Modeling of Tundish Steelmaking Process in Continuous Casting
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Lecture – 19
Energy Conservation Equation

Welcome to the lecture on Energy Conservation Equation. So, you will have that which is also known as conservation of energy equation. So, you know or energy equation in 3 dimension.

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So, if you talk about this equation, so, this is energy equation is derived from the first law of thermodynamics which states that the rate of change of energy of a fluid particle is equal to the rate of heat addition of to the fluid particle plus rate of work done on the particle.

So, as we see this will be rate of increase of energy and that will be because of the heat added which is net rate heat added and then net rate of work done on the fluid particle. So, we will talk about these 2 aspects. So, we will have this will be term that is $\rho \frac{DE}{Dt}$ if you talk about per unit volume, then you will have the net rate of work done. So, work done is normally force times velocity components. So, that way we can have the expression for the net rate of work done.

Similarly, for the heat added we will have the conduction keeping in mind. So, based on that Fourier's law of conduction you will have for the element how the heat element heat is added and how there will be energy rate of heat added to the fluid particle for that a term will come. So, we will add them and then we will equate it to the rate of increase of energy and that will be the equation for the energy conservation equation in 3 dimension.
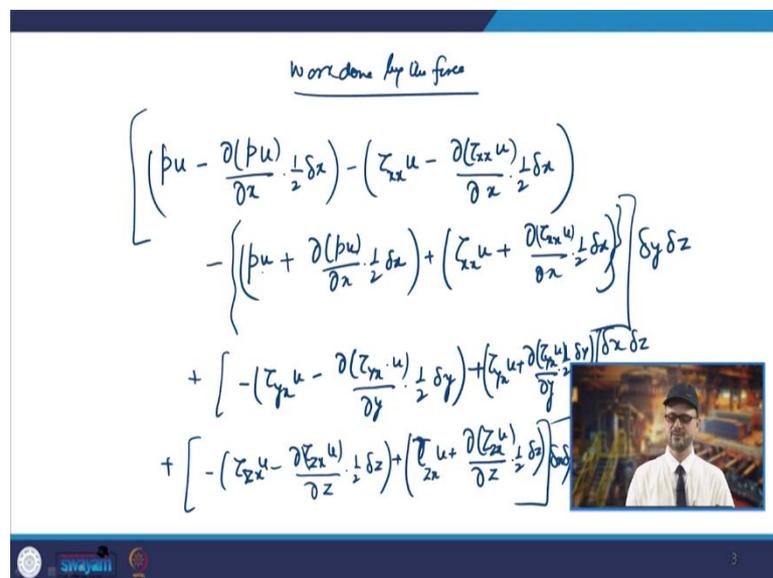
So, if you recall our last lecture where we talked about the term that was for the force. So, in that the term will be multiplied with the velocity component and that will be the work done. So, we will be first talking about the net rate of work done on the fluid particle and we will be talking about the work done by the forces.

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So, work done by the forces will be as we know that work done by the forces force into velocity. So, we will have the. So, we will had the term that is $(p - \frac{\partial p}{\partial x} \frac{\delta x}{2})\delta y \delta z$.

And similarly you had the τ_{xx} term. So, $\tau_{xx} - \frac{\partial \tau_{xx}}{\partial x} \frac{\delta x}{2}$. So, all these terms were there while we are you know arriving at that particular term, now in that each of these term will be multiplied with the velocity component u. So, that will be the work done. So, that term you know if you talked about the work done you know in the x direction, so, that will be coming as you should know so, you had a $p - \frac{\partial p}{\partial x}$ and you had. So, in that we will be multiplying with u.

So, $pu - \frac{\partial pu}{\partial x}$ and then you will have here you will have $\frac{\delta x}{2}$. So, and similarly you will have the another term that will was the τ_{xx} term. So, that will be again τ_{xx} term will also be multiplied with the velocity component and on the left hand side you had the $\tau_{xx} - \frac{\partial \tau_{xx}}{\partial x}$ it will be $(\tau_{xx}u - \frac{\partial \tau_{xx}u}{\partial x}\frac{\delta x}{2})$.

