Fracture, Fatigue and Failure of Materials
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Lecture 41
Crack in Fatigue

Hi there, we are still in the module 2 of this course and today we will be discussing about the presence of crack in fatigue, in this 41st lecture of this course Fracture Fatigue and Failure of Materials.

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So, we have seen in the fatigue behavior of materials so far that even if it is an unnotched fatigue, there is of course a provision for crack initiation that can lead to the final failure. So, now the question will be that, what would be the size of that defect that can be termed as a crack which is potential to lead to the failure of the material? So, we will be talking about the crack size and then we will see that how the crack propagates in the material under cyclic loading.

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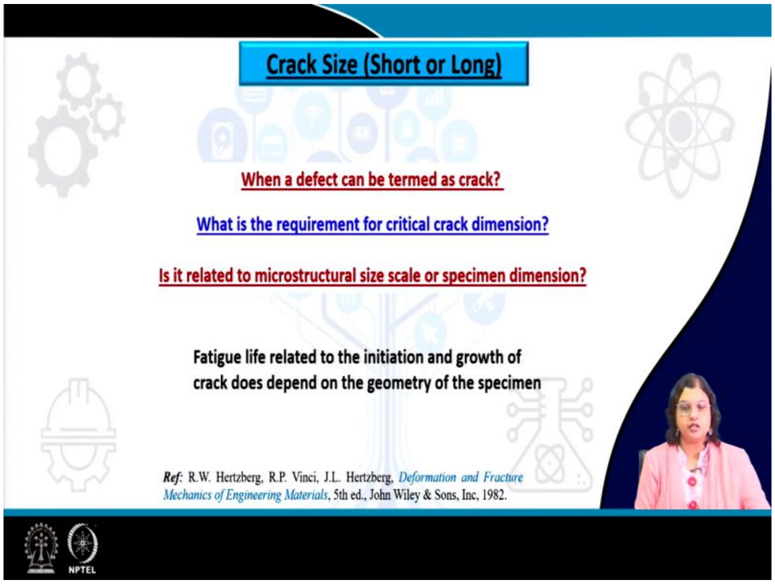

So, first of all there are some discrepancies and there is some confusion about the term crack, when can we term defect as a crack? What exactly a crack is? So, is it related to some microstructural features of the material for example grain size, colony size something like that whenever the defect achieves such kind of length, only then can we term this as crack or if it achieves a length of 1 micrometer or 1 centimeter then we can term defect as a crack.

So, what exactly is a crack, in terms of its relevance for fatigue failure? And also specimen dimension the position of this defects, all this we have seen that this controls the fracture behavior certainly. So, will there be any such effect on crack also? So, what exactly is crack is our main moto to find out and particularly the dimension of the crack that could be used for any particular calculation, is what we need to figure out.

Typically, fatigue life is related to the crack initiation as well as the growth and that also depends on the geometry of the specimen the position of the crack with respect to the specimen etcetera as well as the loading direction and several other factors like that.

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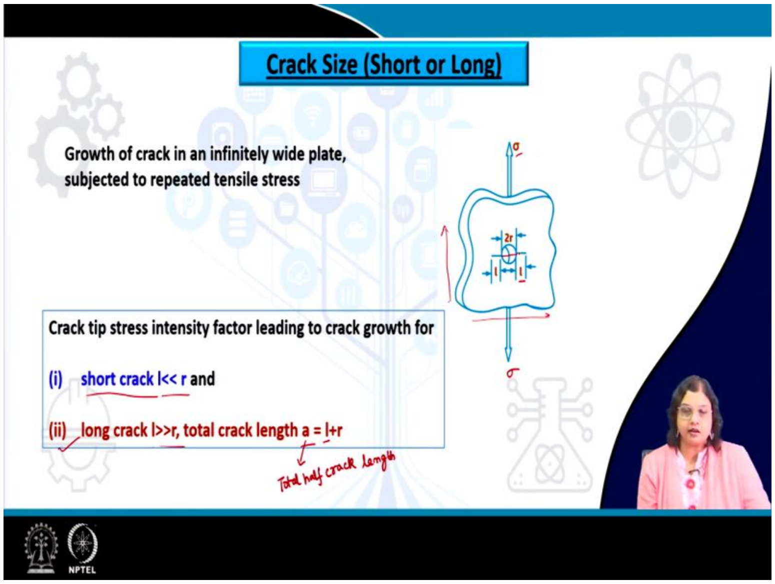

