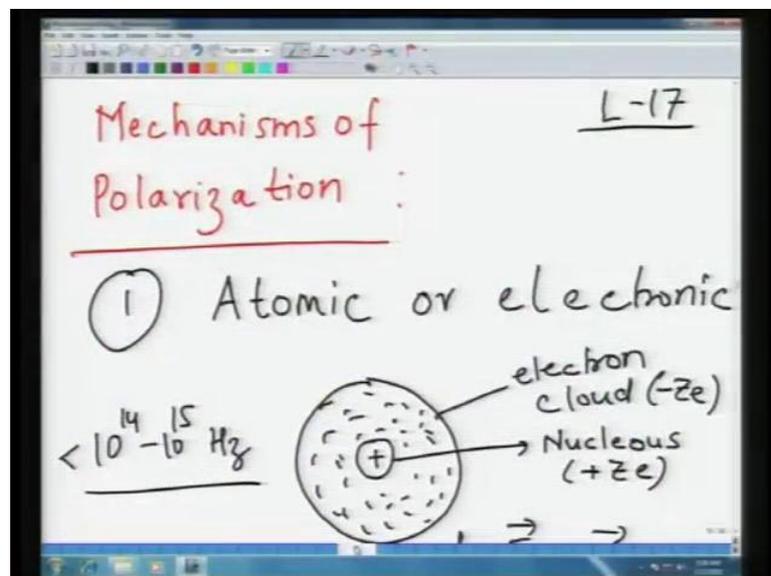


Electroceramics
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Indian Institute of Technology, Kanpur

Lecture - 18

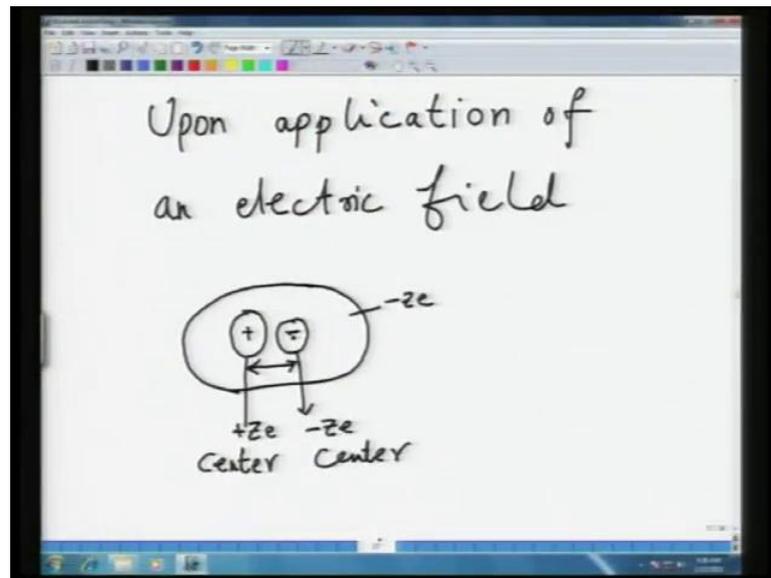
So, good morning to all of you, in this new lecture, so what we will do is that first we will review the last lecture in which we discussed to a basically mechanisms of polarization. So, first of these mechanism is atomic or what we call it as electronic polarization.

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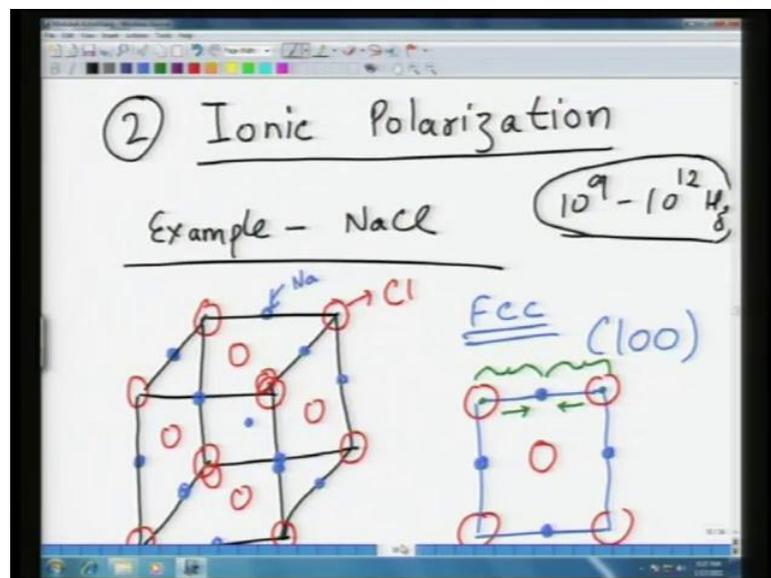
This is basically separation of charges at the very small level that is at the level of atom. What happens is you have shifting of centres of positive negative charges that is shifting of electron cloud with respect to the nucleus. This typically happens at the frequencies, optical frequencies very high frequencies.

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In such a situation this is also, this can also be explained by Maxwell's equation and as a result ϵ_r in this range happens to be equal to n^2 .

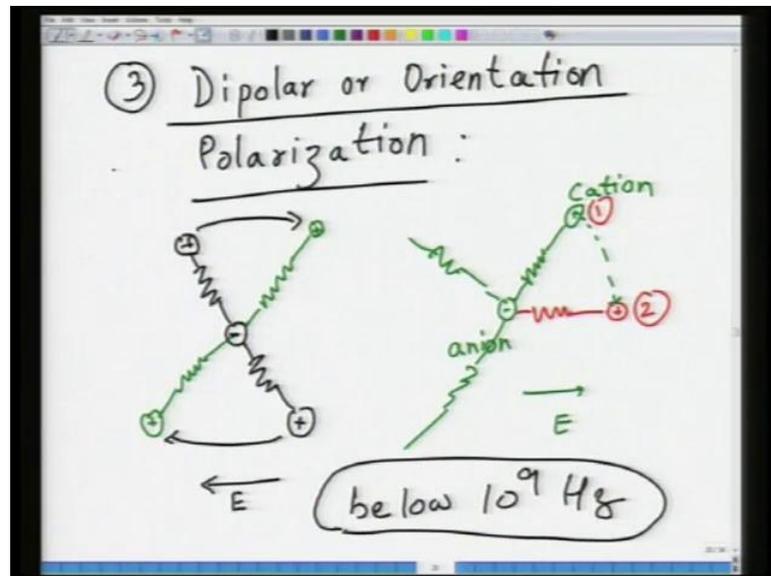
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Then you have ionic polarization, which is separation of ions. Now, this happens in solids which are ionic solids, which have an ions and cations. So, as a result you have shifting of positive and negative ions, with respect to each other. This happens at slightly lower frequencies because of higher mass of entities in volt and as a result time taken is

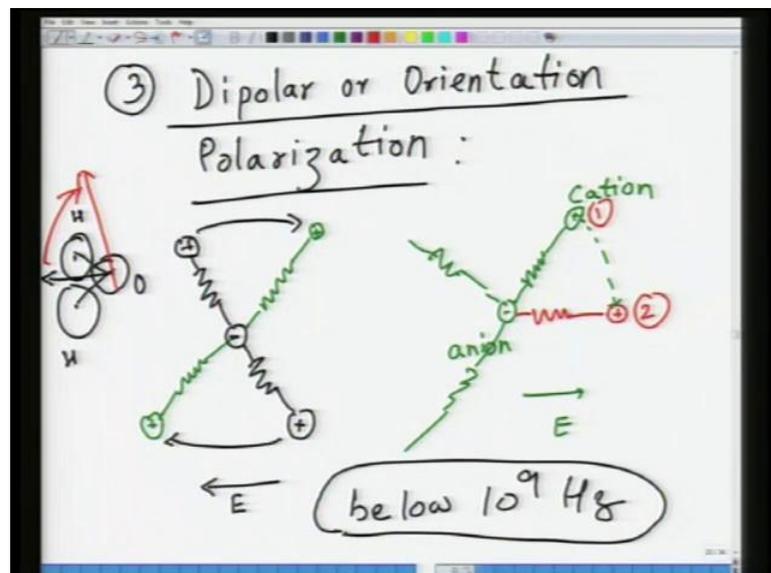
larger and hence the frequencies are in the range of 10^9 to 10^{12} hertz.

