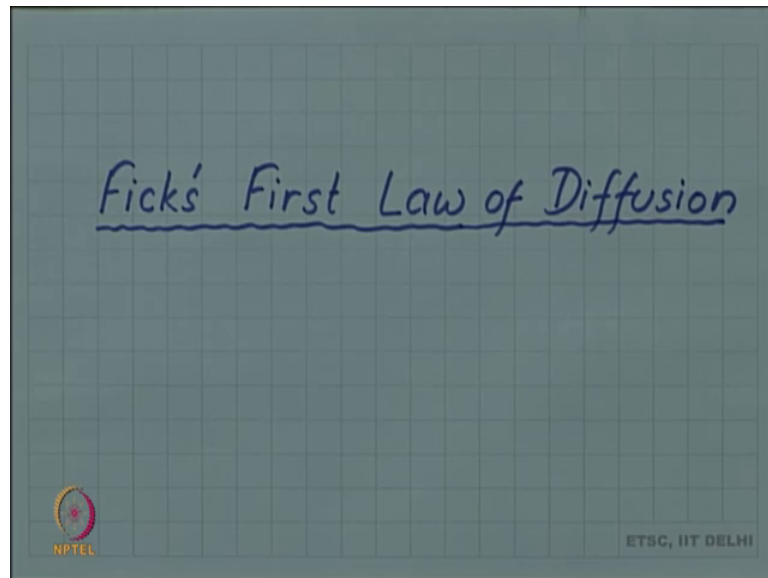


**Introduction to Materials Science and Engineering**  
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**Lecture - 82**  
**Fick's first law**

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An important law in the field of diffusion, we are discussing the topic of diffusion. So, Fick's first law of diffusion, in the last video, we just had an introduction of diffusion in particular we saw 2 examples, one of carburization of gear and another production of NP junction in electronic circuit, which are based on diffusion. These diffusion processes are governed by a 2 important laws both derived by Fick. And they are called Fick's first law of diffusion, and Fick's second law of diffusion.

We will first begin with the first law. What is this fix Fick's first law of diffusion? So, this law can be understood in with comparison with other corresponding laws which we have for heat flow.

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	Heat Flow	Charge Flow	Mass Flow
1) Flux	Heat Flux $q$ ( $J m^{-2} s^{-1}$ )	Charge Flux $j$ (Current density ( $A m^{-2} \equiv C m^{-2} s^{-1}$ ))	Mass Flux $j$ ( $kg m^{-2} s^{-1}, mol m^{-2} s^{-1}$ )
2) Gradient	Temperature Gradient $\frac{dT}{dx}$	Electric Potential Gradient $\frac{dV}{dx}$	Concentration Gradient $\frac{dC}{dx}$
3. Law	Fourier's Law of Heat Conduction $q = -k \frac{dT}{dx}$	Ohm's Law $j = -\sigma \frac{dV}{dx}$	Fick's First Law $j = -D \frac{dC}{dx}$ $C \equiv kg m^{-3}, mol m^{-3}$
4. Material Property	Thermal conductivity $k$ ( $W m^{-1} K^{-1}$ )	Electrical Conductivity $\sigma$ ( $S m^{-1} \equiv \Omega^{-1} m^{-1}$ )	Diffusivity or Diffusion coefficient ( $m^2 s^{-1}$ )
5. Year	1807	1827	1855

Let us have a comparison of heat flow, then charge flow and mass flow. So, let us look at first let us look at the flux. So, what is the flux in heat flow. The flux in heat flow is the heat flux. So, so many heat flux let us call that  $q$  and the unit is so many joules crossing per unit area per second so, that is the flux in heat flow. Now it is known that heat flows due to a existence of a particular gradient. So, what is the gradient of heat flow? What is the gradient which gives us heat flux? So, that gradient we know is temperature gradient.

Temperature gradient let us call that  $d T$  by  $d X$ . So, if there is a uniform temperature in a body, there will be no heat flux. So, only when there is a gradient where then a temperature is changing with position we have a heat flux. We then have a law connecting the flux to the temperature gradient. In the heat flow case, it is the famous Fourier's law of heat conduction. And that law relates the heat flux to the temperature gradient, through a simple expression  $q$  is equal to minus  $kappa$   $d T$  by  $d X$ . So, essentially it is saying that the heat flux is proportional to the temperature gradient, and the proportionality constant  $kappa$  is our material property.

To material property is thermal conductivity, thermal conductivity  $kappa$  which has the units watts per meter per Kelvin. And finally, for our historical record let us write down, the year in which it was discovered and that is this was this is a very, very old of more than 200 years now 1807. Now, so, more than 210 years ago, Fourier established how the heat flux is related to the temperature gradient, and that law we know as Fourier's law of heat conduction. Then came the study of charge flow or the electric current.

So, here the flux is, the flux is charge flux also known as current density. So, this is given a symbol  $j$  units will be ampere per meter square, which is same as actually coulombs per meter square per second. So, you can compare it joules per meter square per second, here it is coulombs per meter square per second so now, the charge is flowing. So, what is the driving force, or what is the driving gradient for this charge flow. So, you know charge will flow only if there is a potential gradient electric potential gradient.

