

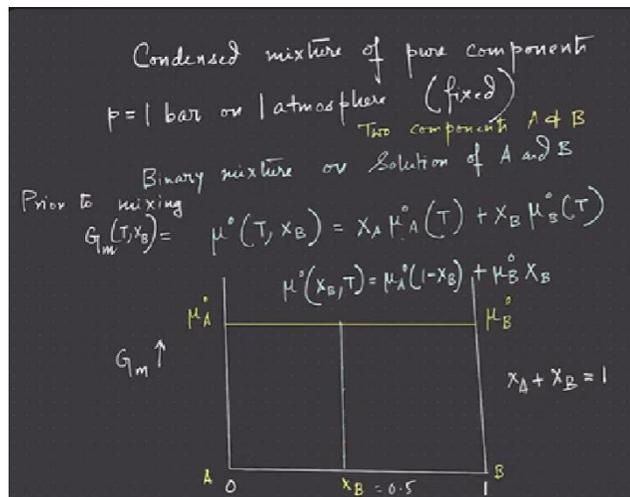
Thermodynamics And Kinetics Of Materials

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Lecture 24

Thermodynamic properties due to mixing

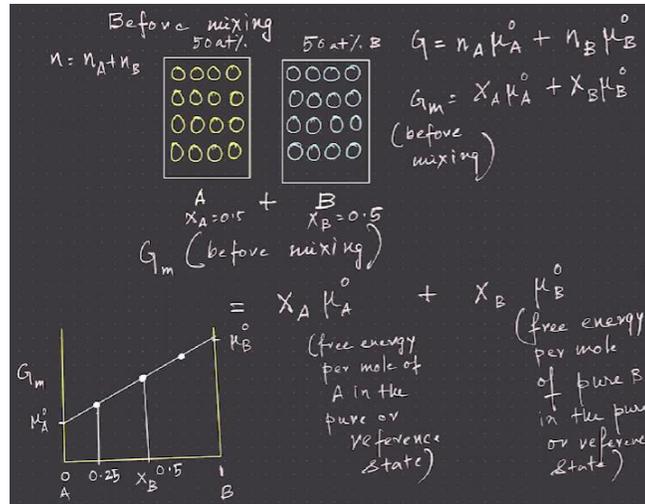
This is the process that I am talking about is before mixing. So, you have before mixing you basically have. So, this is before mixing as you can see here. So, at and again I am looking at a context mixture of pure components. For example, in the liquid state or in the solid state that means the PV work is negligible. So, we are assuming P equal to 1 bar, we are assuming P equal to 1 bar or 1 atmosphere, we are assuming that and we are fixing it.



Now you have two components A and B, two components A and B that are mixed. Now, your binary mixture or solution of A and B. Now before mixing, prior to mixing let us call it prior to mixing, the molar free energy of A and B will be given as $X_A \mu_A^0$ plus $X_B \mu_B^0$. Now X_A and X_B basically is for example the composition.

So, $X_A \mu_A^0$, $X_B \mu_B^0$ is going to be the molar free energy that is prior to mixing. So, basically I have taken some A atoms, I have taken some B atoms, I am trying to put them together and then I am trying to mix them. So, basically before mixing them if you look at that this is what we had. So, this is before mixing. If you look at the process before mixing, you had a box of A atoms and you can imagine them as like some yellow spheres and you had a box of B atoms and you are adding them together.

So, this is before mixing, we are just adding them together, we have not allowed them to mix A and in that case g_m before mixing will be $X_A \mu_{A0}$. Now what is μ_{A0} ? μ_{A0} is the free energy per mole of A in the pure or reference state. So, this is basically free energy per mole of A in the pure or reference state. Reference state may not be pure state, but we are assuming so and then you will have $X_B \mu_{B0}$ again this is free energy per mole of A in the pure state. So, basically this is X_A , X_A and X_B are the concentration.



So, basically if I think of g say which is basically $n_A \mu_A$. So, g is the extensive property plus $n_B \mu_B$. Now divide by n , n is equal to n_A plus n_B and now divide by n . So, g by n becomes g_m equals to now here it is μ_{A0} , here it is μ_{B0} , μ_{A0} indicates pure state, μ_{B0} indicates μ_{B0} the superscript 0. So, please note that the superscript 0 indicates pure state.

Now g_m equals to $X_A \mu_{A0}$ plus $X_B \mu_{B0}$ because I am dividing by n . So, n_A by n is X_A , n_B by n is X_B . So, you have this and this is the g_m before mixing. So, this is before mixing. So, now if you plot this, if you plot them then say for example μ_{A0} is this point and μ_{B0} is this point. Say we are assuming for simplicity μ_{A0} and μ_{B0} to be at the same level then basically if you look at this, this is g_m . Now you see this is your composition X_B . So, if they are at the same level this is your composition say X_B equal to 0.5. Say you have like here you have 50 atomic percent, 50 atomic percent and 50 atomic percent B then basically X_A equal to 0.5, X_B equal to 0.5, X_A plus X_B equal to 1. So, I am taking that composition X_B equal to 0.5. So, X_B equal to 0.5. So, I have my energy of mixing here, right? This is my y-axis. Now since they are at the same level whatever be the composition say X_B equal to 0.25 my energy is here. X_B equal to 0.75 my energy is here.

