An Introduction to Soft Matter
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Lecture No 34
Kelvin Meyer Voigt Model

So welcome back to another lecture on Introduction to Soft Matter. And last time we were discussing some particular sample problem that we have not fully solved, and we were half in the process of solving and I had asked you to, to finish up the problem as part of the self-work. So now I am going to work out the entire problem and you can check whether if you have worked out the answers, you can check whether your answers do match with us.

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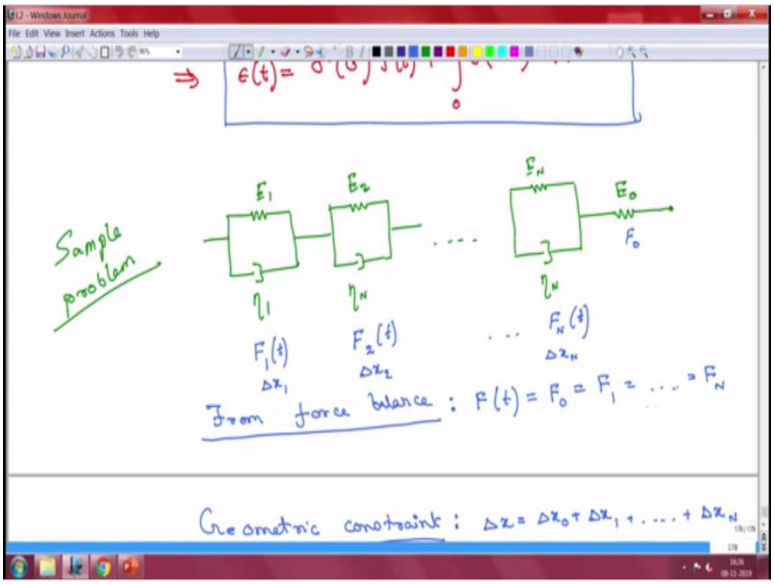

So the particular problem that we had it involved n number of Kelvin Voigt Meyer bodies in series and with the final spring and series as well. And we wrote down that the force balance for this again is very straightforward. It was boils down to the fact that the forces in all of them are actually equal and then we had the, we wrote down the geometric constraint as well.

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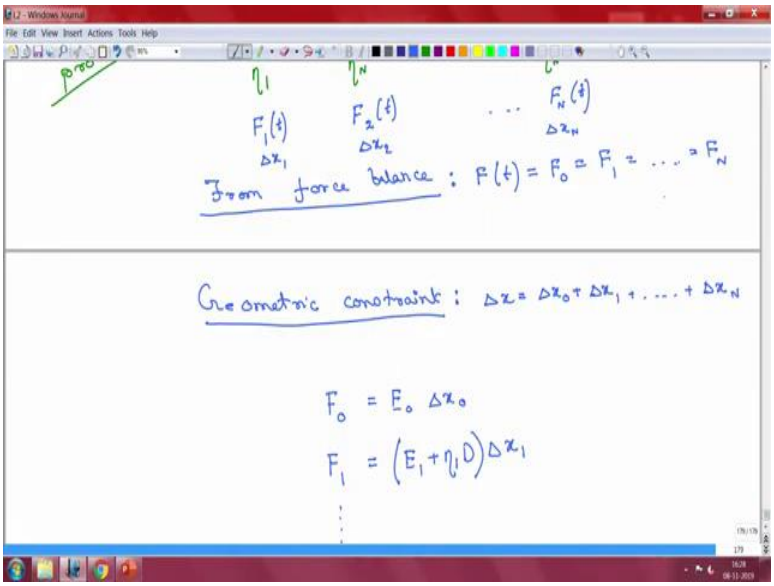

