may be you know diffusion or the mixing over the larger you know bulk, so maybe if you have 1, 2, 3 or 4 and 5 layers, so maybe 1 and 5 layer may mix because of these turbulence. So, that is the basic difference. So, your behavior that that is what it there are radical changes in the flow character you have your flow becomes random and chaotic and you know that regime which you get that is known as the you know turbulent flow.

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So, if you try to see the you know the velocity measurement in case of a turbulent flow then the velocity measurement basically will be like you will have the time here and if your velocity is here; so, what happens that your flow will be, so suppose this is  $u$ , so your actual flow will be like it will be going like this. So, it will be going, it will be fluctuating. So, in normal case you can assume it to be you know fluctuating over a certain mean value. So, that is. So, you will have a steady state mean value that is  $u$ . So, you will your you know this value that is will be in the case of turbulent flow you will have a mean value and you have the fluctuating component.

So, basically at any time  $u(t)$  will be  $u$ , so that will be your mean component and then you will have a fluctuating component, so that is  $u'(t)$ . So, that way the turbulence you know in case of turbulence the flow is basically characterized, and you will have this for all the properties like either as  $u$  or  $v$ ,  $t$  or  $w$  or  $p$ . So, for all that you will have a mean value and

a fluctuating component this component basically you know that component is this component above the mean that is the fluctuating component that is  $u'(t)$ . So, that way you so you will have many statistical descriptors will be used while we try to know more about the you know the turbulent flow.

Now, while in normal case when you you will have two space dimensions for the velocity of pressure now turbulent fluctuations when there are they will have the 3-dimensional character. So, that is the trait about the turbulence. Now, in this case what happens that you have the presence of eddies and if you look at the you know turbulent flow. So, in that flow structures you will have the presence of eddies.

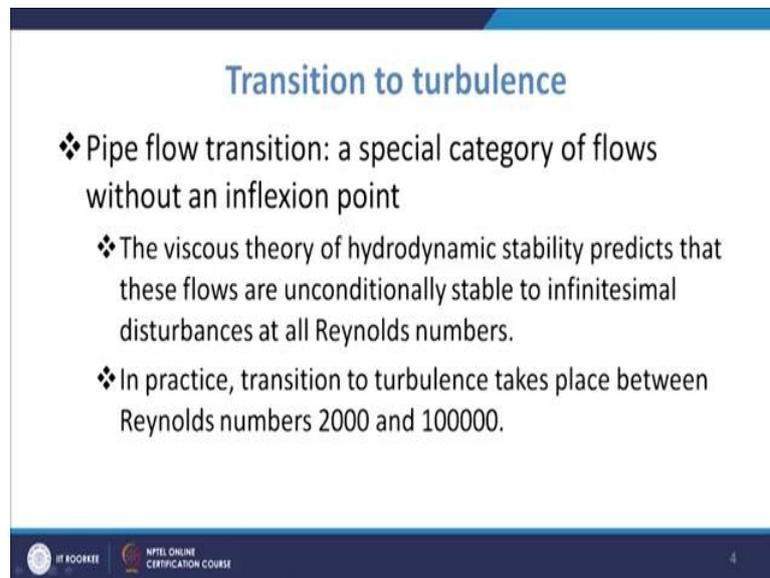
So, these eddies may be of you know smaller dimension or the larger dimension and they can be or in of size, so they can maybe of a smaller size, intermediate size or even the larger size may be to the extent of the length of the vessel or so. So, that way you will have the presence of eddies and in that because of these eddies you know the point which are largely separated in the vessel they also come in intimate contact because of the flow structures in the case of turbulent flow and there will be very high value of diffusive transfer as in transfer of mass, heat or momentum in the case of turbulence. So, that is the basically trait about the turbulent flow. So, you will have the production of or the genesis of very high value of the coefficient of diffusion when we talk about the turbulent flows.

Now, the thing is that in this case you have the characteristic velocity and the characteristic length, and they are of the larger eddies are basically same of the for the larger eddies suppose the characteristic length that is  $l$  will be same as the you know length scale  $l$  of the mean flow. So, similarly the velocity also characteristic velocity for the larger eddies will be same as that of the mean flow velocity  $u$ . So, that is normally there in the case of the turbulent you know flows.

