

**Defects in Materials**  
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**Lecture – 15**  
**Strain**

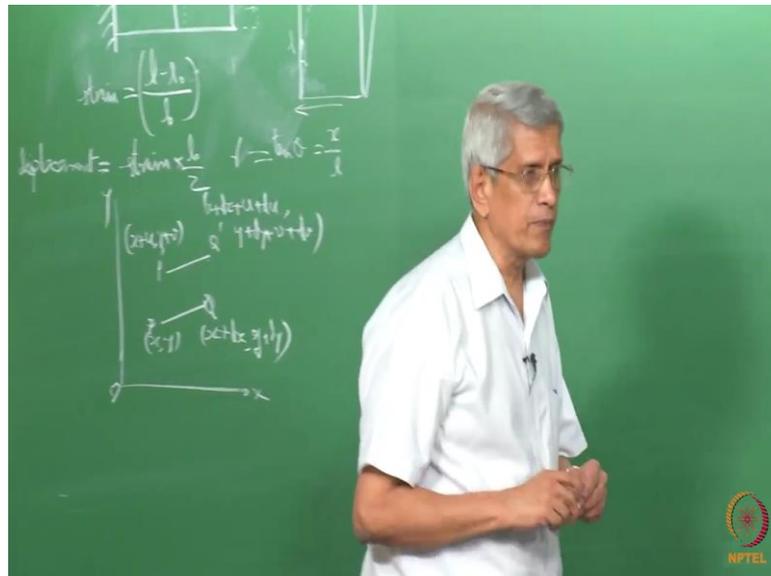
Welcome you all to this course on Defects in Materials; in the last class we have covered basic elementary part of the tensor which is required. Essentially tensor is nothing but an operator which describes the property of the material. So, the property of the material we are wanted to find out essentially we should have an input signal, and an output signal has to come out that is our property is being defined.

So, the input could be like for example, if you apply a voltage we can measure the current density which is there in that direction that gives information about the conductivity of the material or the resistivity of the material. Similarly various instances magnetic permeability; you can think of lot of instances even in electronics or even x ray scattering if you wanted to look at it, we have an input signal we have an output signal which comes what these 2 relate? These 2 relate the property of the material, that property of the material can be if you take with respect to a coordinate system and try to represent it, because that is the over view otherwise everybody will be everyone will be using different type of a coordinate system.

So, if you define with the vectors with respect to a coordinate system, when we look with respect to that coordinate system, then we will have 9 components of the independent 9 components are required to represent the property completely correct or with 9 components the property is represented ok. In the case of materials when we talk about deformation of material, in this case also we use 2 properties one is that stress and strain, these are all the terms which we used that is if you deform a material, and try to pull it we say that that there is a stress which is generated in the material. Both the quantities we call them as tensors, what is the reason for that we will call them instances? But generally the way we think about that is they are like any other quantity like a vector, but that is not the case.

The reason essentially is that; let us take the case of first strain itself, in the case of a strain the strain relates displacement and position of an object. So, position is also a vector displacement is also a vector.

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I put this sample along, and that sample dimension increases to is say this side is being (Refer Time: 03:13) when these get some extension let us take it to be some value, that is from  $l_0$  initial length it becomes a length  $l_0 + \Delta l$  we call it as the strain correct or elongation, this is the terminal elongation is the terminology which we use it.

In this if you take half the length of the sample, what is going to be the displacement? It will be some displacement will come, that is essentially given by the displacement is strain into  $l_0$  by 2 correct? That will be the extension which we are seeing it here, now here its  $l_0$  sorry this is  $l_0$ ; these what it gives is essentially the displacement correct? The displacement equals some quantity which we measure. So, it is what it relates.

So, in terms of vector if you try to represent it position is also a vector in terms of a coordinate system if you using, displacement is also a vector. So, 2 vector quantities are getting related. So, it is a second rank tensor, apart from that what we know of it is this we call it as an elongation. This is one way in which the sample can be deformed or we can bring about the change in shape to the material. What is the other way in which we can bring a change in shape to material that is essentially what we do it is that.

Suppose we take a sample like this, and apply some then a sample shape change in shape correct. So, this angle theta,  $\tan \theta$  will be equal to if the displacement in this direction is  $x$ , and this height is  $l$  and this we call it as the gamma the shear strain. This is the way normally the stress or strain the strain (Refer Time: 05:59) extension or shear strain the shape change we define. But what is essentially going to happen with this process of definition in this way of definition is that, we do not know whether the deformation is essentially only just a shear or it has some rotation associated with it is possible that.

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- (1) *Infinitesimal strain*, where it is assumed that the quantities  $e$  and  $\gamma$  are so small that their squares and products can be neglected. For elastic strains within a crystal, this assumption is a good one, except at points which are only a very few atomic distances from a defect.
- (2) *Homogeneous strain*, where every part of the strained region suffers the same distortion. Large homogeneous strains will be encountered in Chapters



Anyway before we go further there are 2 types of strains which we consider; one these strains when we look at it these are all may be strains may be very large, generally we can consider the strain as an infinitesimal one. When we call a strain is an infinitesimal strain what we mean is that, the value is so small, that product of the strains either longitudinal strain or the shear strain either product among themselves, we assume that they are negligible and can be neglected.

That is the assumption and this is mostly valid or we can consider operates in the most of the cases where the sample is elastically strained. Then another part of the strain which we call it as a homogeneous strain; the homogeneous strain is one in which we look at the strain which is homogeneously applied through the crystal, this strain has lot to do with transformation of the material, like many types of phase transformations we have to consider this like (Refer Time: 07:43) transformation or twinning or examples where this

homogeneous twins; homogeneous shear concept is being employed that I will come to later when I talk about them.

Further present like it let us look at only this infinitesimal strain.

Student: (Refer Time: 07:58) infinitesimal strain.

Yeah.

Student: (Refer Time: 08:02) mention that (Refer Time: 08:03) value for (Refer Time: 08:05) strains accepted points (Refer Time: 08:09).

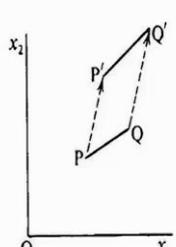
Yeah.

Student: If there is a lattice point which is (Refer Time: 08:16) then this kind of (Refer Time: 08:18).

For example when we take of think of a dislocation; at the core of the dislocation the displacements, which are taking place may be the strain may be of the order of 50 percent. Generally when we use generalized Hooke's law, the assumption essentially is that the linear elasticity it can be applied, and the stresses and strains are extremely small; that is why we make the assumption that this is valid normally throughout the crystal, but around defects that can be strange which can be very large, how do we define an infinitesimal strain.

