Sound and Structural Vibration
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Lecture - 48
Model Average Radiation Efficiency

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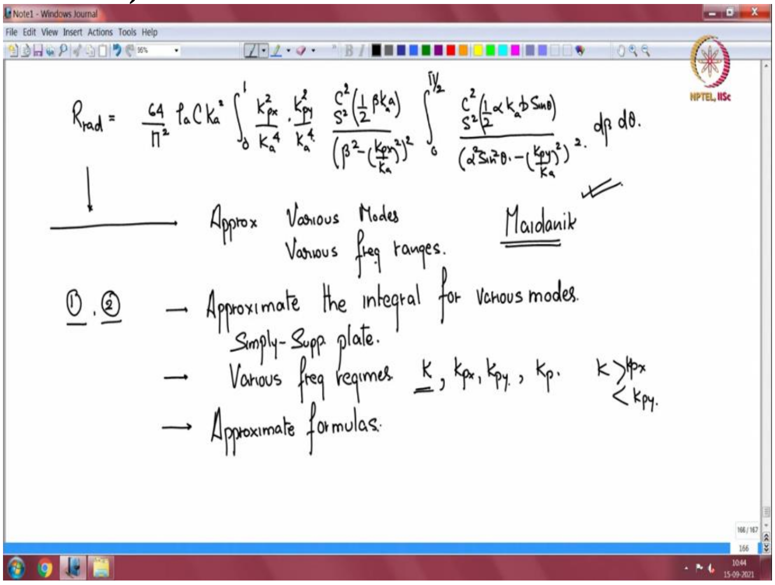

Good morning and welcome to this next lecture on sound and structural vibration. Last time we had arrived at this integral from the paper by Maidanik. From here Maidanik proceeds to approximate as I said last time this integral for various modes this simply supported plate and also in various frequency regimes so, there is the acoustic wave number k there are the plate modal wave numbers k_{px} and k_{py} so, now and the free wave number of the panel which is k_p.

So, now various relations between them whether k is greater than k_{px}, k may be greater than k_{px} but less than k_{py} and so forth. So, all sorts of frequency regimes examine and gives approximate formulas, so, some are quite good and some are not so, good but there are these formulas.

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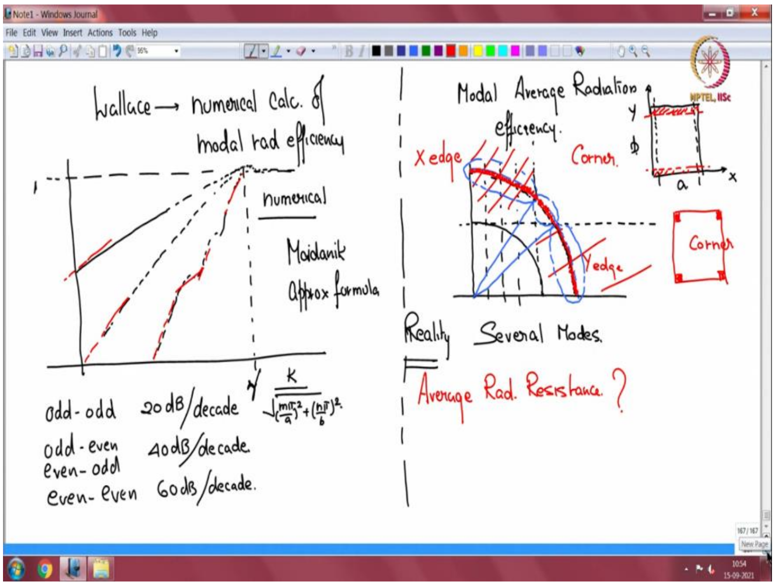

Now, if you recall I had spoken of the work by Wallace who does numerical calculations of modal radiation efficiency or radiation resistance does not matter. So, the picture looks like this, this is the non-dimensional model coincidence γ it is k over $\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ and this is the value 1. So, you get these sorts of curves that approach one and briefly cross 1 and then asymptotically approach 1 or some other mode may behave like this has a higher slope.

And some further higher slope with some oscillations that are related to this cells interacting with each other that I spoke of now, it turns out that if you have odd-odd modes which are the most efficient in radiating their slope at the lower end is about is 20 dB per decade, you have odd even or odd even odd modes it is about 40 dB per decade, decade is 10 times increase in frequency and then if they are even-even it is 60 dB per decade.

That is because of further and further cancellations that happen at the starting point, which is the lower end of the frequency now, so, these are numerical curves as I said now, you can superpose Maidanik's approximate formula on these modal radiation efficiency curves and as I said he does them in regimes. So, you will have one curve here, segment of a curve here or a segment of a curve here then there is a segment of a curve here there is another segment of a curve here.

And so, he approximates in this manner and you can over plot on top of Wallaces numerical curves and you have matches and you have some places the more inaccurate the next portion which I will briefly touch upon is modal average radiation efficiency now, we saw how if we break up the wave number diagram into these regions of modes where the node points are modes then you have an acoustic wave number quarter circle.