So, that will be the term you know on the east face similarly you go and this one on the west face and then you go to the east face. So, you will have $-(pu + \frac{\partial pu}{\partial x}\frac{\delta x}{2})$. So, that will be coming under the big bracket. So, this is one term and further you have the $\tau_{xx}u$.

So, everywhere, you are multiplying with the velocity to have the expression for the work done. So, and $\frac{\partial \tau_{xx}u}{\partial x}\frac{\delta x}{2}$. So, and this all will be if you see all this all together if you talk for the whole component that will be multiplied with the term you know $\delta y \delta z$. So, then you will have the other terms also like you had if you. So, we are talking about the s direction only and we are talking about taking into account all the you know faces.

So, talking about the north and south faces. So, you will have again $(\tau_{yx}u - \frac{\partial \tau_{yx}u}{\partial y}\frac{\delta y}{2})..$

So, that will be there or one term and then similarly you will have a $\tau_{yx}u + \frac{\partial \tau_{yx}u}{\partial y}\frac{\delta y}{2}$. So, this will be all multiplied with the term $\delta x \delta z$. So, this is for the you know north and south face then comes the term for the top and bottom face. So, for that again you will have $-(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z}\frac{\delta z}{2})...$

So, this is multiplied with u and then it will have $\tau_{zx}u - \frac{\partial \tau_{zx}u}{\partial z}\frac{\delta z}{2}$ and then you will have another term for the you know top face. So, you will have a dou of so, you will have first the $\tau_{zx}u + \frac{\partial \tau_{zx}u}{\partial z}\frac{\delta z}{2}$ and then altogether it is multiplied with $\delta y \delta x$. So, this is the expression which we had seen you know where while deriving for the x momentum we had got this expression wherever u is there u was not there in that. So, u are multiplying with these velocity terms.

So, forgetting that you know net rate of work done in the x direction you are going to have a this equation and in this again you can see that you will have some of the terms getting cancelled and like pu and pu will be cancelling. So, there are many terms which will be cancelled and if you know if you solve this if you so, you are getting these you know. So,

you are getting this net rate of work done by these you know surface forces in the x direction.

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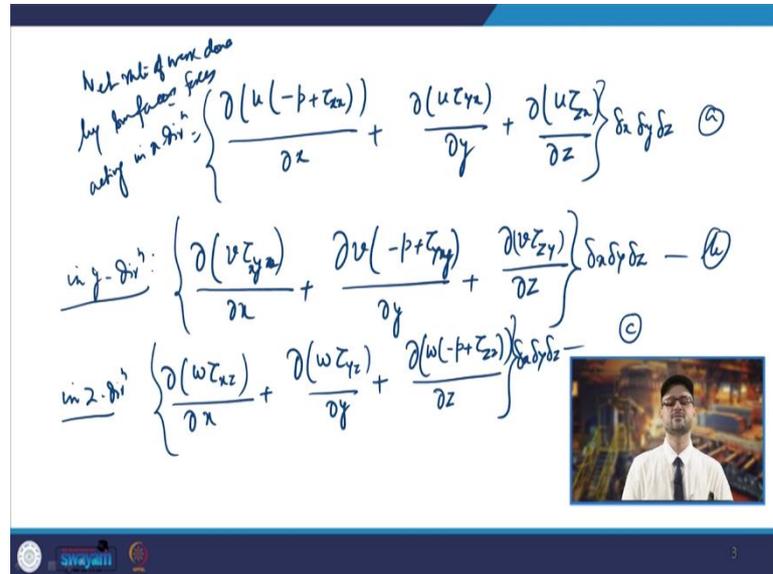
So, that will be $(\frac{\partial(u(-p+\tau_{xx}))}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z})\delta x\delta y\delta z$. So, that term you know this will be coming out and you are getting this you know net rate of work done you know by these surface forces which is acting in the x direction this is defined by these forces.

Now, surfaces stress components are also acting in the y and z direction and they also do work on the fluid. So, you will have those components for the y and also on the z component. So, for that you will have so, this is a. So, you will have another you know net rate of work done that will be. So, this is basically the net rate of work done by surface forces acting in x direction. So, this is in x direction.