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### Infinitesimal strain



$u_1$  and  $u_2$  represent displacement of the point P ( $x_1, x_2$ ) to P' in two dimensions

Some books displacement is represented as  $u, v, w$  in  $x, y, z$  directions.  
Displacement is denoted by  $U = U(u_x, u_y, u_z)$  as well.

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy + \frac{\partial v}{\partial z} dz$$

$$dw = \frac{\partial w}{\partial x} dx + \frac{\partial w}{\partial y} dy + \frac{\partial w}{\partial z} dz$$

$$du = e_{xx} dx + e_{xy} dy + e_{xz} dz$$

$$dv = e_{yx} dx + e_{yy} dy + e_{yz} dz$$

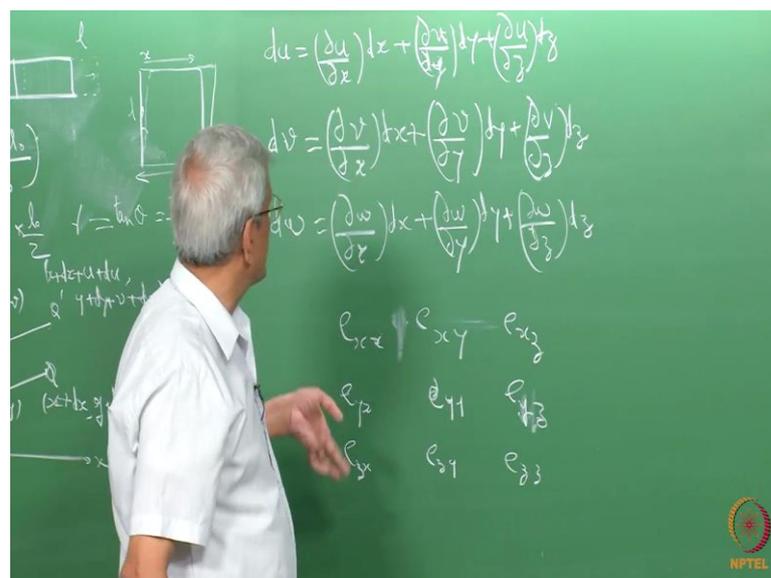
$$dw = e_{zx} dx + e_{zy} dy + e_{zz} dz$$



So, with respect to coordinate system, if you have a point the line that is we take on the object as small vector  $p q$ . By applying some stress there is a deformation is the point  $p$  under the action of a force it moves to  $p$  dash, and this moves to a point  $Q$  dash. Suppose  $x$  and  $y$  are the coordinates of the point  $p$ , the coordinates of  $p$  dash will be this is actually an exaggerated diagram to be very small, it could be  $x$  plus  $u$  and  $y$  plus  $v$   $n$  will be the coordinate, and where  $u$  and  $v$  are the displacements. The  $u$  is very close to  $p$  so that you can write it as  $d x$ ,  $x$  plus  $d x$ ,  $y$  plus  $d y$  then what is going to happen is that this point now it will be  $x$  plus  $d x$ , plus  $u$  plus  $d u$ , plus  $y$  plus  $d y$ , plus  $v$  plus  $d v$  this is how it happen.

Essentially from this what we can make out is that the displacement which is taking place to any of the coordinate you take it to be  $u$ , in that direction has a component from forces which are acting on the other directions as well.

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This is essentially like if you have function  $u$  is a function of  $x$   $y$  and  $z$ , you can write that  $d u$  will be equal to  $d u$  by  $d x$  into  $d x$ ,  $d u$  by  $d y$  into  $d y$ , we can write it in this form correct. In this fashion we can write it for  $d v$  also  $d v$  will be equal to  $d v$  by  $d x$  into  $d x$ ,  $d v$  by  $d y$  into the  $d w$ . In all these terms are nothing but the rate of change of  $u$  as a function of  $x$ ; that means, that that is nothing but an extension that is nothing but a strain which is taking place in the  $x$  direction, the component what is the displacement which is taking place.

Similarly, this  $\frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$  is what does it define? It defines in the direction of  $u$ , but from a component corresponding to whatever it is taking place in the direction of  $y$  correct. So, this can be represented as nothing but components of a strain, this will be a component of a strain which is in the  $x$  direction what is the displacement in the  $x$  direction; the another is displacement in the  $x$  direction contribution from the  $y$  direction, displacement to the  $x$  direction that is in the displacement to the contribution to you from some deformation which is taking place in the is a direct component.

Student: So, for example,  $e_{xy}$ .

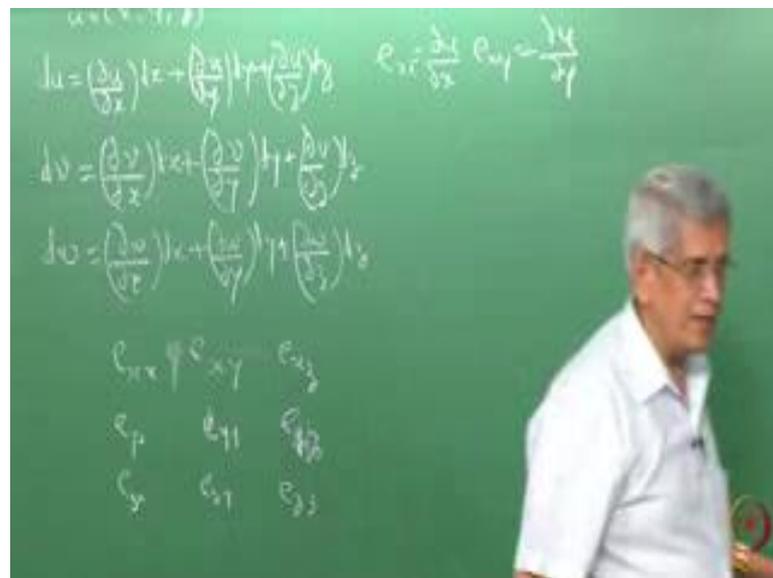
Yeah.

Student: Which is (Refer Time: 13:57).

Yeah.

Student: So, in that the force (Refer Time: 14:59).

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No now we are not talking about the force at all, any component of a strain if you see it that strain essentially is that  $e_{xx}$  is essentially an extension which is taking place in the  $x$  direction look at the.

Student:  $E_x$ .

E x.

Student: Of the length scaled (Refer Time: 14:27).