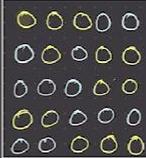
See that this is along the same horizontal line. So, basically the elevation is the same if the elevation because g_m is right from the y -axis. Now if you have μ_{A0} and μ_{B0} at the same level exactly then here or here or here it does not make difference. So, before mixing the energy remains the same means here or here or here. However, it is also possible that μ_{A0} and μ_{B0} are at the different levels.

Say for example, I can think of say some sort of here is μ_{A0} and here is μ_{B0} . So, I have like this. So, when I am mixing just before mixing also there is a difference, right? So, that is also possible. Say for example, let us think of a case like this. So, here you have say some sort of but it is still a straight line.

After mixing

(T, x_B)

$x_A + x_B = 1$



Random Solution

$G = H - TS$

$G_m = H_m - TS_m$

$G_m(\text{after mixing}) < G_m(\text{before mixing})$

$$G_m = \mu_A^0 x_A + \mu_B^0 x_B + \Delta G_{m, \text{mix}}$$

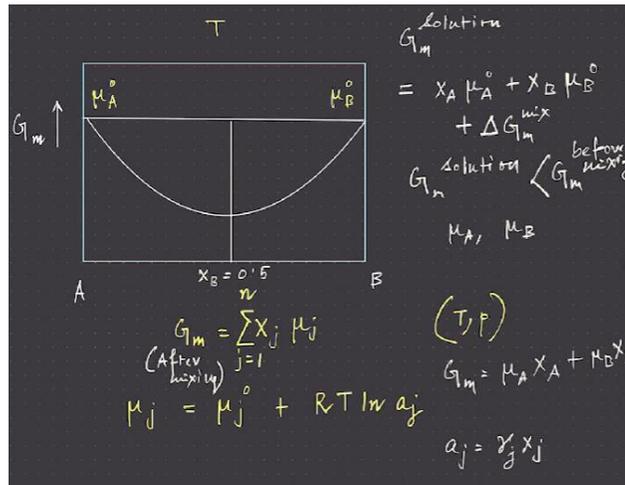
$$= \underbrace{\mu_A^0 x_A + \mu_B^0 x_B}_{G_m(\text{pure before mixing})} + \Delta G_{m, \text{mix}}$$

So, it is now your μ_{A0} it is your μ_{A0} it is your μ_{B0} . Now you can see the more B you add you have slightly higher free energy more A you add you have the slightly lower free energy. But however, that is why for simplicity I drew a horizontal line to show you the concept of mixing. What happens? In mixing after mixing there is a decrease in this energy, the initial energy that you have for the mixture. So, as a result just to make you understand it has been drawn that μ_{A0} and μ_{B0} at the same reference level.

The reference levels can actually be arbitrary μ_{A0} can be less than that of μ_{B0} . However, the important part here is after mixing, right? Important part here is after mixing what happens, right? So, that is why just to avoid any confusion I am please note generally there is a possibility that you can have an axis like this. So, basically generally it is possible to have this and you have say here you are plotting say g_m this is before mixing, g_m before mixing and you have x_B here and x_B equal to 0 means B over A and x_B equal to 1 means B over B and you have μ_{A0} here and μ_{B0} that is understood that is here or otherwise

the other way also is possible like μ_{A0} here, μ_{A0} here somewhere and μ_{B0} somewhere here, whatever you can do. Now, if you have that before mixing it is just given by a line, right? It is given by a line connecting μ_{A0} and μ_{B0} , right? It is just a line connecting that. Now, if you have that you have a line.

Now, if you have a line now you see this is not exactly it is not a horizontal line and as a result for different values of x_B say for different values of x_B say here or here or here you will have some difference, right? Say for example this is for x_B equal to 0.25, okay? x_B equal to 0.25. So, this is 0.25. So, this is the free energy before mixing. Now, say here say x_B is 0.5. So, this is the again the G_m before mixing, right? All of these points that you are lying on this line for different x_B , okay? This is basically a mixture of A and B but A and B have not yet mixed, right? They have not yet mixed to form a random solution, right? So, they have not yet formed a solution I can tell, okay? So, A maintains its distinct A nature and B maintains its distinct B nature, right? You can basically easily distinguish one partition that is completely A, the other partition that is completely B. You have removed the partition but you haven't allowed them to mix.



So, as a result along this horizontal line that connects μ_{A0} and μ_{B0} is where the free energy of the mixture will lie. That is just before mixing. Now, after mixing, now let us consider the scenario after mixing. Let us consider the scenario after mixing. Now, you see that the yellow, so yellow atoms were representing A and the blue borders were representing B.