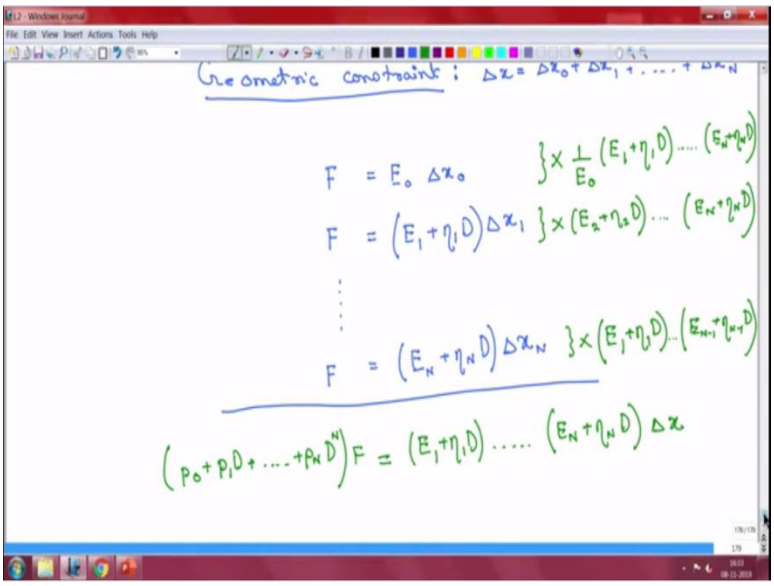

So then, now, we have to write down the individual equations that apply to this case. So let us start with just to simplify the situation, we will just because this spring stands out, I just write the spring equation first. So this equation of the spring is going to be F naught equal to this E naught into delta X naught. Now the equation for all the different Kelvin Voigt Meyer bodies, they are going to be all the same and the force here is going to be given by E1, eta1 times the differential operator into delta x1.

And this is going to be continued all the way till all your N elements are complete. Sorry, the subscript here would be capital N, d delta x capital N. So, this is all the different equations that are there. Now, let us quickly go back and see what we had first establish we said that the force balance equation says that all the different forces are the same. So, I can actually go ahead and drop all these different subscripts.

So, I just go ahead and erase the subscripts we are all left with F on your right hand, left hand side. Now, to make these equal, so we want to be able to add all these deformations together. So, we have to find the right factor for all of them. So, what we are going to do is we are going to multiply this entire part by on 1 by E naught. So, I want to cancel this part out, and then I am going to multiply it with all these different factors E1 plus eta1 D, and this way going all the way till EN plus ETA ND.

Similarly, for the second case, we are going to multiply. By the way, please remember always that when I write it like this, because the equations have become too cumbersome, I do not want to write it on both sides. But here what I am implying is that this factor has to be multiplied on both sides, the left hand side and the right hand side, just because I am writing it on one side do not get confused.

This is just for the sake of space. So, in this particular case, you already have this factor there. So, we can leave that out and you can multiply this entire thing with E2 plus ETA 2D and going all the way till similarly for the last case, you can argue that you are going to multiply it with multiply it with, you start off with the first element and then you will end with N minus 1th element, we are running out of space here.

So we will just make it more compact, E N minus 1 plus eta, actually I am really out of space. So I am just going to compress things here. So we have E N minus 1 plus ETA N minus 1 into D. So I hope you can see what I am trying to do. I am trying to ensure that all the Delta X is now have the same multiplication coefficient in front of it, so that I can just go ahead and add on the left hand side obviously, you have the same forces, so you do not have to worry about that.

So, in that sense, on your when you add all this together, what you will end up getting is on this side is very straightforward. You will have all these different multiplied till the Nth factor and

for F, you are going to have a situation where you are going to have, you are going to have a polynomial again in D, and this time the highest power of the polynomial is going to be. So, in this case, for example, you see, we are going to have n number of, so you will have D n times.

So the highest power of D is actually N, do not get confused here, because here when you multiply on this side, you will have the highest power as D N minus 1, but the first time itself is going to give you D to the power N. So the highest power is D to the power N. And let us just quickly check whether we are going to have any term independent of D and that is that looks to be true. For example, in the first case you have he wants all the EIs multiplied. And that will not have any D operator multiplied with it.

So, here on your left hand side you have now have a polynomial such that, so this has been quite interesting. So, this is the end of the sample problem here, but with this we have seen a number of different examples, we started off with one of the simplest possible Viscoelastic Models which was the Maxwell Model. Now, I want you to remember one thing, so you will just make some important remarks about the kind of models that we have discussed.

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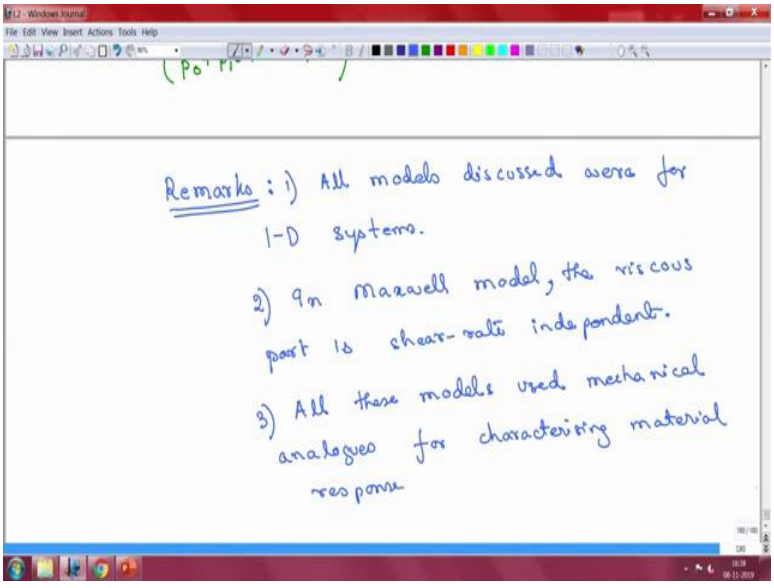