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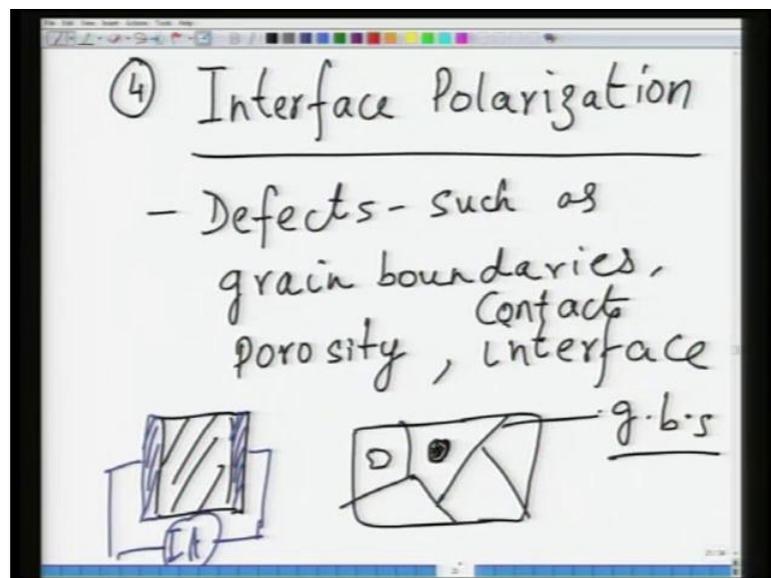
One example which we took was NaCl and third one is was a dipolar polarization, which involves movement of dipoles. Now, this this happens in solids, which have first of all permanent dipole movements. So, just for example, the materials like water etc. Materials like water and any other polarisable solid which has a permanent dipole movement.

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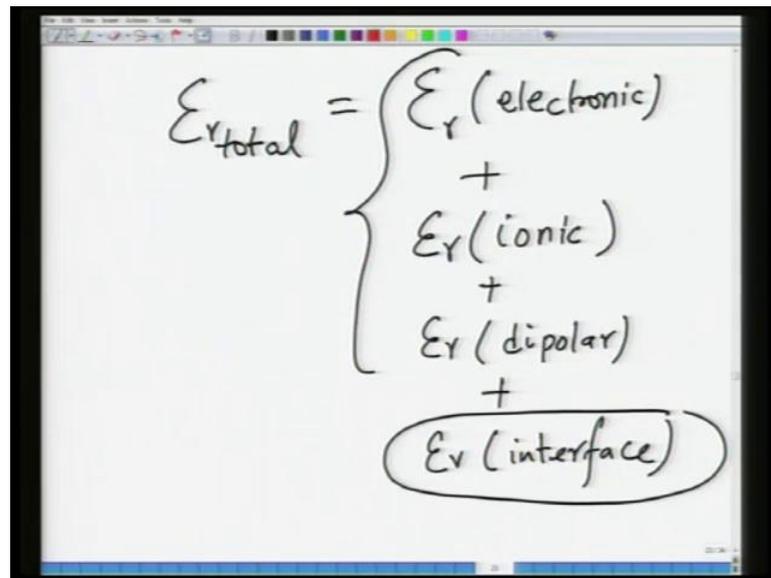
So, water molecule would be something like that. So, so this is let us say hydrogen and this is oxygen so as a result you have permanent dipole movement irrespective of a light field. Now, what happens is when in in in such a solid, it is the alignment of this dipole or vector along the direction applied field. This gives rise to so for so for example, if the field was in this direction, then this whole thing has to rotate in this direction. So, as a result here you have rotation of dipoles, which can happen in the frequency range anywhere between below 10 to power 9 hertz. So, because this involves rotation of higher masses, as a result larger times are required.

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Then you have interfaced polarization which happens because of defects such as grain boundaries, porosity or contacts contact interfaces etc. So, one situation can be you have electrodes and as a result you have polarization of the electrode interface or you can have porosities or any discontinuities in the sample and they also give rise to a charge build up at the interfaces. This is because of this is called as interface polarization, typically happens at very low frequencies. Although in some systems you can observe at about 100 hertz as well but typically it happens below 1 hertz.

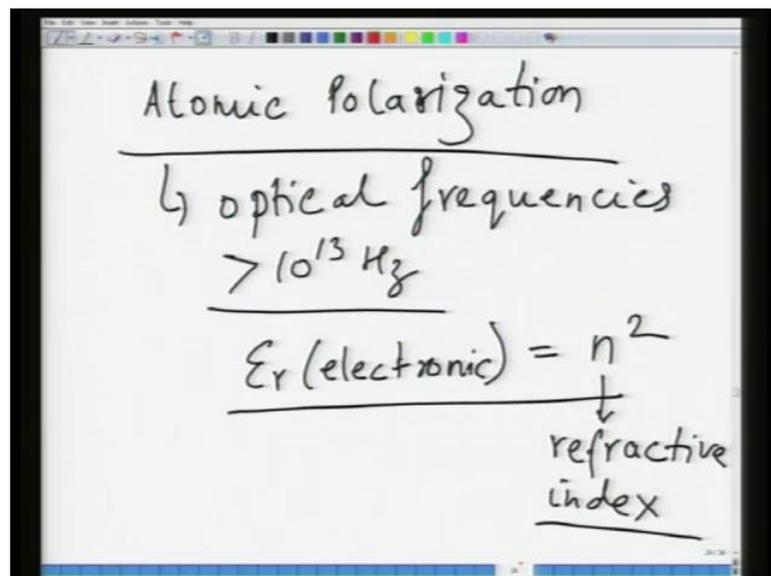
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A whiteboard with a black border showing a handwritten equation. The equation is $\epsilon_{r, total} = \left\{ \begin{array}{l} \epsilon_r(\text{electronic}) \\ + \\ \epsilon_r(\text{ionic}) \\ + \\ \epsilon_r(\text{dipolar}) \\ + \\ \epsilon_r(\text{interface}) \end{array} \right.$. The term $\epsilon_r(\text{interface})$ is circled in black.

So, that total dielectric constant happens to be equal to sum of electronic contribution plus ionic contribution, plus dipolar contribution, plus interfacial contribution. Typically now what we will do is that, we will as I said we will take mathematical treatment of these three, first three but interface we will not take into this course. What I will do is that I will provide you some references, where you can go through is a little t d s.

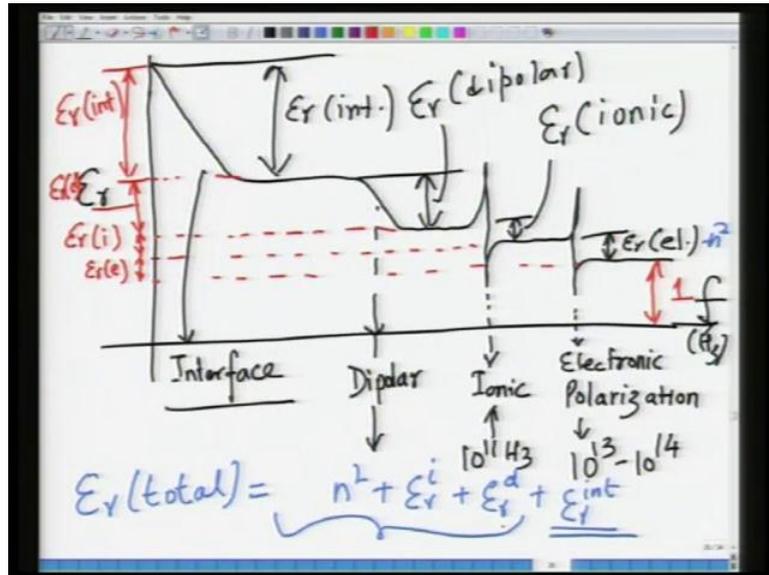
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A whiteboard with a black border showing handwritten notes. At the top, it says "Atomic Polarization". Below that, a horizontal line is drawn. Under the line, it says "↳ optical frequencies" and "> 10¹³ Hz". Another horizontal line is drawn below that. Below the second line, it says " $\epsilon_r(\text{electronic}) = n^2$ ". An arrow points down from the n^2 to the words "refractive index", which are underlined.

But it can be model using other methods, which I will talk about later in the course.

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So, this is how the plot looks like. You have interface, so you start from high frequencies. You have electronic contribution followed by ionic contribution, followed by dipolar contribution followed by now, interfacial contribution. So, this could be anything sum of all these is the total dielectric constant.

