

**Mechanical Behaviour of Materials - 1**  
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**Lecture - 40**

**Strain Hardening: Single Xtal and Poly Crystal Deformation, Taylor Hardening**

Welcome back students. So, (( )) (00:17) as discussed many of the strengthening mechanisms. Now, today, I will be discussing about yet another strengthening mechanism which is again possible because of interaction of dislocations. This time it is interaction of dislocation with dislocations. So, the mechanism that leads to this that comes out of this is called strain hardening.

And, it is one of the primary methods by way because of which is employed on metals and alloys to improve their strength. So, for example, you may have heard that rolled steels. So, why are the steels rolled? Because, it induces a strain in the material and this strain causes dislocations generate dislocation generation. And then, the dislocations interact with each other and because of that the strength increases.

So, what happens in between? That is what we will understand over here. So, to begin with, we will look at single crystal deformation. So, how does this single crystal deformation look like? Now, you remember that we had a single crystal and where we looked at the critical results shear stress. So, once you are pulling it then the once the critical results shear stress for the plane one of the plane reaches then dislocation starts to move in that particular plane.

But then, eventually, other, there are other dislocation slip planes over there. And eventually, when you keep increasing the load, then the stress for other, the stress would reach the critical result shear stress for others slip systems. And therefore, they will also start to come into picture. So, that is why you will always have multiple slip system in a material even though for a single crystal system as you would see.

It, to begin with, we will always have one particular slip system and then eventually others kick in. So, we know that in a material there will be multiple slip system even in a single crystal.

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So, I mean any material system it is dependent on material system not on the whether it is single crystal in a polycrystalline. So, they will always have multiple slip system. Now, as you increase the tensile load so, for the sake of completion, let me just redraw what we had used earlier which was so, this is actually the single crystal. And, there is one slip system which is like this which would be the first one too.

So, we know let us say we know beforehand that this would be the first one to reach the critical result shear stress. So, there will be several of these in parallel. And, you are applying the load like this. Because of this, the crystal would, well I am drawing in a little exaggeration. But, this is what happens. So, there is kind of a rotation because there are parallel slip systems and there slip is taking place on all of these individually.

And, because you are applying a load, so, this will, kind of bend. And, it will also cause other slip systems to get activated. So, first thing is as the tensile load is increased, resolved shear stress on one plane is reached. One plane or system, I should say, sorry, it reaches critical value. So, this one will start to the dislocations would start to move on this. And, this would be the primary slip plane.

Now, as you keep increasing the stress, what would eventually happen is that one because of the rotation and also because we are increasing the load other slip systems would also reach the critical value. So, critical resolved shear stress for other systems is also reached. So, let us say those systems were somewhere like this. Maybe third one was little more (()) (06:52) away. So, it is, the closer it is 245 degrees.

As you would be able to see from the equation, they will be the first one to get activated. And then, other ones which are further away from 45 degrees, they will get activated. And so, the slip systems, so, the dislocations in other planes also start to move.

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And, you can realize that once one if only one pair or one set of planes were activated and dislocations are moving only on them, then all the dislocations are moving parallel to each other. They were just moving out and they were escaping from here which actually led to this rotation. However, once you have other planes, then dislocations would start to intersect and interact with each other and that would lead to a lot more complex phenomenon.

And overall, if you look at the resolved shear stress versus resolved shear strain plot for a single crystal, this is how it would look like. So, now, let me draw. So, this is the x and y axis. And now, I will, on the x axis, you have gamma. And, on the y axis, you have resolved shear stress tau. The plot would look like something like this. And, we will explain the various features of this.

So, overall what you would see is that there are 3 different zones. And, these would be represented and the stage or phase 1, stage 1 and somewhere over here. If we remove the transition, so this is 2nd stage. Let me write. This is 2nd stage. This is stage 1. And, I have left this region because this is more like a transition region. And, this is the 3rd stage. And, you can actually see some distinct features in all these 3 stages.

So, now, let us we will move on to discuss, what are these distinct features that we see in each of these stages?

