Mechanical Behavior of Materials-1
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Lecture - 27
Critical Resolved Shear Stress

Welcome back students. We have looked at the dislocation characteristics. We have also looked at dislocation motion. And we also looked at the fundamentals, which derive or which define the slip system, so on what particular plane in a given material system would the dislocations move.

Next, we are going to look into a very important concept related to dislocations, which is critical resolved shear stress, which is what defines how much or not defines, but this is what will help us determine what will be the stress required for dislocations to move. So let us look into it.

(Refer Slide Time: 01:01)

$$\tau_R = \sigma COS\emptyset COS\lambda$$

Let us assume we have a single crystal. So the single crystal is in the shape of a cylinder like this. And inside this we are looking at one particular slip plate. So the question we are asking is how much stress or load we need to apply say along P. So P is the load here and if we know the area then we know what stress is to be applied. So sigma you can say is to be applied in this direction for the dislocation to move.

We know that the dislocations will move only under shear stresses. So the question is what will be the resolved shear stress along the Burgers vector. So let us say if we were looking at dislocation independently, which is shown in the bottom here, and this is the Burgers vector, then what we are interested is in finding that what will be the total shear stress acting along Burgers vector and on the plane on which this dislocation can move.

So that is what we want to identify. And it has to and here we have given the load in terms of P and we know the area A. Or you can have as well defined it in terms of sigma. So let us say we want to find out the stress. So that we will know what is the stress required for the dislocation motion which in turn would mean the initiation of plastic deformation. So here we know that the angle between normal to slip plane, which is this.

Normal to slip plane is given by N and tensile axis. So this is the tensile axis P and this is the normal, so the angle between them is given by phi. So angle between the slip direction, so let us say this is the slip direction meaning the Burgers vector is like this oriented somewhere over here. So that will define the slip direction. So we want to know what is the angle between slip direction and tensile axis.

And this is the tensile axis, this is the direction of the slip or the Burgers vector, which is where the resolved shear stress must act and this is given by this angle lambda. Now if we know these two angles, we know a lot of information. Let us see how. So now we have area of the slip plane. So if this is the system then the area of the slip plane is, so this is the slip plane now in this orientation.

This oblique case, which would mean that the area of the slip plane which is inclined at angle phi is given by A by cost phi where we know that the original area, this area is A. So this area was A. Now this one is A by cos phi. The load which is or the stress P by A. Stress was acting along this direction. So now we need to find the stress acting along the Burgers vector which is τ_R.

So that would be given by component of load. This would be P exactly in this direction, so cos lambda is equal to P cos lambda. So now we have the load and we know the area and therefore, we are in a position to find what is the stress acting here. And therefore that would be given by resolved shear stress acting along slip direction on the slip plane, on the given slip plane is equal to τ_R.is equal to P cos lambda by A by cos phi which is equal to P by A.

This is very similar to or in fact it is along the same lines where we found or where we did the axis transformation. Here also we are doing the axis transformation and of a particular quantity. And therefore you can see that the form of the equation is similar. Therefore, in the end what we get is this. τ_R.is equal to P by A we know is sigma, cos phi cos lambda.

So what this equation is saying that if you want to find the dislocation stress or the shear stress acting on to this dislocation, where the Burgers vector is along this direction and it is moving on this plane where you are given that sigma or P by A is the stress acting along this direction, then the shear stress on this particular dislocation would be sigma times cos phi times cos lambda.

So that is what we obtain from this equation. So does this mean that once we have any amount of shear stress happening on the dislocation, the dislocation would start to move? The answer is no. You need a minimum shear stress along this direction for the dislocation to move, which is what we call as critical resolved shear stress, which we will get to in a moment. But before that, let us be very clear about couple of things over here and let me put it like this.

(Refer Slide Time: 09:25)

$$\tau_R = \sigma COS\emptyset COS\lambda$$

So we haveτ_R., N which is this one and P. So we see in the image, in the two-dimensional image, it may seem like thatτ_R., P and N are all in the same direction, it need not. Okay, so this is something you have to be very clear about that this τ_R, P and N need not be in the same plane. For example, the Burgers vector could have been pointing in this direction. So this whole thing would have been rotated like this over here.