So, to find out the influence of crack. Let us first consider the growth of a crack behavior in case of presence of a hole on a infinitely wide specimen or component. So, once again we are considering that this is infinitely wide and long as well and there is a circular hole there, which is having the diameter of 2r or the radius of the hole is r. And there are some, because now whole is nothing but a discontinuity and that will have a stress concentration factor.

In the last lecture once again we have seen that the stress concentration factor for a circular hole will be 3. Now, if there is a stress concentration factor and we are applying stress of sigma, then certainly there is a possibility that cracks can initiate. So, let us say cracks are indicating at the two edges of this hole and the length of this crack.

So, far is l small l. So, if such is the case, if we try to figure out the stress intensity factor at the crack tip, we can consider either of the two situation. First thing is the short crack situation, which will be in active when l is much much lesser than the radius of this hole. So, that means that l is very, very smaller almost insignificant compared to the overall diameter of the hole.

This is one scenario, the other could be the long crack situation when the length is actually bigger than the whole diameter. So, in that case we should consider a as the crack length or the total half crack length as the summation of l plus r. So, that means the radius of the hole that is r plus whatever crack has been developed from the edges or edge of the hole that will be also considered.

So, let us see, that how these two situations will be different from each other and how can we still come up to the conclusion that when exactly crack becomes significant particularly for fatigue failure.

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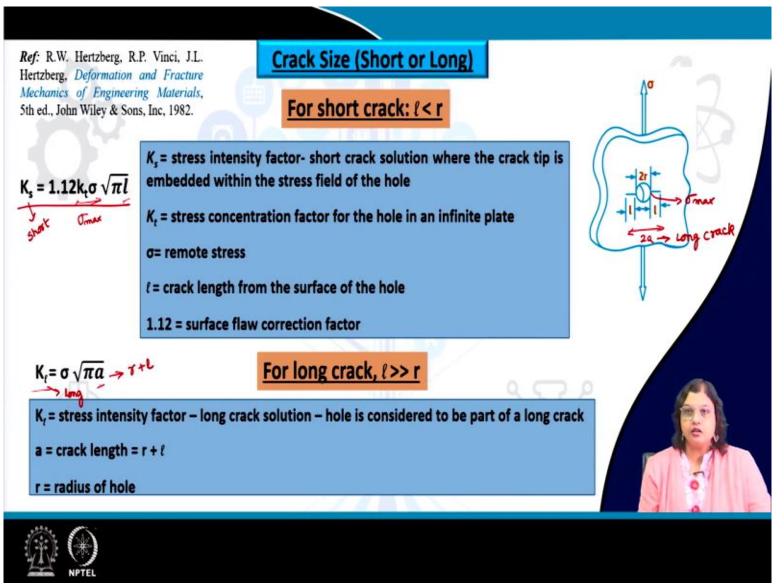

So, for the short crack situation, first of all when we know that l is lesser than r. In that case the short crack stress intensity factor is given by this relation here, $1.12k_t\sigma\sqrt{\pi l}$. Now this 1.12 is coming because the crack is developing from the edge of this hole that is why this factor is coming here $k_t\sigma$ is actually the σ_{max}.

So, this σ is the applied σ and that multiplied with the stress concentration factor k_t is actually the σ_{max} that is there at the tip of whatever defect that is a hole in this case. And we have considered the crack length as l, for the short crack configuration and if we do that then we should be able to figure out the stress intensity factor at the tip of a short crack, that is one of the situation.

The other situation could be the long crack when we know that l is much much greater than r or the radius of this hole, in such case kl or the stress intensity factor at the tip of a long crack. So, l here signifies long subscript l and once again s here signifies short. kl is given by $\sigma\sqrt{\pi a}$. So, that is the very typical relation that we have seen and considering that this is placed at the center of the specimen.