And then you have the free plate wave number quarter circle. So, if we extend these boundaries in this manner so, you know these are let me call them y edge now. So, you have a plate which is x in this direction, y in this direction this has dimension a dimension b. So, these are Y edge modes that means you have a strip on the y edge that radiates here these are Y edge.

Then you have another set you have these sorts of modes where there is a strip along the x axis that radiates so this strip radiates the rest is cancelled. Those are these types of modes, modes here in this zone here where X edge radiators, and then you have corner radiators here and so forth so corner, let me show the corner radiators, corner radiators look like this. Just a quarter cell radiates here the rest is cancelled, so forth corner.

Now, if in reality if a plate is vibrating and redirecting sound it is not individual modes several modes are vibrating several modes are present. So, how to bring in this picture of several modes. So, that means what the k_b circle is here we are right here at a certain frequency we are right here. So, now, if you look at the division here let me use the blue line if I draw this line over here all these are X edge modes all these are X edge modes and all these if I draw this line here all these are Y edge modes.

Similarly, these are corner modes so, you have on the k_p circle on the k_p quarter circle you have a number of Y edge modes a number of corner modes and a number of X edge modes and to begin with they are discrete, in actuality they are discrete, but the thing is that there are infinite dimensions of panels you could choose. So, infinite ways that these nodes could land on this particular line and there is no general way of presenting the picture of panels radiation.

So, what Maidanik did was that suppose there is a continuous presence of modes everywhere there is a mode not discrete points and suppose they carry the same amount of vibrational energy each of them carries the same amount of vibrational energy. So, what is the average radiation resistance at a certain frequency? So, we are trying to find an average. So, let me use another picture here

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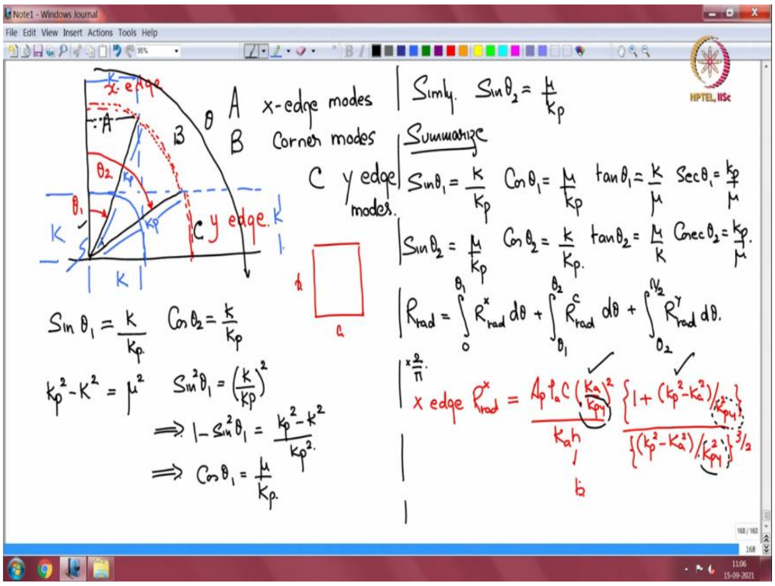

I have the acoustic quarter circle and its bounds then the panel free wave number quarter circle. And if I draw this line here you have along this red portion X edge modes along this portion you have Y edge modes now a continuous distribution, we are going to make it a continuous distribution and then along here you have corner modes below full panel coincidence frequency.

So, now if I join this also so in, I call this the A strip, I call this the B strip, I call this the C strip the A portion has at X edge modes. So, I will use the X edge mode approximate formula derived by Maidanik and that will be made continuous, the modal variable will be made continuous then we have the B strip where we have corner modes. So, there we will use the appropriate approximate formula then lastly, we have seen where you have Y edge modes.

So, we will use the formulas from Maidaniks paper, and we will make the discrete variable continuous, but we need a few special relations here. So, we are going to measure θ in this direction increasing θ will be measured like this, θ increases like this and up till here we will call it θ_1 and till here we will call it θ_2. So, now let us see some relations we have. So, this radius is obviously k_p, this radius is k_p this big radius is k_p.

But this is k acoustic wave number, this is k the acoustic wave number, this is k the acoustic wave number, this is k the acoustic wave number. So, now, let us see $\sin\theta_1$, sin of this angle here. So, it should be a perpendicular by hypotenuse, so, this perpendicular is k hypotenuse is k_p next we have $\cos\theta_2$, so $\cos\theta_2$ is this angle. So, what is that? That is base by hypotenuse base is what? Base is k hypotenuse is k_p.

$$\sin\theta_1 = \frac{k}{k_p}.$$

$$\cos\theta_2 = \frac{k}{k_p}.$$

We will use a substitution which is ${k_p}^2 - k^2 = \mu^2$ and further $\sin^2\theta_1 = \left(\frac{k}{k_p}\right)^2$ which implies that $1 - \sin^2\theta_1 = \frac{{k_p}^2 - k^2}{{k_p}^2}$, which implies that $\cos\theta_1 = \frac{\mu}{k_p}$, similarly will have $\sin\theta_2$ is given by $\frac{\mu}{k_p}$. So, now if we summarise $\sin\theta_1$ is given by $\frac{k}{k_p}$, $\cos\theta_1$ is given by $\frac{\mu}{k_p}$, $\tan\theta_1$ is given by $\frac{k}{\mu}$, $\sec\theta_1$ is given by $\frac{k_p}{\mu}$.

