Modeling of Tundish Steelmaking Process in Continuous Casting

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Lecture – 14
Analysis of RTD Curves

Welcome to the lecture on Analysis of RTD Curves. So, in the past lectures we talked about different aspects during the physical modeling. So, how we have seen that the tracer is injected then you have to draw these using the stimulus response techniques so you have to draw, to trace the concentration maybe in the pulse manner, the tracer is added or it may be added in the step manner.

So, by that you are getting the RTD curve. And we must understand try to understand that how these you know RTD curves are to be you know understood. So, what does it signify, specially we will talk about you know finding the mean residence time then also the mean and you know variance of these residence time, so from these RTD curves. So, that is what our aim will be in this lecture.

So, what we saw? So, once you get the you know RTD curve, so you have to get the mean of the RTD curve.

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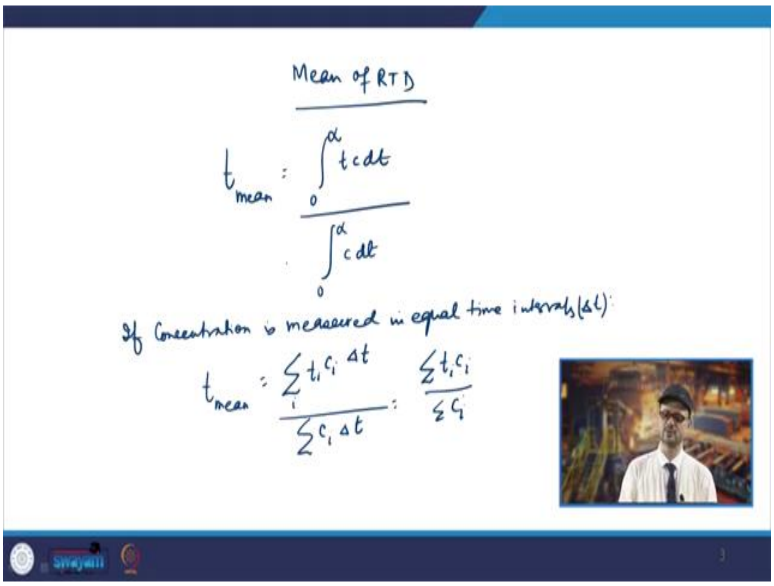

And the mean of the RTD curve, so that can be found by the expression so that we will be writing as t_{mean}. So, t_{mean}will be you know $\frac{\int_0^\infty\ tcdt.}{\int_0^\infty\ cdt.}$. So, as you know that you are having on the abscissa, you have the time axis on the ordinate you have the concentration axis.

So, as you know with time so you will have these $\frac{\int_0^\infty\ tcdt.}{\int_0^\infty\ cdt.}$. So, if you can see the analyze even by dimension wise it will be something in terms of time. So, this time basically will be known as the you know t_{mean}. So, mean residence time. Then if the concentration is measured in the equal time in intervals, so, if concentration is measured in equal time intervals, now in that case you will have equal time interval of Δt, so that is of Δt.

In that case you will have to have the expression for the t_{mean} and this t_{mean} in that case will be $\frac{\sum t_i c_i \Delta t}{\sum c_i \Delta t}$. So, if it the equal time interval is there that is Δt in that case that will cancel, so it will be summation of $\frac{\sum t_i c_i}{\sum c_i}$..

So, this is going to tell you this by this, so once you have these concentration data with time then we can find the mean residence time inside the tundish. So, that is done by this formula. You can find the variance of this quantity.