So, we have seen that now y is equals to 1 and we have considered this whole plus defect everything as a for the long crack. So, that is equivalent to 2a, the whole diameter plus the length of the cracker both the side. So, that is for the long crack. So, instead of l or r, now we are using a here. So, a is nothing but r plus l and we should get the remote stress field active there and we can figure out the stress intensity factor, so that is quite straight forward.

Now up to what point this short crack configuration will be active and up to what point the long crack configuration will be active, this is not only related to r but we also need to figure out that what length of crack or defect can be considered as detrimental of which can lead to initiate the fatigue crack.

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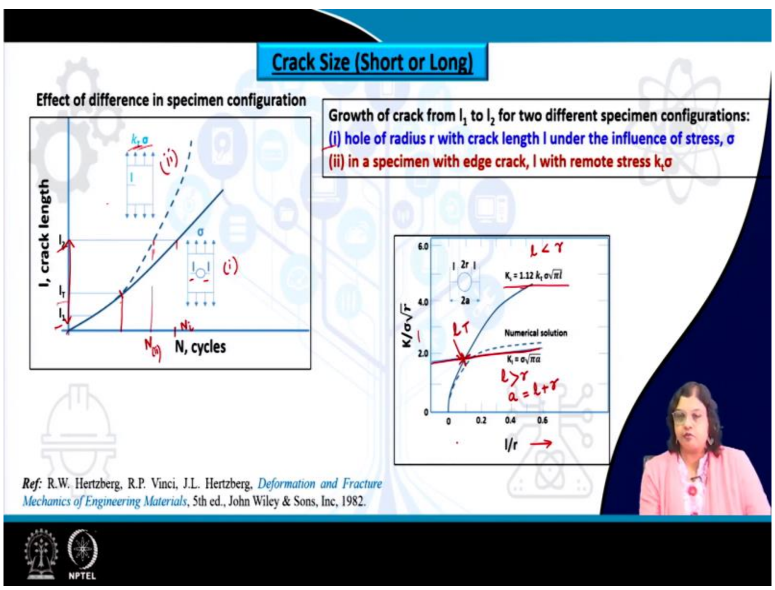

For that, we need to appreciate also the fact that the specimen dimension as well as the position of the crack in the specimen that also dictates the fracture behavior or for that matter crack growth behavior of a material. So, here is an example for the same configuration here let us name this as condition 1, where we have seen that there is a whole at the middle of the specimen and we are applying the remote stress field σ and as a result because of the stress concentration cracks are starting to form at the two edges of this hole.

And if such is the case, we can use the fracture mechanics concept to find out the stress intensity factor at the tip as well and we can find out what number of cycles will be required for the crack length to grow from l_1 to l_2. So, this graph here signifies the variation of crack length with the number of cycles and we can see that this many number of cycles will be required for the crack to grow from l_1 to l_2 based on the short crack configuration that we have seen.

So, this is for condition 1, condition 2 can be a case when we know that there is a crack at the edge. And that is with that there is this remote stress field of $k_t\sigma$, that is active because we are not considering the presence of the hole here but we are putting this k_t into this picture. So, that we can figure out the exact situation and the exact stress intensity factor at the tip of the crack.

If such is the case, then we should be also able to find out the number of cycles that will be required for the crack to grow from l_1 to l_2 and that is given by this dashed line here. Obviously, we can also see that this one is a more drastic scenario condition 2 and that leads to lesser number of cycle that is required for the crack to grow from l_1 to l_2. So, let us name this as N roman (ii) and this one here as N roman (i).

Obviously, the one at the whole at the center with the hole that will require more number of cycles for the crack to grow up to the same extent or same value of l_2. But still the doubt is that there is a difference in the values of N for both this condition. So, how can we figure out what exact crack length is of importance. So, actually if we are looking into this carefully we see that from the initial point up to a certain point this number of cycles that is required based on any of this configuration is same.