Now, as you can see now, yellow and blue have thoroughly mixed and it is a random solution, right? This is random, right? You cannot go to one point and tell, go to an atom

and tell whether it is A or B. You have to look at it and see whether it is A or B and it is almost like in a solution we call it or in a liquid solution we call it like an average atom, right? It is a random solution. It is a random. The solution has become random.

So, random solution. Now, if you remember the statistical interpretation of entropy, the randomness increases, entropy increases, isn't it? Randomness increases, entropy increases. So, if you have a random solution then you have basically an entropy term that is increasing. Now, g is h minus Ts , right? g is equal to h minus Ts . Now, think of this, g_m is also h_m minus Ts_m . Now, think of g_m that is before mixing which is $\mu_A^0 x_A$ plus $\mu_B^0 x_B$ plus I am telling there is some additional Δg_m mix.

Now, this Δg_m mix say has say a Δh mix, h_m mix and there is a Δs_m mix. Now, as you can see due to mixing there is randomness and as randomness increases, right? As the randomness in the configuration increases the entropy contribution also increases. So, Δg_m mix is in general negative and as a result, so g_m after mixing means after this randomness, right? Or after mixing is generally smaller than g_m before mixing. So, then mixing basically reduces the energy of the system, right? Mixing reduces the energy of the system. If g_m after mixing is greater than that of before mixing in general these atoms would not like to mix.

They do not want to mix, okay? At that given temperature. Again, I am looking at a given temperature condition, okay? And say at a given overall x_B , right? T and x_B are given. Now, I am telling why x_B ? Why not x_A ? Because x_A plus x_B equal to 1. So, if I fix x_B , x_A is automatically fixed, right? So, this Δg_m mix, so you have this part.

This is like the pure part. So, pure part, right? So, you can think of this like g_m pure where each part maintains its purity and then you have the purity means, purity means that the distinct characteristic like A atoms you can distinguish as A atoms, B atoms you can distinguish as B atoms. You have a partition, but you haven't removed the partitions to allow them to mix. You have a total system, okay? Containing two subsystems. One subsystem contains A, okay? Like this picture. One subsystem contains A, the other subsystem contains B that's at the same temperature and pressure.

$$\mu_j = \mu_j^0 + \underbrace{RT \ln \gamma_j}_{\substack{\text{due to} \\ \text{interactions} \\ \text{or formation} \\ \text{of chemical} \\ \text{bonds}}} + \underbrace{RT \ln X_j}_{\substack{\text{due to} \\ \text{random} \\ \text{mixing}}}$$

A-B solution
 A-A bonds
 B-B bonds
 A-B bonds

$$\mu_A = \mu_A^0 + RT \ln \gamma_A + RT \ln X_A$$

Due to random mixing

Again, at same temperature, pressure, okay? And the way the subsystems are added is like you have added 50 atomic percent of A and 50 atomic percent of B, but the partition, okay? The partition, it is still impermeable. That is, it is not allowing A and B to mix, okay? Once the partition is removed, now A and B are completely mixing, then you have this g_m , this pure components plus that is before mixing and then you have the Δg_m mix that comes after mixing and that part, basically that part, it basically reduces the energy. So, if you look at this very hypothetical simplistic situation, as I told you, so you had this reference energies, right? You had this reference energies, μ_A^0 and μ_B^0 and you had say some composition, say let us classify some composition here. So, let us specify some composition here and this composition is say x_B equal to 0.5. Now, look at this. Look at the energy before mixing. This is before mixing, this is the energy after. Now, you see after is less than before, right? g_m goes up this way. So, this is before, this is after. So, after is less than before, right? So, it is allowing the mixing, it favors mixing.

So, g_m solution, so it favors forming a solution of A and B, random solution of A and B and it is less than g_m before mixing and as a result of mixing, what has happened? Now, the g_m has now become the g_m after mixing, g_m after mixing, right? This is g_m after mixing. Now, after mixing in the solution, you have for each component, you have the chemical potential, right? So, basically like chemical potential. So, basically the chemical potential now is like μ_A and μ_B . Now, g_m is now $\mu_A x_A$ plus $\mu_B x_B$. Previously, it was $\mu_A^0 x_A$ plus $\mu_B^0 x_B$.

So, $\mu_A - \mu_A^0$ and $\mu_B - \mu_B^0$ is the excess part, okay? Is a relative change, relative partial molar free energy. Is that relative partial molar free energy, right? So, this is the idea and if you look at this, if you look at this, you have μ_J , see you have μ_J . So, I am defining μ_J as this, of component J goes to μ_J^0 . μ_J^0 is the standard or reference step plus $RT \ln \gamma_J$, right? You remember the last lecture, I have defined A

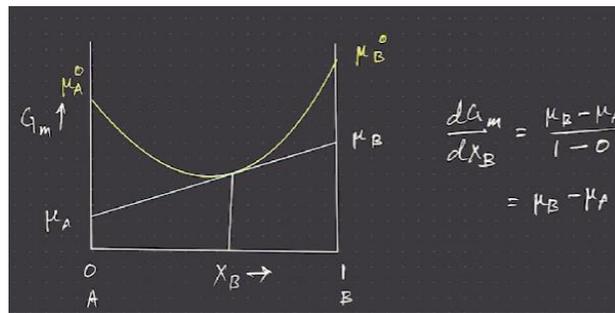
J. A J is the activity of J in the solution, activity of J in the solution, okay? Now, you see that this A J, as I told you, A J is nothing but gamma J x J.