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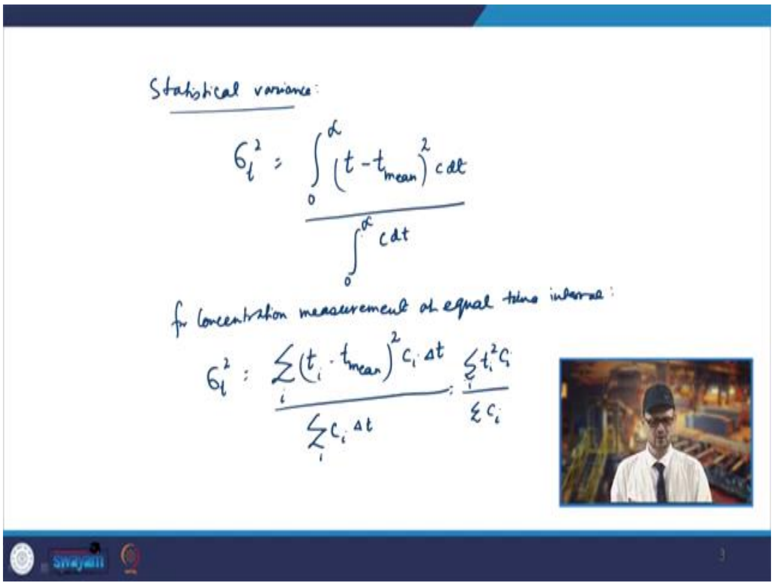

So, this variance so that will be statistical variance, so that talks about the spread you know spread of the this residence time about the mean. So, that variance is calculated that is spread of the you know residence time distribution about the mean. So, that is found by σ_t^2.

So, as we know we have to take the difference of the individual value with the mean value, and then it taking the square, and then further you are taking the I mean we are further you know taking the ratio. So, that will be $\frac{\int_0^\infty (t-t_{mean})^2 cdt}{\int_0^\infty cdt}$. So, t_{mean} what you calculate using the you know earlier expression. So, this way you calculate the variance of the residence time distribution about the mean.

Now, here also if you have the equal time interval, so, for concentration measurement at equal time interval; so, in that case again you give the term Δt, so that will be σ_t^2 it will be again $\frac{\sum (t_i-t_{mean})^2 c_i \Delta t}{\sum c_i \Delta t}$. So, that way you know as Δt is equal so, that will be anyway cancelled, so it will be $\frac{\sum t_i c_i}{\sum c_i}$.

So, that is what we have seen. So, this is the t_{mean} and this is your so that was mean time and in this case you have this as the you know the variance that is computed here. So, that you have the square term. And this way you compute these mean and variance of the RTD curve, when you get these values experimentally or by you know any other means like simulation or so.

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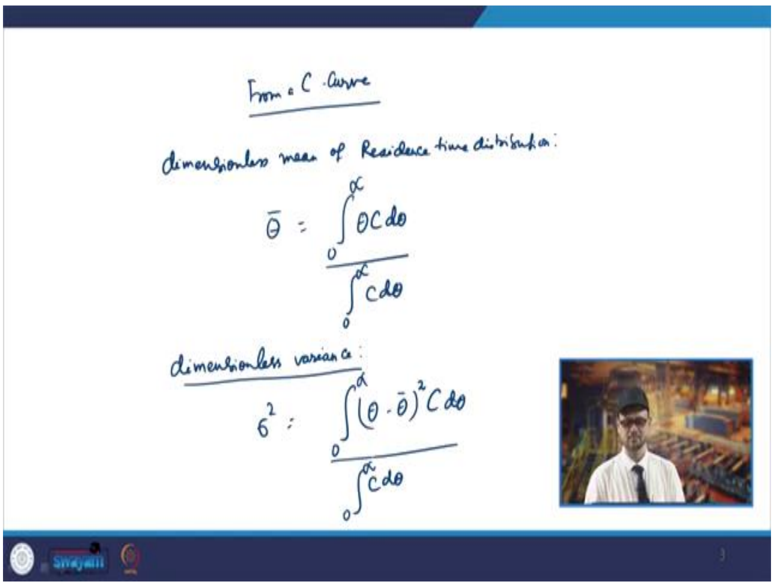

Then if you are talking about the C-curve, so in the case of C-curve, so, from C-curve, so, in that you will have the dimensionless you know mean of residence time distribution. So, you can calculate the dimensionless mean of residence time distribution. So, in that case you have dimensionless term which will be coming to picture.