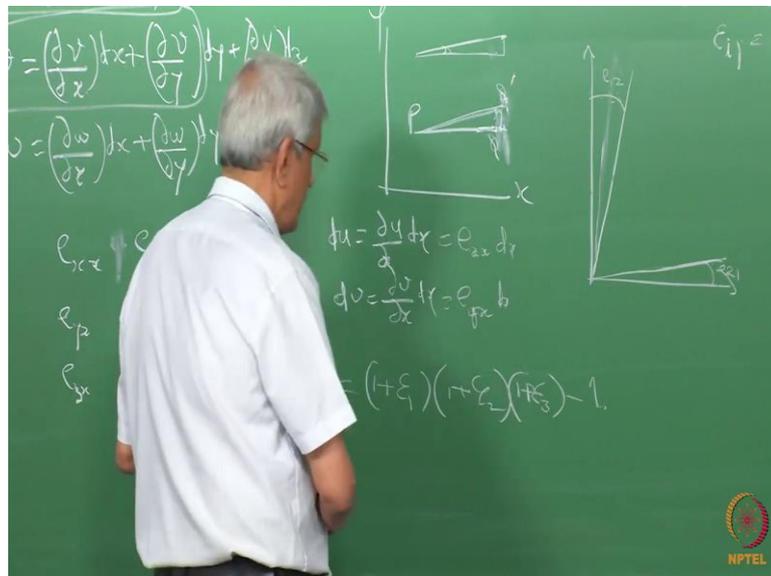
Length scale then that is displacement which is taking place in the x direction.

Student: (Refer Time: 14:32).

Another is contribution to displacement in the x direction could have come from something which is I know by a shear strain, you can have a contribution to displacement in the x contribution to you the displacement in the x direction we can give it correct.

Student: (Refer Time: 14:53) the length scale is scaled along x (Refer Time: 14:57) divided d u when you consider is essentially a displacement of x in the x direction, v is the displacement which is there in the y direction, that we use the displacement in the x direction, but that will have a contribution from strain which is taking place in that direction, plus some part of a strain which is also contributing that could be a shear strain that direction.

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So, that is what we essentially write it as; I am just writing only these components e y z, e z z these are all these terms e x x will be d u by d x, e x y equals. That is essentially with respect to some change in the exposition of position, what is the displacement to u some as a function of position in y what is it is contribution to u that is the strain in that

direction that is  $e_x$   $y$  we can define it this way. And in fact, you will find that there are many representations are being used, one has to be very careful about the type of representation  $x$   $y$  and  $d z$  which is being used, this there are lot of confusions are there in the literature, especially with respect to stress.

So, essentially what we have we can have 9 components of strain or possible correct. Each of this component may not be equal also they can have all different values, so it is possible, but to understand it a little bit better let us just consider a 2-dimensional case.

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$$du = e_{xx}dx + e_{xy}dy$$

$$dv = e_{yx}dx + e_{yy}dy$$

when  $dy=0$

$$du = e_{xx}dx$$

$$dv = e_{yx}dx$$

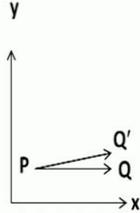
$e_{xx}$  measures extension per unit length of PQ measured along  $x$ -direction and  $e_{yx}$  anti-clockwise rotation of PQ

Similar explanation can be given for  $e_{yy}$  and  $e_{xy}$  as well

In the present form  $e_{ij}$  do not completely represent strain

$$e_{ij} = \frac{1}{2}(e_{ij} + e_{ji}) + \frac{1}{2}(e_{ij} - e_{ji}) = \epsilon_{ij} + \omega_{ij}$$

$$\epsilon_{ij} = \frac{1}{2}(e_{ij} + e_{ji}) \quad \omega_{ij} = \frac{1}{2}(e_{ij} - e_{ji})$$





In the 2-dimensional case what happens is that then  $d u$  will be depending upon  $d u$  by  $d x$  only this part of the equation essentially only these 2 have to be considered correct.

Suppose we assume that the sample itself is a just vector  $p Q$ ; and this vector  $p Q$  deform to  $p Q$  dash it becomes; then what are the contributions which can have because since we are considering it is a vector which is going to be in this direction,  $y$  direction is  $d y$  is going to be 0 correct.

So, then this equation turns out to be  $d u$  will be  $d x$ ,  $d v$  equals  $d v$  by correct. In this if you look at it, these represents only just  $u$  (Refer Time: 19:06), the elongation now it has reached this point right  $p$  remains the same  $Q$  has reached this point; that means, that some component of extension is there along the  $x$  direction, that component is being given by  $e_x$  into  $d x$ . What is the component which is going to be there from here to

here, that is being given by  $e_{yx}$ . That component itself we can consider it as if there is it is only a line, it would be considered as a rotation. I can just consider this line as taking it from here to here extended this much and give a rotation to it then also this can be reached.

Student: Sir.

Yeah.

Student: (Refer Time: 19:51) transforming to  $Q$  and (Refer Time: 19:52).

No here it is essentially what we are taking it is that the  $pQ$  is becoming  $pQ$  dash,  $p$  is fixed.

Student: Sir, then there is no (Refer Time: 19:05).

Which one?

Student: So, then there is not (Refer Time: 20:08).

No  $pQ$  see it is like this you take  $pQ$ , it reaches a point  $pQ$  dash. If it extends to  $Q$  dash you will have some displacement which will be there in this direction right.

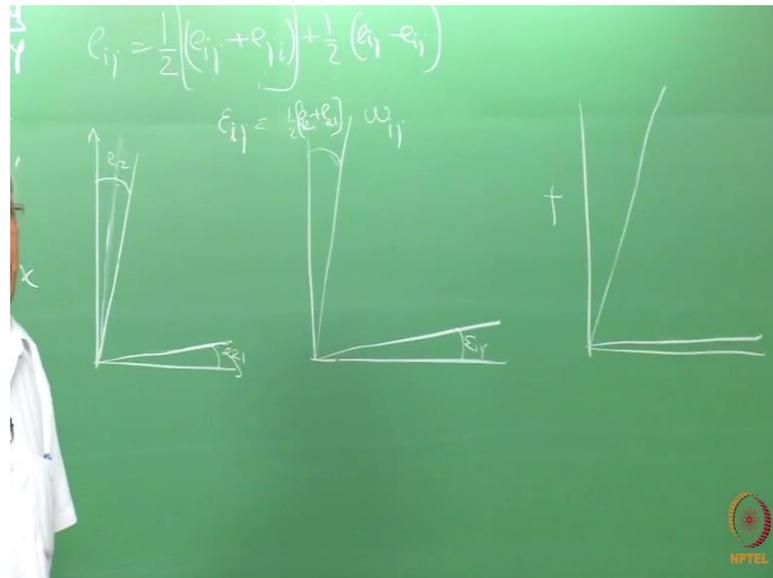
Student: if it is goes beyond (Refer Time: 20:06).