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So, first thing is that 3 stages. And remember, this is single crystal.

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So, in stage 1, what we see is that, as you increase the strain, the stress increases. But, very soon, it reaches our saturation value and remains constant after that which is expected because what is happening here is that there is only once parallel set of planes where dislocations are moving and they are not increased. The density is not increasing. They are, their number remains constant. And, they are just escaping from the surface.

So, there is no strain hardening taking place. So, this is when slip system with lowest CRSS gets activated and what we observe is that there is saturation which is almost no strain hardening because all the dislocations are eventually escaping away. So, here basically only one set or one parallel set of planes have got activated. Now, in the 2nd stage, what we will observe is that more than one slip system gets activated which means that there are more and more dislocation interaction taking place.

And, you remember their forces that we talked about when there are more dislocation, so, there is force from one dislocation to another dislocation and when they are more than, when they are several, so, there are forces acting on each of them. And therefore, if you are applying a stress for a dislocation to move basically what you are trying is to overcome all these internal stresses.

So, a larger amount of internal, larger stress is required to overcome these internal stresses. And hence, we see a very sharp rise in shear stresses. Strain hard, almost linear strain hardening is absorbed. And, the main mechanism of strengthening is pileup which was discussed like by (()) (14:46) in with respect to grain boundaries. But, here also, once there is intersection of dislocations and then there are some immobile regions.

And, the dislocations cannot pass through that. And therefore, they kept piling up over there. And therefore, there are several such you can say knots or locks where the dislocations cannot pass through and they keep piling up over there and which eventually means that much higher (()) (15:12) stress is required for dislocations to keep moving. And, it is at this stage that dislocation tangles meaning now that you can see it.

They are intersecting like each other something like this. And, some of it will form a loop other will remain there. And therefore, the dislocations do not remain independent of each other. They begin to get tangled with each other. So, that kind of tangling of dislocation starts to take place which results also result in dislocation cell structure. So, at this stage, you would also expect to see some amount of cell structure getting formed.

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And, these are very well documented if people have done careful experiments with single crystal and they did find these different stages and whatever important key points that we are writing down here, they have all been observed in majority of the systems. Then, the 3rd stage, so, let us just, let us come back and take a look here. So, this is the stage 1 where almost no strain hardening is taking place.

When we say strain hardening meaning there is stress required with increasing strain is not increasing. So, stress is almost constant. While in 2nd stage, we see almost linearly increasing. And, in the 3rd stage, what we see is that thus stress strain hardening is decreasing. There is a

decreasing tendency for strain hardening. So, initially, there is a large rise of resolved shear stress with amount with small amount of strain increase.

And, later on that rise becomes smaller and smaller and smaller. So, the strain hardening becomes strain hardening rate becomes lower becomes smaller and smaller with increasing strain. So, one of the chief characteristics of this stage 3 that it is characterized by decreasing rate of strain hardening. Now, this, why is this decreasing rate of strain hardening taking place?

So, it is taking place because some amount of dynamical recovery is also taking place along with deformation which releases part of the strain. And, regarding dislocation, the cross slips starts to take place which also releases some of the stress which would have been caused because of the pileup. So, the stresses are so high that so cross slip requires a little higher stress because it is on a different plane which is not the lowest stress plane.

So, because of the increased stress, cross slip is now more convenient. And therefore, cross slip becomes more common. And, when it takes place, then it releases the stress created because of pileup.

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Another point that has been observed or another observed phenomenon that has been observed is that behaviour in this stage is temperature dependent. Meaning, if you go to higher temperature, then strain hardening rate is lower. If you are at lower temperature, then strain hardening rate is higher. So, this suggests that it is temperature dependent. And, what dislocation phenomenon is temperature dependent.

It is the climb. So, it clearly indicates that climb is also active in this stage. And, we saw that in the previous stage dislocation tangles were beginning. Here, it is confirmed that this dislocation tangle or entanglement becomes even more pronounced and intersection of dislocations is the chief strain hardening mechanism. So, this is how the dislocations qualitatively, this is how the dislocations lead to strengthening in the single crystal.