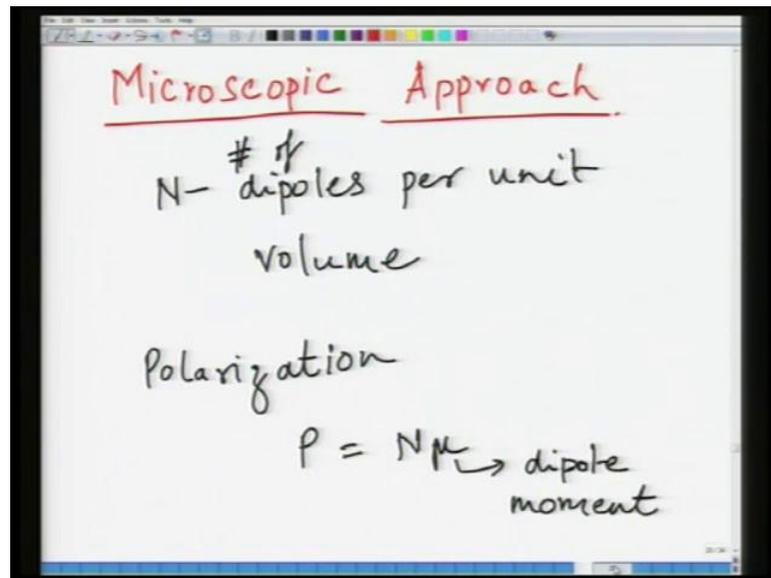
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Material	ϵ_r	n^2	Probable Mechanism
C (Diamond)	~5.7	5.85	Atomic
Ge	~16	16.73	"
NaCl	5.9	2.37	Atomic + Ionic
H ₂ O	80	1.77	at. + ionic + dipolar

Note: A small diagram of a water molecule (H₂O) is drawn next to the H₂O entry, showing an oxygen atom (O) bonded to two hydrogen atoms (H).

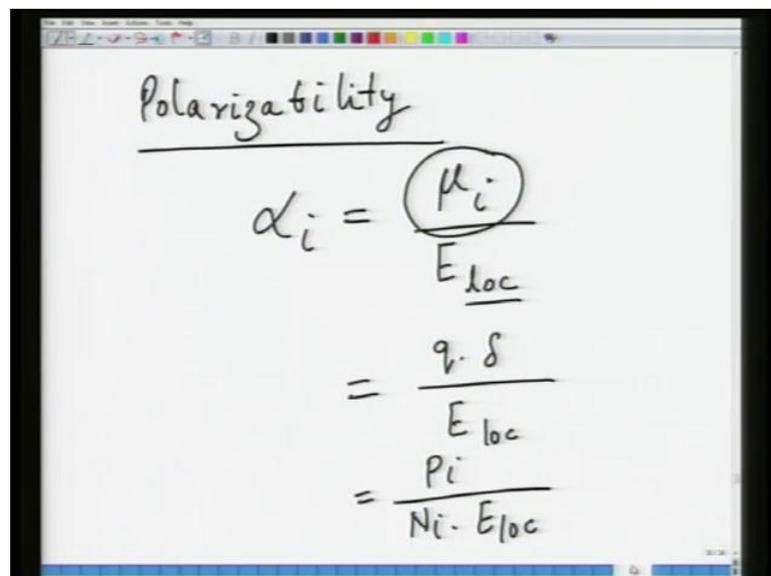
Then we looked at certain values of various systems.

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Finally what we did was, we took a microscopic approach and we worked for a system having N dipoles. Dipole movements per unit volume and from that what we determined was a quantity called as polarizability.

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This polarizability is a microscopic quantity. It is an atomic property or local property rather and this is given as α_i and this is equal to your dipole moment divided by the local field. So, μ is equal to αE . So, μ is dipole moment is related to electrical field by a microscopic quantity α .

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The whiteboard shows the following equations and labels:

$$P_i = N_i \alpha_i E_{loc}$$

Below P_i is the label "Macroscopic". Below α_i is the label "Microscopic".

$$\chi = \epsilon_r - 1 = \frac{P}{\epsilon_0 E_{loc}}$$
$$\chi = \frac{N_i \alpha_i}{\epsilon_0}$$

This again, this alpha is also alpha, which could be sum of various alpha.

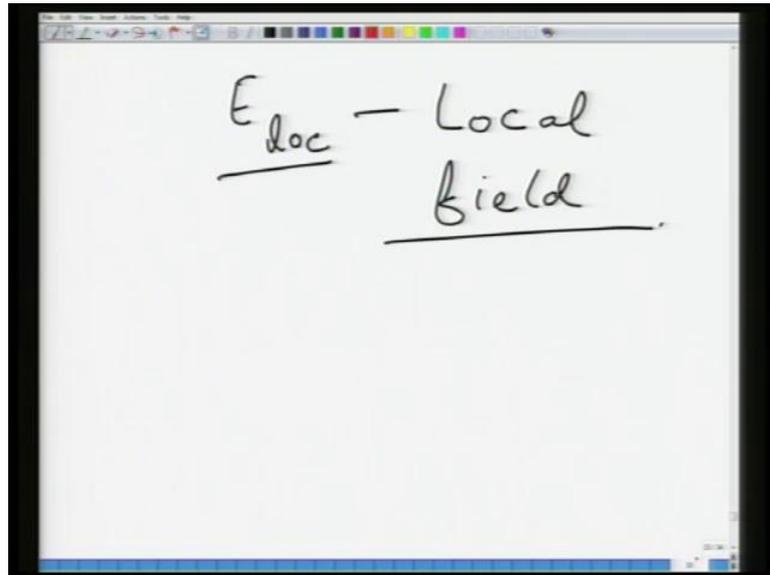
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The whiteboard shows the following equation:

$$\alpha_i = \sum \alpha$$
$$= \alpha_{el} + \alpha_{ionic} + \alpha_{dipolar} + \alpha_{int.}$$

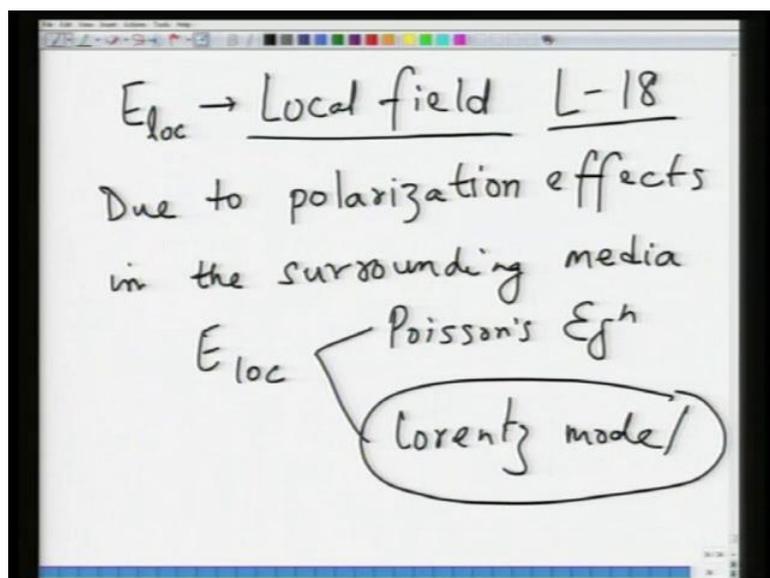
So, you can have electronic. You can have ionic, you can have dipolar as well as you can have interface. What we will do is, that now, we will establish formulism's for each of these alphas, except interfacial alpha before we do that however we would look at, we would look at the origin of what is called as a local field.

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What happens in reality is that we apply a field E , but the field experienced locally by the solid or the atoms inside the solid is rather different. It happens to be different because of the presence of surrounding medium, which exerts, which modifies this applied field as a result the local field is slightly different. That is what we are going to do now. So, this is lecture number 18.

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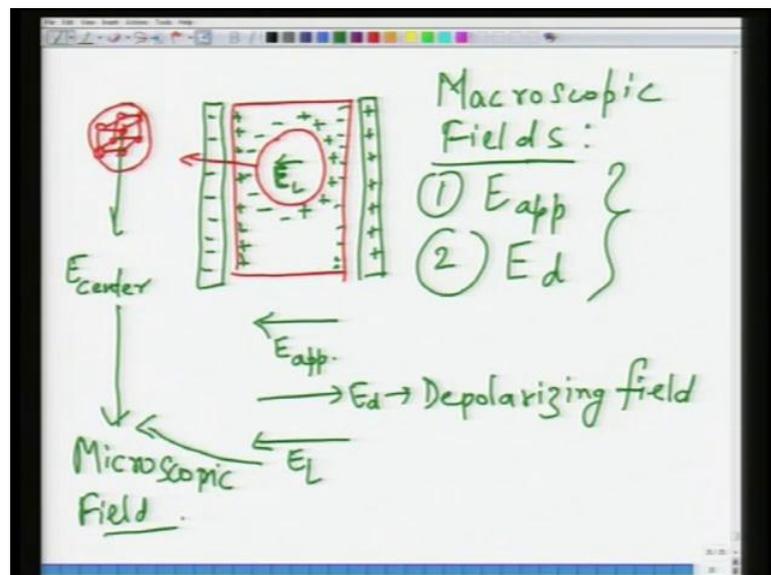


So, this local field, we define it as E_{loc} and this could be due to polarization effects. In the surrounding media it could be either around a dipole or a molecule or whatever

basically the entities which which which make up this dielectric. So, what happens that since, the since on a microscopic scale the material is electrically neutral so what you would expect is the overall field in the material to be equal to 0. When we start looking at individual dipoles and molecules, we find that this is not the case and this local field is basically nothing but sum of applied field with lot of other fields and this you can in principles solve using Poisson's equation. Which basically couples charged density and applied field, but if you start doing it from Poisson's equations it becomes very complicated and combustion.