Yeah.

Student: Ok.

That is what we are considering it is a vector  $pQ$ , the position of the vector after some elongation is in this direction. Then the question which comes is that we can find out what is a component of extension in the  $x$  direction that is essentially nothing but a longitudinal strain correct. And what is going to be the component in the  $y$  direction, but that component could be as well as just a simple rotation it could be. So that means, that this strain what we try to represent it  $yx$ , it could be a shear strain or it could be a rotational also it is something that it need not completely represent whereas,  $e_{xx}$  definition is very clear whereas, the all the other terms when  $x$  is not equal to  $y$ , then there is a problem it could represent anything. What is the way in which we can separate between rotations and shear pure shear how it can be done.

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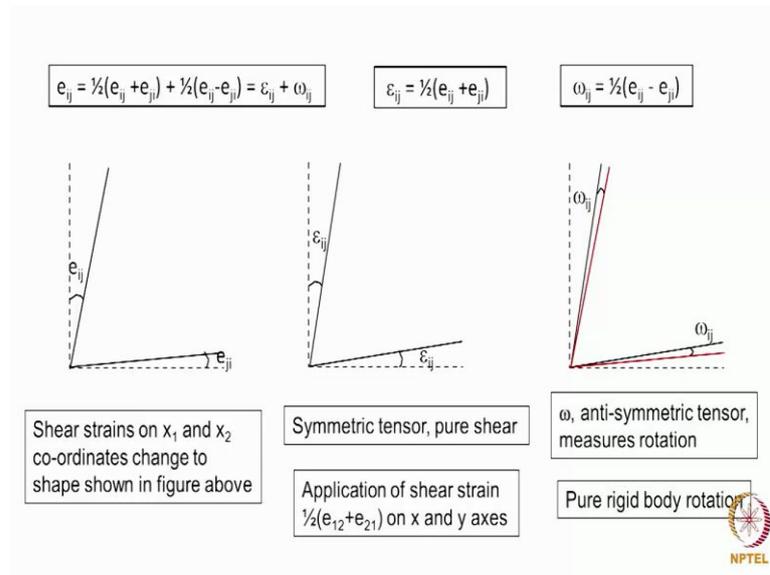


That is if you take any component  $e_{ij}$ ,  $i$  not equal to  $j$ , this can be written as half  $e_{ij}$  plus  $e_{ji}$  that this sum total essentially is becoming  $e_{ij}$  only; essentially is actually a shear  $e_{ij}$  and this part of it nothing but only a rotational component which is essentially the body rotation. So that means, that whenever we look at a display whenever we see a displacement from one position to another, and the displacement is changing as a function of position then only we can see that there is a strain correct.

If the displacement is uniform throughout the sample irrespective of the position; that means, that there is no strain only the sample has moved from one position to another position. If the sample has with respect to sample as moved from here, there is a displacement you assume it to be something like  $\delta$ , any point if you consider from this point it has displaced by  $\delta$ , and from this point also it has displaced by  $\delta$  right. So that means, that the displacement is uniform throughout the sample, and it is independent of the position then there is no strain.

So, strain comes only then the displacement is a function of position. So, now, we can consider it in we can understand looking at a geometrical construction.

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And the another important fact also is that when we take terms like this  $e_{ij}$  and  $e_{ji}$ , we had initially now we had 9 components are there in this case, now if we take each of this term to be equal to half of  $e_{ij}$  plus  $e_{ji}$ , then the this part becomes a symmetric tensor correct. Half diagonal values will have some other value, but these terms are all going to be that that is a diagonal terms have can have different values, but half diagonals terms will all have the same value correct each one of them; is it clear.

So, these we can look at it with a geometrical construction. So, the geometrical construction what we have taken is, here if you see that dashed lines represent the now I can draw it also. You assume that these are all the coordinate system which we have chosen, I apply a strain which is going to be take 1 2, these are all the 2 components of the strain which is being applied with respect to. Then one which has a shape which is maybe whatever that you take assume to be a square, now it has change the shape to a different shape the sites of it correct. This if you take the product of this that not product if you take the half of the sum of these 2 strains, and then try to do it essentially what will happen is that, it is equivalent to as if changing it symmetrically by a strain half into  $e_{12}$  plus  $e_{21}$ , this is equal to  $\epsilon_{ij}$  the same the same way we have deformed it correct.

This is different components of the strains are there all the 9 components are different. So,  $e_{12}$  is this component,  $e_{21}$  is this component so this coordinate has moved to this

position this coordinate has moved. The same operation we perform by taking the strain to be the symmetric part it, then it is essentially by the same half into  $e_{12}$  plus  $e_{21}$  we have moved this has shifted to this position, maybe where sheared vector.

Now, to this now if you give a rotation what is the rotation which we give it? We have  $e_{ij}$  plus  $e_{ji}$ , this rotation what it will do it is that which was initially in this position now it will rotate it to this position this will be rotate it. So, essentially similar to what has happened here the same shape change has been achieved in that. That in fact, with respect to a figure which is being shown on the slide you can understand it much better. Here you can make out from the shape that; obviously, that  $e_{ij}$  what is being taken is larger than  $e_{ji}$ .

Now, the symmetric part essentially half into  $e_{ij}$  plus take it. So, this vector has been essentially sheared that is from which is parallel to it is sheared and brought into this position, the same position it has been brought and that is what is shown by the black line; what is the red line show is what is the rotation which is being given. Now if you look at both of them is similar to the original position. So, essentially quite often when we look at the general 9 component of the strain which we consider, actual shear which is required with respect to a symmetric case if we consider, it is only a shear part of it other strain which is not shear the strain which it undergoes, is being given by the symmetric component and the other anti symmetric component is part of it is just a rotation.

This actually happens in the case of a normal simple shear which we do it, what do we do like this case which we have considered here is that this square, we have applied a shear stress and it has sheared and this is the strain which we call it. This strain we generally call it as an engineering shear strain, this engineering shear strain is nothing but equal to with respect to a tensorial strain if we consider, it is corresponding to some pure shear plus some part of it is actually a rotation, this clarity should be there in this concept.

Quite often when an engineering strain we apply, actually part of it may be the pure shear which is occurring, other part maybe just a rotation or the physical significance of it is that if you try to deform a material up to a certain extent it can accept the strain by compression, expansion, shear pure, shear and all when that cannot be done it will try to

twist or rotate. This happens in most of the bodies, that is when we look at deformation of a polycrystalline material; at the middle of it mostly strains will be there closer to the grain boundary when we come, when the matching between the boundaries have to be maintained and suppose the boundaries on the other side is very hard, the only way it can accumulate is by just a rotation rigid body rotation. But rigid body rotation is also equivalent to a strain, but with respect to or whether the material has really undergone a strain no correct.