Now if you find in y direction. So, in y direction again you will have dou of you know v tau. So, it will be p term will go with this side. So, it will be $(\frac{\partial(v(\tau_{yx}))}{\partial x} + \frac{\partial(v(-p+\tau_{yy}))}{\partial y} + \frac{\partial(v\tau_{xz})}{\partial z})\delta x\delta y\delta z$.

So, similarly in the z direction you can have the expression that is $(\frac{\partial(v(\tau_{xz}))}{\partial x} + \frac{\partial(v\tau_{zy})}{\partial y} + \frac{\partial(v(-p+\tau_{zz}))}{\partial z})\delta x\delta y\delta z$.

. Now if you look at this so, the summation of all these forces you know this work done. So, if you total rate of work done will be the summation of these you know 3 cases a, b and c. So, total rate of work done you know per unit volume on the fluid element and that will be you know summation of all these in the forces. So, if you all these terms in these 3 equations and then you have to divide it. So, if you have to you have to find the per unit volume in that case this volume term will go.

So, you will once you will add you will have that will be your total rate of work done per unit volume on the fluid particle. Now that will be consisting of few terms like you will have one is $\frac{\partial(-pu)}{\partial x}$ then $\frac{\partial(-pv)}{\partial y}$ and you will have a $\frac{\partial(-pw)}{\partial z}$. So, one is that term and another term will be $\frac{\partial(u\tau_{xx})}{\partial x}$, $\frac{\partial(u\tau_{yy})}{\partial y}$. So, that will be another term. So, you will have you know different terms together. So, the first term which will be segregating will be $\frac{\partial(pu)}{\partial x}$ with negative sign $\frac{\partial(pv)}{\partial y}$ with negative sign and $\frac{\partial(pw)}{\partial w}$ with negative sign.

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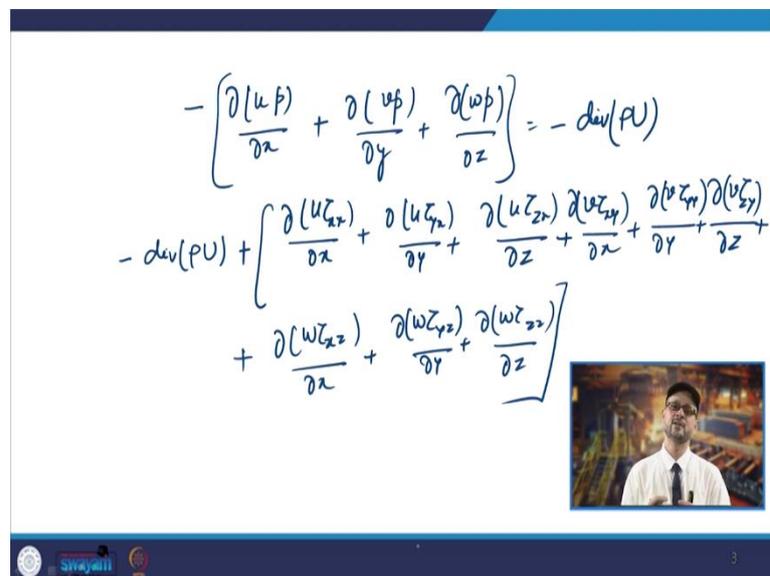
So, if you sum these 3 terms so, you will have you know term. So, this term comes as $-[\frac{\partial(pu)}{\partial x} + \frac{\partial(pv)}{\partial y} + \frac{\partial(pw)}{\partial z}]$. So, you can write this as minus of $div.(pu)$. So, that way this is one part of this summation of the term.

Now next part so, did here we have taken the pressure term collected together. So, this is all the term which is having the pressure. Now if you talk about the total you know rate of

work done taking all the terms. So, that becomes minus of $-div.(\rho u)$. plus now if you recall these terms like dou of u tau xx by dou x dou of u tau yx by y dou y so, that will come.

So, you will have a $\frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z}$. So, that was for the first term similarly you have another you know 3 terms coming up. So, that will be taking a $\frac{\partial(u\tau_{xy})}{\partial x} + \frac{\partial(u\tau_{yy})}{\partial y} + \frac{\partial(u\tau_{zy})}{\partial z}$. So, that is the next 3 term and then the last 3 terms will be you know. So, this is this is this term plus this way. So, that term come came here and then you have a $\frac{\partial(u\tau_{xz})}{\partial x} + \frac{\partial(u\tau_{yz})}{\partial y} + \frac{\partial(u\tau_{zz})}{\partial z}$.

