Statistical Thermodynamics for Engineers
Professor Saptarshi Basu
Indian Institute of Science, Bengaluru
Lecture 26
The Uncertainty Principle

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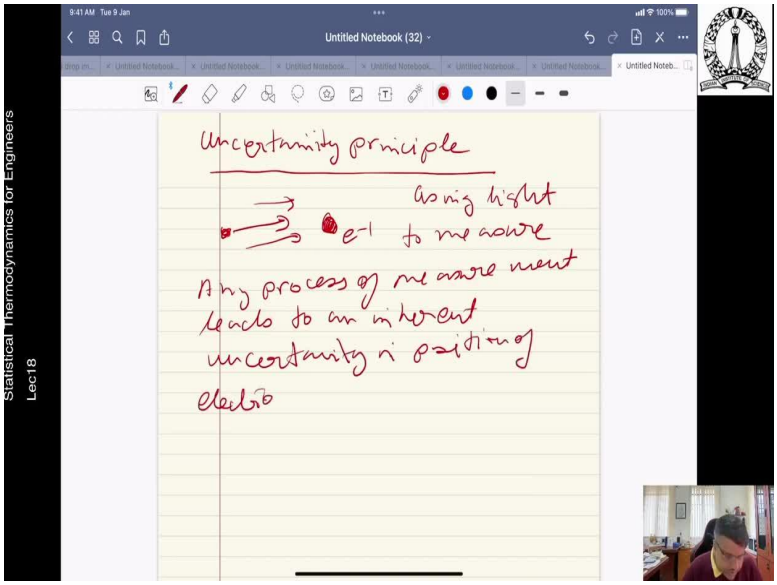

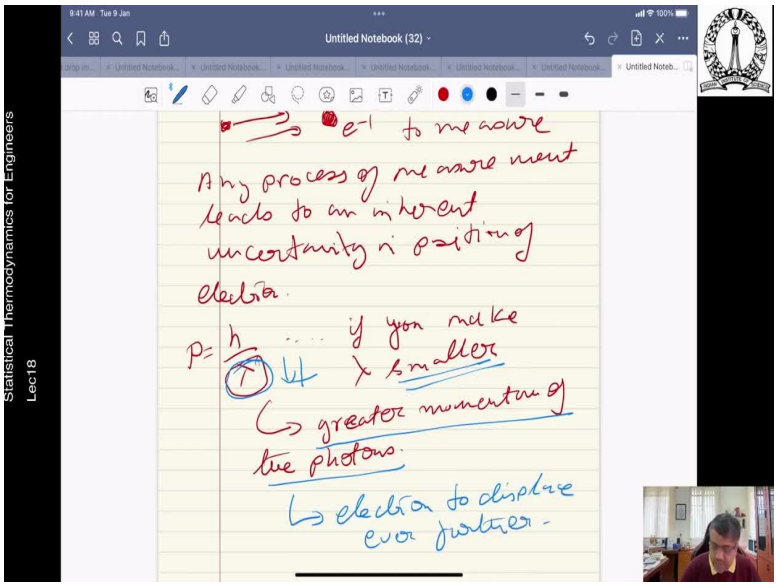

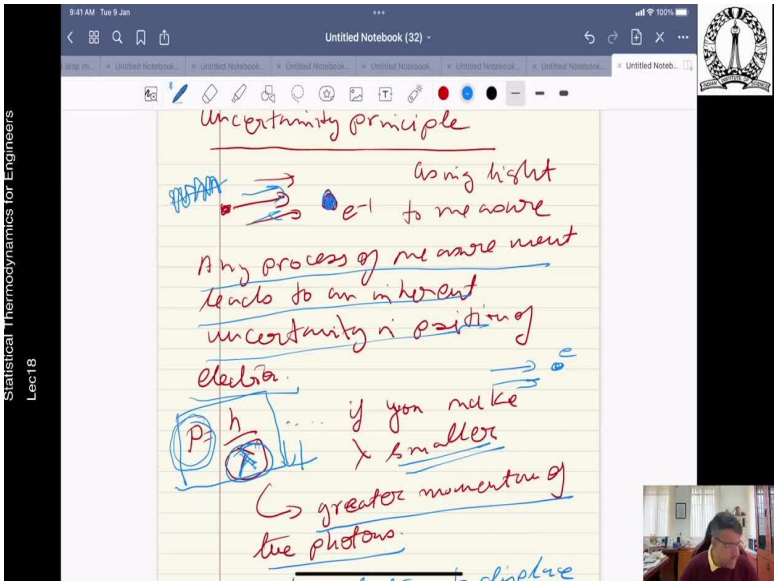

Welcome to lecture number 18. So, in this particular lecture, we are going to start with the uncertainty principle. So, what is basically an uncertainty principle? For example, if you want to normally we want to measure something, we use light for example. Say we want to measure the position electron, so what you do is that you shine a photon on this electron, so use using light to measure. So, now in the process of measurement leads to an inherent uncertainty in position of electron.

So, if you look at now the expression that p is equal to h by lambda keeping the momentum. So, if you make lambda smaller that means you progressively use light of smaller and smaller wavelength, this leads to needs a greater momentum of the photons. So, if you make lambdas smaller and smaller, say you are making it smaller and smaller lambda being smaller and smaller, greater is the momentum of the photons. And this means that this will cause the electron to displace even further. So, you understand the dichotomy here. You are using light to measure say the position of the electron.

So, what you want to do? You want to make the light smaller and smaller and smaller so that you can measure the position of the electron more precisely. So, the wavelength of the light is being made smaller and smaller. So, you want to go like this then you want to decrease the wavelength even further. So, but any process what will happen is that that photon is getting bombarded on the electrons, so the electrons position is going to be moved because of the impingement by the photons, so you wants to avoid that.

So, what do you want to do is that basically he wants precision which means you have to make lambda smaller, but the moment you make lambda smaller the momentum of the photon increases. And so that means it is hitting the electron, if the electron is here, the

photon is hitting the electron with a higher momentum that means, it will cause more displaced. As a result of that, just by making the photon wavelengths smaller and smaller is not going to serve the purpose of this particular case.

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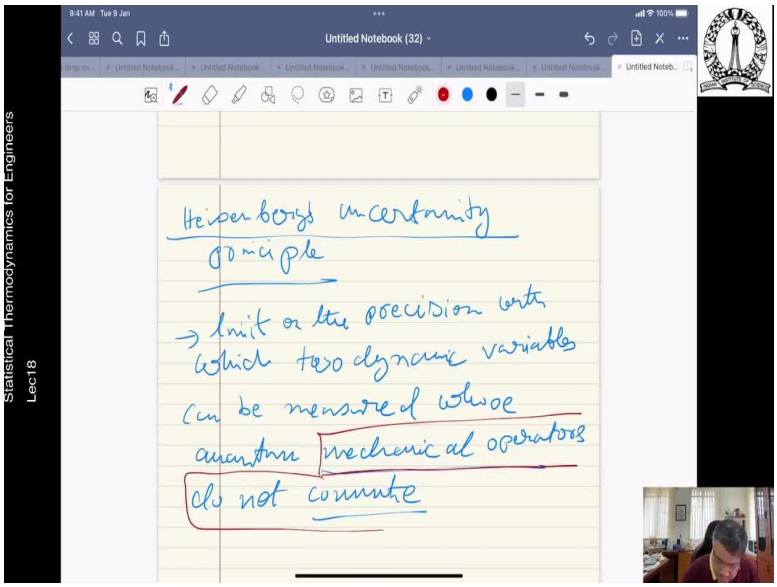

So, this sets the stage for Heisenberg's Uncertainty Principle, stage Heisenberg's Uncertainty Principle. Again, a cornerstone of quantum mechanics the Heisenberg uncertainty principle. So, what Heisenberg's Uncertainty Principle states is that it is basically a limit on the precision and the precision with which two dynamic variables can be measured. So, two dynamic variables can be measured whose quantum mechanical operators do not commute, whose quantum mechanical operators do not commute.