So, if we write it in this particular format if we try to change it, then what we are able to see now is that; actual strain which is taking place the material which is being strained that is the either it is a shear strain or whether it is a pure strain, that plus rigid body rotation is the. See when we look at the displacement of a position, the displacement could take place due to your straining of the sample, due to shear of the sample plus by a rotation finally, what we measured is only this displacement.

Student: (Refer Time: 31:51).

We are we do not know what the components are. So, what we are trying to do it is that here in this expression if you try to see it, this is what we are looking at it is with respect to some position change, what is going to be the displacement which is taking place in that that is all which we are looking at it.

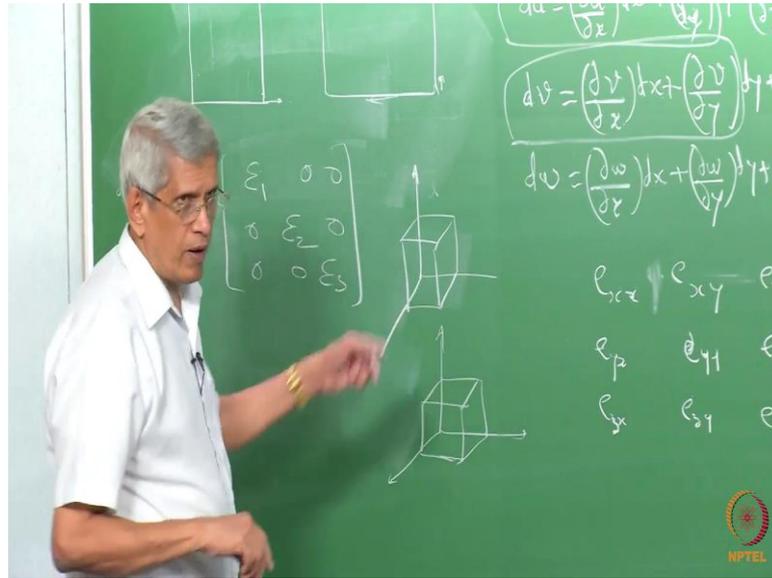
Student: (Refer Time: 32:09).

That displacement could have the components which are corresponding to it, from this expression it is not becoming very clear what it is going to be, but each has got different values. So, what is the jugglery mathematical jugglery which we can do it? Is by taking it as a symmetric and an anti symmetric part of it, we are able to separate this into rigid body rotation plus pure strain part of it.

Student: Sir, so the symmetric part of (Refer Time: 32:42) pulling along pulling equally along a diagonal (Refer Time: 32:48).

See the symmetry part of it pulling equally along a diagonal, there are many ways in which can I will just come to that.

(Refer Slide Time: 32:59)



If I take a square apply a pure strain an extension, that is why the pure strain you find that there is no rotation part comes; the rotation part is coming for all the half diagonal elements only, half diagonal elements we cannot say whether it is a shear or a rotation or it is a combination of both that is what we are trying to do by this mathematical operation.

This we can give some elongation in this direction, some elongation in this direction, it becomes a rectangle correct. Now what we are trying do it is that apply a stress in this direction that is your stress in this phase, and the shear stress in this phase, then what it will happen? The shear stress in this, this is essentially going to deform these 2 a parallelogram will come the same parallelogram, but then now the components if you look at it what are the components which you have pure elongation plus some shear is also there, both have been combined together to get a parallelogram.

By along that is this is the axis which you have chosen to represent or the coordinates which you have chosen to represent the sample, with respect to this we can choose another coordinate (Refer Time: 34:43) with respect to it, but perpendicular to each other to which we can apply some strain, elongation in one direction and perpendicular direction if we give compression the same shape change could be achieved.

This shape change is essentially equivalent to giving just only pure strain, but no shear strain correct.

Student: Sir, but (Refer Time: 35:19) in original (Refer Time: 35:21).

Yeah.

Student: (Refer Time: 35:23) diagonal and rotate it.

Yeah.

Student: So, this is what will get (Refer Time: 35:26).

That will be a parallelogram sites equal.

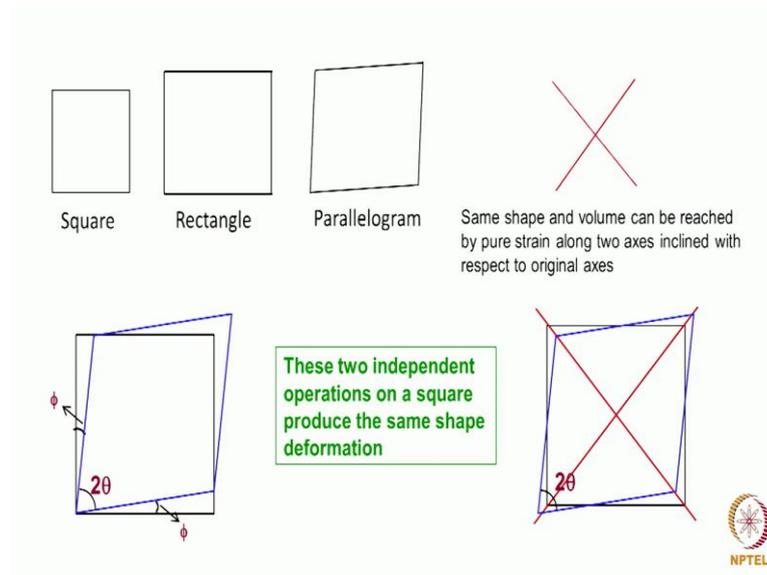
Student: So, here sites are not equal.

Sites will not be equal.

Student: (Refer Time: 35:36).

So, some angle difference will come, for an infinitesimal that type of strain which we consider it will be almost the same close to it.

(Refer Slide Time: 36:03)



So, essentially what we are trying to do now is that here deformation which this square has undergone involves pure elongation a pure strain, plus a shear strain; and that same shape change can be introduced into the sample by applying a pure strain along some other axis correct. So, that those axis we call it as principal axis, this is just a schematic

diagram just to tell that how this can be achieved; the lower half if you look at it here what we have taken is that essentially a pure shear has been applied, by this pure shear what we have done is that is whenever we apply a shear what happens that is going to be a change in shape of the sample no change in volume.

Whenever we apply pure strain there is going to be a change in volume will be there, if it is going to be different in all the directions. If we apply a compression in one direction and the tensile in an another direction, and adjacent it in such a way that there is no volume change then the same shear strain whatever is the shape change which has been brought about here has been achieved by tensile along tensile in this direction, and compression in this direction it could be achieved right.