Now, when we talk about the other scales or the other properties, suppose if you go for those when the turbulent is very small, suppose turbulent if you talk about the Reynolds number. Now, in the Reynolds number you have the ratio that is inertia force by viscous force. So, when your inertia force and viscous force will be somewhat nearly equal in that case Reynolds number is close to 1, so that tells that your viscous force is quite high. So, it is something that that is why it is close to about 1. Otherwise when you go in the upstream, so that is your depending upon, so if you take suppose the  $\frac{ul}{\eta}$ , if the  $u$  is 1 meter

per second,  $l$  is 1 meter and your  $\eta$  in normal case becomes  $10^{-6}$  or so, so that in that also you see that the value of the Reynolds number becomes  $10^{-6}$ . So, that is how you know the, so that will be inertia dominated kind of flow.

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The slide is titled "Transition to turbulence" in blue text. It contains three bullet points, each preceded by a diamond symbol (❖). The first bullet point states: "Pipe flow transition: a special category of flows without an inflexion point". The second bullet point states: "The viscous theory of hydrodynamic stability predicts that these flows are unconditionally stable to infinitesimal disturbances at all Reynolds numbers." The third bullet point states: "In practice, transition to turbulence takes place between Reynolds numbers 2000 and 100000." At the bottom of the slide, there are logos for "IIT ROORKEE" and "NPTEL ONLINE CERTIFICATION COURSE" on the left, and a small number "4" on the right.

So, we will come to this further if you try to know about the turbulence. So, now, you will have the transition to turbulence and in the case of a different kind of flows you will have a Reynolds number which will say that here the transition takes place and you know in the case of pipe flow transition you will have the category of flow without inflexion point.

So, basically there are two cases there may be the presence of inflexion point when you have the transition to turbulence or you may have the case without the you know the presence of these inflexion points. So, that way you will have, so that depends upon the different situations.

Now, the viscosity of hydrodynamic stability predicts that these flows are unconditionally stable to infinitely disturb to disturbances at all Reynolds number in case of pipe flow. And in the case of pipe flow, the transition to turbulence will be taking place between Reynolds number 2000 and 100000. So, basically normally what we might have studied about the transitions in case of these you know pipe flows.

Before that when we are talking about the turbulence. So, in that we talked about these smaller eddies and the larger eddies, and the you know the larger it is basically they will

be dominated by the inertia effects and the viscous effects are negligible in that case. So, you know, so normally this larger eddies which are there, so these are basically inviscid, are effectively inviscid.

So, basically if you look at the values  $\frac{ul}{\eta}$  so in that case if you take  $u$  and  $l$  and what we saw, so if you are taking that length as a characteristic length and we are taking larger eddies into account in that case those the Reynolds number becomes very high, so inertia force is dominating. So, in that case your the flow is inviscid, viscous forces are negligible, their effect is negligible. So, that is why we take it as the inviscid.

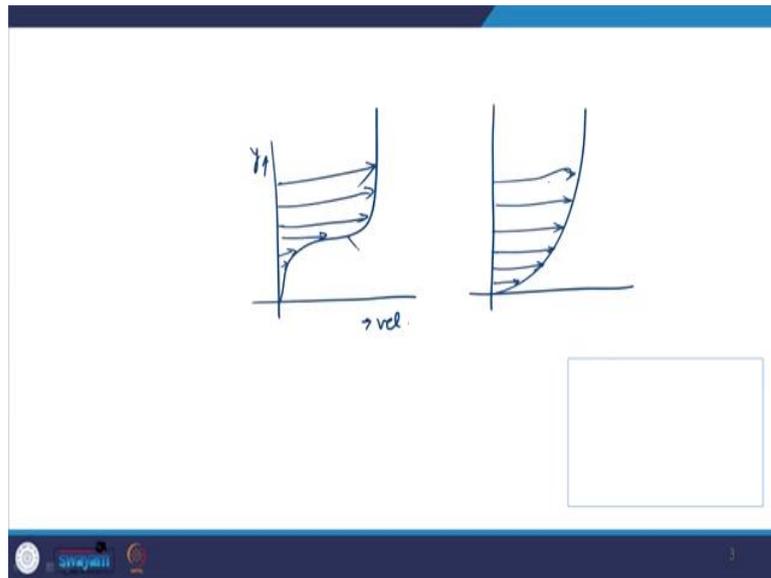
Now, in those cases your angular momentum is conserved and that is why that leads to you know I mean during that vortex stretching this angular momentum is said to be conserved. So, that will you know lead to the rotation rate to further you know increase and the you know radius of the cross section will decrease.