So, that will be defined by $\bar{\theta}$, and that you will be calculating as $\frac{\int_0^\infty \theta c d\theta.}{\int_0^\infty c d\theta.}$ So, same way in that case you have C dt and here you have capital C so that is your dimensionless turn. So, then you calculate these as $\theta c d\theta$. So, that will be the dimensionless mean.

Similarly you have the calculation of the you know variance so, dimensionless variance. So, dimensionless variance basically will be again based on the relation. So, that will be sigma square, so it will be $\frac{\int_0^\infty (\theta-\bar{\theta})^2 c d\theta}{\int_0^\infty c d\theta}$, so this will be 0 to infinity. So, that ways you know for the continuous function.

Now if you take for the equal time interval, so, for measurement at equal time interval.

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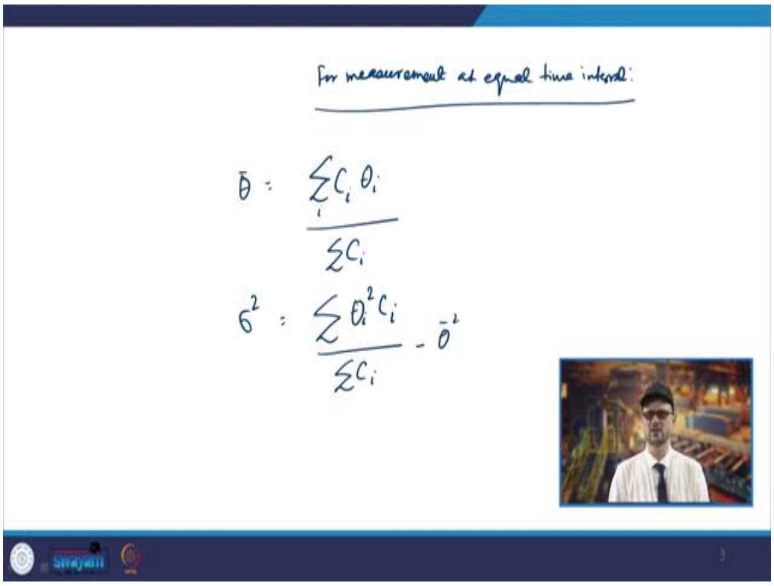

So, in that case your you know equation will be like you have $\bar{\theta}$. So, that will $\frac{\sum t_i \theta_i}{\sum c_i}$. So, that way you will have the $\bar{\theta}$, theta average. And similarly you will have the variance. So, that will be $\frac{\sum \theta_i^2 c_i}{\sum c_i} - \bar{\theta}^2$.

So, this way you know we calculate these you know mean as well as the variance quantities in the case of the RTD curves. So, so we can understand it using one example. So, suppose you have one example where there is concentration measurement, so that you can understand using that example.

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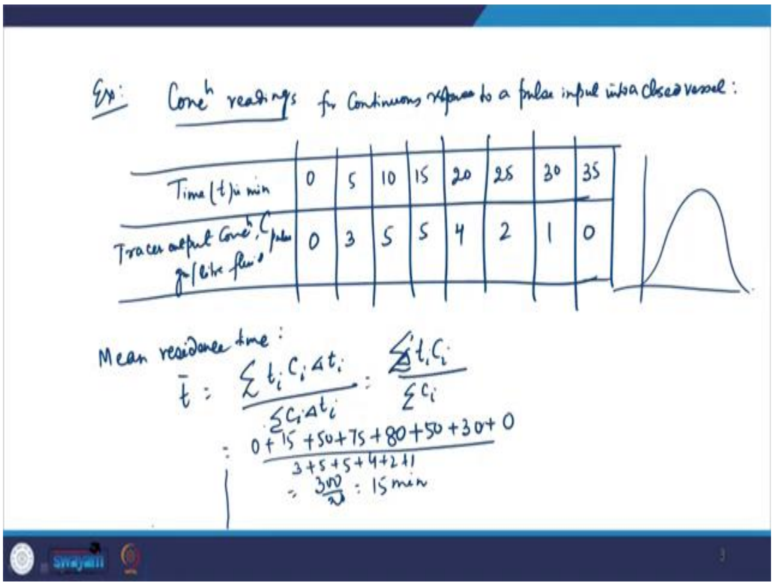

So, the concentration readings are plotted for you know continuous response to a pulse input into a closed vessel. So, you know for that one which is used as a chemical reactor, so for that there are readings. And you will have the reading of the concentration at different times. So, you have time you know t that is in minutes and then you have the you know tracer output that is given. So, there is a tracer output concentration that is C pulse. That is in the term of gram per liter fluids.