But the axis now if you look at it, with respect to here the shear stresses are acting along these axis, now in this particular case the axis are perpend at 45 degrees with respect to it correct. So, what I wanted to indicate from this is that even though we will have a in a general case we will have a shear strain as well as tensile or compressive strain, choosing an appropriate coordinate system which we call it as a principal axis along which if we apply just dilatation, we can bring about the same shape change.

Student: (Refer Time: 38:12) the example that we solve in parallelogram.

Yes.

Student: The sides are not equal.

Yeah.

Student: (Refer Time: 38:20).

Yeah.

Student: These 1 by definition will (Refer Time: 38:24) these 2 operation.

These 2 operations will make it there the axis maybe some other axis you have to do it.

Student: (Refer Time: 38:31) at the same parallelogram (Refer Time: 38:32).

Yeah.

Student: We have to just (Refer Time: 38:35) this 1 (Refer Time: 38:25).

The axis may not be exactly, but now here the x are 45 degrees with respect to it, there may be some other rotation which may have to be given. Essentially at some angle with respect to a coordinate system which we have chosen we can apply just dilatation.

Student: (Refer Time: 38:52) simple dilatation and (Refer Time: 38:52).

Get the same that dilatation has to be will be 1 tensile in one direction tens and compression in the other direction. So, that is what in the case of a square here what is going to happen is that it will be exactly 45 degree.

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**Symmetric strain tensor**

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} e_{11} & \frac{1}{2}(e_{12}+e_{21}) & \frac{1}{2}(e_{13}+e_{31}) \\ \frac{1}{2}(e_{21}+e_{12}) & e_{22} & \frac{1}{2}(e_{23}+e_{32}) \\ \frac{1}{2}(e_{31}+e_{13}) & \frac{1}{2}(e_{32}+e_{23}) & e_{33} \end{bmatrix}$$

Volume and shape change

Diagonal elements-tensile or compressive strain ; off diagonal – shear strain

$$\begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} = \begin{bmatrix} e_{11} & 0 & 0 \\ 0 & e_{22} & 0 \\ 0 & 0 & e_{33} \end{bmatrix} + \begin{bmatrix} 0 & 1/2(e_{12}+e_{21}) & 1/2(e_{13}+e_{31}) \\ 1/2(e_{12}+e_{21}) & 0 & 1/2(e_{23}+e_{32}) \\ 1/2(e_{13}+e_{31}) & 1/2(e_{23}+e_{32}) & 0 \end{bmatrix}$$

Strain = Dilatation + Pure shear components

Volume change

No volume change only shape change

NPTEL

So, that is what in this particular slide what I had just taken is only the symmetric part which I have taken, that the symmetric part shows what is the strain which the sample is undergoing and the diagonal elements represent nothing but that dilatation dilatation and the half diagonal elements represent just a pure shear component or this is called as the dilatation is called as a pure strain component.

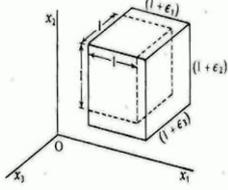
So, when a pure dilatation takes place there is going to be a volume change, when pure shear components are going to be there then no volume change, but only a shape change and in this case when both component all the components are going to be present, we will have a volume change and a shape change right.

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Symmetric strain tensor can be represented as containing only pure strain components or only pure shear components.

$$\begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix}$$

referred to principal axes



Volume change  $\Delta = (1+\epsilon_1)(1+\epsilon_2)(1+\epsilon_3) - 1 = \epsilon_1 + \epsilon_2 + \epsilon_3$

**During transformation, principal axes can rotate but remain orthogonal**

$$\begin{bmatrix} \epsilon_x & \gamma_{xy} & \gamma_{zx} \\ \gamma_{xy} & \epsilon_y & \gamma_{yz} \\ \gamma_{zx} & \gamma_{yz} & \epsilon_z \end{bmatrix}$$

$\gamma$  are usually engineering shear strains.  $\gamma_{xy} = 2\epsilon_{xy}$ . This array does not form a tensor



So, essentially from this (Refer Time: 40:22) can understand is that any symmetric strain matrix could be returned in terms of just pure strain this is also called as a diagonalization in matrix. Symmetric matrix can be to a diagonal elements then when we do that roots of that matrix are going to be the solution to this question correct. So, here now epsilon 1, epsilon 2 2, epsilon 3 3 are going to be values which are going to be different. So, if we choose a cube with such a shape, and apply this principal strains in those directions what will happen to the cube? The cube will become rectangular parallelepiped right.

That is what the shape which is going to take what is the volume change it is nothing but if it is a dimensions are of unit dimensions of further cube initially, then the volume change delta will be.

So, what are your strain to tell is that when we have a symmetric second rank strain tensor matrix is there with 6 components which are present, choosing an appropriate axis we can convert this into just pure strain along 3 axis, with which the same shape and volume change could be achieved.

In such a case then the matrix becomes these diagonal elements are represent only just pure strain along these principal axis. If you choose a sample with the coordinate system corresponding to these principal axis, and if you take a cube and allow them to undergo the this strains along this direction, then what is essentially is going to happen is that the

cube will try to become nothing but a rectangular parallelepiped the volume change which is going to take place is essentially nothing but the sum of  $\epsilon_1$  plus  $\epsilon_2$  plus  $\epsilon_3$ , since the other cross product terms are going to be negligibly small.

Generally whenever we consider its a strain in any direction any coordinate system if we choose it, suppose I choose an arbitrary coordinate system and take a cube of a material, and try to deform them what is the shape this will take? Because it can have both on each of the faces it can have shear as well as a strain, all the faces there will be going to be a change which is going to take place from here to here, this face will undergo a change from here to here then this face will undergo a change this face will under a change I am not drawn it correctly, but all of them will undergo a change, now it will become something like a parallelepiped that is what I had shown their 2-dimensional case which is a parallelogram which it comes correct earlier, but when we take it along a principal axis then this shear strains are not going to be there.