So, this is you know all together all these this is the total rate of work done that will be you know on the fluid particle by all these stresses. So, by the surface stresses that is coming you know here. So, per unit volume calculating. We have remove this term $\delta x\delta y\delta z$. So, if you look at the term. So, we have already you know we have seen. So, this term is in a net rate of work done on fluid particle this we have achieved you know here. Now we are going to find the value of the energy flux due to heat conduction.

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So, if you are trying to find the energy flux due to heat conduction. So, for that we will have the heat flux vector q will be taken.

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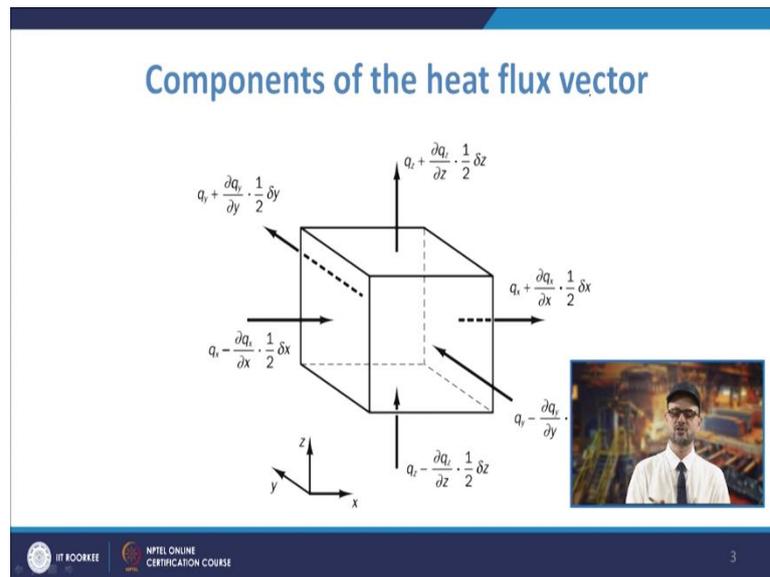
And if you take this heat flux vector q so, it has 3 components q_x, q_y and q_z. So, you know and we are going to have the concentration of all these 3 components q_x, q_y and q_z and we are going to have the net you know rate of heat transfer to the fluid particle because of the conduction because of the heat flow in the x direction.

So, that will be given by the difference between the rate of heat input. So, you have the rate of heat input. So, from the west face as well as it from the east face. So, you have to have the difference from of the heat you know due to the heat flow, in x direction so, we have to take that difference on these 2 faces. So, if you take that it will be $q_x - \frac{\partial q_x}{\partial x}\frac{\delta x}{2}$ and that will be you know and it will be subtracted with $q_x + \frac{\partial q_x}{\partial x}\frac{\delta x}{2}$.

So, that is your so that will be you know that will be your heat net rate of heat added. So, you are adding and them something is going out. So, net rate of heat added will be this minus the amount which is going out. So, you know we will have the difference of you know the this minus this and that will be multiplied with the area on which it is acting. So, if you talk about the you know west face. So, you know west and east face into consideration. So, in that case your rate of heat loss across the face e that will be $q_x - \frac{\partial q_x}{\partial x}\frac{\delta x}{2}$.

So, that will be from the east face and from the west and then going towards the east. So, you will have $q_x + \frac{\partial q_x}{\partial x}\frac{\delta x}{2}$ and altogether it is multiplied with the area that is $\delta x \delta y \delta z$. So, if you take this value that becomes equal to $-\frac{\partial q_x}{\partial x}\delta x \delta y \delta z$. So, this is you know net rate of heat transfer in the x direction, similarly you will have the net rate of heat transfer in the y direction as well as in the z direction.

So, in y direction it will be $-\frac{\partial q_y}{\partial y}\delta x \delta y \delta z$ and in z direction you will have the $-\frac{\partial q_z}{\partial z}\delta x \delta y \delta z$. Now the total you know rate of heat added to the fluid particle will be the sum of these 3 terms and if you want to have the per unit volume then you have to remove these volume term. So, total rate of heat added to fluid particle per unit volume now that will be addition of these 3 and we have to divided by volume.