So another model which is simpler model is called as Lorentz model. So, basically this problem of local field can be solved using Poisson's equation or Lorentz model. We will take this second one, which is of simpler approach and which gives you reasonably good agreement with what is observed. So, what we will do is that we will take a model dielectric.

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So, let us say this is our dielectric material and this dielectric material let us say, we take out a group of atoms away from the material. So, let us say we take this volume of material as sphere, we create a spherical cavity. So, let us say this is the material. So, we have taken this out and and the material could remain, it could have some structure inside it. So, basically you can have atoms sitting here. So, a group of atoms is taken out in the form here. So, this gives rise to now when you have, when you take out this sphere

away from the material this gives rise to creation of charges on the surface. So, you have positive charges let us say, on this side depending upon the direction of light field and negative here and so you have applied field in this direction this is E_{applied} .

So, this E_{applied} you would have typically the plates this side. So, plates would be positively polarized charged. These would be negatively charged. So, your E_{app} is in this direction consequently you will have surface charges, minus minus minus minus minus minus here on the surface of dielectric and plus on the left side. So, as a result you have applied field in this direction, you have a creation of another field which is called as E_d which is the depolarizing field.

This is due to build up of surface charges and when you take a cavity in it, when you take a volume of material out create a cavity inside, then the surface charges on the surface of the cavity would be opposite to the what you see on the surface of the material. So, as a result you have a applied field, which is in this direction minus x in this case. To oppose that you have to a now, this applied field called is causes polarization in the material, to oppose this polarization you have a inbuilt field which is called as depolarizing field, which is in the opposite direction.

Now, what about this cavity? This cavity produces field which is called as Lorentz field. The direction of this Lorentz field would be equal to E_L in the direction of light field. So, this would give rise to what is called as E_L Lorentz field and another field which is there is at the centre of this material which is called as E_{centre} because of presence of surrounding atoms. So, basically you have these four different kinds of fields and all these four different kinds of fields are not at the same scale. There are some there are some fields which are macroscopic in nature, there are some fields which are microscopic in nature and you can see which of these are macroscopic and microscopic.

So, macroscopic fields are of course, one is the E_{applied} which is a macroscopic and then second is the depolarizing field, which is the E_d , these two are the macroscopic field and your E_{centre} as well as Lorentz field both of these are microscopic fields. Considering the dielectric as a continue this approach is built up so this total field so this E_{local} now is nothing but sum of all these fields.

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$$E_{loc} = (E_{app} - E_d) + E_L + E_{center}$$

For isotropic materials,

$$E_{center} = 0$$
$$\underline{E_L = (E_{app} - E_d)}$$

So, we had E_{app} minus E_d depolarizing field plus E_L Lorentz plus E_{center} . Now, for let us say for isotropic material. Now, this was shown by Lorentz himself that for isotropic material and most of the polycrystalline material can be considered as isotropic. This E_{center} can be taken as 0. So, this cancels out, so what you have now is? You have, you are now left with E_L and E_{app} minus E_d . So, the question is how to work out these two? This is what we will do now?

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$$E_{app} - E_d = E - \frac{P}{\epsilon_0}$$

$\left(\frac{V}{d}\right)$ ← assuming $\epsilon_r = 1$

normal component of polarization

$$P \cdot n \, dA = P \cos \theta \, dA$$

Now, we know that E_{app} minus E_d which is the macroscopic field, which is basically equal to nothing but the E field which is experienced macroscopically by the dielectric or capacitor. This is equal to something called as E_{naught} and this is what we experience. When we apply a potential to the dielectric and this E_{naught} is nothing but E_{app} minus P divided by ϵ_{naught} . Assuming ϵ_r was equal to 1 and this E is nothing but V by d . So, basically what we have done is this E_{app} or rather we can let us not make it very confusing, let us just make it E_{app} minus E_d was equal to E_{naught} minus P divided by ϵ_{naught} , where P is nothing but the P by ϵ_{naught} is the depolarizing field. This comes from your Maxwell equations.

Now, another field which is left now is E_L . So, how to work out the E_L ? For this we take the shape of the spherical cavity. This is the spherical cavity and now, if you go to previous picture. So, we are looking at this way which means the positive charges are up negative charges are down, so looking at the direction of E_L . So, for this you have let us say, this is the centre so what you have here is. If I so this is the spherical cavity which has charges around it. Each of the now, this you can divide this spherical cavity into smaller areas. So, let us choose a segment dA here. For example, so this would be something like this, making the angle θ of the thickness angular thickness $d\theta$.

Similarly, here θ and $d\theta$ and let us say, the radius of this sphere is r . So, if this is the element dA small dA both of them then the normal component of polarization of this small element, which gives rise to what is called as surface charge density because polarization as we know it is surface charge density. So, the polarization is in this direction because the material is polarized along this direction. So, basically what and this is let us say is the normal vector, normal to the area dA . Since, you have dA 's all over the place, you will have normals varying all over the different direction. So, as a result polarization makes an angle θ with respect to so this angle would also be θ . So, polarization makes an angle θ to the surface normal of area dA and each of these gives rise to a field which is dE_L and what you get here now, is $2 dE_L \cos \theta$.

This is the only field which survives the other components cancel each other as you can see from the symmetry because for for each segment on the upper half of this sphere. You have another segment at the bottom half, now the only thing that you have to be careful about is a sign. So, what happens is that when you take for this for a segment on this side will have polarization on this side. As a result you have a component to this this

direction component that direction. What happens is that these two components the horizontal components cancel each other, because because of sign is now negative and as a result you have only the vertical components surviving.

So, what happens is now you can write this normal component of polarization. So, $P \cdot dA$ this is equal to $P \cos \theta dA$. What you now, what you can do is that in order to find out dE you can invoke what is called as Coulomb's law. So, if you apply Coulomb's law from that you can calculate field.

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Apply Coulomb's Law,
 Radial field intensity

$$dE_r = \frac{P \cos \theta dA \rightarrow \text{charge}}{4\pi \epsilon_0 r^2}$$

 Total field component = $\frac{2dE_r \cos \theta}{}$

$$E_L = \oint_{\text{sph}} dE_r \cos \theta$$

$$= \oint_{\text{sph}} \frac{P \cos^2 \theta dA}{4\pi \epsilon_0 r^2}$$

Apply Coulomb's law and then you get radial field intensity which is given as dE_r and this is equal to $P \cos \theta dA$. $P \cos \theta dA$ would be the charge and you know that Coulomb's law is F is equal to Q_1 and Q_2 divided by $4\pi \epsilon_0 r^2$ and if you want to calculate E then you take one of the charges on the other side. So, that gives you electric field because F by Q is the electric field and then you get Q divided by $4\pi \epsilon_0 r^2$. So, this is your charge and r is the separation distance which is as you can see from this sphere.

So, if now for as as we said that for each dA in the upper half, you have another dA in the negative in the in the lower half but due to due to the due to the change in the polarity of charges what happens is the horizontal components cancel each other. The vertical components survive so the total total field component is $2dE_r \cos \theta$.

These survive so E^2 so this $E \cdot L$ can be so overall $E \cdot L$ can be calculated for a sphere of $E \cdot L \cos \theta$ and this is equal to if you replace the magnitude for dL then this becomes $P \cos^2 \theta dA$ divided by $4\pi \epsilon_0 r^2$.

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Handwritten derivation on a whiteboard:

$$dA = 2\pi r \sin\theta \cdot r d\theta$$

$$dA = 2\pi r^2 \sin\theta d\theta$$

$$E_L = \int_0^\pi \frac{P \cos^2\theta \cdot 2\pi r^2 \sin\theta d\theta}{4\pi \epsilon_0 r^2}$$

$$= \frac{1}{3} \frac{P}{\epsilon_0}$$

$$E_L = \frac{P}{3\epsilon_0}$$

The diagrams show a sphere of radius r with a small segment of thickness $r d\theta$ and width $2r \sin\theta$.