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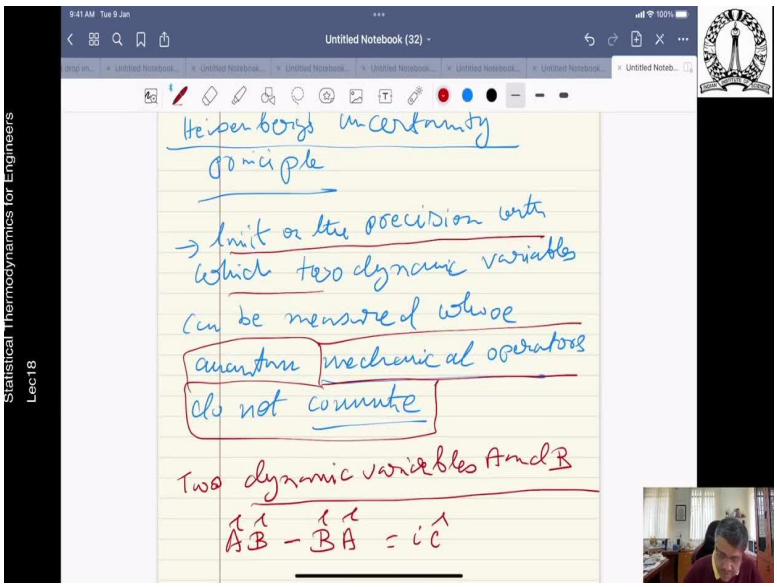

So, let us see two dynamic variables A and B. So, A hat, B hat minus B hat, A hat. So, these are two dynamic variables A and B and these are their operators given as say c hat, they do not commute they are quantum mechanical operators which is A hat and B hat they do not commute. So, there is a limit on the precision with which two dynamic variables can be measured where the quantum mechanical operators do not commute. So, that two dynamic variables A and B, A hat, B hat are there corresponding quantum mechanical operators. So, if they do not commute this is what we can write.

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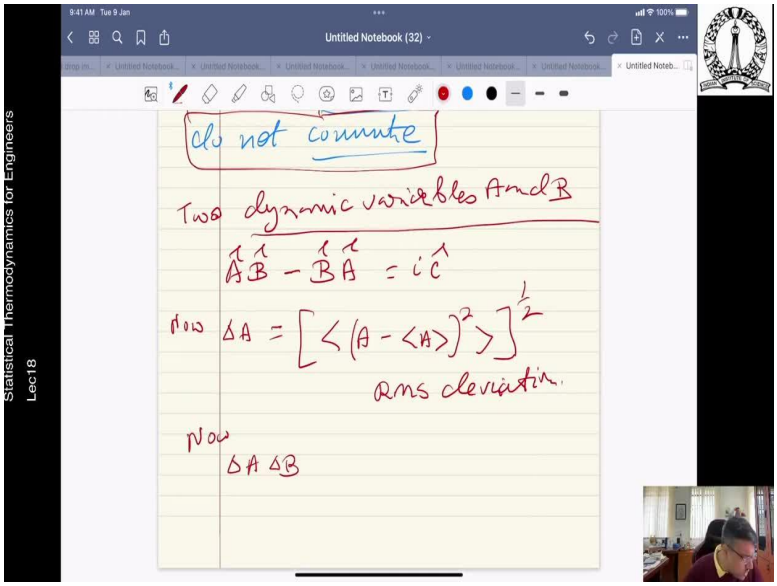

Now delta A equal to is a some kind of RMS deviation. So, you saw what this? This is the dynamic variable minus the average of that particular dynamic variable, square of that then you take the average and then you take the square root of the, this is the RMS deviation. So, therefore, now delta A into delta B, delta A into delta B is basically the deviations, the RMS deviations of those two quantities that you are trying to measure.

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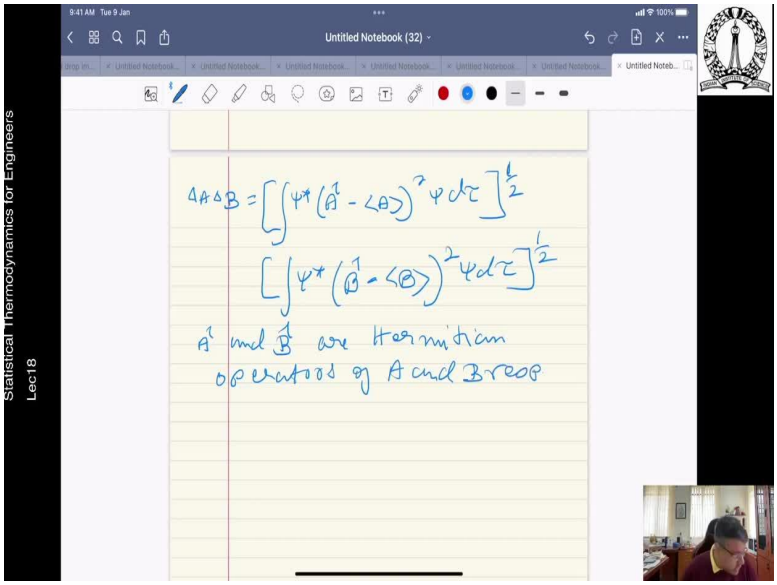