Similarly, $\sin\theta_2$ is given by $\frac{\mu}{k_p}$, $\cos\theta_2$ is given by $\frac{k}{k_p}$, $\tan\theta_2$ is given by $\frac{\mu}{k}$ and $\csc\theta_2$ is given by $\frac{k_p}{\mu}$. So, now, how is this average radiation resistance going to be computed

$$R_{rad} = \int_0^{\theta_1} {R^x}_{rad}\, d\theta + \int_{\theta_1}^{\theta_2} {R^c}_{rad}\, d\theta + \int_{\theta_1}^{\pi/2} {R^y}_{rad}\, d\theta$$

and then the whole thing is over π / 2.

So, we will multiply by 2 / π in the end. So, now, let us see this the X edge radiators the approximate formula by Maidanik is of this form

$$\frac{A_p \rho_a c \left(\frac{k_a}{k_{py}}\right)^2}{k_a h} \frac{\left\{1 + \frac{\left({k_p}^2 - {k_a}^2\right)}{{k_{py}}^2}\right\}}{\left\{\frac{\left({k_p}^2 - {k_a}^2\right)}{{k_{py}}^2}\right\}^{3/2}}.$$

This is an approximate formula from that big formula that we derived last class. Now in here k_a is the acoustic wave number which is fixed at a frequency k_p is the panel free wave wavenumber fixed at a frequency the only varying thing is k_{py} the y component of k_p it varies with θ. So, k_p will be made a continuous variable as a function of θ. So, let us see now what we do?

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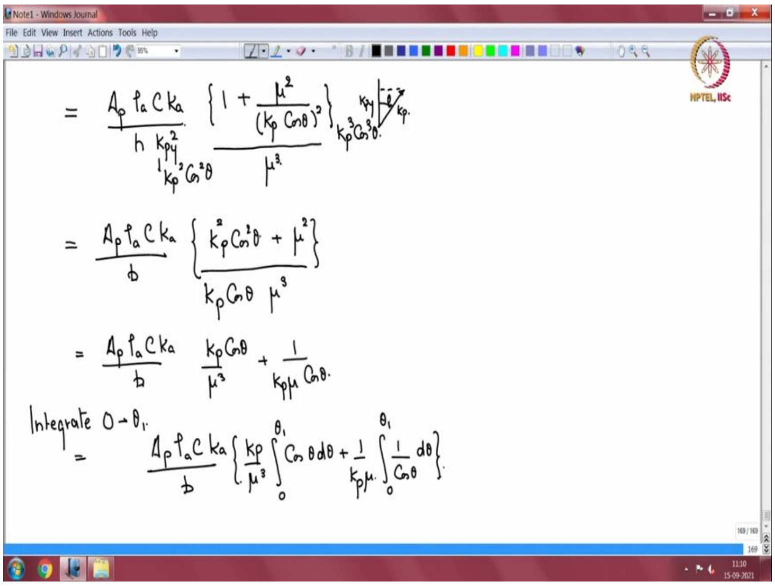

So, as a result, what will happen is that the formula will become

$$= \frac{A_p \rho_a c k_a}{h {k_{py}}^2} \frac{\left\{1 + \frac{\mu^2}{\left(k_p \ \cos\theta\right)^2}\right\}}{\mu^3} \ {k_p}^3 \cos^3\theta.$$

${k_{py}}^2$ is also ${k_p}^2 \cos^2\theta$ so, as a result we will get I am going to use b instead of h now,

$$= \frac{A_p \rho_a c k_a}{b} \left\{\frac{{k_p}^2 \cos^2\theta + \mu^2}{k_p \ \cos\theta \ \mu^3}\right\},$$

$$= \frac{A_p \rho_a c k_a}{b} \left\{\frac{k_p \ \cos\theta}{\mu^3} + \frac{1}{k_p \mu \ \cos\theta}\right\}.$$

So, if we now integrate this from 0 to θ_1. So, what happens we have

$$= \frac{A_p \rho_a c k_a}{b} \left\{\frac{k_p}{\mu^3} \int_0^{\theta_1} \cos\theta \, d\theta + \frac{1}{k_p \mu} \int_0^{\theta_1} \frac{1}{\cos\theta} d\theta\right\}.$$

We are out of time. So, I will close the lecture here and continue from the next class. Thank you.