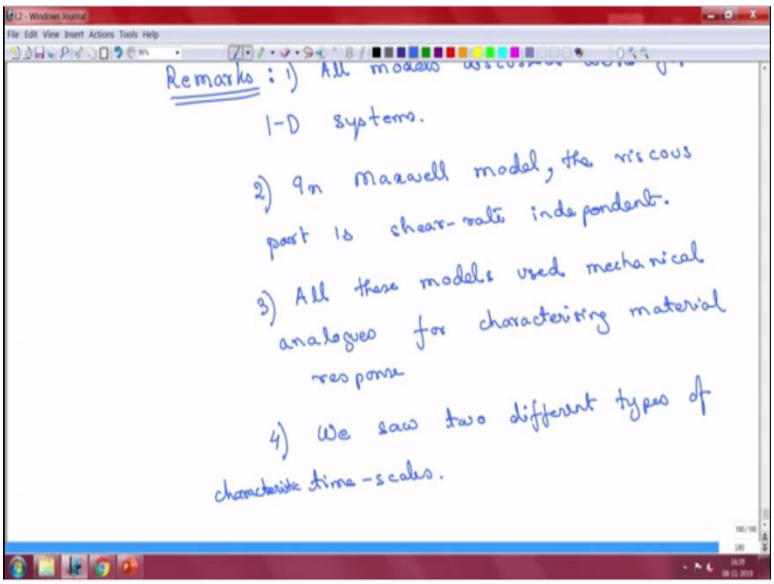

So, all our discussion was always about 1-dimensional models. Because we were calculating delta X, you have to remember that the way we formulated the entire model was, is valid for a 1-dimensional case. So, all models discussed were for 1-dimensional, 1-dimensional cases. So if you have to create models for 2-D and 3-D they include they are actually generalizations of this. But this is, this is an important starting point, but these were all 1-dimensional systems.

Even if you have any number of elements, that does not mean imply that you have generalize it to other dimensions, we are still subject to the restriction that we are discussing only 1-dimensional cases. In Maxwell Model, so and, so in a sense, the viscous part of the Maximal Model is for a Newtonian fluid in a sense. In the sense that the viscosity is not shear rate dependent.

So the viscosity et cetera, were assumed to be constants at all different shear rate. Or maybe I just reword this sentence yet. So, so to just write this, so the second point is that the Maxell model where we discussed please remember that the constants are actually, E1 and etas are actually constants. So those are not viscosity dependent. So, in Maxwell Model the viscous part is shear rate independent.

So, ETA is not a function of gamma dot. So while the Viscoelastic the Maxell model does represent a good viscoelastic model. It will not be appropriate if you want to model of sheer thin thinning or shear thickening behavior. The third important point is in a Maxwell model, the

spring and the dashpot represent phenomenological objects in the sense that there is no real spring or real dashpot in the system.

The spring represents a storage of energy. The dashpot represents a dissipation of energy. Their analogs, their analogs derived from the continuum world and being applied to the continuum world. So, microscopic features of a fluid are not taken into account in the model itself. We have not made any assumption inherent assumption about one of the fluid actually looks like at the atomistic scale.

So the molecule while the continuum approach is nice in a certain sense that it does not rely necessarily on the microscopic details. It does miss out on some, so there are later on we will see that or in a more advanced course you will see that these models there are other models which are better equipped to handle some of the real world fluids. So, all these models used mechanical analogous to model or analogs for characterizing or all these models using mechanical analogous for characterizing material response.

And we saw that for complicated systems you can end up having different type of characteristic timescales. So, there are timescales associated with the relaxation phenomena, where we then call it the relaxation time skills. And then there are characteristic timescales associated with the three phenomena where we would call them the retardation timescales.