Now, dA how can you calculate dA ? If you have this sphere like this, the radius is r and we are looking at this small segment of thickness $d\theta$ and we know that this segment goes like this. So, the area of this segment would be $r \sin \theta$ into because you have the angle θ here. So, this would be $r \sin \theta$ because this is r . So, this is $r \sin \theta$ and this would be the another side would be $r d\theta$. So, this dA would be equal to $r^2 \sin \theta d\theta$. So, if you expand dA it would look like this. So, $2\pi r^2 \sin \theta d\theta$. So, $r \sin \theta$ into $2\pi r d\theta$ and this would give us $2\pi r^2 \sin \theta d\theta$. So, this would be basically what it would be is like this, $2r \sin \theta$. So, if I modify this as $2r \sin \theta$ into $r d\theta$.

Now, you replace dA in the previous equation. So, your $E \cdot L$ becomes equal to or you integrate from 0 to π because upper half is symmetric, because you are taking from θ from this sorry, so θ goes from here to here. So, that is 0 to π . So, this becomes $\int_0^\pi P \cos^2 \theta \cdot 2\pi r^2 \sin \theta d\theta$ divided by $4\pi \epsilon_0 r^2$. So, this becomes a manageable equation. You can solve it easily and this gives rise to what is solution is $\frac{1}{3} P$ divided by ϵ_0 . So, this Lorentz field $E \cdot L$ is

P divided by 3 epsilon naught. So, this is the Lorentz field that we have got. So, what we have got is we have got hang of most of the fields.

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The image shows a whiteboard with the following handwritten equations:

$$E_{loc} = (E_{app} - E_d) + \frac{P}{3\epsilon_0}$$

$$= E_0 + \frac{P}{3\epsilon_0}$$

$$P = \epsilon_0(\epsilon_r - 1)E_0$$

$$E_{loc} = E_0 + \frac{\epsilon_0(\epsilon_r - 1)E_0}{3\epsilon_0}$$

$$\boxed{E_{loc} = \frac{E_0}{3}(\epsilon_r + 2)}$$

So, the local field now is E local is now E applied minus E d plus P divided by 3 epsilon naught and we have said that this E applied minus E d is what is felt by the by the capacitor. So, let us say this is equal to something called as E naught plus P divided by 3 epsilon naught and we know that P is equal to epsilon naught epsilon r minus 1 into E E naught. What is felt by the now, this is a macroscopic equation. So, we can plug that back in here and as result we get E local equal to E naught plus epsilon naught epsilon r minus 1 E naught divided by 3 epsilon naught. So, these cancel each other and what you get is... So, if you just rework this equation what you get is E naught divided by 3 into epsilon r plus so plus 2.

So, you got this expression for E local in terms of the field which is experienced by the dielectric macroscopically. So, you can see that the macroscopic field which is experienced by the dielectric is different from the local field. So, all the local field is different from the macroscopic field, which is measured due to application of applied voltage.

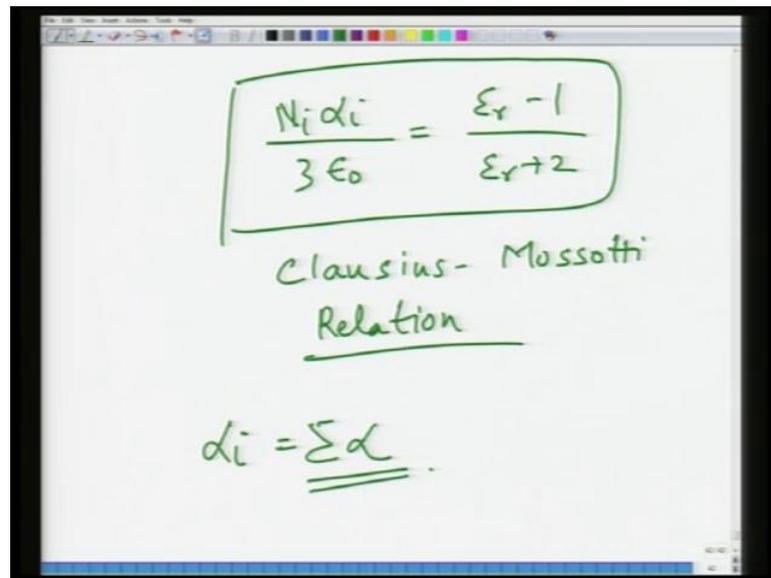
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$$E_{loc} = \frac{E_0}{3} (\epsilon_r + 2)$$
$$P = N_i \alpha_i E_{loc}$$
$$= N_i \alpha_i \frac{E_0}{3} (\epsilon_r + 2)$$
$$P = \epsilon_0 (\epsilon_r - 1) E_0$$
$$\epsilon_0 (\epsilon_r - 1) E_0 = \frac{N_i \alpha_i}{3} E_0 (\epsilon_r + 2)$$

Now, if you combine now, this so you get either E_{local} is equal to E_{naught} divided by 3 into $\epsilon_r + 2$ and we know that P in terms of polarizability is $N_i \alpha_i$ into E_{local} from the previous lecture. So, here we are relating polarization to polarizability and the local field. This is what we established in the last lecture. So, this would be equal to $N_i \alpha_i E_{naught}$ by 3 $\epsilon_r + 2$. Now, again we put the expression for P so this is the expression for P in the microscopic quantities.

What is the expression for P in the macroscopic quantities? P is also equal to $\epsilon_0 (\epsilon_r - 1) E_{naught}$. So, if we equate these two equations top and bottom. So, what we get is $\epsilon_0 (\epsilon_r - 1) E_{naught}$ is equal to $N_i \alpha_i E_{naught}$ by 3 into $\epsilon_r + 2$. So, what will happen now is E_{naught} will cancel each other, and if you just work them around work these all the parameters around. So, the objective is to bring the microscopic parameters on one side macroscopic on the other side. So, what is the microscopic parameter? α_i is a microscopic parameter, ϵ_0 is the microscopic parameter, well it is a fundamental quantity N_i is the microscopic parameter. What is the macroscopic parameter? Macroscopic means something which you can measure macroscopically and this is ϵ_r . So, when you take all these when you rearrange the parameters what you get is N_i .

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The image shows a whiteboard with a green border. At the top, the Clausius-Mossotti relation is written in green ink and enclosed in a green rectangular box:
$$\frac{N_i \alpha_i}{3 \epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2}$$
 Below the box, the text "Clausius-Mossotti Relation" is written in green ink and underlined. At the bottom, the equation
$$\alpha_i = \sum \alpha$$
 is written in green ink, with the summation symbol and the Greek letter alpha underlined.

Alpha i divided by 3 epsilon naught is equal to epsilon r minus 1 divided by epsilon r plus 2. This equation is a very useful, very famous equation which is called as Clausius Mossotti relation and what this equation does is, this equation relates the microscopic parameters for a dielectric material to the macroscopic parameters by taking use of a modified field, which is called as a local field. Field, which is experienced locally by the dielectric rather than the overall field, which is experienced by the dielectric.

So, the the the the what it what it does is, if you know epsilon r from the measurements you can measure you can calculate alpha i for a given system given the fact that what you would know is N_i for a for a system. Now, this alpha i as I said before is again sigma alpha. So, this is so net polarizability alpha would be sum of electronic ionic and dipolar, as we have looked before you can apply as a special case.

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As a special case

For gases.

$$\epsilon_r - 1 \ll 1$$
$$\epsilon_r + 2 \approx 3$$
$$\frac{N\alpha}{\epsilon_0} = \epsilon_r - 1 \approx \chi$$

This equation to the gases so for for gases this epsilon r minus 1 is much smaller than 1. So, what will happen is such a case if it is much a smaller than 1 so and at lower pressures it becomes epsilon r plus 2 will become approximately equal to 3 and as a result N alpha divided by epsilon naught will become epsilon r minus 1 and that is equal to chi. So, this is equal to what we got earlier. So, for gases this equation modifies to the previous form that we got. So, what now we will do is that so we will just summarise this part before we move on to the next part. So, what we have done is we have studied the various mechanisms of polarization and from that that point onward we looked at a microscopic quantity, which is called as alpha, which is the polarizability. Now, this polarizability is something which you cannot measure directly.