So, you can clearly see that as the strain is increasing, overall the strength, the resolved shear stress required for the deformation is increasing. Meaning, strength of the material is

increasing. So, that is for single crystal. Now, also, let us also briefly take a look at how this thing will happen in a polycrystalline material.

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So, we already know that if you were to look at the polycrystalline material, then, but here, what we see is only the 2nd and 3rd stage. So, there is somewhat 2nd or linear point and then there is that ever decreasing strain hardening rate. So, in brief, what we can say is that there is only stage 2 and stage 3 deformation taking place in polycrystalline materials. So, why is there no stage 1 over here?

And, it should not be surprising and it should not be very difficult to answer. It is because we will not have only 1 set of plane at any time. So, only 1 set of planes cannot deform. Let us understand this why it happens. So, in our polycrystalline material, let us say I draw it schematically like this.

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So, you have grains which are contiguous. And, the material is contiguous. And, there is 1 grain. And, very next to it, there is another grain and so on. So, therefore, the material looks something like this. Now, let us say if only 1 set of planes were deforming into neighbouring arrays. So, we will pick 2 grains from here. Let us draw it separately. Let us say that the 2 grains look like something like this.

And, this is the bar interface or the boundary bit. And, this is the, another grain. So, let us say that 1 set of plane which can deform is such that deformation is taking place along these lines in this one and along these lines in this one. So, now, what will happen? If we allow this kind of deformation, what will happen is that there will be steps getting formed over here. But, these set of planes cannot fill this gap.

Therefore, eventually, what we will see is that and this one will have, this will remain plane because no deformation takes place over here. And, what it eventually tells you is that there would be gaps created between the grains if only 1 set of planes for deforming in a polycrystalline materials. So, clearly this is not possible. You need minimum some number of planes systems to be deforming simultaneously to be able to accommodate or to ensure that there is contiguity of materials.

Otherwise, there will be voids created at very small strains itself. Now, looking at this itself you would be able to say that there should be minimum of 2 independent slip system in a 2D (( )) (27:15) 2D material system.

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So, if theoretically there were 2D material system, there must be at least 2 directions. So, if there, 1 set is like this and if you had a corresponding set another one like this, then it would have been able to take care of all the gaps at the interface. And, there would have been no problem of contiguity. Similarly, if you people have shown that if you are since we are, have materials in 3D, so, in for 3D materials, you need minimum 5 independent slip systems.

So, overall, what we see is that in the upper when you are deforming polycrystalline materials, it shows only the stage 2 and 3. And, it is obvious that it cannot show stage 1 which is dominated when you have only 1 set of planes deforming in the single crystal. So, this poly crystal is similar to the single crystal with only stage 2 and stage 3. Then, we also looked at why stage 1 kind of scenario cannot exist.

Because it will lead to contiguity of materials, there will be gaps created. And, since we know that it does not happen there must be contiguity. And therefore, there must be at least 2 independent slip system in a 2D material system and at least 5 independent slip system in a 3D material system. So, now, that qualitatively explains the role of dislocation in the deformation of single crystal and polycrystalline material.

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Now, let us get a little bit of into quantitative values to understand how the dislocation causes strength increase in the strength. So, for that, now, we will go back to our equation for interaction of dislocations. And, you would see that from there we will be able to derive what is called as Taylor hardening. So, if you remember, we had dislocations, one like this. And, let us say another one is somewhere on another plane.

So, we said that this is a distance  $y$  and some distance  $x$ . What are the forces acting on this? Now, from that forces, we will be able to understand how much force you would be able, you would need to apply externally. So, whatever force is there you would need to apply externally

opposite of that to be able to overcome that forces if you want to move the dislocation in the other way or in the direction of the applied stress.