So, that is considered as the transition crack length. So, from there actually the crack behavior or the crack growth behavior changes completely at the stress situation, stress intensity factor changes drastically. And we consider this intersection of this Junction as the l_T value, transition length. We can also understand this based on the short crack and the long crack estimation. So, here the solid curve here signifies the short crack situation for the same kind of configuration when we know that there is a hole and the cracks of length l are initiating from both the edges.

So, in this case we have seen that l is actually less than r and we can get something like this. On the other hand, if we are talking about the long crack situation when we know that l is greater than r and we consider the total cracked length as l or the length of the crack plus the radius of the hole then this straight line here actually is giving us the long crack solution.

So, we are plotting the K/σ√r on the y axis and l/r on the x axis. So, if we do that based on this long crack differences in the values for the long crack and the short crack scenario, we can see that there is once again an intersection point and that intersection point is actually what is l_T or the transition crack length and that is of significance.

So, up to this point actually we are supposed to get the same values if we are using even for the short crack two different kind of configuration the long crack or the short crack one also will give us the same values of this y-axis with respect to the x-axis for both the long and the short condition.

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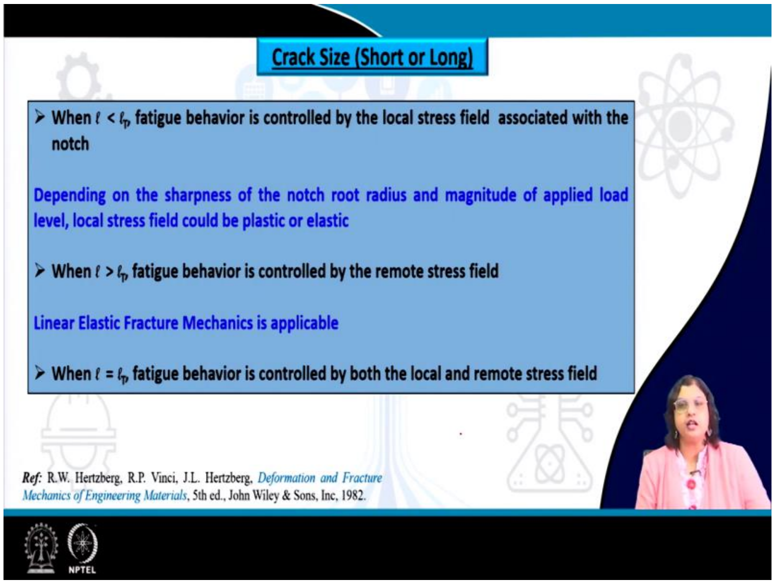

So, based on this this l_T is what is now more significant and when the crack length is actually lesser than this l_T values is transition crack length value, then the fatigue behavior is primarily controlled by the local stress field. Whatever stress field that is there at the localized position at the tip of the notch that is controlling the fatigue behavior in case the crack length is less than the transition crack length.

On the other hand, so this is particularly dependent on the magnitude of the applied load level and based on that this local stress fill could be plastic or elastic. On the other hand, when this crack length is more than l_T at the transition crackling then the fatigue behavior is controlled particularly by the remote stress field. So, then the linear elastic fracture mechanics condition will be totally applicable.

And we have already seen that how the stress intensity factor based on this linear elastic condition will be applicable. And we can do the rest of the assessment based on that. And when this l is exactly equal to this transition crack length then the fatigue behavior will be controlled by actually both this situation. So, there will be a mix mode kind of condition where both the local and the remote stress field will be active in determining or dictating the fatigue behavior of the material.