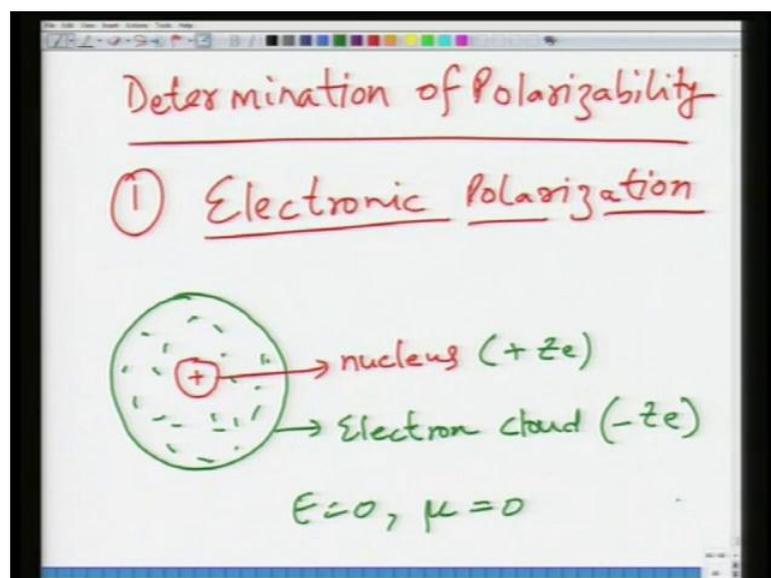
So, as a result you need parameters to measure it and those parameters are macroscopic parameters. We also introduced a concept of local field, which is the field which is locally experienced by the dielectric material. When the field is applied so you have an external field which is due to the external voltage is applied to the system. So, as a result this external field but this external field is different and local field is different. Why this local field is there because the external field causes the polarization. Now, polarization so when you measure the field at a particular point, not only you have polarisation because of applied field but also because of the surrounding media which is present around this particular piece of particular point in the material.

So, then you so then we established various different kinds of fields. So, you have applied field you have what is called as depolarizing fields. So, applied field depolarizing field put together make what is called as E_{naught} , which is the macroscopic field which is experienced because of application of voltage and then you have what is called as a Lorentz field which is the internal field because of charge build up on the surface of this is sphere which you took out. So, assuming that it is, it was so on each sphere at every place in the material you will have a Lorentz field. So, we we then worked out what is the Lorentz field.

Then each centre which is the centre at the, which is the field at the centre of the material that was taken as equal to 0 because of assuming that materials are isotropic and then we calculated what is called as local field which is different from the applied field. This E_{local} is helpful in then relating the microscopic parameters, to the macroscopic parameters. So, we have seen how the polarization takes place and we have also included introduced a concept a local field and tried to relate microscopic and macroscopic parameters of a for a dielectric material.

Now, what we will do is that we will calculate this alpha which is the polarizability for each of the mechanism that we talked about. So, what we will do is that we will we will first calculate so.

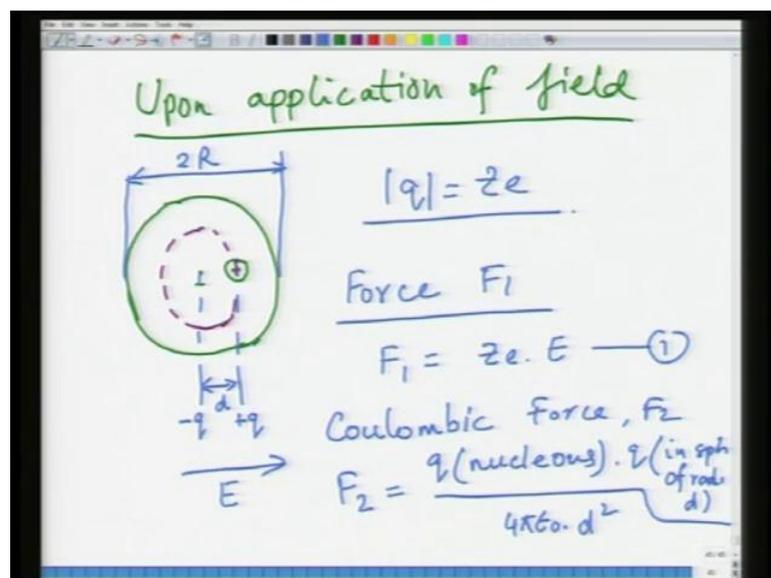
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Let us do determination of so first we will take the case of electronic polarization and this is the simplest. Now, in case of electronic polarization assume the atom to be a perfect sphere. So, this is the centre of atom, this is the nucleus and around it what you have is this does not look like a perfect sphere. I will have to draw it again better so you have a nucleus you have the surrounding electron cloud. So, nucleus of charge plus $Z e$ and electron cloud will have minus $Z e$. So, in the absence of electric field the centre of this electron cloud matches with the centre of nucleus. So, as a result in the when E is equal to 0 μ is equal to 0.

Now, what happens when you apply the field? When you apply the field, the centre of this positive charge moves with respect to centre of negative charge.

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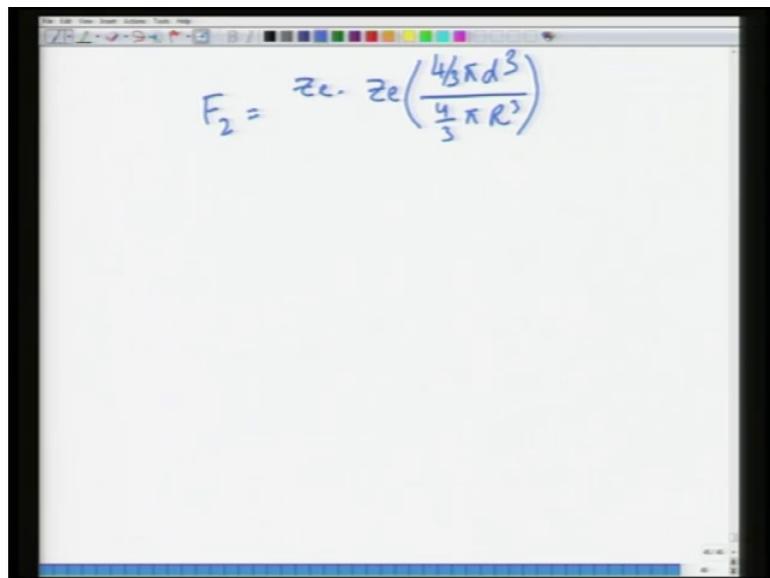


So, so let us say upon application of field so that so we have, so this is the centre. So, let us say this is negative and the positive is somewhere here. So, this is your centre of positive charge. So, if I just draw another circle so these two guys have moved by a distance d . So, this would be minus q this would be plus q and we know that q is equal to $Z e$. Let us say this whole thing is equal to $2 r$ and and we apply the field in the applied field. So, so when you apply the field E then the force which is created on the system. So, force F_1 which is due to application of electric field can be given as. So, F_1 can be given as $Z e E$. Let us say this is equation number one. Now, if you have force one

which is first force is because of creation of applied field, which sent separates the centre from positive charge, they will move by a distance d.

So, when the move by distance d you have a Coulombic attraction force and this Coulombic attraction force because of opposite charges. So, this Coulombic force F_2 is given as F_2 is given as q of nucleus multiplied by q of electrons, within this sphere of radius d. So, q in sphere of radius d or q in the distance d and this would be divided by $4\pi\epsilon_0 d^2$.

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$$F_2 = ze \cdot ze \left(\frac{4/3 \pi d^3}{4/3 \pi R^3} \right)$$

So, this F_2 can be written as $Z e$ multiplied by again $Z e$ and this would be $4/3 \pi d^3$ divided by $4/3 \pi R^3$, so within that distance the the number of electrons enclosed.