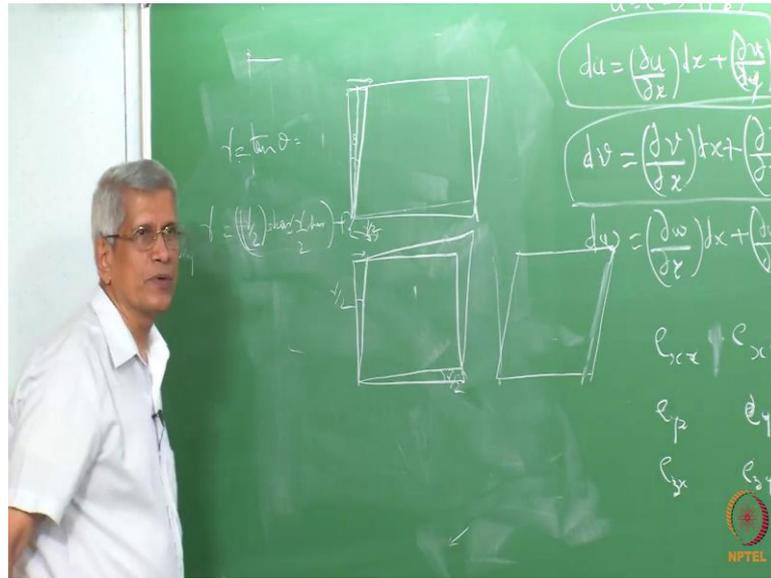
But generally if you look at the strain matrix which is represented, in many textbook which you can see it that  $\gamma$  the engineering strain which is being measured, will be shown as the matrix which will be shown consisting of this is the form in which it will be shown.

Here these terms are all engineering terms engineering's a shear stress, but quite often technically if you look at it these stresses, if it is a symmetric matrix this is going to be each of the term will be nothing but half of this  $\epsilon_{ij}$  plus this one that will be written. But strictly speaking if you look at it that is not correct because it can have some rotational components also. So, that does not represent the pure strain tensor, the strain tensor has to be written in this form only the strain tensor is being written.

Student: (Refer Time: 46:26) we which is what that (Refer Time: 46:31).

These what here that a because when you write this technically it looks like that same, but there is a problem which is going to be there is that engineering stress and strain, and the actual shear stress there is a slight difference is there.

(Refer Slide Time: 46:58)



The difference which it comes is because of this, if I take a sample deform it this is the angle theta. So, this tan theta is nothing but the engineering strain correct. Suppose you assume that this is a square ok.

Student: (Refer Time: 47:24).

This square itself can be applying a strain which is equal to half this value, in this direction and half this value in this direction, I can bring about this is just a pure strain shear strain only, that if this is gamma this will be gamma by 2, this is gamma by 2 both these directions this shape has been.

Student: Sir (Refer Time: 48:10) both of them together.

Both of them.

Student: Together.

Good together simultaneously in this. So, this is that component of the strain in this direction shear strained this is the component of the strain in this direction correct.

Student: (Refer Time: 48:24) inherently assuming that gamma x y and gamma y x also.

Irrespective of whatever be the value if you take half they will be symmetrical. Now, what we have done essentially is that only one strain this is only the strain which we

have applied it. That is we have taken a sample and given a deformation in this direction, we are trying to find out what is the strain which is going to be there in that direction; this strain this engineering strain what I am trying to tell you is how it is related to pure strain, how this shape change can be achieved let us look at it. This shape change can be achieved by giving a half by 2 shear strain in this direction, in this direction and shear strain of  $\gamma/2$  by  $\gamma/2$  in that. So, it has achieved this shape.

Now, if I give this shape your rotation of  $\gamma/2$  in this direction, what it will happen? It will now come back to this shape it will reach correct similar to what is has happened here.

Student: But, I am not sure if (Refer Time: 50:00) come back to (Refer Time: 50:01).

Which will come back?

Student: It has to.

It has to absolutely there is no problem about it, this if you see this is by  $\theta$  it has been rotated this is by  $\theta$  correct by some angle  $\phi$  we have and this.

And now if I give another rotation of  $\phi$  after this shape has been reached, if I give a rotation of  $\phi$ , this part of it come down to this position original position now it reaches. Now what is the difference? This is just a shear strain this plus this if you take it is equivalent to the same, but both of applied in 2 different directions. So, they are not the same tensor really you understand that magnitude looks like the same, takes out to be magnitude turns out to be  $\gamma/2$  plus  $\gamma/2$ ; but the directions in which each of the  $\gamma$  is being apply  $\gamma/2$  is being applied are in 2 different directions.

So, that is what the actual pure strain is the shear strain is, but engineering strain is only in one direction we are a doing the measurement. So, we cannot replace with engineering strain writing it to draw matrix like this, it is not a correct way to do it mathematically it has to be written as.

Student: Sir (Refer Time: 51:34) why do not we just use the (Refer Time: 51:35).

No, what I am telling that the original strain only should be used, the reason essentially is that many books they give that matrix in this form one should not be confused with that that is all.

Student: (Refer Time: 51:49) no confused; I am confused with this actually (Refer Time: 51:52).

With what?

Student: In this notation I will just did not understand which 1 is (Refer Time: 51:58) now in this case all are (Refer Time: 52:02) that is (Refer Time: 52:08).

We should not depend on the pure strain in the sample when you go you rely upon what is being defined with respect to the axis, what we are defining it go with respect to that only. The reason essentially is that here if you take this gamma engineering strain is nothing but equivalent to giving (Refer Time: 52:35) any particular direction, gamma by 2 yeah shear gamma by 2 is shear in another direction plus a rotation of gamma by 2 you understand that.

So, both does not become equal. So, use only the pure if you wanted to find out just the shear strain part of it, use the strain what is the definition which is given by the strain tensor only has to be used.

Student: So, can (Refer Time: 53:17) this symmetric product (Refer Time: 53:19) in a symmetric point (Refer Time: 53:28).

No that is.

Student: (Refer Time: 53:30).

These diagonally elements give just the pure strain there are two.

Student: (Refer Time: 53:42) pure strain can I (Refer Time: 53:44) separating between (Refer Time: 53:46).

It is a pure shear strain.

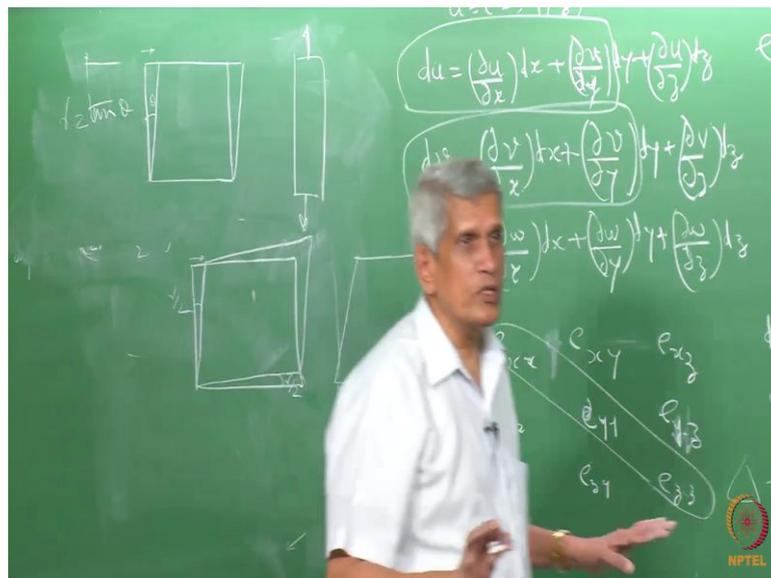
Student: Pure shear strain.