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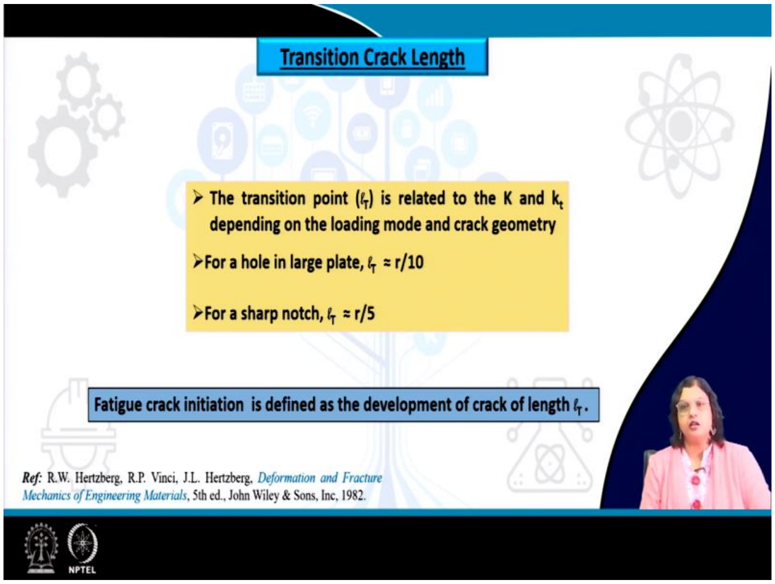

Now, this transition crack length is what is important? And this transition point or l_T is related to the stress intensity factor as well as the stress concentration factor and along with that the loading mode crack geometry all this will contribute to or will influence the value of this transition crack length for a hole in a large plate that we have seen this l_T value or the transition crack length is something like r/10. So, $1/10^{th}$ of the radius of the hole, that is what is the crack length that is important as the transition crack length.

So, if a value less than r/10 is obtained that means, that the short crack kind of situation will be there. On the other hand, for a sharp notch this is equivalent to the l_T value or the transition crack length is just r/5. So, twice that of the previous condition of a hole in a large breath a sharp notch is this transition length is just a 1/5th of the radius of the hole. And fatigue crack initiation is defined as the development of crack up to a length l_T. So, that is why l_T is so important.

And again we have seen that up to this l_T both the long crack and short crack condition will have same value. Even for the for any particular situation with the based on the change in the geometry up to l_T value, we are going to get the same number of cycles that will be required for the crack to go up to that level. So, this is what is significant, if we are talking about the presence of a crack for a fatigue.

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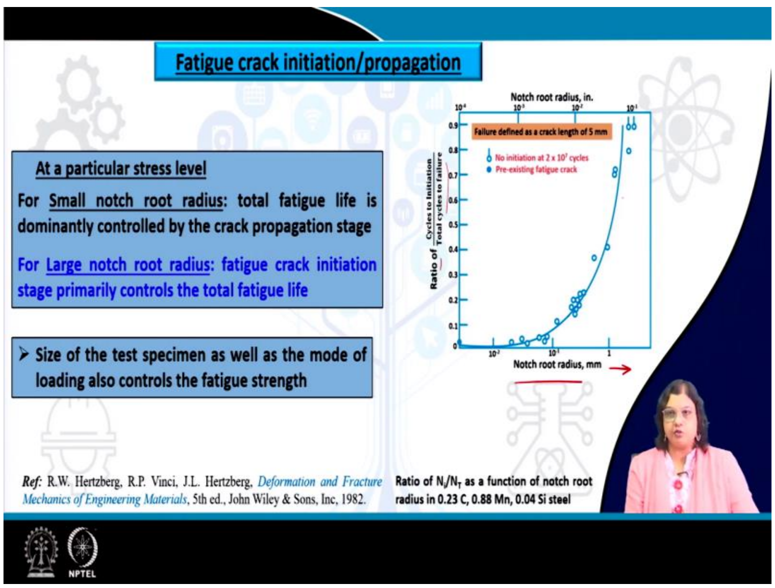

If we are talking more about the fatigue crack initiation or propagation particularly at any distinct stress level for small notch root radius, total fatigue life is controlled by the crack propagation. Typically, fatigue is controlled by both the crack initiation as well as propagation. So, it is a summation of the crack initiation life and the propagation life that gives us the total fatigue life.

But so far we have seen when we are talking about the high cycle or the low cycle fatigue we have considered that it is a smooth specimen and there is no dominant notch or crack present there. So, in such cases we consider that the crack initiation is the most distinct or the most detrimental part and majority of the life will be used up in just initiating the crack. Once the crack initiates it just leads to fracture by very quick propagation in an unstable manner.