So, suppose you have the values which are given to you. So, suppose at you are taking at interval of 5 units: 0, 5, 10, 15, 20, 25, 30, and 35. Suppose this way you are monitoring these tracer output concentration at these times and the values which you are getting is at 0 it is 0, then you have 3, 5, 5, then you have 4, 2, 1, and 0. So, this way suppose your concentration readings are coming when you are measuring it, so there are many measuring devices, you have conductivity meters are there by which you can measure. And there are many wet data stick is there, so there are many devices which are used for measuring these output concentration. So, you can find the mean residence time.

So, the mean residence time so that will be in a $\bar{t}$. So, it will be you have as you know that you have the Δt of 5 minutes. So, you have $\frac{\sum t_i c_i \Delta t_i}{\sum c_i \Delta t_i}$. So, as Δt is constant. So, you will be writing a $\frac{\sum t_i c_i}{\sum c_i}$.

So, now you will be multiplying these and then you will be adding. So, if you know add, so that way $t_i c_i$. So, you are adding, so that will be first will be you know 0 then you have you know 15 plus so you will have 0 plus 15; so that is 15. Then you have 50, then you have 75, 80, then you have 50 plus 30. So, this is coming and then you have 0; and if you add the concentration.

So, so that is $t_i c_i$. And then you are having the concentration you know summation, so that is 3 plus 5 plus 5 plus 4. So, this will be 3 plus 5 plus 5 plus 4 plus 2 plus 1. So, it will be 65 plus 75 you know 140, 220, 270 and this is 300. And if you this is 13 for you know 17, so this is 0, this is 300 by 20. So, you know you are getting the value of you know 15. So, 15 minute; 15 minute is you know the mean residence time that is what you are calculating. This 15 minute time is the mean residence time when you have you know such are the readings. So, 15 minute will be the mean residence time.

Now what we see in this case? Normally, you also can find the variance of also the mean residence time; variance about the means the variance of the RTD curves can be drawn you know in this case. And also you can plot the RTD. So, you have at 0 it is 0, then at 5 you will have 3. So, you can have that value also. And what you see that you will get some similar kind of curve you will be getting. So, at the different times you will have the values coming. And this way you can have the value of the, you can have the plot of the RTD curve.

So, depending upon, so what happens that when you do the experiment? In that case you have you can either you can measure manually or you have the you can connect it with the computer you get the readings. These readings are basically you know stored in computers. So, you can have it in store in excel or in any you know graph plotting software. And then from there you can draw the curves. So, that is how you know you draw. So, you can further calculate the variance over this you know the mean then you will be taking the difference of the value from the mean. So, that way and accordingly you can have the calculation of variance also.

Then we will talk about the other model which is a conceptual model and that is also used for the analysis of these response techniques. So, that is you know that is known as the tank series tank in series model. So, many a times we feel that it is like tanks which are you know which are the where the mixing is taking place and they are connected in series. So, in that case the response of the tracer which is there at the different you know places. So, that you know, how that response will be coming, how they will be responding to the changes in the concentration. So, that is to be seen.

So, if you talk about this tank in series model.