$$\begin{aligned}
 G_m &= \sum_{j=1}^n \mu_j X_j & \Delta G_{m, \text{mix}} &= X_A (\mu_A - \mu_A^0) + X_B (\mu_B - \mu_B^0) \\
 dG_m &= \sum_{j=1}^n \mu_j dX_j \\
 \sum_{j=1}^n X_j &= 1 & X_i &= 1 - \sum_{j=1, j \neq i}^n X_j \\
 G_m &= \mu_i + \sum_{j=1, j \neq i}^n X_j (\mu_j - \mu_i) & \sum_{i=1}^n dX_i &= 0 \\
 \left(\frac{dG_m}{dX_j} \right)_{j \neq i} &= \mu_j - \mu_i & dX_i &= - \sum_{j=1, j \neq i}^n dX_j
 \end{aligned}$$

Now, you see if I now write individually for the component J, so this is for component J. So, for example, for component A, it will be $\mu_A^0 + RT \ln \gamma_A + RT \ln X_A$, okay? Which you can write as $RT \ln (1 - X_B)$. But see, this is due to random mixing, and this part is due to interactions or due to interactions between other atoms. So, A is interacting with B, so this part comes due to interactions between A and B and formation of chemical bonds. So, what type of chemical bonds A and B? Say, if I have AB, if I have an AB mix, right? AB solution, then what are the chemical bonds that can form? You can have AA bonds, you can have like pairwise, if I consider only pairwise bond formation, then you can have AA bonds, you can have BB bonds, and you can also have AB bonds, right? So, all of these bonds can form and you have this bonding part and you have, and these bonds means these are the interactions, right? This formation of chemical bonds and this $RT \ln \gamma_A$ or $RT \ln \gamma_J$ for the component J, basically this $RT \ln \gamma_J$ contributes to the or basically is due to the interactions between A and A, A and B and B and B, right? So, these interactions are basically resulting from the formation of chemical bonds like AA, BB and AB bonds, right? And there is this $RT \ln X$, which is just due to random mixing, okay? Now, think of this as I told you, so G_m mixing, after mixing you have $\mu_A X_A + \mu_B X_B$.

So, if you look at, if I tell you ΔG_m , so this is another thing you have to understand, ΔG_m mix is $X_A \mu_A - \mu_A^0 + X_B \mu_B - \mu_B^0$, where μ_B and μ_A are basically the potentials of A and B in the solution, right? Chemical potentials of A and B or partial molar free energy of A and B in the solution and μ_A^0 is for the pure

form and or the standard form and μ_B^0 is for the partial molar free energy in the pure form of A or μ_B^0 is in the pure form of B or the standard form of B, right? So, you have, so basically you can write this way, you can try to derive this way, G_m is summation of $\mu_J X_J$ over all components N, okay? You can call all components say I equal to 1, 2, so you have say C components, okay? C components because N I am talking about total number of moles, right? So, this way I am telling you have C components, so J equal to 1, 2, C, component 1, component 2 like that component C, okay? So, up to component C. So, $D G_m$ as you know is $\mu_J D X_J$ because the other part is basically 0 because of Gibbs relation, right? $X_J D \mu_J$ is 0, right? $X_J D \mu_J$ is 0. Now, $D G_m$ is $\mu_J D X_J$ and summation X_J equal to 1, X_I is 1 minus summation X_J where J equal to 1, 2, C, J equal to 1, 2, C and J not equal to I, right? Obviously, it cannot be equal to I. So, 1 minus summation of all the other X_J 's. So, G_m is μ_I plus J equal to 1, 2, C, so I put C here, okay? J equal to 1, 2, C, J not equal to I, $X_J \mu_J$ minus μ_I , right? It is μ_J minus μ_I .



So, $D G_m$ by $D X_J$, okay? J not equal to I is μ_J minus μ_I . See, this is something that I have drawn when I did the tangent construction, right? We basically did this. I just want to reprise. So, if I show you when I did this for binary system, what we told that, okay, let us have this curve. Let us have this diagram where you have plotted G_m this way and X_B this way such that this is 1, that is pure B and this is 0, pure A and then I got a free energy curve.

So, just drawing a free energy curve like this, right? So, it comes here, it comes here. So, basically this part, this point is μ_A^0 , this point is μ_B^0 and this is after mixing. So, if you look at a tangent to this, if you want to draw a tangent line to any composition, you guys do any composition, you want to draw a tangent line. So, I will just push it and make it a tangent at some composition, say at this composition, at this composition, okay? So, at this composition, it is a tangent. Then the intercepts are, this is μ_A , this is μ_B of the solution.