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$$F_2 = \frac{Ze \cdot Ze \left(\frac{\frac{4}{3}\pi d^3}{\frac{4}{3}\pi R^3} \right)}{4\pi\epsilon_0 d^2}$$
$$= \frac{Z^2 e^2 d}{4\pi\epsilon_0 R^3}$$

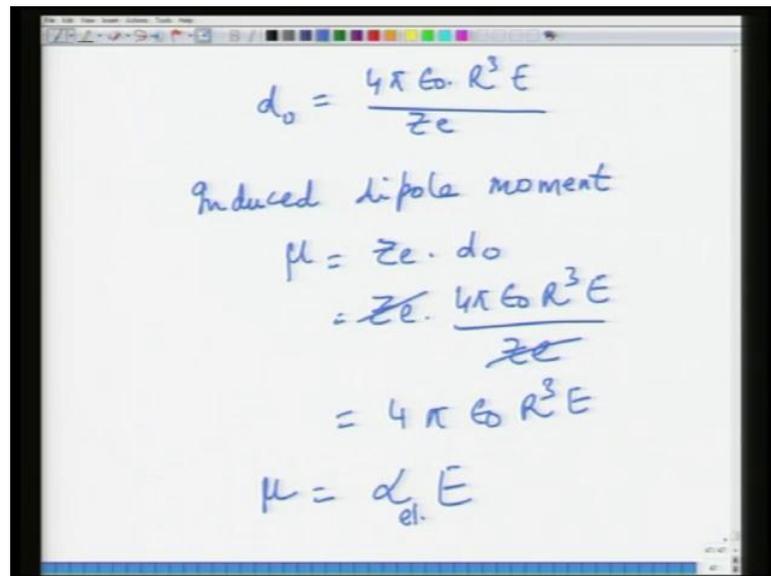
At equilibrium,

$$F_1 = F_2 \quad \& \quad d = d_0$$
$$ZeE = \frac{Z^2 e^2 d_0}{4\pi\epsilon_0 R^3}$$

So, this F_2 can be written as so first was q of nucleus Ze multiplied by the charge the electrons inside the distance d . Electronic charge inside the distance d so that would be Ze into $\frac{4}{3}\pi d^3$ the a volume of the radius of d volume of the sphere of radius d divided by the volume of the whole electronic sphere and this will be divided by $4\pi\epsilon_0 d^2$. So, this would be further simplified as $Z^2 e^2 d$ you have one d left because 2 d 's cancel each other, divided by $4\pi\epsilon_0 R^3$.

Now, at equilibrium F_1 will be equal to F_2 and let us say d is equal to at equilibrium distance which is called as d_0 . So, when you do that so ZeE is equal to $Z^2 e^2 d_0$ divided by $4\pi\epsilon_0 R^3$. So, these one of the Ze 's cancelled here d will be d_0 and this gives rise to if you work it out.

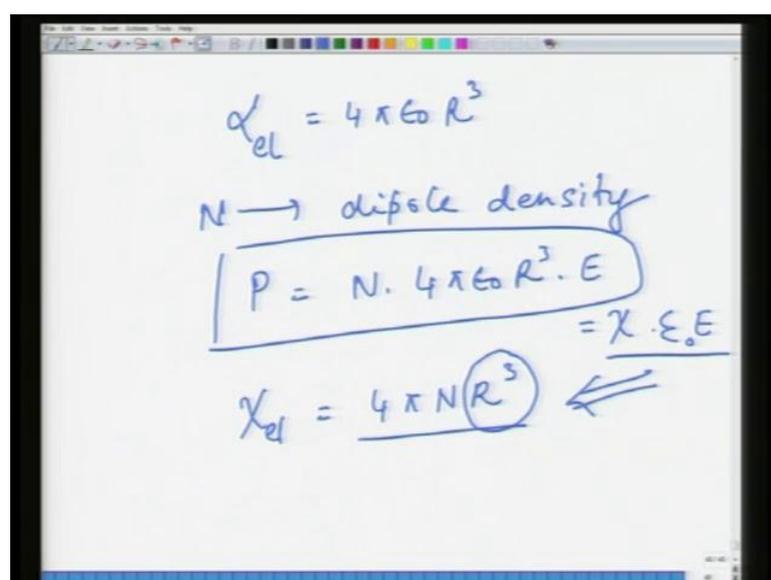
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A whiteboard with a black border containing handwritten mathematical derivations. At the top, the equation $d_o = \frac{4\pi\epsilon_0 R^3 E}{Z e}$ is written. Below it, the text "induced dipole moment" is written. The next line is $\mu = Z e \cdot d_o$. The following line is $= Z e \cdot \frac{4\pi\epsilon_0 R^3 E}{Z e}$. The next line is $= 4\pi\epsilon_0 R^3 E$. The final line is $\mu = \alpha_{el} E$.

This will be d ought to be equal to 4 pi epsilon naught r cube E divided by Z e. So, this will be the equilibrium separation distance. So, the induced dipole moment induced dipole moment, would be mu which would be equal to Z e d ought. So, this would be Z so Z e dot 4 pi epsilon naught R cube E divided by Z e. Z e Z e cancel each other. So, what you get is 4 pi epsilon naught R cube E and and you know that mu is equal to alpha E.

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A whiteboard with a black border containing handwritten mathematical derivations. At the top, the equation $\alpha_{el} = 4\pi\epsilon_0 R^3$ is written. Below it, the text "N → dipole density" is written. The next line is $P = N \cdot 4\pi\epsilon_0 R^3 \cdot E$, which is enclosed in a hand-drawn box. To the right of this box is the equation $= \chi \cdot \epsilon_0 E$. The final line is $\chi_{el} = \frac{4\pi N R^3}{\epsilon_0}$, with a double-headed arrow pointing to the right.

So, this is alpha electronic so what it gives you is that so alpha electronic becomes $4\pi\epsilon_0 R^3$ and if let us say, if N was the dipole density number of dipoles per unit volume and polarization becomes $N \times 4\pi\epsilon_0 R^3 \times E$. So, this the polarization that you get. So, what this equation tells you the implications of this this equation are pretty important. The implications of this equation are that larger the atom is more the polarizability of the material is which means more the material can be polarized. So, this the first conclusion. Secondly from this you can also find out what is the susceptibility. So, susceptibility, electronic susceptibility would be $4\pi N \alpha$ because P is equal to, you can equate it to $\chi \epsilon_0 E$. So, this gives rise to what is called as susceptibility.

So, so basically higher the radius of the atom is, higher the susceptibility and the polarization of the material would be. You have and this also shows that you have a linear relationship between the polarization and electric field, at least for this particular mechanism. You can also relate from this the polarize, you can also compare the polarizabilities of various types of atoms. For instance your the polarizability of alkali and rarer alkali rarer alkali atoms is alkali earth atoms is more than the d atoms because of shielding of electrons, shielding of nucleus by the electrons in d type of materials. So, as a result as an as an P type of materials tend to have higher polarizability as compared to d type materials.

So, what we will do do now is, we will just look at some of the values and on the observation is since R for anions is typically larger than R for cations.

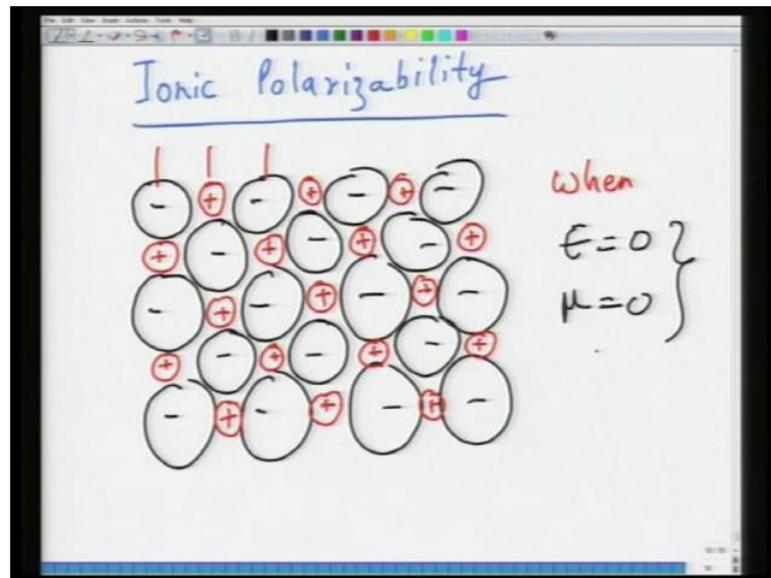
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$R(\text{anions}) > R(\text{Cations})$	
Values of a few atoms	
	$\alpha (\times 10^{24} \text{ cm}^{-3})$
F	1.2
Cl	3
Br	4.5
I	7
Li	0.03
Na	0.3
K	0.9

Anions tend to be more polarisable compared to cations, so for some values for for a few atoms, so fluorine so this alpha is into 10 to power 24 centimetre cube. So, this would be 1.2 chlorine, 3 bromium bromine 4.5 and then iodine is 7. So, you can see as the size increases the polarizability increases. Now, for alkalis lithium had 0.03, sodium has 0.3, potassium has 0.9 so again the number goes up, but you can see that cations have much smaller number as compared to anions and that because of bigger size of the anions as compared to cations.

So, anions in general are more polarisable as compared to cations and this would be this is more important from the point of view of solids, which have mixture of anions and cations.