But in case, the pressure with the presence of a defect there up to the length of the l_T this total fatigue life is controlled also by this notch root radius. In case of this small notch root radius which means sharp crack propagation is what is very important. And on the other hand for large notch root radius which means for blunt crack this fatigue crack initiation stage is primarily what is important.

Because initiation there will take a lot of time considering that it has a blunt notch root radius. And this shows actually this graph here shows, the ratio of cycles to initiation to cycles to failure and on the x-axis we have the notch root radius. So, we have seen that as the notch root radius is increasing, we require this higher and higher ratio of the cycles to initiation divided by cycles to failure.

So, that means that as the notch root radius increases more and more number of cycles will be required for the initiation to maintain this ratio to a very high value. Size of the test specimen as well as mode of loading also controls the fatigue strength and we have also seen this in several occasions that how different geometry, different size, thickness all these are very much relevant for the overall fracture mechanics and failure in general.

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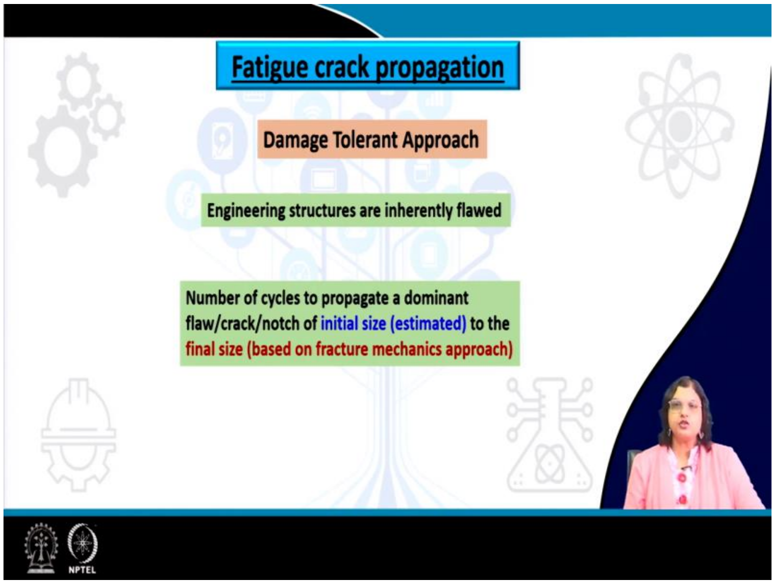

So, now that we know that there could be cracks that are present. Let us talk more about the fatigue cracked propagation behavior. So, basically fitty crack propagation all this theory are based on the damage tolerant approach what it considers at the very first place is that engineering structures are inherently flawed.

So, that means that there may not be severe notches like we have considered for the case of fracture toughness estimation but there can be still some defects and that can lead to crack initiation under cyclic loading and that may propagate to final failure, catastrophic failure. So, let us begin with the fact that there is existing some kind of defect which is relevant considering this transition crack length and so on.

So, if such is the case, then what we need to find out is the number of cycles to propagate a dominant flaw, this could be a crack or a notch or any other kind of defect from the initial size. Now, this initial size can be only estimated based on some kind of non-destructive testing, or some kind of microscopic analysis we can figure out that if there is any defect, what is the size of this defect.

Let us start with a size of a_0 for example and what number of cycles are required for this kind of defect to grow up to a certain level or the final crack length that is what is important. Now, the final crack length can be determined based on the fracture mechanics approach that means that the final crack length is the one that is the critical value that leads to failure.

We can determine that and we can find out that how many number of cycles will be required to achieve that kind of length and based on that the life of the specimen can be determined.