So, for this kind of scenario, we looked at and we saw that stresses look like and over here we saw that this is equal. This is the distance in along x direction. So, for the x distance, we know this is 1y, somewhere over here we have 2y and so on. And here, we have minus 1y minus 2y and so on. Now, this is how the forces are acting on this. The other dislocation let us say this is a fixed dislocation for some reason just for the purpose of understanding the amount of force required for movement of dislocation.

So, let us say this is the fixed dislocation. And, this is free to move. Then, this is the force acting on it. Now, if we want to move this dislocation in some or we want to stop this dislocation from movement or we want to move it in any other direction, then clearly whatever force is acting on it, we must apply more than that. And, if you want to be able to completely move it as per the applied stress or as per your desire to say, then you must apply them with a minimum of this much stress either this much or this much in the other 2 directions.

And just to make things clear that this value is actually, let me, equal to this value in positive sense and this one is equal to negative sense. The earlier time, the last time, I drew this. If you go back, you would see that there is a little bit of discrepancy this one is lower than this. But, that was a schematic. And because I could not draw it as accurately as it is supposed to be, so, just keep in mind that the value of this force and this force is same in the positive direction.

And, this force and this force is also same in magnitude but just in opposite direction. And, this is your tau, this is the x. And, where does this maxima or minima occur? And, if you would be, if you look at the equation carefully, you would be able to find out that this minima or maxima occurs at 0.414y or equal to minus 0.414 y. So, if we look at the force equation that we looked at earlier, F equal to tau into b is equal to Gbx x square minus y square times b by 2 pi 1 minus nu.

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$$\tau_c = \frac{Gb}{8\pi(1 - \nu)h}$$

Now, if I want a critical tau which is this one, then I need to calculate it at x equal to 0.4141 or the minus of this, minus 0.414y. And, it is not very difficult to show that it will come out to Gb by 2 pi 1 minus nu y and the rest of the factor would come out to approximately 1 by 4. So, this is Gb by 8 pi 1 minus nu y. And, if we call this distance y as h because y is represents some many of the places something different.

So, the tau critical can be written as Gb by 8 pi 1 minus nu h. Now, this, what is this stress signify? This stress signifies that you need to apply at least this much stress shear stress to move this dislocation in either direction. If you are able to apply this much shear stress then this dislocation can be moved in any direction a little bit higher than this because this is the maximum stress that is acting on it because of the presence of another dislocation.

So, if there is another dislocation and you still want to be able to move this dislocation, you need to apply at least this much of shear stress. So, that is what this equation tells us. Now, let us move on. We can have this is just the, there we need not be just one dislocation. In a material, there will be lot more dislocations. So, what do we do? Or, how do we calculate that? Not very difficult.

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$$\tau_0 = \frac{Gb\sqrt{2}\sqrt{\rho}}{8\pi(1-\nu)}$$

$$\tau_0 = \alpha Gb\sqrt{\rho}$$

So, let us say, we have, so, here, we will make some assumption that the dislocations are slightly arranged. So, it is something like this. And, let us say the distance from here to here is

h. Similarly, the distance from here to here is h. So, if I were to find the dislocation density, so, let us say, I take this area. So, this is 2h by 2h or 2h square and number of dislocation that we have over here is 1 single dislocation and 1 by 4 into 4.

So, there is total of 2 dislocation in this area of 2h whole square. So, if I were to calculate dislocation density row dislocation then it will come up to 2 by 2h square which is 1 over 2h square. And now, if I want I can also write h in terms of the dislocation density, so, it is like this. And now, I can insert this h because this is h I will not be able to calculate for individual dislocation.

But, yes, as a dislocation, we know in general the dislocation density of the material at a given state. It can also be calculated by various means. So, I would note rho. So, I can use this to insert or approximately calculate the value of h. And therefore, tau c would become Gb which was, sorry, which was already given us Gb by 8 pi 1 minus nu h. Now, instead of tau c, I will now call it tau naught which is the stress.