So, note we saw different types of timescales, characteristic timescale sorry so I will add that characteristic timescales. So, now that this So, we saw that for an N body system for example, a Maxwellian model where you have n number of elements, maybe the in one case we replaced we said that the viscosity tends to infinity and then we saw that there will be n number of whatever is the order of the polynomial that many number of characteristic timescales will be there.

So, there exists a generalization of this idea, where we have a discrete set of characteristic timescales, we say that in a in a more general sense, it is possible that a fluid can have an entire spectrum of timescale. So, the timescales themselves go from a discrete variable to a continuous variable.

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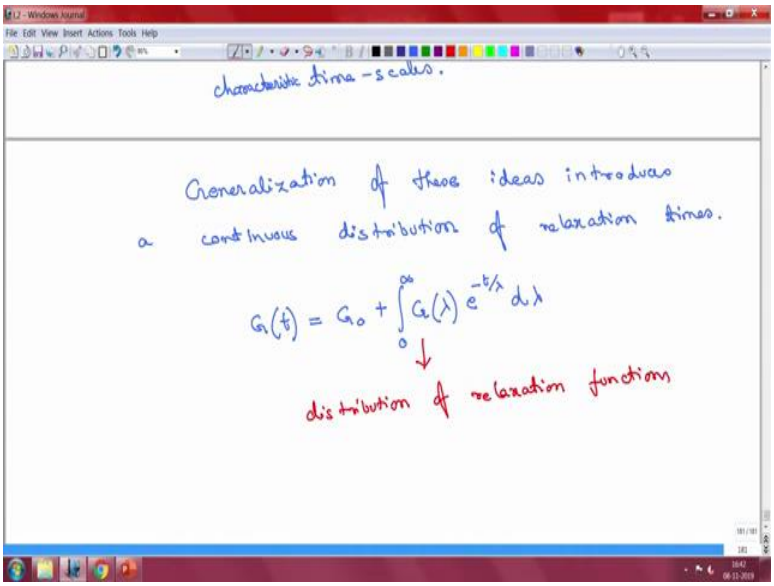

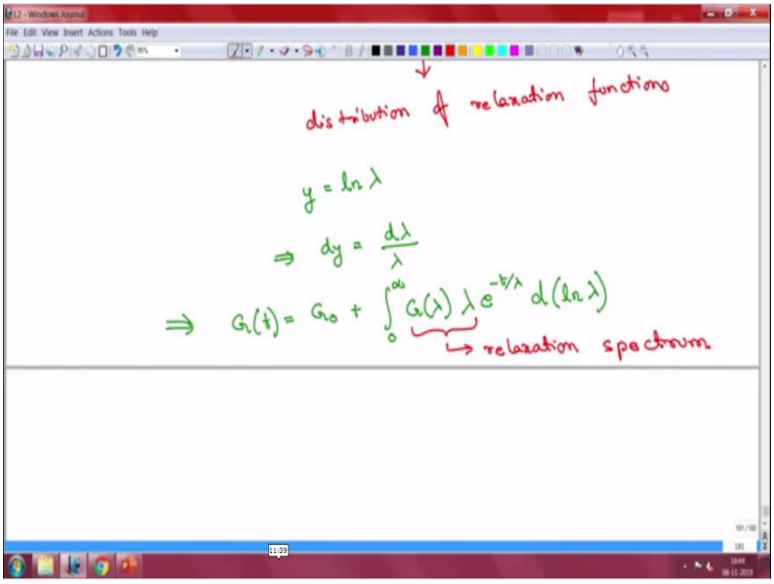

So, generalization of these ideas of the timescale ideas introduces a continuous distribution of relaxation times and in that kind of situation you will write it as this Gt can be written as some G naught plus 0 to infinity some another function G lambda. This is please note that this is a different function e to the power minus t by lambda d lambda. And in this case G lambda is also called the distribution of relaxation functions.

It is customary in polymer science and engineering to not use a linear scale, but to use a logarithmic scale. And in that case, what we would have to do is we have to have to introduce another variable, which is now. So, this the lambda is the lambda is the time to variable the characteristic time scale associated here and you can now take the logarithmic of the logarithm of lambda and let us say why is that variable. So, this implies that your dy is equal to d lambda by lambda.

So, if your previous expression now, if you substitute it there this will become G naught plus integral 0 to infinity G lambda and now I am going to transform it into the derivative instead of being the derivative of lambda I want to take the derivative lambda log of lambda. So then I will just transformed the variable here, this is dy or you can instead of dy you can just write, log of lambda. Either way, it is the same thing.