See, this is μ_B^0 , see this is higher, μ_B is less and μ_A is less and you can also see

that this is your G_m and this tangent basically gives you $dG_m dx_B$, the slope of this tangent line is $dG_m dx_B$ which is $\mu_B - \mu_A$ by 1 minus 0 which is equal to $\mu_B - \mu_A$.

$$G = n G_m$$

$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, P, n_{j \neq i}} = \left[\frac{\partial (n G_m)}{\partial n_i} \right]_{T, P, n_{j \neq i}} \quad \frac{n_i}{n} = \frac{n_i}{\sum_i n_i} = x_i$$

$$n_1, n_2, n_3, \dots, n_r$$

$$\rightarrow n_1, x_1, x_2, x_3, \dots, x_r$$

So, if you want to do it for a ternary system, quaternary system, how do you do that? So, you basically see $\partial G_m / \partial x_j$, so from this derivation, you can easily see just by partitioning this x_i and x_j where j is not equal to i , I can write $\partial G_m / \partial x_j$ I get is $\mu_j - \mu_i$, right? And remember, dx_i is equal to 0, dx_i equal to 0, right? So, if you look at $\partial G_m / \partial x_j$, okay? So, it is $\mu_j - \mu_i$ and dx_i is equal to 0, so because you have $\mu_j dx_j$, right? So, $\mu_j dx_j$ is there. Now, if you have that, you have $\mu_i dx_i$ and $\mu_j dx_j$ and x_i is 1 minus some x_j and dx_i is minus some dx_j , right? Because the sum of dx_i equal to 0, right? And again, here this j indicates j is not equal to i , right? So, we can write this way and so we can write it for any number of components, the same curve that we have shown in the previous lectures, okay? And you can derive it in a very general form like you can start with the definition of μ_i since it is the chemical potential of component i which is the derivative partial derivative of free energy that the extensive free energy with respect to the change in mole number of component i , keeping mole number of component j not equal to i that is all other mole numbers of all other components constant, temperature constant, pressure constant. So, now g is nothing but ng_m , right? g is nothing but ng_m , g_m is the molar free energy of the solution, g_m is the molar free energy of the solution and n is the total number of moles of some of all the components, right? So, and so if that is so and n_i / n is nothing but x_i and so from n_1, n_2, n_3 , if you're mapping this way, okay? And say there are r components, so I am talking about r components, so if you do that and if you continue to derive, so you have $\partial g / \partial n_i$ which is $\partial n / \partial n_i g_m + n \partial g_m / \partial n_i$, we have done this and g_m comes in $n \partial g_m / \partial n$ and now if you have j equal to 2 because see this is for i th component, now you are going from 2 to r , right? One component you have already considered here, right? So, i you have considered here, now you are going to 2 to r and this is $\partial g_m / \partial x_j$, $\partial x_j / \partial n_i$, now $\partial g_m / \partial n$, basically this term $\partial g_m / \partial n$, right? Equal to 0, right? ∂

gm del n, n is the total mole number, right? Del gm del n equal to 0 and del n del ni equal to 1, right? So, this term goes to 0, so you have now this term, okay? You start from j equal

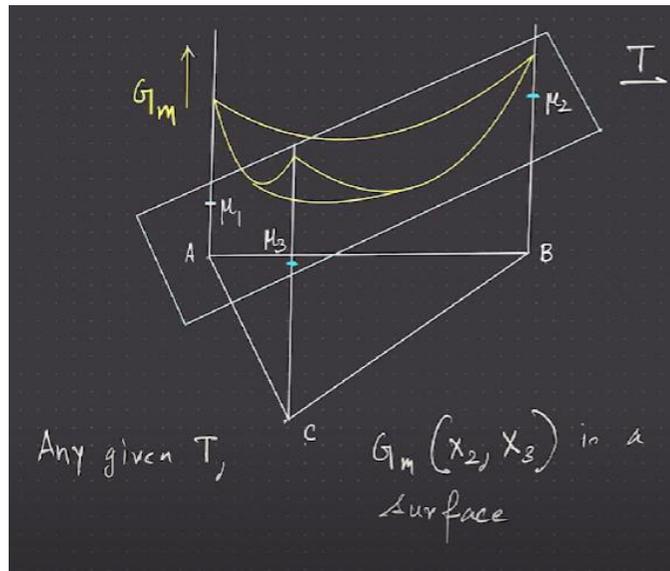
The image shows two panels of handwritten mathematical derivations. The left panel shows the derivation of the chemical potential μ_i as a function of the total number of moles n and the mole fractions x_j . It starts with $\mu_i = \left(\frac{\partial G}{\partial n_i}\right)_{n_k} = \left(\frac{\partial n}{\partial n_i}\right) G_m + n \left(\frac{\partial G_m}{\partial n_i}\right)_{n_k}$. This is then expanded to $\mu_i = G_m + n \left(\frac{\partial G_m}{\partial n}\right) x_i + n \sum_{j=2}^r \left(\frac{\partial G_m}{\partial x_j}\right)_{n, X_k} \left(\frac{\partial x_j}{\partial n_i}\right)_{n_k}$. A note at the bottom states $\frac{\partial G_m}{\partial n} = 0, \left(\frac{\partial n}{\partial n_i}\right)_{n_k} = 1$. The right panel shows the definition of the Kronecker delta $\delta_{ij} = \frac{\partial x_j}{\partial n_i} = \frac{\delta_{ij} - x_j}{n}$ with conditions $\delta_{ij} = 1$ when $i=j$ and $\delta_{ij} = 0$ when $i \neq j$. It then presents the generalized formula $\mu_i = G_m + \sum_{j=2}^r (\delta_{ij} - x_j) \frac{\partial G_m}{\partial x_j}$ and applies it to a binary system ($r=2$) where $x_1 + x_2 = 1$, yielding $\mu_1 = G_m - x_2 \frac{\partial G_m}{\partial x_2}$ and $\mu_2 = G_m + (1 - x_2) \frac{\partial G_m}{\partial x_2}$. For a ternary system ($r=3$) where $x_1 + x_2 + x_3 = 1$, it shows $\delta_{i2} = 0$.