So, that way you will have that kind of flow structure which will be you know coming up in the case of the turbulence and you know you will have the and also what happens that you will have the mean flow, you will have the larger eddies, you have a smaller eddies. So, they will be deriving the energy from the mean flow, so that way and also the turbulence is said to be self dissipating also. So, those terms we will be discussing you know later.

Now, coming to the you know the turbulence and the transition to the turbulence if you look at. So, this we discussed that for the pipe flow that was the case where and for different kind of flows you will have the in different way you will have the you know transition taking place and we are discussing that when we talk about the you know velocity profile where that will be susceptible to the different kind of flows and creating the instability.

So, the inflexion point which we were discussing that you will have you know when you may have the you know instability and the velocity profiles may be like if your velocity profile goes like this, so basically this is the point of inflexion. So, this is your  $y$  and this is your velocity.

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Similarly, you know when you have the, you can have the transition of the turbulence also, transition to turbulence even without the inflexion point, so your situation may go like this. So, that maybe in this case you know you have this is your inviscid instability. So, in that case you know viscous effects are negligible and this is your viscous instability. So, this is basically the you know velocity profile which you see. So, this is with the presence of inflexion point and this is without the presence of the inflexion point. So, that is what is normally happening when we have the transition to turbulence you know taking place.

Now, that is basically for the different you know there may be different kind of flows, you have jet flow, you have also the, so in that jet flow and then you have the mixing or you have the you know many cases, so mixing or flow behind the wake and all that. So, in all those cases you will have you must have the idea about how you know the turbulence is takes place, how there is you know how I suppose in jet.

So, it will be moving this way moving fluid and that will be interacting with the stationary fluid which is there, so accordingly you will have a deformation and all that. So, these needs to be you know we expect that you have some understanding of these phenomena and that will help you basically, in understanding basically the terminologies which will be coming later related to the turbulence.

Now, coming to some descriptors of the turbulent flow. So, when we as we discussed that in the case of turbulent flow you have a mean component and one is your fluctuating

component. So, you know, so what we do is normally the mean  $\Phi$  of the flow property, so you will have the flow property  $\varphi$  and this will be defined as the  $\Phi + \varphi'(t)$ . So, that way we define.

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**Turbulent flow properties**

- ❖ Time average or mean
  - ❖ The **mean**  $\Phi$  of flow property  $\varphi$  is defined as follows:
 
$$\Phi = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi(t) dt$$
  - ❖ This definition of the mean of a flow property is adequate for steady mean flows.
  - ❖ The time average of the fluctuations  $\varphi'$  is, by definition, zero:
 
$$\overline{\varphi'} = \frac{1}{\Delta t} \int_0^{\Delta t} \varphi'(t) dt \equiv 0$$

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So, basically for the for the turbulent flow you know for any property for turbulent flow when we talk about any property of  $\varphi(t)$ , so this  $\varphi(t)$ . So, that will be defined as the mean value. So, that is  $\varphi$ , so and plus you will have the fluctuating component of  $\varphi'(t)$ . So, that way your this is how the property will be there in the case of the turbulent flow.

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For turbulent flow

$$\varphi(t) = \Phi + \varphi'(t)$$

Time average or mean

$\Phi$ :  
turbulent KE per unit mass:

$$K = \frac{1}{2} (\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$$

$T_i$ : Average rms vel. divided by reference mean flow velocity

$$T_i = \frac{(\frac{2}{3} K)^{1/2}}{U_{ref}}$$


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Now, the thing is that for as far as this fluctuating part is there. So, you know for this the if you take the time varying component and if you take its value, so there, so basically its time mean will be 0 for the fluctuating component. So, so you will have the other, there are different ways to express this and one is the time average or mean. So, for any property you know  $\phi$  the mean  $\phi$  that mean will be denoted as, so that will be basically, so that is shown here. So, your this mean of the flow property is defined as we are taking  $\frac{1}{\Delta t} \int_0^{\Delta t} \phi(t) dt$ . So, that way we define the mean of the flow property.

And then you have the you know fluctuating component also. So, you may you know we should take that limit towards the infinity you know, but then you will have, but you have to take the  $\Delta t$  you know cautiously because you have certainly a size of there is a limitation on that limitation is put by the size of the eddies. So, that you know slowest variation will be because of the largest eddies. So, accordingly you will have to take the  $\Delta t$ .