So, you will have $-\frac{\partial q_x}{\partial x}$ similarly you have a $-\frac{\partial q_y}{\partial y}$, $-\frac{\partial q_z}{\partial z}$ and this can be written as minus of $div q$. So, this is the $-div q$, now we will use the Fourier's law of heat conduction and it will be relating that heat flux to the temperature you know gradient. So, if you use the Fourier's law of heat conduction so that can be that rights as $q_x = -k\frac{\partial T}{\partial x}$. Similarly we write $q_y = -k\frac{\partial T}{\partial y}$ and $q_z = -k\frac{\partial T}{\partial z}$.

So, you can write this $q = -k.gradT$. So, normally you write $q = -k.gradT$. So, if you see minus. So, this term becomes $-k\, div(.gradT)$. So, $-div q$. So, $-div q$ $div(.k\, gradT)$ and this minus term comes here. So, it will becoming cancelling so, you will have $div(.k\, gradT$. So, that is how you know this $-div q$ can be written as $div(.k\, gradT$ and now you will get the energy equation.

So, now if we have to find the expression for the energy equation now we have to equate this with the term. So, as we see you had this is the rate of increase of energy of the fluid particle and that will be equated to the addition of these 2 terms and we have to define this energy term. So, that is your specific energy E of the fluid and it will be the summation of. So, if you talk about the specific energy E of the fluid.

So, that will be basically summation of the internal energy or thermal energy you know that is. So, and then you will be adding. So, that is your $\frac{1}{2}(u^2 + v^2 + w^2)$ and also you will have the gravitational potential energy.

So, this term gravitational potential energy that we normally take into account in the right hand side and we take it as the source term. So, you will have this will be the sum of i plus, you will have the kinetic energy and all that and then you will have the gravitational p. So, what we do, now we are equating these equation gravitational force will be taken as the source term on the right hand side and you are adding the 2 terms on the right hand side. So, your equation becomes $\rho \frac{dE}{dt}$. So, that is rate of change of that term you know that is what rate of increase of the energy of the fluid particle.

Now, that we will be taking so, and you will have the source term coming out on the right hand side that is S E and E will be basically internal energy $i + \frac{1}{2}(u^2 + v^2 + w^2)$. So, you will have the that is internal energy that is I and also you will have this sum of the kinetic energy. So, that also term will come here. So, these terms will come in this term and in this side you will have the gravitational potential energy.

Now, this $\rho \frac{dE}{dt}$ that will be the summation of you know these terms. So, you will have this is the thermal term and this side you had got the these work done. So, you will have the

first term as $-div(\rho u) + [\frac{\partial(u\tau_{xx})}{\partial x} + \frac{\partial(u\tau_{yx})}{\partial y} + \frac{\partial(u\tau_{zx})}{\partial z} + \frac{\partial(v\tau_{yx})}{\partial x} + \frac{\partial(v\tau_{yy})}{\partial y} + \frac{\partial(u\tau_{zy})}{\partial z} +$

$\frac{\partial(u\tau_{xz})}{\partial x} + \frac{\partial(u\tau_{yz})}{\partial y} + \frac{\partial(u\tau_{zz})}{\partial z}$.

So, that will be the term which we had earlier you know derived then you will have the terms that is for the thermal term that is energy term that was $div.(k\ gradT)$ and then you will have this source term. So, that is you know for the other type of you know energy that is the gravitational potential energy we take this term as the source term. So, we are taking that so, on the right hand side.

So, this is the energy equation you know which is which is to be used when we and which is used for solving and forgetting the you know requisite you know outputs of interest which we are you know interested in while solving the problem. We can have even the we can take it for a ρ of you know you can take the internal energy and the kinetic energy together also I mean we can separate it.

So, $\rho d(\frac{1}{2}(u^2 + v^2 + w^2)$. , so that way now and $\rho\frac{di}{dt}$. So, internal energy also for that also there may be terms taken together, but then we should be knowing about these terms which will be used in our coming lectures.

Thank you very much.