to 2 to r, you have del gm del xj and del xj del ni, then basically you get del xj del ni which is a Kronecker's delta, this is again, what I am doing is I am expanding this proof, okay? To the two systems more than binary, more than two components, so it becomes delta ij, delta ij is Kronecker's delta, remember delta ij equal to 1 when i equal to j, okay? And equal to 0 when i not equal to j, right? When i not equal to j it is 0, so basically what you get is mu i equal to gm plus j equal to 2 to r, r is the number of components, okay? R means you have starting from component 2 you have up to component r and you have delta ij minus xj del gm del xj, right? Del gm del xj is the generalized loop, right? It is the, remember it is better to write instead of writing this del gm del xj, I do not like this because del gm del xj is, I mean sometimes it is acceptable but it is better to write b gm del xj or actually this one of the components, so basically yeah, it is better to write this way here because these are all independent, these are all means x1 plus x2, say if I think of that, if I think of x1, if I want to fix x2 that is not possible because x1 plus x2 equal to 1 for a binary system, right? So, basically these are not all independent, right? So, basically if I look at x1 plus x2 plus x3 equal to 1, so that is why this partial in general for multi component systems is possible because x2 and x3 are independent, right? Not x1, right? One of them is not, so basically if you look at multi component systems where the number of components is greater than 2, then this is fine because one of the x's is a dependent variable, all other x's are independent, right? But now let us look at a binary, so here I am giving for the same formula I am talking about binary, generalization, so this is the generalized formula, this is generalized. Now, if you have this generalized formula, then for a binary system, binary system means containing two components, you write i equal to 1, so mu 1 equals to gm and now j equal to 2 to r, so you have anyway r equal to 2, right? You have two components only, right? You have two components only.

$$\begin{aligned}
 \eta &= 3 & X_1 + X_2 + X_3 &= 1 \\
 \mu_1 &= G_m - X_2 \frac{\partial G_m}{\partial X_2} - X_3 \frac{\partial G_m}{\partial X_3} & \text{Assume } X_1 & \text{is dependent variable} \\
 \mu_2 &= G_m + (1 - X_2) \frac{\partial G_m}{\partial X_2} - X_3 \frac{\partial G_m}{\partial X_3} \\
 \mu_3 &= G_m - X_2 \frac{\partial G_m}{\partial X_2} + (1 - X_3) \frac{\partial G_m}{\partial X_3}
 \end{aligned}$$

Now, if you have put i equal to 1, so if you see i equal to 1, so and if you look at that, you do not have, if i equal to 1, you start with 2, right? So, if there is no other, so j is always 2, so j starts with 2, so delta 1 2 is equal to delta 1 2 will be equal to 0, right? And so there is a minus, so it's 0 minus xj, j again, j actually r here is equal to 2 and it starts from 2, are you seeing that? So, basically if it is so, then this is 2, so for a binary system this is 2, this is also 2, so you have 2 here and since i is 1, 1, 2, this is 0, so you have minus x2 dgm, so here you see, I cannot write del gm del x2 keeping x1 fixed because x1 plus x2 equal to 1, right? You have to write dgm dx2, right? So, and the other part, say if I put i equal to 2, now you see delta 2 2, delta 2 2 equal to 1, delta 2 2 equal to 1, so 1 minus x2 dgm dx2, so you get both the intercepts mu1 and mu2. Now, if you look at a, so basically as I have done here, so mu A and mu B or mu1 and mu2. Now, if you put n equal to 3, apply that formula, now there is an interesting part that see x1 plus x2 plus x3 equal to 1, so I can now tell x2 and x3 are independent, so when I write del gm del x2, what I mean is x3 is fixed and when I write del gm del x3, then x2 is fixed, right? x1 is the dependent variable, x1, so 1, so if I write this way, so x1 is dependent, so x2 and x3 are there, so x2 when I do del gm del x2, x3 is fixed and when I do del gm del x3, x2 is fixed, so now if you have that and x1 is dependent, so again you have gm minus x2 del gm del x2 minus x3 del gm del x3, mu2 will be gm plus if you apply the formula, if you see this, you have 2, 2, 3, right? R equal to 3, 2, 2, 3, so delta, so for example for mu1, so this is 0, so that's why minus x2 del gm del x2 minus x3 del gm del x3, but here when you do 2, then in one case it is 0, so delta ij is 0 for this case, for this term, right? When mu, when i equal to 2, but this term is not 0, see there are, so you have one intercept at mu1, mu2 and mu3, now if you have that, you have mu1 that you got, again it's a ternary system, so x2 and x3 I can take as independent, right? x1 is dependent, I cannot take 2 as dependent, right? x2 plus x1, since x1 plus x2 plus x3 equal to 1, I can take any 2 to be independent, it can be 1 and 3, it can be 2 and 3, we have chosen 2 and 3, okay? We have assumed, so assume x1 is, so in this case what I've done is I assumed x1 is dependent variable, but you can also assume x2 is dependent variable, then x1 and x3 are independent, okay? You can also assume x3