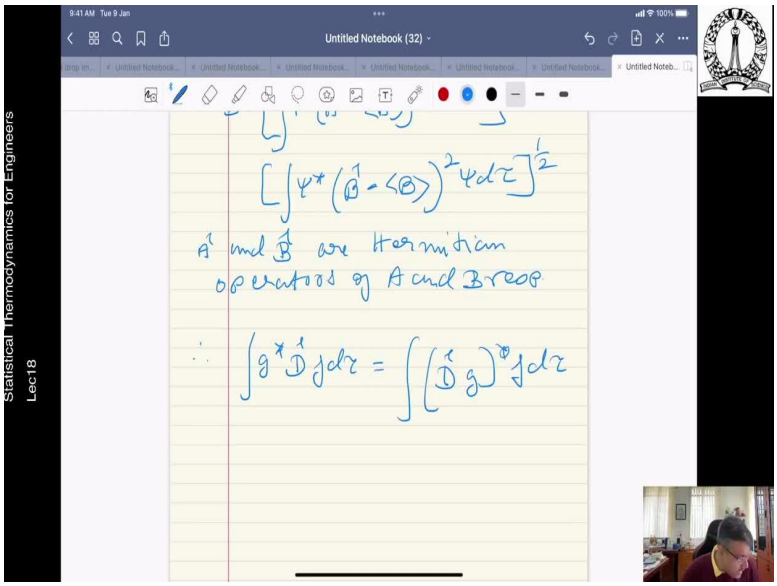

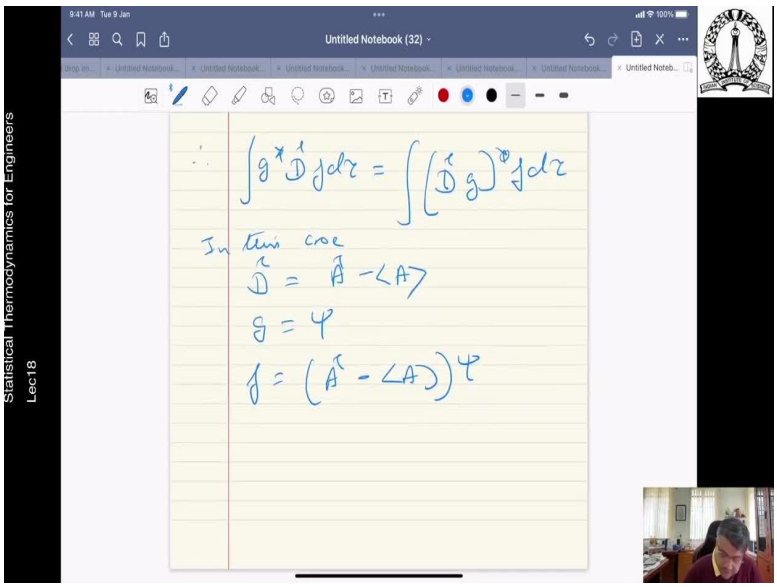

So, maybe we should go to the next page because it is a longish expression. Delta A into delta B is equal to Psi star, A hat bar square chi d tau(())(7:36). Alright that is what it is because it is acting this is the operator that is acting, acting on the on the wave function. So, and similarly other expression is Psi star B hat delta B square d tau raise to the power half. So, A hat and are Hermitian operators A and B respectively. You can see that this is what it is.

Now, if the operators are Hermitian we can do cool thing here(())(8:43) here. So, say this is the arbitrary, this is a rule, so if tau equal to (())(8:57) f tau for the Hermitian operators this should be valid. Now, let us assume in this case D hat is A hat minus A bar, and G let us assume is the wave function, and f is nothing but A hat A close star, with the acting on the wave function.

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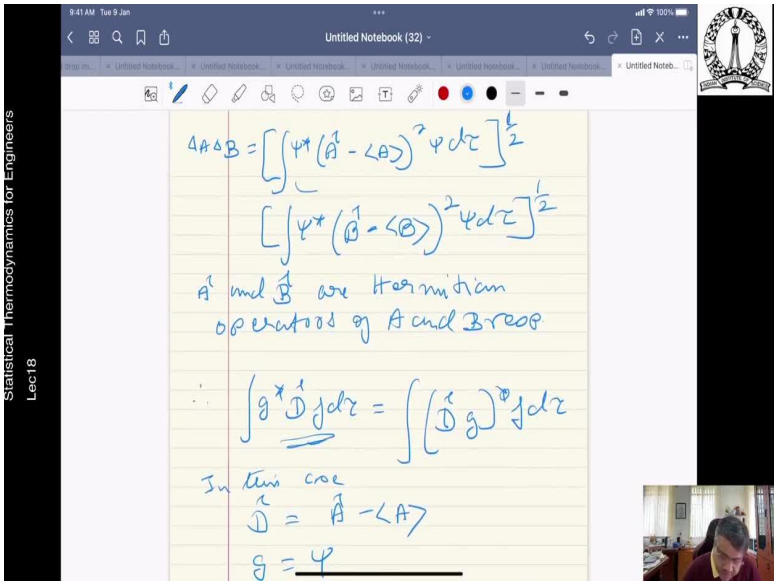

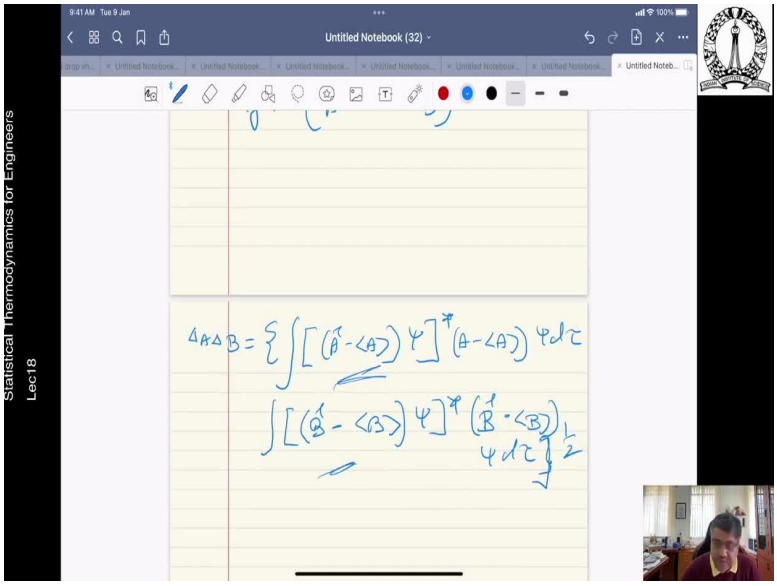

So, therefore, once we get into these rules all set. So, delta A into delta B given as A hat (())(9:53) star A (())(10:01) d tau (())(10:07) to (())(10:25) raise to the power, that thing is raise to half. So, you got this expression this comes like acclaim the Hermitian operators rule. So, we have just used this rule to go from here to here, so that is really nice.