And therefore τ_R, P, N would be here and the, sorry the P and N would be here and the $\tau_R$and P would be like this. So in the general case, τ_R P and N would not lie in the same plane, need not lie on same plane. And the corollary for this is that phi plus lambda need not be equal to 90 degree. In fact, that would be a very special case where phi plus lambda is equal to 90 degree.

Even lambda it may in some orientation of this drawing, it will look like lambda is equal to 90 degree. So let us be very clear about that too that now lambda need not be 90 degree. Of course, in some particular case it can be 90 degrees. So with this we have now resolved or found a relation to find what would be the resolved shear stress acting on a slip, on a dislocation with a given Burger vector which is oriented where the slip vector or the Burger vector is along this given direction.

So we know all this vector with respect to or in terms of the angle. So the Burger vector is oriented with respect to P at an angle lambda and the plane normal is oriented with respect to P at an angle phi. And like you said this isτ_R. But just because the shear stress is being applied, it does not mean that the dislocation would start to move. We need some critical value for the dislocation to move. So what is that critical resolved shear stress?

(Refer Slide Time: 12:17)

So this is the minimum value ofτ_R. So τ , this will be called τ_{CRSS}. So minimum value of $\tau_R$that must be applied for dislocation to move. So this is minimum value or a threshold value. The critical resolved shear stress is a minimum or critical or critical threshold that τ_R must reach before the dislocations can start to move. And what it implies is that if τ_R is less than this critical value, then there is shear stress on the plane but it is not enough.

And also there is force on the dislocation, but dislocation does not move. So τ_R must be equal to or greater than this critical value before the dislocation starts to move. τ_R, now what is τ_R? It is a variable which is dependent upon applied stress, geometry of the crystal and orientation of the slip. And so we saw it is dependent on sigma, it is dependent on cos phi and cos lambda.

On the other hand, τ_{CRSS} the critical resolved shear stress, it would be a fundamental property of the material just like the yield strength. So yield strength is something we described with respect to polycrystalline materials. But in single crystalline material and for a given slip plane, we will define it in terms of τ_{CRSS} or τ_{CRSS} shear stress. So $\tau_R$is a variable. So it keeps changing, not τ_{CRSS}.

Tau CRSS is one quantity, while tau R is a variable quantity. It is dependent on applied stress, geometry of the crystal, and orientation of the slip plane. So even inside a given geometry there may be different slip planes. So it will also depend on that while Tau CRSS is a fundamental property, is a fundamental material property and a constant.

(Refer Slide Time: 16:40)

$$\tau_R = \sigma COS\emptyset COS\lambda$$

And this τ_{CRSS} can be said to be is single crystal equivalent of yield strength. In brief, when we keep increasing the load or the tau R keeps increasing, and when it reaches, this tau R reaches a critical value which is what we call now asτ_{CRSS}, then dislocations start to move and yielding takes place. So the overall relation for τ_{CRSS}would still be defined like this.

τ_{CRSS} is equal to sigma cos phi cos lambda, but then this would be a particular value and this is one particular value where the dislocation start to move. So it is for the given phi N lambda whatever the sigma comes out to be that would be the stress at which the dislocation start to move. And it would, in fact this sigma would be determined using this value τ_{CRSS}.

And for all other values what we will say is that tau R whatever value we get from this relation is equal to the resolved shear stress. So τ_{CRSS} as you have seen is a critical value where the dislocation start to move. But then it will probably also remind you of another parameter that we had studied earlier, I have looked into, which was called the Peierls–Nabarro stress, tau P-N.

And that was also the stress at which the dislocation start to move. The question is what is the relation between this CRSS, the τ_{CRSS} and the τ_{PN}.

(Refer Slide Time: 18:52)

So we want to understand what is the relation between τ_{CRSS} and tau P-N. So what do you think? Are they the same thing? Are they something completely different or they have no relation amongst each other? So it so happens that both these quantities reflect the minimum shear stress. Why we are actually discussing it here is for this reason. That both these quantities, minimum shear stress to overcome lattice resistance.