It represents the stress required for the material to deform. So, for a given dislocation density, what is the stress that is required to deform the material? And, this would be given by Gb by 8 pi 1 minus nu. And, instead of h, I will write 1 by 2 root 2 1 by rho. And, this can be approximated as alpha Gb root rho which is called the Taylor hardening relation. So, let me just write down what it is saying.

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$$\tau_0 = \alpha Gb \sqrt{\rho}$$

It is giving the shear strength of material as a function of dislocation density. And, what it is saying is that as you keep increasing dislocation density, shear strength of the material keeps increasing. So, this is exactly what we mean by strain hardening. When you have more the strain more the dislocations more is the strength of the material. And, you will look at a single crystal sorry polycrystalline material.

So, we know the stress strain diagram looks like this. See, if you compare the original yield strength versus this point and let us say you deform it up to this point and then release the stress. Then, it will come back. And, once you, if you do the tensile test again, this will become

the new yield strength of the material. So, why has the yield strength increased? Because, as the strain increases, dislocation density increased.

So, the strain is increasing there which leads to dislocation density increment. And therefore, strength the yield strength of the material increased. And, people have also plotted normalized shear strength of the material against the dislocation density and what they observed is something like this. So, if you look at y axis  $b \sqrt{\rho}$ . So, this is the dislocation density and the normalized shear strength of the material.

People have obtained a straight line relation. So, it clearly confirms the Taylor hardening relation. So, this is, this clearly establishes the strain hardening. Now, we will go through some example, to be able to appreciate this even better.

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So, here, the first example is that you are given the 2 positive dislocations are located in 2 parallel glide planes which are 10 nanometer apart. At what value of separation in x direction, so, in this direction, they experience maximum and minimum force? So, there is force between the 2 the dislocations or there will be some interaction and because of that, they will either attract or this, repel each other.

So, the question is, at what value of x, they will experience maximum and minimum force? And, if you go back to the notes, you would see that the maximum force occurs at x equal to  $0.414y$ . But, y is 10 nanometer which means 4.14 nanometer. And, minimum occurs at x equal to minus  $0.414y$  which is minus 0.414 nanometer. So, maximum and minimum are here, just the science, well that they are positive or negative, but magnitude wise the forces would remain same.

And in fact, both of them would happen to be attractive. So, both of them will try to attract it and eventually form a array like this. This is what we saw when we looked at the interaction of dislocations with similar sign when they are in different planes. So, this is Example 1.

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Now, let us look at still another example. So, here we are given that in a material, 100 dislocations are observed in TEM in an area of  $10,000 b^2$  square. So, what is the shear stress

required to move dislocation through this material? So, TEM is basically assuming that it is showing the representative dislocation density. So, let us look at what would be the representative dislocation density value.

It is equal to number of dislocation which is  $100 \times 10,000 b^2$  which is equal to  $1 \text{ over } 100b$ . Now, let us look at the relation for shear strength. We know it is equal to  $Gb \sqrt{8 \pi (1 - \nu)}$ . Instead of  $\sqrt{\rho}$ , we can write it as  $\sqrt{\rho}$  is equal to  $1 \text{ over } \sqrt{100}$  is 10 and sorry, this is again  $b^2$ . So, under root  $b^2$  becomes  $b$ , so,  $\sqrt{2}$  into  $1 \text{ over } 10b$ . And therefore, the  $b$  gets cancelled over here.

And, what we remain  $8 \text{ into } 10$  becomes 80. So, this becomes  $G \sqrt{2}$  by  $80 \pi (1 - \nu)$ . So, this gives you the shear stress required to move dislocation. And, if you had higher dislocation density, you would see that the shear stress required is also higher. So, this clearly establishes strain hardening. So, over here, we have, so, assume that strain increasing strain causes increase in dislocation density. And, that part is still to be shown or understood.

And, that we will look at in the next lecture. But, assuming that increasing the strain increases the dislocation density we have clearly seen qualitatively and quantitatively that the strength of the material should increase. So, with that we will come to end of this lecture. And next time, we will look at how movement of dislocation with strain causes increase in the dislocation density. Thank you.