And now, this particular function is usually denoted by H or it is also called. So, this particular function now G lambda, lambda is regarded as another function and this is now called the relaxation spectrum. So, this is nothing but now your function of a slightly different function derived from the previous case.

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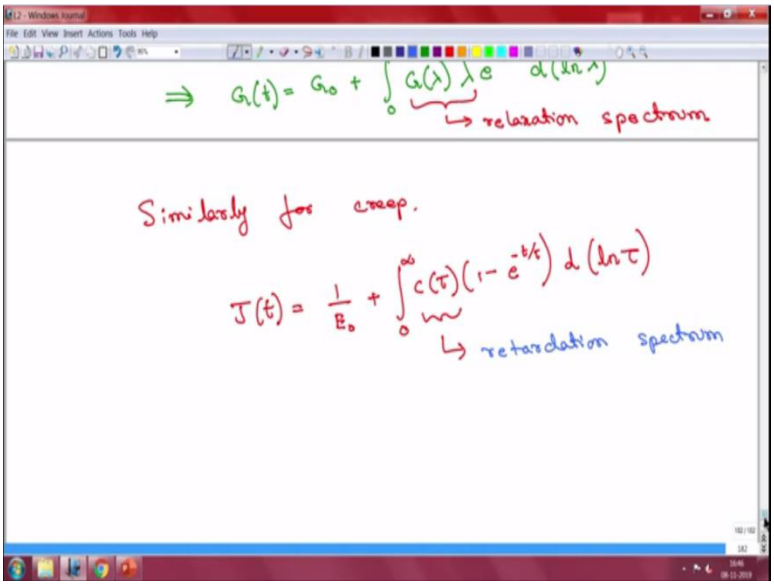

And similarly, for Creep this generalization leads to an expression of the form where Jt becomes some 1 by some constant plus 0 to infinity, C tau, 1 minus e to the power, so I am just using a

different variable here because I have used lambda just differentiate the two cases and d log of tau and then this particular function here, this is also called the retardation spectrum. Now, once you go into this retardation spectrum and to this generalization, you can no longer use a finite combination of springs and dashpots to model this kind of a system.

So, this is so, we have seen that we started with a 1-dimensional system and we went towards we started off with a very simple system and we went on and make made a system more and more complicated by adding more elements. And finally, we have general we came up with an idea of how the relaxation timescale itself varies and depends on a number of timescale that has a number of characteristic timescales built into it. And we finally generalize the idea of the characteristic timescale to get this idea of the relaxation and the retardation spectrums.

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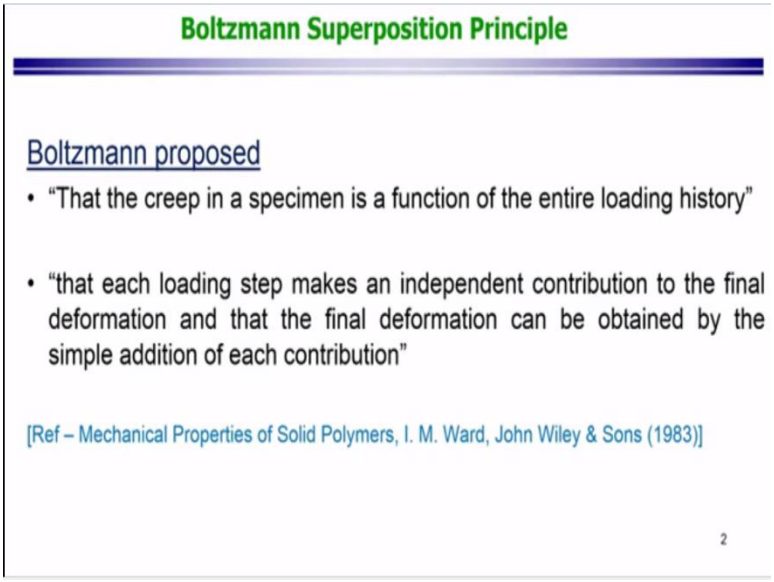

So at this point, what I like to do is I would like to introduce another important principle called the Boltzmann Superposition Principle. Now, both this particular definition or this particular variation of the Boltzmann Superposition Principle is taken from the book Mechanical Properties of Solid Polymers by I. M. Ward, which is a John Wiley and Sons publication. The particular edition that I have is from 1983.

And here, what this book says is that the Boltzmann's Proposition was that and that the creep in a specimen is a function of the entire loading history, that is number one. So, you can see that

Boltzmann is bringing in the idea of memory right here, that you have memory built into the system is very elegantly put. And then at each loading step, each loading step makes an independent contribution to the final deformation and the final deformation can be obtained by the simple addition of each contribution. .