Electric potential gradient  $dV$  by  $dX$  and the law connecting them is the well-known ohms law, which we can write as  $j$  is equal to minus  $\sigma$   $dV$  by  $dX$ . So, the material property involved here is the electrical conductivity, electrical conductivity  $\sigma$ , and the unit is Siemens per meter, or you can call it ohm inverse meter inverse. And this law was discovered 20 years later in 1827. And in fact, the motivation for this law or the inspiration for this law for ohm came from the study of Fourier law of heat conduction.

Since he saw that in heat conduction there is a flux of heat which is proportional to the temperature gradient, he asked what should be the flux and the gradient corresponding gradient in the case of charge flow. So, he could relate these two in 1827, 20 years after the Fourier's law of heat conduction.

Now, we come to our interest that is the mass flow so, here the flux is mass flux which we also call  $j$ . So now, this is mass per unit area per unit second, sorry yeah per second. So, kg per meter square per second, or sometimes mass can also be written in the units of mol. So, you can have mol per meter square per second so, that is mass flux. Now it was Fick who derived this law, and Fick again was inspired by heat flow and charge flow, he asked the question, that what is the driving force for mass flux. And he found that it should be the charge gradient, sorry the it should be the concentration gradient.

So, the corresponding gradient is the concentration gradient, let us write it as  $dc$  by  $dX$ . And Fick just by analogy so, this is a demonstration of power of analogy in success of analogy in science. So, just by analogy he could write so, Fick's first law that the mass flux should be nothing but proportional to the concentration gradient, the proportionality constant he called  $D$ . So,  $D$   $dc$  by  $dX$ . So, the property property  $D$  here was named diffusivity or diffusion sometimes also known as diffusion coefficient, the units of this is meter square per second. And this one was in 1855.

So, you can see another 28 years passed before Fick could establish his famous first law, that the mass flux is proportional to the concentration gradient. So, this will be an important law for us in the study of diffusion. And remember that since this is if you see the unit of mass diffusivity meter square per second, this indicates that C for our C should always be in the units of a per unit volume. So, it should be either kg per meter cube the concentration should be volumetric concentration kg per meter cube or moles per meter cube.

We saw in the phase diagram we used atomic percent or mole percent or weight percent as our x axis of the phase diagram and that was also the units of concentration that unit is fine, but if we want to use Fick's first law there we we should use the concentration in per unit volume, then only we will get a consistent D which will be in meter square per second.

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**FICK'S FIRST LAW**

$j = -D \frac{\partial C}{\partial x}$

$j = \text{Mass Flux}$

Mass flows from high concentration to low concentration regions.

$D = \text{Diffusivity or Diffusion coefficient.}$

$[D] = \frac{[J]}{\left[\frac{\partial C}{\partial x}\right]} = \frac{\text{kg m}^2 \text{s}^{-1}}{\left(\frac{\text{kg/m}^3}{\text{m}}\right)} = \text{m}^2 \text{s}^{-1}$  }  $C \text{ in } \frac{\text{kg}}{\text{m}^3}$

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So, let us look at a little bit more about this Fick's first law. J is equal to minus D del C by del x, j is mass flux. So, essentially by flux you mean so, let us let us have an area, across this area, there is a concentration gradient. So, let us say that there is a concentration, which is changing, and then we are looking so, if there is a high concentration on this side, and a low concentration on this side. So, mass will cross this area in this direction, from high concentration to the low concentration. So, mass flows from high concentration to low concentration regions.

So, you can see from here that if mass is flowing to the right this is our x axis. So, the flux is in the positive x direction, this is what we will call a positive flux, but to have the positive flux so, means if j is positive if j is positive, then  $\frac{\partial C}{\partial x}$  the concentration gradient we have a higher concentration on the left-hand side and a lower concentration on the right-hand side. So, this gradient  $\frac{\partial C}{\partial x}$  is negative. So, this is negative, and since we want to associate the proportionality constant with the material property, and it will not be nice to have a negative sign with the material property.

So, by convention that that is the reason why we have put a negative sign. In fact, that is the reason why. So, this negative sign has no any fundamental derivation or anything this is just a desire by the scientific community to keep their conductivity the thermal conductivity, the electrical conductivity and now the mass diffusivity as positive quantities. If you want to keep them as positive quantity we have to have minus sign in all these equations.

So, that is the reason for this minus sign. So, this D if we want positive, then this minus sign should be there to make D positive. So, D is diffusivity or diffusion coefficient. I have already written the units for diffusion coefficient; you can see the consistency of that that the dimension of D will be the dimension of j divided by the dimension of the concentration gradient.

And if we write it in terms of units so, we said that j is kg per meter square per second. And concentration gradient I insisted that if we want D to be meter square per second concentration should be the volumetric concentration. So, in this case it should be kg per meter cube, but since it is a gradient one more meter in the denominator comes. So, if you work this out it will be simply meter square per second.