In τ_{PN} also we saw that there is there are the energy values and we derived from that or we can derive from that energy values by that by taking the first differential we can find the shear stresses. And those shear stress if you look at the amplitude, if your shear stress, overall shear stress is less than amplitude, then it would mean that you will not be able to come out of the valley.

And therefore, the minimum shear stress that you need to apply is equal to the amplitude. And we that is what we called as tau P-N. So that is also the minimum shear stress that you need for the dislocation to move against the lattice resistance. And here also we look at the tau P-N is a shear resolved shear stress along the dislocation, but it will not move unless it is over and above some critical value which is what we call asτ_{CRSS}.

So below this it is as good as saying that it is inside the valley and when you apply stress higher than this critical value, then the dislocation start to move. So what is the relation that is the question? And the answer is that actually both of them are, both of them represent the same phenomenon, the same I would not say phenomena, but both of them represent the same quantity.

However, there is a slight difference between the two and it is given here. So tau P-N is actually a theoretical quantity. So you see we obtained that based on theoretical model and in that theoretical model we have not taken into, at least until now we have not taken into account temperature. So tau P-N is a theoretical energy or sorry theoretical stress that is required for dislocation to move.

And also that it is a theoretical value only for zero Kelvin, because we have not considered the temperature part at least in the equations that we have dealt with. On the other hand, τ_{CRSS} is the experimental quantity and therefore, whatever temperature you are doing the test, it represents the resistance at that particular temperature. And therefore, it is a temperature dependent quantity.

If you do it at very low temperature versus at very high temperature you would get different values of τ_{CRSS}. So that is the main difference between the two. However, both of them represent the same thing that is the lattice resistance of the lattice resistance against the movement of the dislocation. So let me summarize what we what I just mentioned. Tau P-N, is a theoretical quantity while τ_{CRSS} is an experimental quantity.

τ_{PN} is given for zero Kelvin while τ_{CRSS} is obtained for a given whatever temperature you have done. So that was the second difference. So this is the relation and we must keep this in mind because, it is not that we are talking about two different things. So with this understanding, now let us move on to understand another aspect about this resolved shear stress.

(Refer Slide Time: 24:30)

$$\tau_R = \sigma . m$$

$$\sigma = \frac{\tau_{CRSS}}{m}$$

And we have looked that there is a factor cos phi into cos lambda. Now this factor of cos phi by cos lambda this can be termed as m or equal to so cos phi into cos lambda, which is also called as Schmid factor. Now if you look at the relation again that we had obtained, so τ_R is equal to sigma times cos phi cos lambda or m and if it happens to be CRSS then sigma is given by τ_{CRSS} by m.

Meaning that this stress that you need to apply for the dislocation move on a given plane would be lowest when m is highest. And when is m highest, when cos phi times cost lambda is highest. So people have done experiments, where they have changed the values of phi and lambda. So you get, here this is done where phi has been changed from 0 to 90 degrees and cos lambda was kept constant so that your cos phi to cos lambda values changes continuously.

And what you can see is that, when the cos phi cos lambda value is the angle is somewhere close to 45 degrees that is when you would get the lowest value of the yield strength. And if you were to draw the Schmid factor values, then it would actually be the inverse of this. So this will be highest over here. So this is highest value for m and at the same point you would get lowest value for sigma.

So the planes which are oriented close to 45 degrees would give you the or there you would need the lowest stress, the tensile stress for the dislocation to move. And this is in line with what we have already observed that when you apply the tensile stress then along the 45 degree planes, you get the highest shear stress.

So this is you can see corollary of that, because the shear stresses are highest over there, that is where that is why the planes oriented close to those 45 degree planes would show the lowest value of yield strength. And a corollary of this we can look at this is like this. So when can we get the highest value of the cos phi cos lambda? When the angles are lowest.

So for maximizing, so we will show quantitatively that this lowest value of yield strength and highest value of Schmid factor would actually be obtained for 45 degree. So for maximizing Schmid factor, angles phi and lambda must be minimum. And it is minimum. So far we have assumed that they can be in different planes. So phi can be in one plane and lambda can be in another plane.