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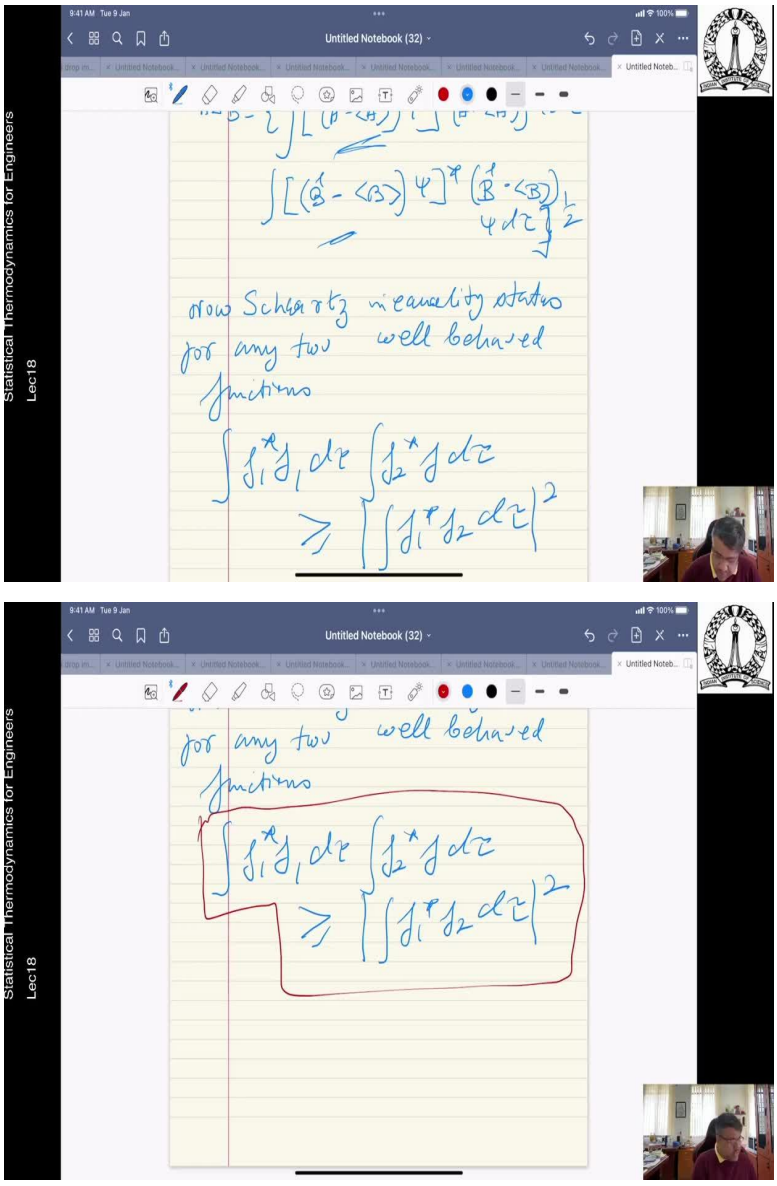

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Untitled Notebook (32)

$\Delta A \Delta B$

$$\Delta A \Delta B = \left[\int \psi^* (\hat{A} - \langle A \rangle)^2 \psi \, d\tau\right]^{\frac{1}{2}}$$

$$\left[\int \psi^* (\hat{B} - \langle B \rangle)^2 \psi \, d\tau\right]^{\frac{1}{2}}$$

$\hat{A}$ and $\hat{B}$ are Hermitian operators of A and B resp

$$\int g^* \hat{D} f \, d\tau = \int (\hat{D} g)^* f \, d\tau$$

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$$\Delta A \Delta B = \left\{\int \left[(\hat{A} - \langle A \rangle)\psi\right]^* (A - \langle A \rangle)\psi \, d\tau\right.$$

$$\left.\int \left[(\hat{B} - \langle B \rangle)\psi\right]^* (\hat{B} - \langle B \rangle)\psi \, d\tau\right\}^{\frac{1}{2}}$$

now Schwartz inequality states for any two well behaved functions

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Untitled Notebook (32)

$\psi \, d\tau \Big\}^{\frac{1}{2}}$

now Schwartz inequality states for any two well behaved functions

$$\int f_1^* f_1 \, d\tau \int f_2^* f_2 \, d\tau \geq \left|\int f_1^* f_2 \, d\tau\right|^2$$

Now Schwartz inequality states for any two well behaved functions this should be valid, f1 f1 d tau into f2 star f2 d tau should be greater than equal to the mod f1 star into f2 d tau square. So, this should be the validity of the Schwartz inequality. So, Schwartz inequality is important.

So, we saw that we use that the general Hermitian operator to take this side of a quantity which is basically nothing but the product of the total differential of standard RMS deviation in dynamic variable A and B and we have substituted it and we have passed it therefore, in this particular form, in this particular form like this. Now Schwartz inequality sates that for any two well behaved functions this is what is what we are going to get. So, this is Schwartz inequality.

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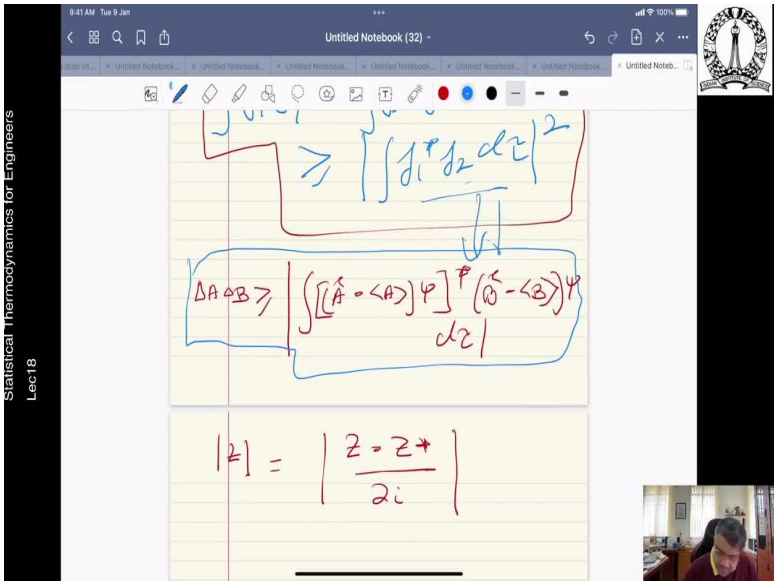