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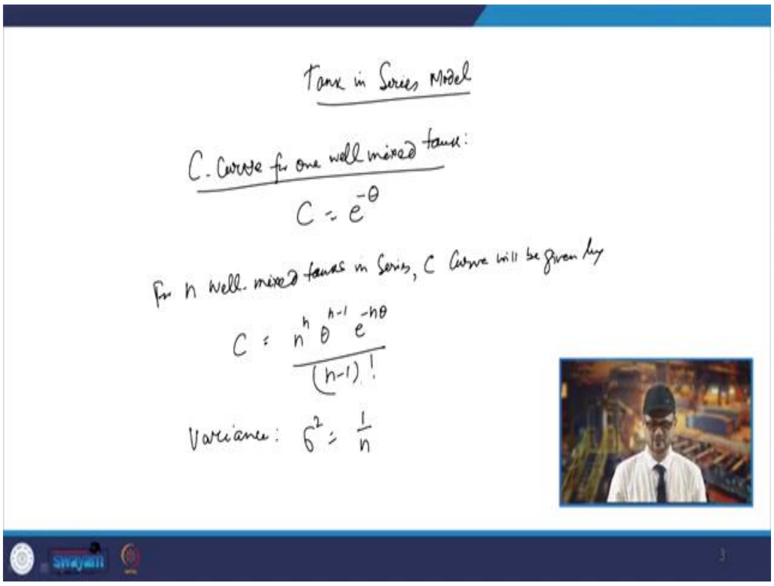

So, your tank in series model is there. Now in that case what we talked is that you can have the tanks connected in series and you have these you know tanks as the well mixed tanks. So, they will be giving you know, they will be giving the tracer responses. And they are similar to that is given by the dispersion model. So, you know any type of such model can be used to correlate the experimental data you know which is obtained.

So, you will have these some model results and then you will guess see that they expand from the experiments also you get that kind of RTD curves, when you have the you know outlets which are far away. So, when you have a vessel where the input is coming at one place and then you have different places where you have to measure the output. So, it is like you can it is like the tanks which are connected in series. So, in those that number of tank that numbered tank you will have the tracer response in a different manner.

So, if you recall for a for one tank. So, this is the if you recall that if you go for the C-curve, so C-curve for one well mixed tank. So, as you know that you recall that the for the well mixed flow the exponential you know that curve was seen. So, in that case then mean that will be represented by C equal to $e^{-\theta}$.

Similarly, if you have you know n well mixture tank and they are in series connected together. So, for those n well mixed tanks you know in series, so for those n well tanks in series so C-curve again will be given by will be given by the equation. So, for that you have C equal to $\frac{n^n \theta^{n-1} e^{-n\theta}}{(n-1)!}$

So, this is you know for when you have to have n well mixed tanks in series. You can predict the value of the concentration you know according to this and if you take the variance of these curves. So, variance of the curves say if you know determine it will be $1/n$. So, you know these experimental data which you get. And you have the you know critical C-curve. So, you both can be matched and your results can be validated for the different tanks. And can be even said that and may be an non integer also.

So, this is you know one parameter model. And here the parameter is that the number of tanks; so, based on that n and now if you go for the large number of n. So, if you take c equal to 1 it will lead to the same value, if you take c as I mean n as 1 if you take n as 2, so it will have a different value. What we see that as you go, as you increase the value of n; so as the value of n is increasing it becomes more and more symmetrical in shape.

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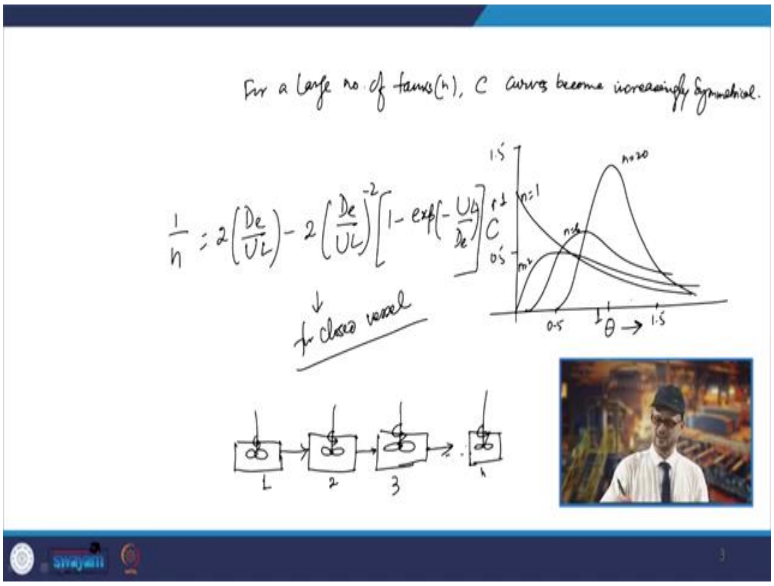