But if you want the total to be minimum, then these two must become together or it must be in same plane. Therefore, it will be minimum when all, it will be absolute minimum only when τ_R, P, N lie in same plane.

(Refer Slide Time: 29:30)

And when that is the case that would mean that phi plus lambda is equal to 90 degrees and hence maximizing cos phi cos lambda you can now apply the simple mathematical rules to maximize the cos phi times cos lambda is same as maximizing 1 by 2 sin 2 lambda because now they are related by this relation. And it implies that 2 lambda is equal to 90 degrees or lambda is equal to 45 implies phi is also equal to 45 degrees.

So you see, we have mathematically obtained that Schmid factor would be highest when phi is equal to lambda is equal to 45 degrees. And at that time at that particular configuration you will have the lowest yield strength values.

(Refer Slide Time: 30:49)

Thus lowest stresses are required for yielding of planes close to 45 degrees. Now here we have taken configuration such that some planes were coming at 45 degrees. However, in practice depending on what orientation of the crystal you take, it need not be necessary that some planes are actually oriented close to 45 degrees, which means there is no slip plane at 45 degree orientation.

So what it means is that all the planes closer to the 45 degrees would be the ones that would this is what this drawing is showing will lead to the lowest yield strength or these are the planes where plastic deformation or dislocation motion would begin. If at all there is a possibility that there is a slip plane at 45 degrees, then of course the dislocation motion would take place over there.

But if not, then the ones closest to 45 degrees would be the ones where dislocation motion would take place. And as I mentioned earlier, just few minutes back that this is also a corollary of what we had seen earlier that when you apply normal stresses, the maximum shear stress is at 45 degrees. So this is a artifact of this. So now that we understand this now let us okay I have missed t here.

So now we are in a position to look at certain examples. So let us try to solve some problems related to this. Okay, so one point I missed that I wanted to say which is the topic here hard orientation versus soft orientation. So the orientations which are close to this region would be called a soft orientation meaning you need less stress for the dislocation to move along those planes.

While the orientation which would be over here would be called hard orientation and in fact at 0 degree or 90 degree those would be called very hard because you cannot, no amount of stress, the tensile stress can make dislocations move on a plane, which is just normal to it. Because the simple reason is that there is no component of load which would be stress which can lie on this particular normal to it.

So that is the reason that makes it hard orientation. Okay, so with that now let us try and solve an example.

(Refer Slide Time: 33:29)

So the question is determine the tensile stress that is applied along the 1 bar 1 0 axis of a silver crystal to cause slip on 1 bar 1 bar 1 0 bar 1 1 system. So basically, this is telling you the plane on which the dislocation is and this is telling you the slip vector. It is not the Burgers vector, just a slip direction. So Burgers vector would be probably A by 2 011, 0 bar 1 and it is silver.

So we already know it is the FCC system. And the stress is being applied around 1 bar 1 0. So just for the sake of completeness, let me draw the overall configuration, it would look something like this. So this direction is 1 bar 1 0. And there is a plane, which is 1 bar 1 1. And let us say there is a dislocation over here. And let us say the Burger vector is like this. So this direction, would happen to be 0 1 bar 1.

So that is the overall configuration that you have been given. This is the normal which would be 1 bar 1 bar 1 and we have said that this is the, so let us say this is the phi and this is the lambda. So we know that the angle between Burger vector and the normal is the phi and the angle between the normal, sorry angle between the normal and the Burger vector or the slip direction is lambda and the normal and the loading direction is phi.

So this is phi and this is lambda. So next what we need to do is find the phi and lambda and therefore cos phi is the by taking the cos of these two vectors and this is simple this we have 1 into 1 plus -1 into -1 plus 0 into -1. And on the denominator we will take the square root of this 1 square plus 1 square actually -1 square plus 0 square. And the other one is -1 square, sorry this was just I have not written it very neatly here.