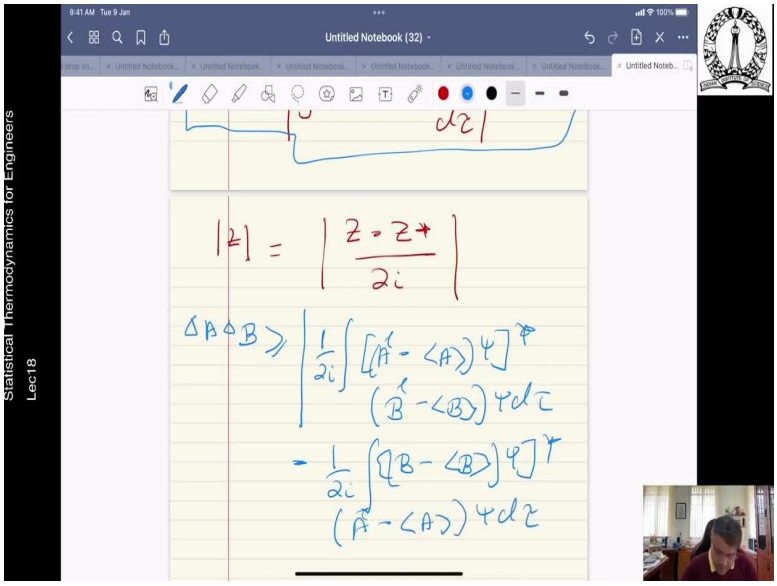

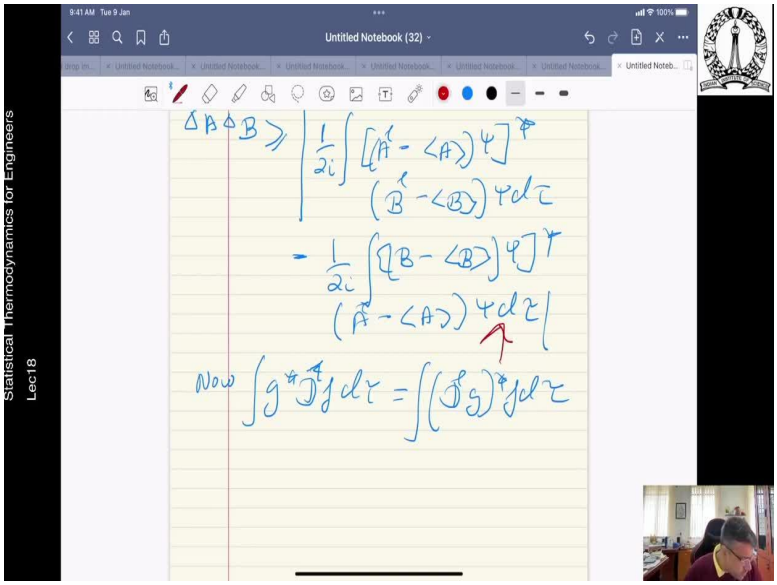

So, therefore, delta A into delta B must be greater than or equal to mod this is in bracket(())(13:10), then this becomes star into B hat this does not require the star d tau. So, all we can say is delta z equal to delta z this is also true. So, what we can say over here is basically we have used this Schwartz inequality to get to this particular form and this is just a definition of set modulus of set which is basically the quantity within bracket, we will show it a little bit what it is. So, delta A into delta B must be therefore greater than or equal to 2i (())(14:18) B tau minus 2 1 by 2i, integral B star and we will put a modulus here.

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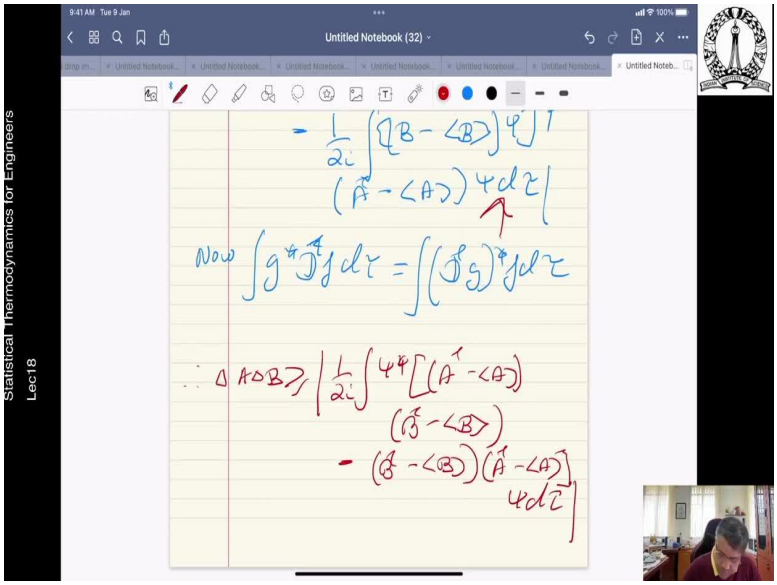

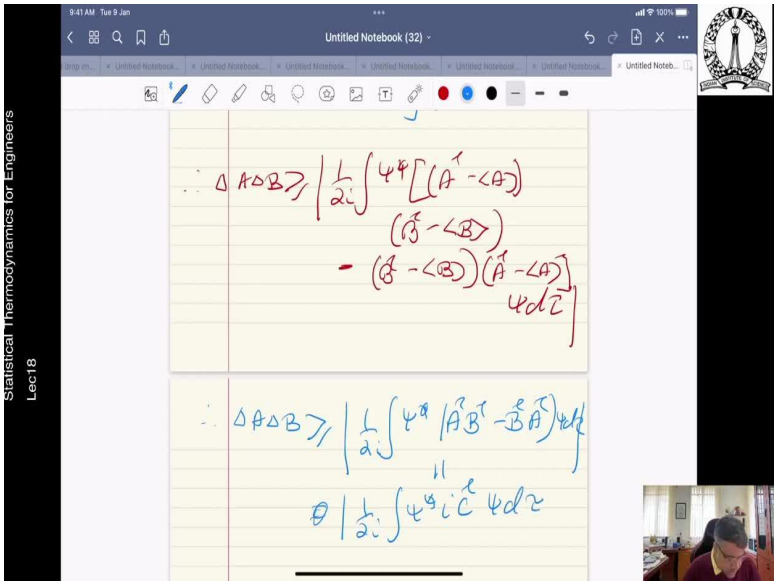

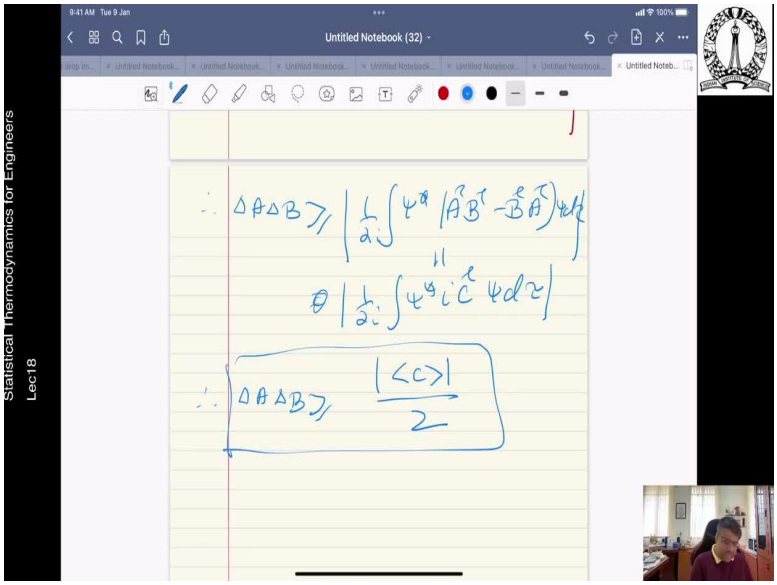

Now tau equal to this is what has been here(())(15:28). Now so if we move it further now therefore, delta A into delta B is greater than or equal to 1 A A bar into B minus A A bar into B times d tau. Now further delta A into delta B therefore this part is equal to okay. Therefore, delta A delta B is greater than, so it is greater than or equal to C bar by 2 that is what you get. So, that is what the inequality between two dynamic variables is formed and mechanical operators do not commute and it is.

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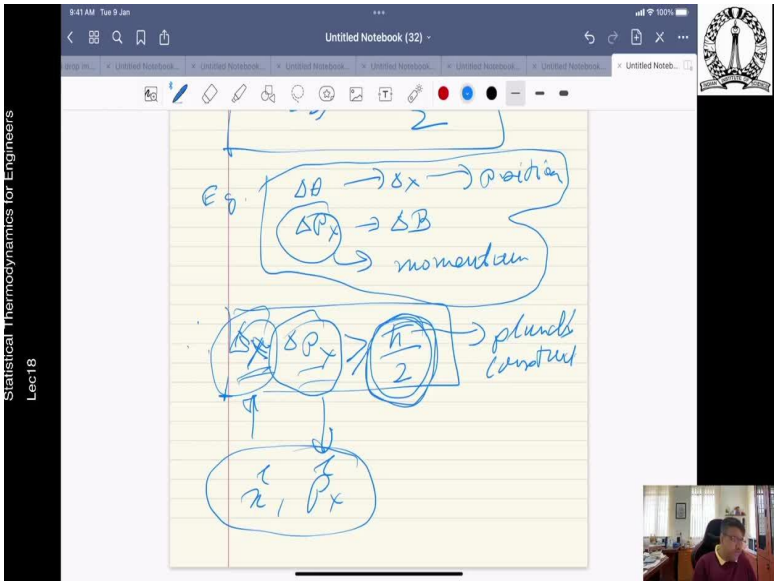

Now let us see the example, let us take an example where delta A is basically delta X delta P X is delta B therefore, this is the momentum and this is the position, the error in position the uncertainty in a position and the uncertainty in this is given in this particular fashion. So, now, so in this particular case this is left as an exercise, one can show that delta X into delta P X is greater than or equal to h bar by 2 which is given this is the Planck's constant. So, that means, if you want to reduce the error in say delta X you want to measure the position more accurately that means, this becomes smaller and then this has to become larger for this to be valid, has to be greater than the Planck's constant.