as dependent variable, so then x_1 and x_2 are independent, right? You can assume anyway, now you have that, then you have these three equations, three equations, μ_1 , μ_2 and μ_3 , these are basically the equations for the intercepts, right? And if you look at this gives triangle, see if you look at this gives triangle in three component system, you have three components A, you have three components A, B and C, okay?



And if you look at it, this is at a fixed temperature, okay? Again, we are looking at a fixed temperature and this is a condensed phase, I don't care about pressure, pressure is not an axis, so G_m is plotted, the G_m now has become a surface, right? It's a function of, G_m is a function of x_1 , x_2 and x_3 but x_1 is dependent variable, so G_m is a function of, I can tell that x_2 , x_3 and T . Now, T I have fixed, T I have assumed, so at this T , G_m is just a function of x_2 and x_3 , now I have plotted this, so you can see this is your triangle where A, this is 100 A and this is say 0 A and this is say 100 C and this is 100 B, now if you look at this axis, if you look at each of this axis, so you have this axis and then you have another axis here and you have another axis here, right? There are three axis, now if you look at the surface, this is your surface, the free energy surface, right? This yellow guy, this is your free energy surface, now at some composition, again the composition, if you have to specify composition, I have to specify, so this is, so what I am telling is at a given T , at any given T , you can write G_m as a function of x_2 , x_3 , right? Now, this is a surface, now if it's a surface, if it's a curve, you have a tangent line, if it's a surface, right? It's a three-dimensional surface, then you have a two-dimensional tangent plane, right? So, you have this blue guy that this one is the tangent plane through a suitable composition set x_2 , x_3 , okay? Now, if it is so, this tangent plane is making intercepts, say for example, if you

see that this is the intercept, so if you look at it, it's a slightly perspective joint, so you have like, if you look at this, this is gone through like the surface and if you look at that, that this plane has got this axis at μ_1 , the A axis at μ_1 , the C axis at μ_2 and the B axis at μ_3 , okay? So, you have A, B, C, so or you can write this also, this is also fine, you can tell this is μ_2 and means it's just a convention and say this is μ_3 , C is 3, so this is μ_3 , so this blue point that you can see here, right? The blue point that you can see here or the blue point that you can see here or here, these are the intercepts on this axis, on this axis, these are the intercepts by the plane and these are the intercepts, one intercept here, μ_1 , so for the A axis, for the B axis, it is μ_2 and for the C axis, it is μ_3 , right? These are the, these intercepts are the chemical potentials of the solution at a given composition and you can set x_1 as x_2 , x_3 , okay? x_1 is automatically determined because I have assumed x_1 to be dependent value, okay? So, basically you can, so what we are telling here, again, let us go back to this, this is a generalized form, okay? Generalized form for the expression of μ and μ_s are nothing but the chemical potentials and this chemical potentials of component I and as you can see, if you want to visualize, if you want to visualize for greater than 2, then it becomes, for example, for component 3, you have a surface, you have a tangent plane, for component 4, you have a hypersurface, you have a hyper tangent plane, so basically, it's very difficult to visualize but using this approach, you can always get μ_1 and μ_2 as a function of g_m and x_2 , right? So, that's the idea. So, if you look at this, so basically, this is a very generalized way, okay? It can apply to n equal to 2, n equal to 3, n equal to 4, n equal to 5, any number of n and you can get all these μ_s , right? μ_1 , μ_2 , μ_3 , μ_4 , μ_5 .