It should be just 1, there is no minus sign. So this is 1 square plus -1 square plus -1 square and therefore it comes out to be 2 by root 2 into root 3 equal to 2 by root 6 and cos lambda will come out to, so now we are taking the cos between 1 bar 1 0 and 0 bar 1 1. So this is 1 into 0 plus -1 into -1 plus 0 into -1. And this one we already know is 1 square plus -1 square plus 0. And this one is also the same.

It is 0 square plus -1 square plus -1. Actually it is 0 bar 1 1 square. And therefore, this is 1 over root 2 into root 2 equal to 1 by 2.

(Refer Slide Time: 38:58)

$$\frac{\tau_{CRSS}}{COS\phi COS\lambda} = \sigma$$

So we know that sigma is equal to $\tau_{CRSS}$by assuming that this is the stress, normal stress that will cause the dislocation to move we have used CRSS and the CRSS is given to be 6. Therefore, this is 6 by 2 by root 6 into 1 by 2 and this comes out to be 6 root 6 is equal to 14.7 mega Pascal.

So if we have a dislocation with slip vector along this direction and this is to be gliding on 1 bar 1 1, bar 1 bar 1, then you need to apply a normal stress of 14.7 mega Pascal along 1 bar 1 0 for it to move the dislocation. That is what this example tells us. So you can see this is very helpful in helping us understand which particular dislocation would move and what is the stress required.

Next, we will look at an example, to understand of the various slip planes in a system which one would get activated. So here until now in this example, we are looking at one specific slip plane, but the fact is that there can be more than in fact, there is a lot more slip planes as we will see.

(Refer Slide Time: 40:31)

So you are, in this example you are given that a single crystal of copper is deformed in tension. So again we are applying tension and only that here it is in a little different direction. So this is again FCC and this direction is given us 112. And here you are not given any specifics of the plane. You have been asked to calculate Schmid factors for all the possible slip systems.

And for that we need to remember what are the possible slip systems in FCC. So 111 plane and 111 110 type of directions. And you are also, the second part is you are also given if the critical resolved shear stress is 50 mega Pascal, what is the tensile stress at which the material will start to deform plastically. So first we will look at which one has the highest Schmid factor and then for that we will calculate the tensile stress that would be required to make the dislocations move.

So we will begin with the place, so what are the possible slip planes. So you here we can assume that basically there are slip planes like this. I have drawn this it will become clear to you like just in a moment.

(Refer Slide Time: 42:33)

So what are the slip planes and the possible slip direction? So, **so** we will find cos phi, we will find cos lambda and then we will find Schmid factor and eventually we will find sigma. So this is the table and here we will use first the 111 plane and very easily you can show that slip directions possible for this are, slip directions I have written not the Burgers vector because we will have to add that factor of A by 2.

But that is immaterial from the point of view of calculating cos phi and cos lambda. So the possible directions are bar 1 1 0 or you can also take the negative bar 1 0 1 and 0 bar 1 1. So I will show it for two systems and then I will just note down the values for other systems. So over here we will take the cos phi which will be between 111 and 112.

And we know the relation cos phi equal to h 1 k i, h 1 h 2 plus k 1 k 2 plus l 1 l 2 divided by h 1 square plus k 1 square plus h 1 square under root times under root x 2 square plus k 2 square plus l 2 square. The equation that we used in the previous example. So I will not go through that but I will just write the values and it will be 2 root 2 by 3. This will be constant because this is constant here.

And the cos lambda would be different. So you would see that 112 and bar 1 1 0 if you take the dot product, then it is 0. What it means is that this slip direction lies in the plane normal to 1 1 2. So the bar 1 1 0 is normal is like this. And which is not a good news in some sense. 0 and here we will get root 3 by 6, root 3 by 6. So again, I am not going through the equation.

You would use the same equation that we used in the previous example. And you can find cos phi. And similarly, you have found cos lambda. And then you take the product to find Schmid factor. So this is 0, this is root 6 by 9. This is root 6 by 9. I am not finding the sigma values. I will do it only for the one which comes out to be the lowest.