So, that this is left as an exercise you should find out what are the operators basically you have to find out X hat and P X hat operators and we already know what those operators are can be those operators, using this operators you are supposed to find out that what will happen to delta X delta P X but the overall answer that is given is still the h by 2, is the answer.

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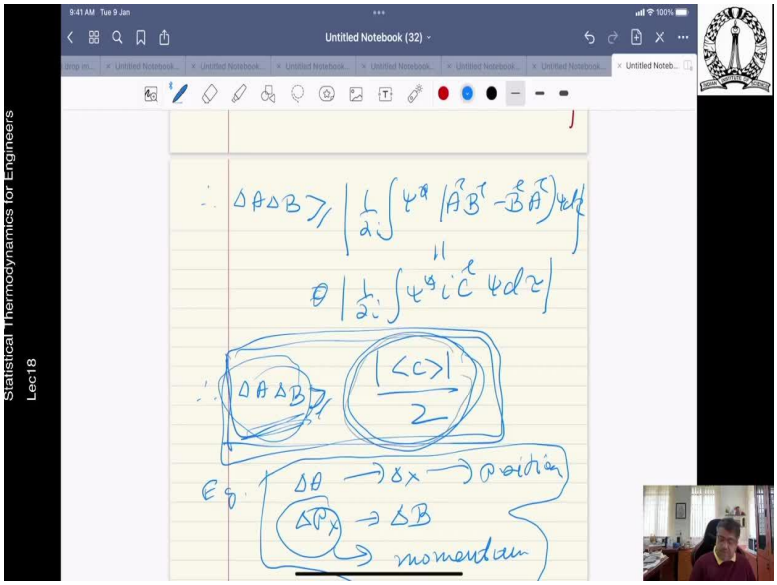

So, what we have seen over here in this particular exercise is that we have used non-commutative quantum mechanical operators that we have shown that this is the most generalized expression which we do want it is dynamic variables and these are basically RMS deviations of those quantities, they cannot they must be greater than equal to some value. So, this is not zero this is sum value.

Similarly, like this is not zero this is some value. So, you cannot measure something with what we call a great deal of certainty because the precise position cannot be measured, if you want to measure the precise position then the other position mean the error of the uncertainty in your momentum for example, will be huge, just to keep track of this inequality. So, this is a important piece of information that we have now here.

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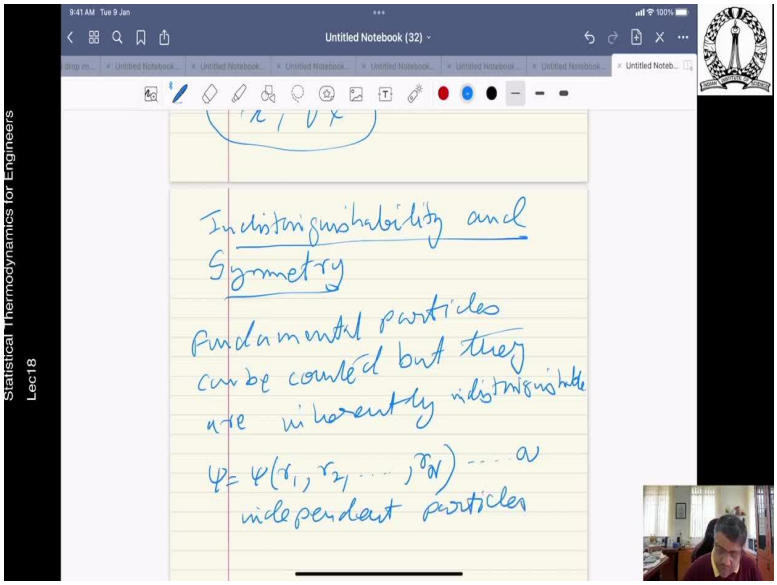

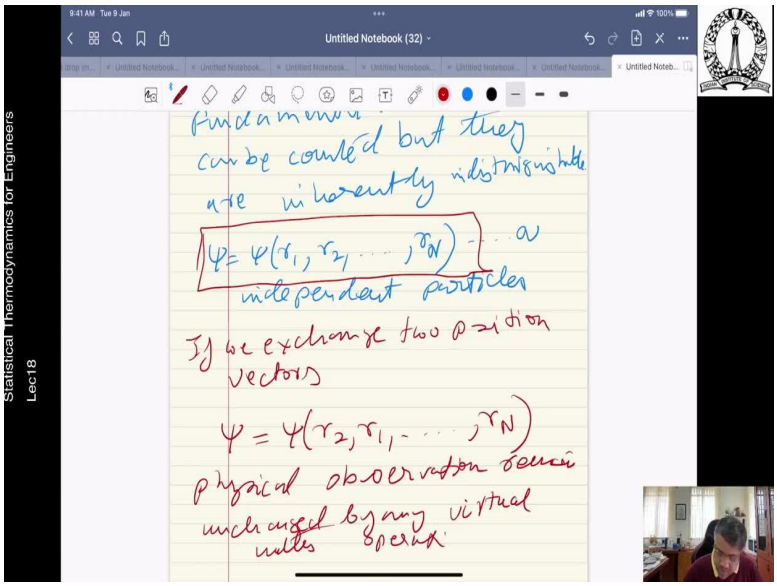

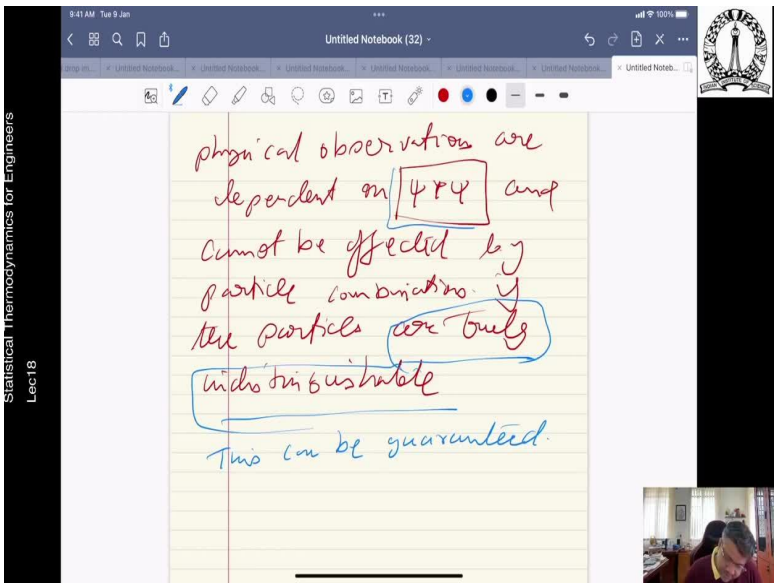

Now, let us take a look at two other things one is indistinguishability and symmetry. So, fundamental particles can be counted but they are inherently indistinguishable, for example, Psi equal to Psi r1, r2, which are N independent particles, so Psi is like this. So, if we exchange two position vectors that means you interchange, so Psi becomes equal to r2, r1 dot dot dot upto r n. Physical observations remains unchanged by any virtual math operation.