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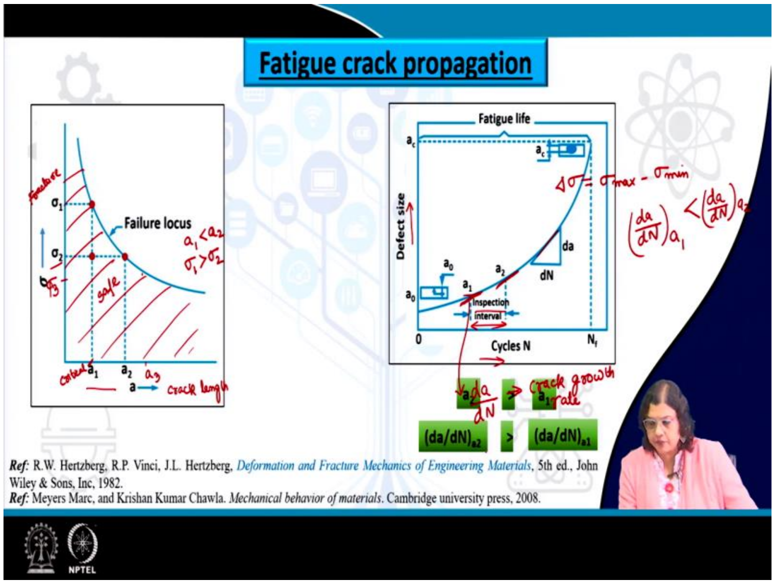

Now if we want to do that there are some criticality that we need to consider. For example, let us say this one here signifies the variation of sigma with a or the crack length. Now this signifies that if we have a crack of length a_1 that is the critical crack length which means that it can lead to fracture then the critical value of stress that is required for fracture to materialize in a component with a crack of length a_1 is σ_1.

So, this is the fracture strength for a critical crack length a_1. On the other hand, if there is a crack of length a_2 that means, if a_2 is the critical crack length, then the fracture strength under such condition will be σ_2. Obviously, since a_1 is less than a_2, σ_1 should be greater than σ_2, lesser is the size of the crack, more stress it can withstand.

So, that means that the fracture strength will be higher for shorter cracks. Overall this is a failure locus that we can see and if we are talking about even higher crack length of let us say a_3, we can figure out what should be the fracture strength for crack length a_3. So, let us say this is σ_3 and so on.

So, that means if we are having certain crack length, we can figure out that what is the safe zone within which we can have no failure. So, if we are applying stress in this hatched section there will be no failure. Now as I mentioned that we consider here for this damage tolerant approach that materials are inherently flawed which means that the next thing that we target to find out is the number of cycles for the growth of the crack.

So, let us see here, that what it shows is the defect size on the y-axis, which means the crack length on the y-axis and number of cycles on the x-axis. So, this is a_1 here is the initial crack length and that can be determined based on some inspection. We need to figure out that how many number of cycles will be required for the crack to grow from a_1 to a_2 and this can be figured out based on the slope. So, slope of this actually signifies the da/dN.

So, this slope is what is da/dN or the crack growth rate. Now, this has been done for a particular stress condition. So, let us say stress is σ, if we are continuously applying Δσ, which is nothing but σ_{max} - σ_{min}, which means that we are applying cyclic loading of the maximum stress value of σ_{max}, minimum stress value of σ_{min} and then stress amplitude mean stress everything you can calculate.

If such is the case, then there is an initial crack of length a_1 and that can grow to a_2 in N number of cycles in certain number of cycles and we can figure this out based on this slope here as shown here as da/dN. Another interesting thing that we can also observe from here is that this da/dN or the slope for a_1 versus a_2.

If we look into this in a more closer way we will see that da/dN for the condition a_1 is actually lesser than da/dN for the crack length a_2, which means that the longer the crack length faster it will grow at any particular stress level. So, we are not varying the stress level or anything just the, if the crack length is increasing then the crack growth rate will be higher and higher.

So, you know that is actually a double detrimental effect considering the fact that crack any way will lead to fracture and because of the stress concentration at the crack the stress concentration is also directly proportional to the crack length. So, higher the cracked length longer the crack is more is the value of stress concentration factor that means that the crack will grow faster and faster. So, that is something of very much of concern for the case of fatigue and we need to understand this.

So, that is what is shown here that if a_2 is greater than a_1 then da/dN for a_2 condition is greater than da/dN for the a_1 condition.