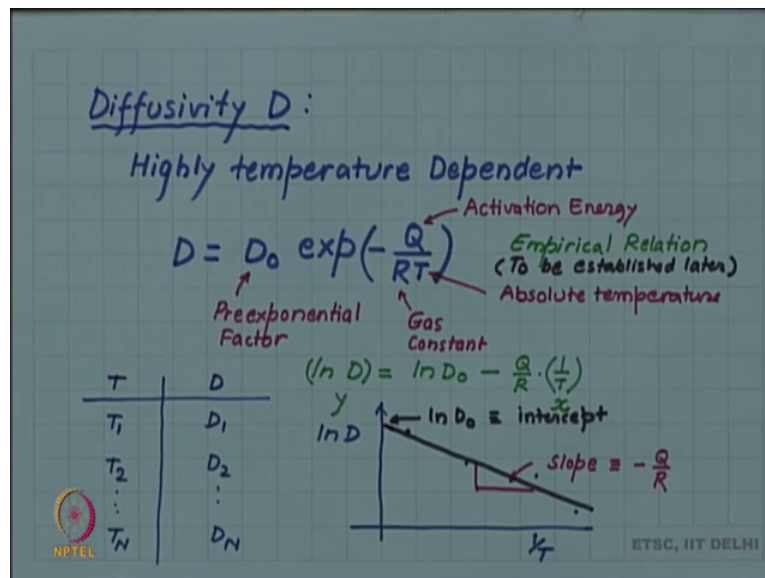
So, you want D to be meter square per second. So, we insist concentration to be in kg per meter cube. Of course, kg is not required I means not essential it can be moles per meter cube, but then the flux should also be in moles per meter square. So, the flux the unit of mass in flux and the unit of mass in the concentration should be consistent. So, if this is mole this also should be moles if this is gram that should also be in gram and so on.

So, this is the mathematical form, and essentially actually Fick did not direct derive it, and there is no such derivation for this first law. But we we should be we will see later in this course, that we will be able to derive this law from the first principle in by atomic

movement. But at the moment we will not do that we will just take this law as given. So, and this will be our flux to concentration gradient relation, this is our Fick's.

First law it is interesting to note that Fick was actually a physiologist he was not an engineer interested in carburization or not or neither in doping of semiconductors, he was interested in the diffusion of oxygen in blood. So, and that was the reason why he was interested in diffusion problems and he came up with this nice relationship.

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One more point about diffusivity let us talk about so, this diffusivity D is temperature dependent highly temperature dependent. And it follows an exponential law D is equal to D naught exponential minus Q by R T. So, since D naught is a constant which comes before the exponential. It is known as pre-exponential factor. R is the well-known gas constant, and T like in all thermodynamic equations, it is absolute temperature and Q is a constant which is known as the activation energy.

so in fact, so, this this again we will see that when we will try to derive the Fick's first law from the first principles of atomic jumps, then in the process we will also derive this temperature dependence of diffusivity. But at the moment we are taking it purely as empirical relation. So, we can call it for the moment an empirical relation that is based on experiments empirical relation, but we can note to be established later by atomic mechanism of diffusive jumps.

Also, we can see that this empirical relation can actually be established by experiments. So, if we have experimental data let us say of as a temperature, if we do an experiment diff diffusion experiment on different temperature, and if we measure the diffusivity.

So, if we have a temperature  $T$  diffusivity  $D_1$  and temperature  $T_2$  diffusivity  $D_2$  and so on, if we have a table like this, then it is possible to get the pre-exponential factor and  $Q$  or is impossible to verify whether this relationship is true or not by plotting you can see we will get a straight line if we simply take log of this equation. So, if we take the log, then we will have we will we will have  $\log D$  is equal to  $\log D_0$  minus  $Q$  by  $R$   $1$  by  $T$ . So, you can see that if I plot  $\log D$  as my y axis, and  $1$  by  $T$  as my x axis, then I should get a straight line.

So, let us do that if x axis is  $1$  by  $T$ , and the y axis is  $\log D$  in my plot and if I plot this data of diffusivity versus temperature. So, then these data should actually give me a straight line. So, since it is an experimental data, all of them may not be exactly on a line, but it should follow a line so, I should draw a best fit line to a get this relationship. So, you can see now that the intercept intercept of this line will give you, that is if  $1$  by  $T$  is  $0$ , then that is  $\log D_0$ . So,  $\log D_0$  can be found from intercept, and the slope of this line should be nothing but minus  $Q$  by  $R$ .

So, since  $R$  is a constant gas constant, you can find the activation energy  $Q$  from an experiment like this. So, in fact, experiments like this is what means if you haven have a temperature diffusivity data, and if you are able to fit a straight line to  $\log D$  versus  $1$  by  $T$ , that gives you the reason to believe that  $D$  should have an exponential relation like this, and that is why we will call this an empirical relation. But later on, when we talk about atomic mechanism of diffusion, we will see that this comes out from atomic model of diffusion also.