So, the physical observations are dependent on star Psi and cannot be affected by particle combinations if the particles are truly indistinguishable, the particles are truly indistinguishable. This can be guaranteed we understand this. So, the particles as we say that even if we exchange their positions, we exchanged their positions. Because they are indistinguishable so there is no change in fiscal observations and nothing changes, everything remains the same and since it is dependent on this, this is not affected.

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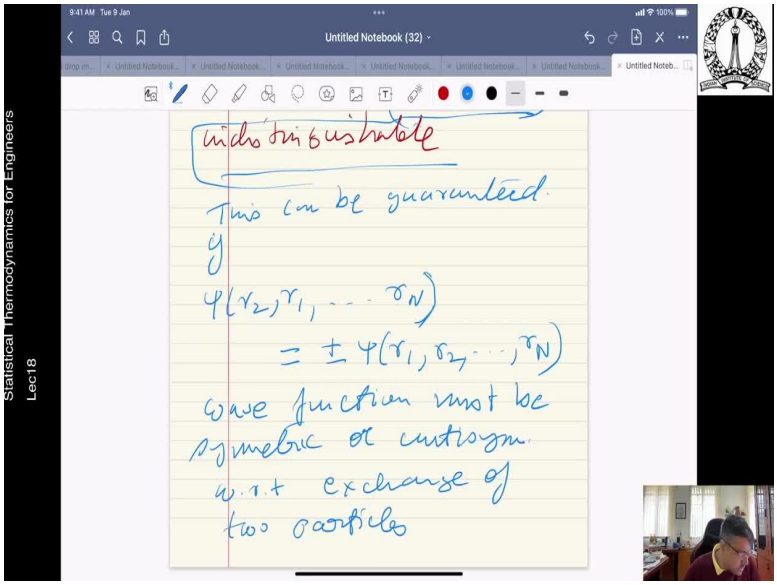

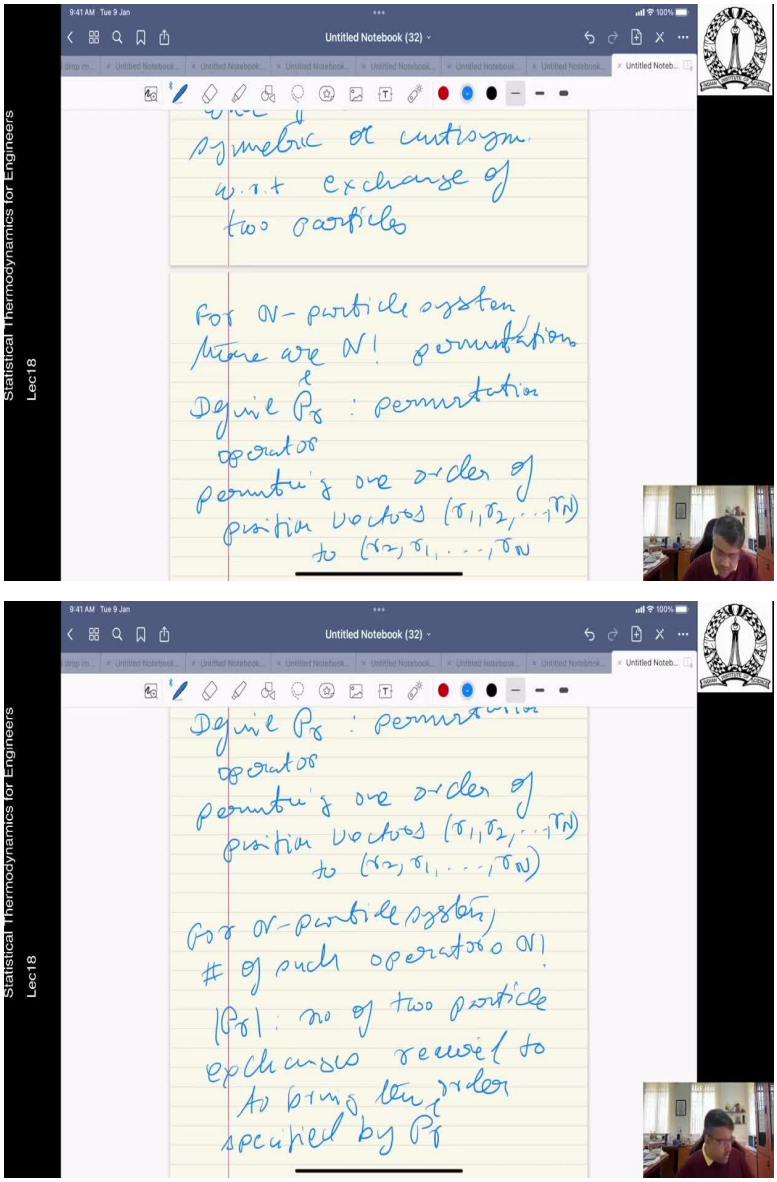

So, this can be guaranteed if Psi r1 continued up to N is plus so equal to plus minus Psi r1 up to N. So, wave function must be symmetric or anti-symmetric. So, the wave function must be symmetric or anti-symmetric with respect to exchange of two particles. So, for N particle system there are N factorial permutations permutations that are possible. So, if we define Pr hat the permutation operator permitting one order of position vectors r1, r2, rN to r2, r1, … rN. For N particle system number such operators N factorial. So, delta P r the number of two particle exchanges required to bring this bring the order specified by Pr hat.

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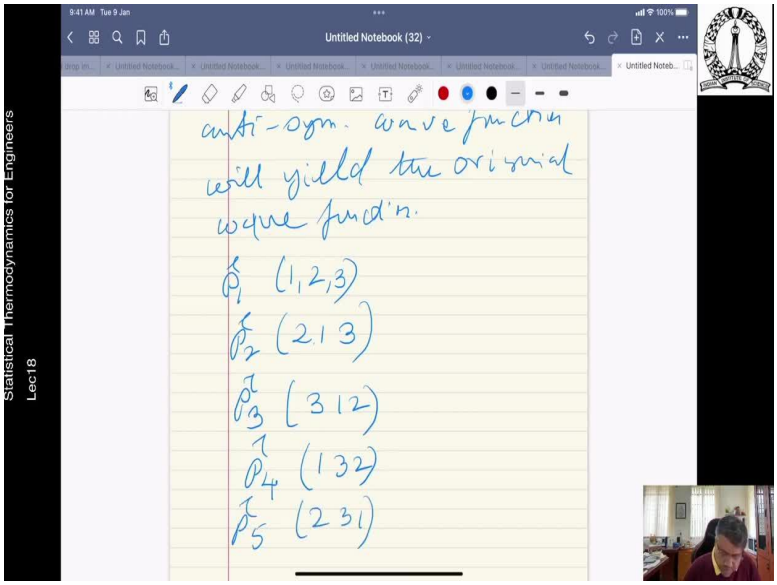