Now, if you take the time average value of the fluctuations. So, by definition the time average value of the fluctuation will be taken as 0. So, if your the fluctuation which we have seen in the first case that you have the mean value and then fluctuation taking place, so basically it is assumed that the if you take the time average of this value it will be coming to 0. So, that is why you have a mean value and then you have the time average value. So, we write normally when we do not take the  $t$  into every time for  $t$  notation, so you will be writing  $\phi$  equal to mean  $\phi$  plus,  $\phi'$ , so that way we are writing you know these values.

Now, apart from that you know you have other properties. So, other properties will be your now this fluctuation component of the suppose velocity we are taking into account. So, this fluctuation component will also be leading to the energy also. So, the total kinetic energy that. So, these fluctuating parts which will that give rise to the kinetic energy they are the turbulent kinetic energy known as.

So, you will have the turbulent kinetic energy. So, this turbulent of kinetic energy then you know per unit mass. So, this will be by the respective velocity fluctuations and that will be equal to  $\frac{1}{2} (\bar{u}'^2 + \bar{v}'^2 + \bar{w}'^2)$ , So, that way this term because of this fluctuation part which is squared. So, that gives you basically the turbulent kinetic energy you know per unit mass and this is basically to be taken into account and we deal with the turbulent flows modeling in that case this will be used.

Similarly, you have based on this you define the turbulence intensity. So, that is basically you know defined, and turbulence intensity will be the average you know rms, so it will be depending upon the you know rms velocity. Now, before that we need to know what is the you know variance and rms.

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The slide is titled "Turbulent flow properties" and contains the following text and formulas:

- ❖ Variance, root mean square and turbulence kinetic energy
- ❖ Fluctuations  $\phi'$  about mean value  $\Phi$  are the variance and root mean square

$$\overline{(\phi')^2} = \frac{1}{\Delta t} \int_0^{\Delta t} (\phi')^2 dt$$

$$\phi_{rms} = \sqrt{\overline{(\phi')^2}} = \left[ \frac{1}{\Delta t} \int_0^{\Delta t} (\phi')^2 dt \right]^{1/2}$$

There is a small inset image of a man in a white shirt and tie, possibly a lecturer, in the bottom right corner of the slide.

So, you know first of all the we are talking about the variance, so the variance we will be we define as  $\overline{(\phi')^2} = \frac{1}{\Delta t} \int_0^{\Delta t} (\phi')^2 dt$ . So, that is how the variance is calculated. You will have the fluctuation part, it is it is square, and then we are dividing by  $\Delta t$ . if you are integrating over  $\Delta t$ .

So, this way we are getting the variance and the rms value of this fluctuating part, so that  $\phi_{rms}$  that will be the square root of this. So, this is your you know square root. And basically what happens that before that we need to understand that the  $u'^2$  or  $v'^2$  or  $w'^2$  on all these components so, they give rise to the some of the you know stress terms.

So, you know, so that is why and these are and these stresses which are because of these fluctuation components we call it as Reynolds stresses that is what we discussed. And then we defined this turbulent kinetic energy that will be  $K$  equal to  $\frac{1}{2}(u'^2 + v'^2 + w'^2)$ . And the turbulence intensity, that is the average rms velocity divided by the reference mean flow. So, this will be you know average rms velocity divided by reference mean flow, so mean flow velocity.

So, if you take reference mean flow velocity as we  $u$  reference, so what you do is you can derive it through this  $K$ , so that will be  $\frac{2}{3}K$ . So, so  $T_i$  will be  $\frac{(\frac{2}{3}K)^{1/2}}{U_{ref}}$  because you will have a one velocity component, so you will have  $\frac{3}{2}$ . So,  $\frac{2}{3}$  you multiply and, so it will be velocity component that is your rms velocity and then it will be divided by the  $u$  reference.

So, this way you define these you know turbulence intensity and this will be one of the you know parameters which will be used when we are going to give the boundary conditions in the case of the flow where the turbulence will be used.

Apart from that you have even the terms like the moment of the these different fluctuating variables, so that will lead to the terms like  $u'$ ,  $\overline{v'}$  or  $\overline{w'}$  that leads to these you know stresses. So, so that those terms those will be the additional shear stresses. They will be coming up you know when we deal with turbulence.

So, when we go to deal with the Navier-Stoke for that Reynold averaging Navier-Stoke equations, where we take this turbulence flow into account in that case we will see that these terms will also come into picture that will be the extra Reynolds, that is shear stress that will be you know coming up.

So, accordingly now you know these terms need to be understood and that we will see now you know more of these terminologies in our you know coming lectures.

Thank you very much.