(Refer Slide Time: 46:01)

Next plane that we will select is, so there are four 111 type planes. And so we will need to do this four times. And the directions here are 110 101 and 011. So you can see that these two are dot product is 0. So this is indeed a vector that lies on this plane. And then we will calculate the cos phi, which comes out to same value and the cos lambda comes out to root 2 by 3, root 3 by 2, and root 3 by 6. So the overall Schmid factor comes out to root 6 by 18.

(Refer Slide Time: 47:07)

And the third plane we will select is 1 bar 1 1 and the three directions are 110 bar 1 0 1 and 011. And here also this is a little different, okay. So I have made a mistake here, there is no 2 here actually. So this one is also root 2 by 3. And the values are root 3 by 3, root 3 by 6 root 3 by 2. And the Schmid factors come out to root 6 by 9, root 6 by 18, root 6 by 6. So we have done for three planes.

(Refer Slide Time: 48:05)

Now the fourth one is little bit you can say very different 1 1 bar 1 and the four directions are bar 1 1 0. Now what happens here is that 1 1 bar 1 is at 90 degrees. So the cos phi comes out to be 0. So now from here itself, we can say that the m would be 0. But anyways, we will go through the motion and find out the values.

So these are the lowest Schmid factor and certainly we are not interested in this which is the highest Schmid factor that we see over here. And you would see that this value is highest. Similarly, we get another root 6 by 6 which is this. So these are the slip systems where the Schmid value is highest and which would mean that the yield strength value would be lowest and if you calculate what you would see is that 122 mega Pascal, 122 mega Pascal.

And just so what this means is that if this is 0 no matter how much stress you apply, this will not deform. So this will, I will just say N.D. meaning not deformed, N.D., N.D. Similarly, we had obtained 0 over here. So this is also non-deformable plane or system. So this one is non-deformable and just for the sake of completion it is not required here.

But just because we are here I will for the sake of comparison, I will put the values of how much stress would be required for dislocation to move in this slip system, you would see that it will come out to 184, 184 and this one will be, this mid factor value is very low 184, 367. So these two have the lowest amount of tensile stress required for dislocations to move.

So for this particular orientation, we have four different planes and on the four different planes we have again three different directions and for that we get these different values. Now for again what is what would be useful here, if I put in the values in terms of orientation rotation or angle between this 111 plane and the tensile direction.

So what you would see is this one would come out to 19.5 degree, this one will come out to 61.8 degrees, this one is also coming out to 61.8 degrees and this is coming out to be 90.0 degrees. So actually I have drawn is a little bit incorrect. So there are two planes which are at 61.8 degrees. So something like this and something like this. On the other hand, we have a plane at 90 degrees. So it is like this.

And one is at 19.5 degrees. So this one is, so clearly the stresses required for this plane and for this plane, in fact this one is undeformable. This is the last one where the Schmid factor is come out to be 0 meaning no component of load will lie in here, which is what we know. And therefore there is no amount of stress can make the dislocations moving this plane. Here this one you can make it move.

Only that you would need much higher stresses and which is clear over here and in fact one of them still would be non-deformable. For these two systems, we see that we do get these are close to 45 degrees, which is what we expect, which is where the shear stresses would be highest.

But then we also have to look at the direction and looking at the direction we can clearly see that there are two direction in this one, bar 1 1 1 plane and bar 1 1 plane where the yield stress required for moving the dislocation is lowest. So these are our preferred or you can say orientations that we are looking for where the dislocations would start to move at the lowest tensile stress.

Or in other words, if you were keep pulling this particular system if you keep pulling along 1 1 2 direction, then the first dislocation to move would be on these two systems and in those two particular directions, where the dislocations would start to move and then it will cause the plastic deformation to begin in the system. So that gives you a understanding of how to find the dislocations that would start to move.

So in within a given slip system which particular dislocation system starts to move and what would be the stress required given the critical resolved shear stress. So and we also realized that critical resolved shear stress is something equivalent of a yield strength in a polycrystalline materials. So these are the some important concepts that we got to learn today. And we also solved some problems with that. So with that, we will come to end of this lecture. Thank you.