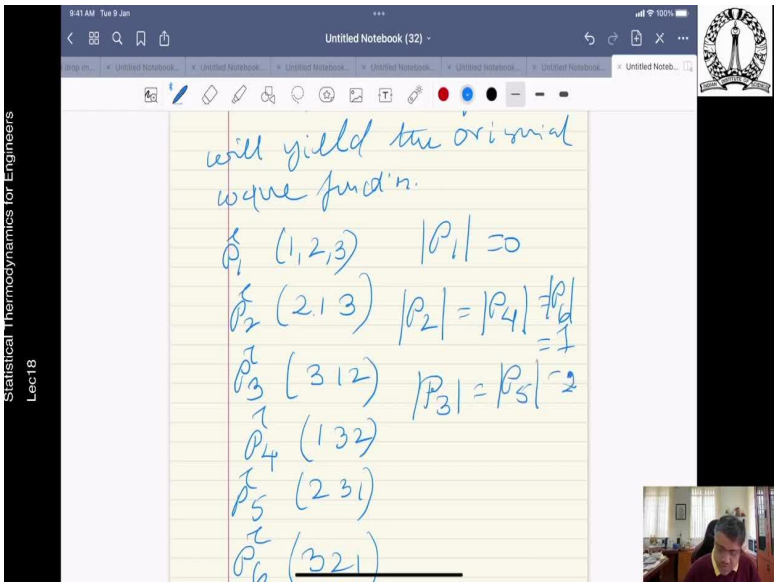

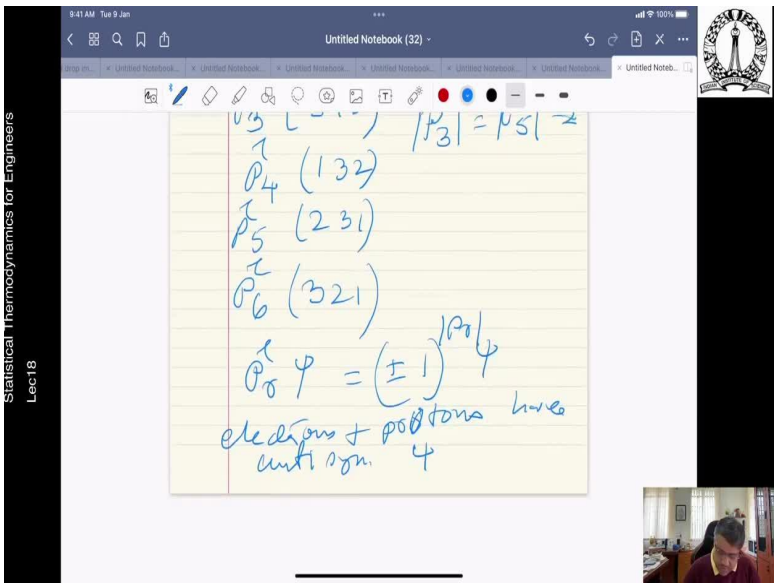

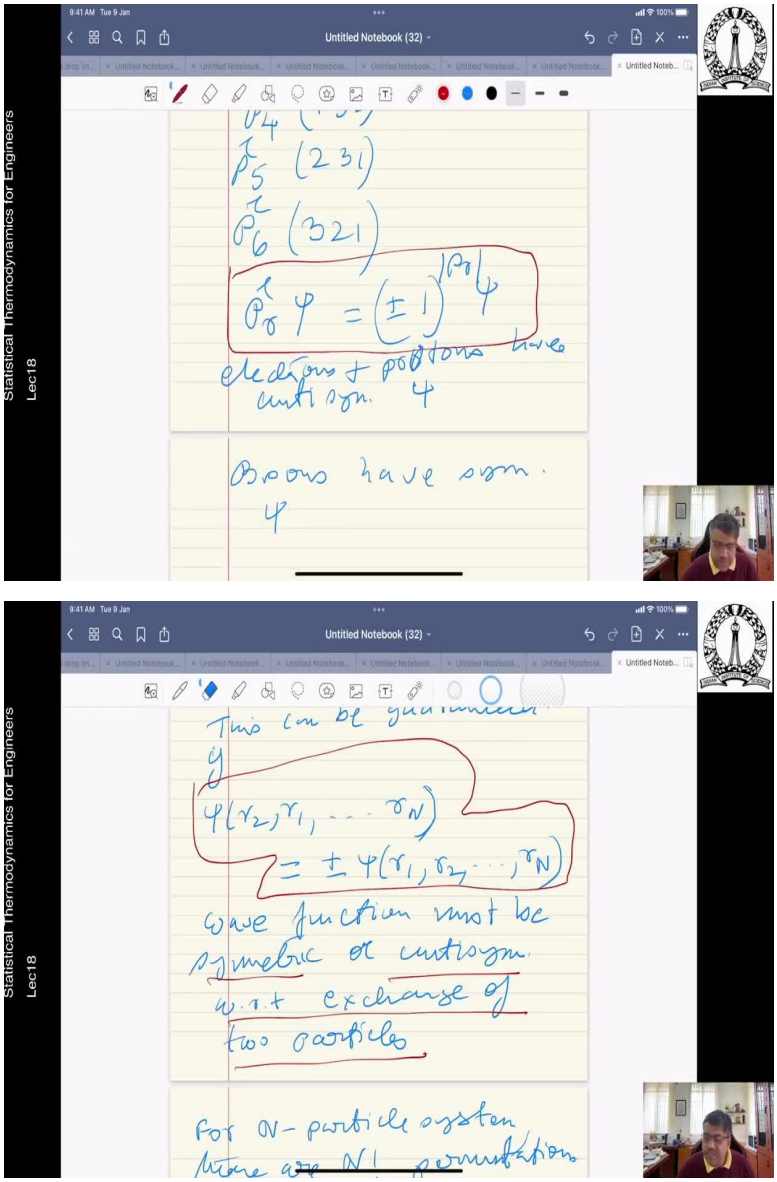

So, two successive exchanges of two particles having heavy anti-symmetric wave function will yield the original wave function. So, say P1, let us say any combination 1, 2, 3, where P2 is 2, 1, 3, then of course P3 which is 3, 1, 2, and P4 1, 3, 2, P5 is 2, 3, 1, I am just giving an example a random example over here, P6 is 3, 2, 1. So, delta P1 is obviously equal to 0 delta P2 is equal to delta P4 equal to delta P6 these are all equal to 1 and P3 is equal to P5 is equal to 2.

So, in order in other words Pr is acting on a wave function will give you plus sign delta Pr, the electrons and protons have anti-symmetric wave function that is bosons as symmetric wave functions. So, this is the general rule, so you can see that what it will yield when you actually have indistinguishability and we have this is a weight must be either symmetric or anti-symmetric with respect to exchange of two particles. So, we finish our lecture on

uncertainty principle and symmetric and anti- symmetric wave function, next class we are going to do for this exclusion principle and few other things in details.