Only thing, remember that if it is a 4 component system, then 3 components are independent, one of them, 3 components like x_2 , x_3 , x_4 are independent and x_1 , say, we can assume to be dependent, so we cannot have all 4 as dependent, right? When we are expressing in terms of mole fraction, right? But this is a generalized formula. But if you can see, if you apply this generalized formula, you get for the binary system, the common tangent that the not common tangent construction, you get the tangent intercept method, so you get the intercepts, right? You get the intercepts in the binary system, okay? Excuse me, common tangent will come when we will discuss phase equilibrium, okay? We haven't yet discussed phase equilibrium in binary systems, which will soon, although we have discussed equilibrium between subsystems and there's a same idea that will apply, but I haven't shown it in the form of free energy composition diagrams, but this is a very, very important diagram that you should remember, okay? And this is a diagram that I will apply when I also explain the phase equilibrium subsequently, okay? Not right now, but subsequently. So, only thing that I wanted to tell you that this approach that which starts with the definition of chemical potential, right? It starts with the definition of chemical potential and then G , I write as NGM and then we go on, right? ΔN , ΔN_i and $N \Delta G_m$ ΔN_i and then ΔG_m ΔN_i is again split into one is like ΔG_m ΔN and ΔN

del Ni and then there is this, this is the chain rule, okay? When I am having Xi, Xi is the dependent variable and then for other independent variables, J equal to 2 to R, these are the independent variables, I have this del Gm del Xj and del Xj del Ni and since this is del Xj del Ni, this is basically del Xj del Nj, del Ni. So, as a result, you can see that there is a delta Ij automatically that comes in and that's why you have this delta Ij, this is called Krone-Kirch delta, okay? This is also called, this is known as Krone-Kirch delta. It's a very useful mathematical function, you can actually see that it's an identity matrix basically.

Delta Ij equal to 1 when I equal to J or 0 when I is not equal to J, right? And as you can see here, you have this expression which becomes basically 1 minus X2 here and 0 minus X2 here, right? So, this expression as you have is multiplied with the generalized slope, right? With respect to X1, not X1, X2, X3, X4 and so on, right? Del Gm del Xj, right? When I am writing del Gm del Xj, I am telling not the dependent variable but other independent variables are fixed, right? For higher order multi-component systems, okay? For example, a higher order system here is N equal to 3. I have assumed X1 is dependent and X2 and X3 are independent. So, when I am writing del Gm del X2, understand that X3 is fixed, not X1. Dependent variable cannot be fixed, right? And when I am writing del Gm del X3, X2 is fixed, right? So, this is the idea. Okay, so now I give an example of ideal gas mixtures, okay? In ideal gas mixture, what happens? If you have ideal gas, you have all these components which behave, all the components in the ideal gas mixture will not interact.

IDEAL GAS MIXTURES

Mixing process occurs at a given temperature T and pressure P

Note: No interaction between particles in an ideal gas

Each component of ideal gas in the mixed state behaves "as if" it occupies the entire volume

Dalton's laws of partial pressure

$$p_k = X_k P \quad P - \text{Total pressure}$$

$$P = \sum_{k=1}^C p_k = \sum_{k=1}^C X_k P = P \sum_{k=1}^C X_k = P$$

$$dp_k = -\bar{S}_k dT + \bar{V}_k dP$$

$$= \bar{V}_k dP$$

(∵ mixing process is isothermal)

There is no interaction between particles in an ideal gas, right? There is no interaction between particles, right? That's what we have assumed. No interaction between particles in an ideal gas and mixing process occurs at a given temperature and pressure. Again, we have fixed the temperature and pressure, no interaction, that is Ln gamma i term, that gamma term, that RTLm gamma term goes to zero, right? Because there is no interaction. So, each component of ideal gas in the mixed state behaves as if it occupies the, so if there

is no interaction, each component of the ideal gas will behave as if it occupies the entire volume, right? Each occupies the entire volume, right? Because see, there is no interaction. Now, if there is no interaction and I have these different components, now each component acts as if, because the other, each component, these components, molecules or atoms are not interacting with the other molecules as a result, they act as if they occupy the entire volume, right? Now, and it is again, if you think of Dalton, now if you remember Dalton's laws of partial pressure, then what does it tell? The partial pressure of component K in an ideal gas mixture is proportional to x_K times P, where P is the total pressure and x_K is the mole fraction of component K.

So, partial pressure of component K is mole fraction of component K times the total pressure, right? Partial pressure equals mole fraction into total pressure, that's the Dalton's law of partial pressures, right? And if you see, $P_K = x_K P$ if you have C components, $\sum_{K=1}^C x_K = 1$, okay? So, P_K is nothing but $x_K P$, right? From this relation, right? So, P_K equals nothing but $x_K P$ and P is the total pressure which is fixed. So, total pressure is coming and some x_K , $\sum_{K=1}^C x_K = 1$, which is nothing but 1, right? This is equal to 1. So, this is nothing but P, right? $P = P$. So, as you can see here that this, so this proves that this is a consistent relation, right? And now, if you have that, you know, $d\mu_K$ or dG_K bar is a partial molar Gibbs free energy of component K, which is equal to $d\mu_K$ is equal to $-\bar{S}_K dt + \bar{V}_K dP$, right? You remember the bar because it's the partial molar entropy of component K plus $\bar{V}_K dP$. So, this comes from dG, so we already know the relation $dG = -SdT + VdP$.

Okay, so in the next lecture, I will continue this. In the next lecture, I will continue this and this ideal gas mixture stuff.