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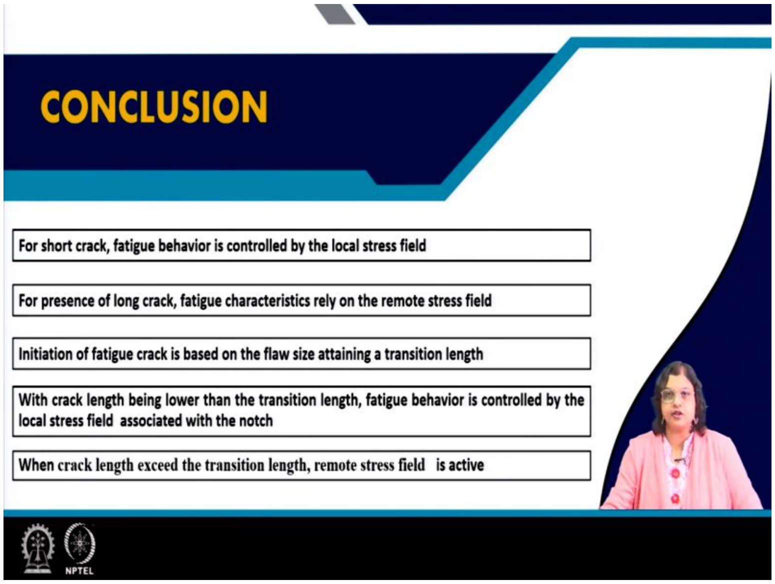

So, let us come to the conclusion of this lecture with the following facts that we have seen that for short crack fatigue behavior is particularly controlled by the local stress field. And for the case of long crack on the other hand fatigue characteristics rely on the remote stress field and this long crack or short crack is actually dependent on the transition length.

And particularly fatigue initiation occurs when this flaw size attains the transition length. With crack length being lower than the transition length fatigue behavior is controlled by the local stress field associated with the notch. On the other hand, when the crack length exceed the transition length then the remote stress field gets active.

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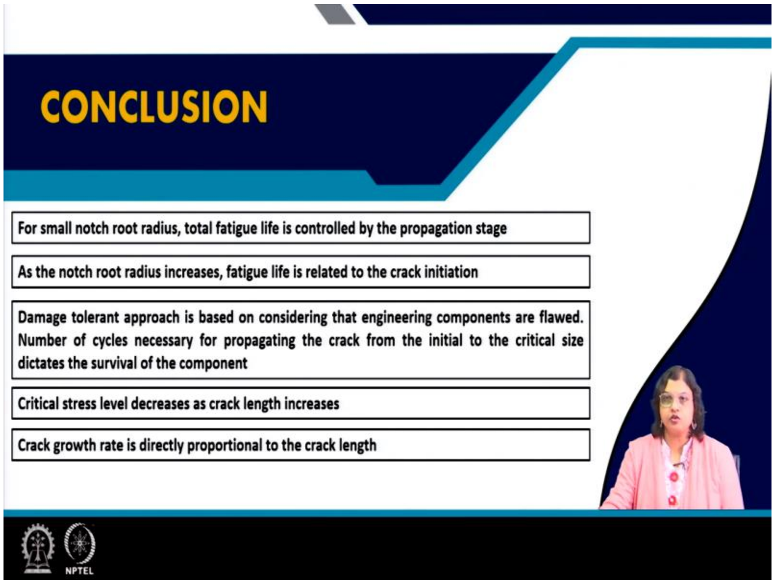

For small notch root radius total fatigue life is particularly controlled by the propagation stage. When we are talking about the notch root being increased, then the fatigue life is related to the crack initiation for the particularly for the blunt crack. And damage tolerant approach is considering the fact that most of the engineering components are typically flawed.

And in that case, we need to find out the number of cycles that is necessary for propagating the crack from the initial to the critical size, that dictates the survival time of the component. And this critical stress level we have seen that that decreases as the crack length increases. We have longer and longer crack that means, we will it can survive for lower and lower value of fracture strength.

Crack growth rate on the other hand is noted to be directly proportional to crack length which once again means that as the crack is getting longer, it is more prone to grow further and that means that the speed of the crack growth will keep on increasing as the crack attains higher and higher length.

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So, following are the references that has been used for this lecture. Thank you very much.