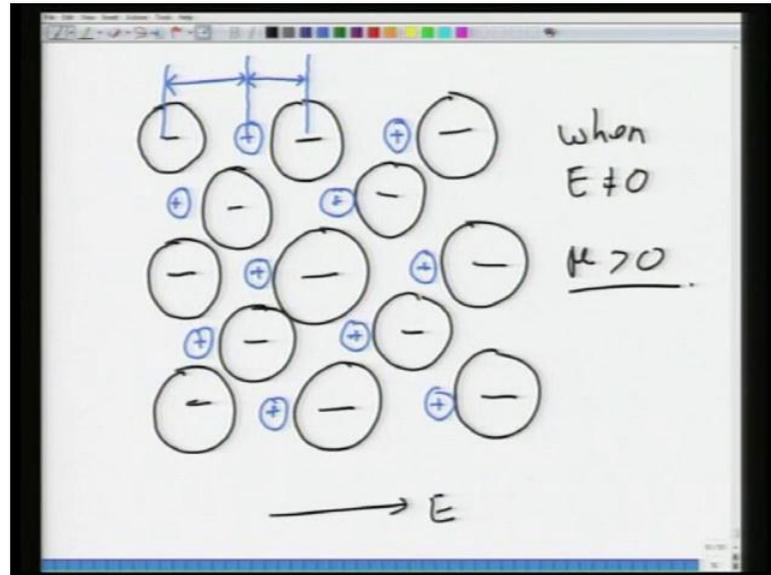
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Now, the second mechanism for this we will take is ionic polarizability. So, in case of ionic polarizability, this happens in ionic solids, where you have presence of cations and anions. So, in the absence of electric field let us again take the case of sodium chloride. So, you have so these are let us say, again you have so these are your chlorine ions. So, there is the 0 0 one face of sodium chloride and then your sodium ions will go in the octaheatal site and at least on this face on the x centres. Now, each of this sodium and chlorine ion make a dipole, but the direction of each of the dipole is equal and opposite as a result when E is equal to 0 μ is equal to 0.

Now, what happens when you apply electric field so as a as a result of symmetry when you apply there is no electric field. When there is no electric field, the net dipole movement is equal to 0 because all the components cancel each other. So, when you apply electric field, what happens is, so let us say the picture gets modified a little bit. So, what I will do is that I will keep the anions fixed and move the cations.

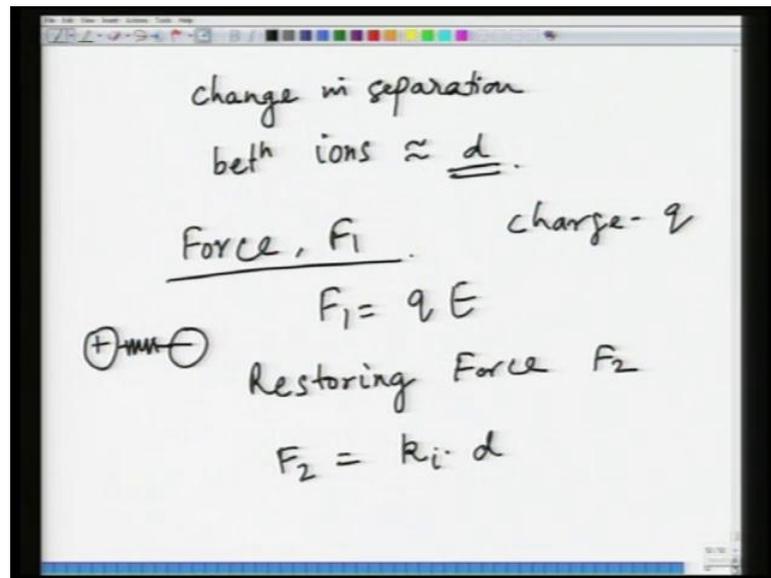
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I will just make this a little bigger in order to be little bit more clear. So, these are anions now, what happens is if the field is applied in this direction which is the E then the cations move to let us say this site, we forgot to draw another anion here. So, all these ions will move closer to so and even here. So, as a result what has happened is you have the you have two different distances between the positive and negative ions. So, this is let us say one distance this is another distance. So, as a result of unequal separation you have a net dipole movement created inside the material. So, when so when E is not equal to 0, mu is greater than 0. It is finite dipole movement so this illustration probably explains that much better. So, basically what happens is that you can say that the distance between you can take, the distance to be increasing or decreasing.

It does not matter because ultimately it is a question of relative increase. So, so let us say this now whatever the whatever the distance is the distance increases by a certain magnitude on one side and decreases by a certain magnitude on the another side. So, let us say this increase or decrease is defined as so this change in separation between ions is defined as d. So, we in we are interested in calculating the magnitude of this d.

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So, first when you apply electric field then the force F_1 , which increases the which increases or decreases the distance between the ions. Let us say the ions are charge q so charge on the ions is q plus or minus q . So, the this F_1 would be equal to qE , which is the same force due to field. Now, in the ionic systems, if you remember we earlier introduced that ionic systems when you have positive and negative charges, then they tend to have a bond between them and this bond is let us say, this bond is like a spring. So, this spring tends to restore this these two ions back to equilibrium position. So, so the restoring force the restoring force is given as F_2 and this F_2 is given as $k \cdot d$. Let us say k_i , for a given kind of system k_i . Where k is this spring constant which depends upon the bond and d is the distance. So, so so this we have done assuming that that the bonds between two ions act like springs. So, again at equilibrium we would like to equate them.

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At equilibrium
 $F_1 = F_2$
also $k_i = \gamma \cdot d_0$
 $q \cdot E = \underbrace{\gamma \cdot d_0 \cdot d}_k$
 $d = \frac{qE}{\gamma d_0}$

F_1 becomes equal to F_2 and also what is k ? So, k is a spring constant, which is a physical quantity and that is called as modulus of elasticity. So, if γ was the modulus of elasticity and d_0 was the separation distance and this k_i becomes equal to γd_0 and at equilibrium $F_1 = F_2$ and $d = d_0$. So, again we match the two equations so that $qE = \gamma d_0 d$. So, this k . This spring constant refers to the equilibrium distance. So, what you will have in this expression is γd_0 multiplied by d , so a multiplied by d . So, basically this is k and this is the distance. So, this gives you the magnitude for d and this becomes qE divided by γd_0 .

This d is not the equilibrium separation so this would not be equilibrium this is just a magnitude of increase or decrease between the distance. So, this d_0 is nothing but related to lattice parameter or equilibrium distance between the ions. Now, from this d you can calculate what is μ .

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The image shows a handwritten derivation on a whiteboard. It starts with the equation $\mu = q \cdot d$. This is followed by $= \frac{q^2 \cdot E}{\gamma d_0}$, where the fraction is circled. An arrow points down from the circled fraction to the text $\alpha_{\text{ionic}} = \underline{\underline{\alpha_i}}$. Below this, the final equation $P = N \mu = \frac{N q^2 E}{\gamma d_0}$ is enclosed in a rectangular box.

As you know $\mu = q \cdot d$, which is a dipole moment and this is $q^2 E$ divided by γd_0 . So, again this particular term without E becomes α_{ionic} or α_i . So, this and as a result the polarization becomes, if dipole density was again N per unit volume. So, this will become $N \mu$ and this would be $N q^2 E$ divided by γd_0 and this would be the expression for P . So, again what this equation tells you is that, smaller the higher the modulus of elasticity is the less polarizable the material is and that makes sense, higher the modulus of elasticity which means higher the bond strength. If the bonds are stronger they are not going to be stretched or compressed that easily. So, as a result the polarization of the material goes down. So, so so I will give you some values for this particular.

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A handwritten table on a whiteboard showing dielectric constants (epsilon_r) for various compounds. The values are: ZnO - 4.6, ZnS - 5.1, ZnSe - 5.8, CdS - 5.2, and CdSe - 7.0. A large curly bracket on the right side groups all these values under the label epsilon_r.

ZnO - 4.6	} ϵ_r
ZnS - 5.1	
ZnSe - 5.8	
CdS - 5.2	
CdSe - 7.0	

So, for example, zinc oxide 4.6, zinc sulphite 5.1, zinc solenoid 5.8, cadmium sulphite 5.2. So, of course, this analyses also does not take care of any cov,alency. So we have to be we have to take care of that. It is a very rough guide, but it works reasonably very well. Cadmium solenoid 7 so basically the here these values are epsilon r values. So, they are increasing so so they are increasing as you go within the for the for the zinc you go from oxygen to sulphur to selenium. What basically it means is that the bond strength is going down. So, as a result the polarization is increasing. So, what we have done is we have covered we have covered the origin of local field. We on the basis of that, we have come up with an equation which is called as Clausius Mossotti equation relating the microscopic and macroscopic parameters.

Then we did the evaluation of electronic and ionic polarizability which are dependent upon parameters like for example, electronic polarizability dependent upon the size of the atom and ionic polarizability is dependent upon the bond strength. So, in the next class we will take up the dipolar polarizability, which is a little bit more comprehensive in terms of its analysis.

Thank you.