Pure shear strain.

Student: And it is coupled with rotation (Refer Time: 53:52).

When we do an engineering strain measurement, pure shear that shears simple shear when we do it normally this I will come back to it that; simple shear is what we do in an engineering application.

(Refer Slide Time: 54:10)



Just take a sample that is if it is a 2-dimensional object, apply a homogeneous shear then this gets deformed to this shape and this is what we define this angle theta equals tan theta, this is how we define an engineering strain. This engineering strain is a combination of pure shear plus their rotation.

Student: Now (Refer Time: 54:45).

That is what is this that is why whenever we wanted to talk with respect to the strain which the sample is undergoing the rotation part has should be removed.

Student: (Refer Time: 54:57).

Rotation part is the sample is undergoing only a rotation; it is being sheared and rotated. So, what we do measure in these ones is only the displacement from that we are calculating it. So, the displacement always inherently has got from displacement if you

try to look at the strain shear strain, that has got the rotation part plus an actual shear part is going to be there that is that is the one which I have to be always careful about it

Student: (Refer Time: 55:29).

Whereas if we took  $\epsilon$  with respect to any extension in these directions, this is no rotation; so whatever you do is the engineering's a tensile strain and compressive strain becomes equal to.

Student: Just (Refer Time: 55:46).

Just the tensile strain you can use that, but not the shear that is what essentially wanted to tell them.

Student: (Refer Time: 55:55).

Ok. So, what why we looked at it in this whole lecture of about an hour that is what we measure is simple engineering strains which we measure either a elongation, from that we can get the tensile or compressive strain and shear strain which we can measure; then those values now what we look at it from that we can get that strain matrix we can find out the components of it, that is because the shear strain part of it always has got both the rotation as well as pure shear which is going to be there.

See if you look at the matrix which we have written here only the off diagonal elements we are taking this sum, where is the diagonal elements remain the same there is no problem with that. So, that is because the shear is the part where we have this rotation path also comes. So, we are pure simple shear need not mean that an atom has undergone some deformation, it may have undergone some displacement and either the total displacement which it has undergone could be some due to pure shear well some due to rotation.

So, considering it as a pure strain which the sample is undergoing, then it is going to be only what the pure shear as well as the pure strain which the material undergoes, that is given by only this symmetric strains strain tensor; the other part of it essentially the rotation part of it.

Student: Sir (Refer Time: 57:58).

Yeah.

Student: Sir the rotational matrix also (Refer Time: 58:01).

Yeah.

Student: (Refer Time: 55:08).

Yeah.

Student: Even (Refer Time: 58:10) that is dependent on the (Refer Time: 58:11).

Yeah.

Student: So, if (Refer Time: 58:15) sample (Refer Time: 58:16) different points you have different (Refer Time: 58:16). So, the rotation with the local see this is not.

Yes, that is what it happens when you deform a polycrystalline material, when you take a polycrystalline material and deform the interior of the grain undergoes just pure strain whereas, the goes to a grain boundary it undergoes rotation. The rotation in the interior the center of the grain will be almost 0, but the maximum rotation will be there. So, that is going to vary.

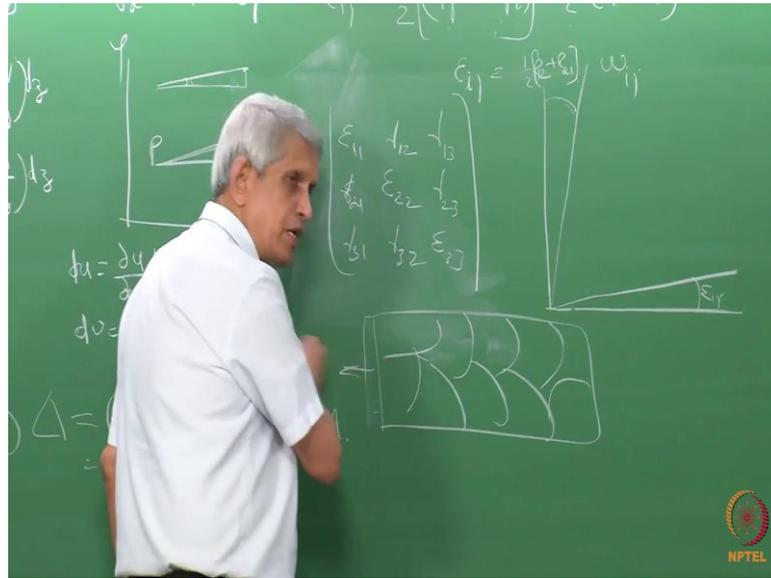
Student: (Refer Time: 58:52) it is a varying unit.

It is a varying unit.

Student: It is not like a (Refer Time: 58:54).

No, but what has to happen is that this has to adjust itself in such a way that the total deformation remains that same for the sample.

(Refer Slide Time: 59:08)



That is if I have a sample which contains different types of grains, if I have try to give some extension along in it in the, at some strain that I am loading it.

Then all the grains have to undergo equal deformation and then only it will be uniformly it will be the length will be increasing if some grain is so hard that that does not happen what will happen. Though sample will failed that is what can happen ultimately that is what it can, if that should not happen what it should do is that the one which is not able to undergo the elongation they may try to rotate and adjust it.

So, every grain will have a component of epsilon  $i j$  and mega  $i j$ , this together will define the total strain which you can call it as  $e_{ij}$  I will write it the total corresponding to the externally applied it this has to change. So, this strain to be generated in each one of them (Refer Time: 60:30) systems will be activated depending upon whatever is the direction which is favorable falsely to hooker in that sample you understand that. So, that is how the deformation in a polycrystalline material will take place is it clear.

Student: (Refer Time: 60:48).

So, essentially the whole exercise of or the whole aim of this lecture itself is to find out or to have a clear understanding of actual strained and the engineering strain which contains rotation, and how these things can be separated when we can get that information. And essentially we have 9 components independent components will be

there, and that itself can be divided when we take the rotation part out then separate the rotation and the pure shear then we can write it in terms of a symmetric tensor, then we will have 6 components of strain are going to be there. Out of the 6 components of strain if you find out one of them we can fix it, others will all be dependent on them. So, that is why that  $\phi$  independent the slip systems, which we talked about will come because of them.

We will stop here now.