So, it as it is found that for a large number of tanks that is n. So, C-curves become increasingly symmetrical.

So, you know if you see the first curve will be just like exponential one, so it will starting from 1 and it will go for well mixed flow. But as the n is increasing, so that tends to be more and more symmetrical. So, that can be seen. And for the closed vessel you know it if you try to see that, if you that can be understood by having the analysis of these plots.

So, what is seen is that if you have c as this and if you have a θ on this line. So, for n equal to 1 it will go like this. So, this is for n equal 1. Now the thing is that if you have n equal to 2, so that will move like this. Similarly if you have you know for example, if you have n equal to 6, so it may go like this. And if you have n equal to 20, so it will move like this. So, this is n equal to 2, this is equal to 6, this is n equal to 20. So, this has been reported in the literature.

So, you know what happens that if you see this when it is n equal to 1 this is basically this is how it starts from here, that is 1 and you have 0.5 and then this will go as 1.5. Similarly, you have theta as some where 1 here, and then you will have this side you have 0.5 and this side you have point 1.5 or so.

Now this both these tank and series model as well as we have studied about the dispersion models where we have got the expression for the you know for c. So, depending upon the values dispersion numbers and all that d by u l and all, if you compare so if you see both these models they give rise to similar type of curve that is C-curve. And if you compare these two models so you can have you know, after comparison you can see that you will get $\frac{1}{n} = 2(\frac{D_e}{UL}) - 2(\frac{D_e}{UL})^{-2}[1 - exp(-\frac{UL}{D_e}]$.

So, this is how you for the closed vessel. So, this is you know for closed vessel, this is the correlation which is you know which is coming you know once you compare once you take into consideration the models of the tank and series model; tank and series model and all these other dispersion models. So, the you know and that also. So, we will also see that how these you know what these indicate.

So, basically when you have connected in series when your tanks are basically connected in series. So, basically you have this is the first tank. So, you will have this tank here, so this is having mixing here and this is mixing then, you know it is connected to the second tank. So, here again this is this is also rotating. So, you will have first this will have second, so that where you will have the third one. So, all these are connecting. And then that way it will go to the n-th tank. So, you will have n tank which is connected in series and you know.

So, if you see that for this tank you know here the mixing is more. So, mixing has a dispersion has dispersion is more so your it starts from here. However, if you go to this tank, so in this tank it has started it has gone like this. For the third one it will it will be delayed from here. And it will go on delaying as you go towards the larger value of n.

So, that is indicative of the other things also like you have different reasons. If you see here you have completely well mix joined starting, whereas if you go to the other regions so in this there is no dispersion so you have a certain component like the plug component. So, you have mix component as well as plug component. So, those things you know they are can be analyzed using these curves and they have certain meaning.

So, as you move you see that it is getting delayed. So, this plug and the part is you know getting apparent here, whereas in this part plug part is not seen, because here is pure mixing I mean well mixed flow is more prominent in this case. So, so that way you can have the analysis of the curves in different manners.

So, we will talk about the other models also you have the combined or mixed models so, that with that model also we can analyze these RTD curves. And in the coming lecture we will talk about, so the analysis of these RTD curves for you know by taking into account the different zones inside the tundish. So, you will have the; so this way you have you are seeing the plug zone. So, you will have the mixed zone and similarly you will have dead zones. So, all these things will be discussed as we. So, we will have another you know apart from this tank in series model also you have combined models, so that we will discuss in our coming lectures.

Thank you very much.