So, this is basically a mathematical statement that which we have sort of already discussed in the case of the superposition of responses. And we will just reformulate that for the case of Boltzmann because we will see that it leads to another interesting generalization, so that is why I am discussing it again.

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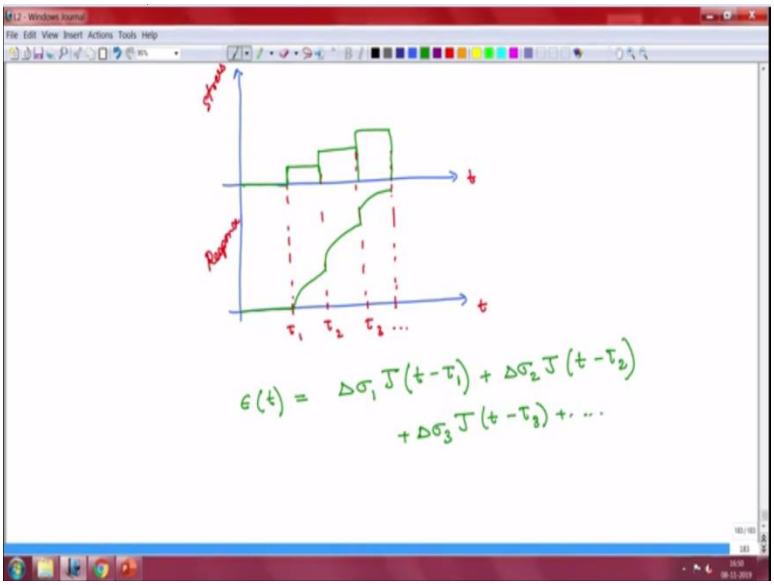

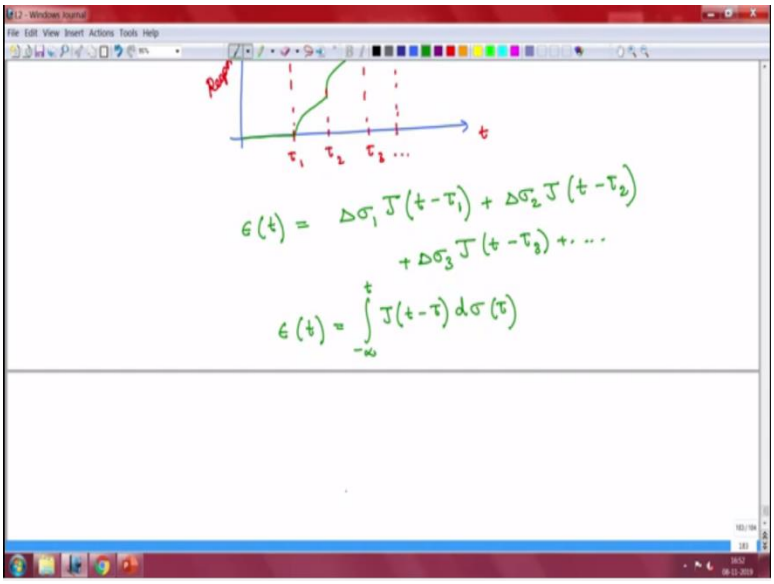

So let us say so I am just trying to put draw what are said in words there. So let us say you have the stress on one axis and this is your input and you are measuring the response. So the response is going to be the strain. And these are the time axis. Let us say you have different points which will denote as tau 1, tau 2, tau 3, etc. and too many points.

So let us say the stress, the stress is such that it is being applied such that you are applying some unit stress between this interval and the next interval you are increasing it to some other value and so on. So, if we have to consider the Creep response here, the Creep response will look as if right now it is 0. And here it will increase because of the stress that you have put, then it is going to increase again now, it is going to be at slightly different function because you have changed your stress.

So, there is another delta sigma that is you have added and then you added another delta sigma here so it will change. If you have to put this into an equation format, if you have a loading program like this, what Boltzmann Superposition Principle is saying that the total strain or the response is now going to be my individual responses summed? So, let us say you are the earlier delta sigma as the stress here that is multiplied by Jt minus tau 1.

This is your Creep compliance, which is shifted now, so that the origin is at tau 1 and similarly, you are going to add the other cases. So on. If you generalize this idea, then you will realize and you generalize this idea is such that you go on making this smaller and smaller. So, these small

time steps into smaller and smaller time steps, then basically you can have you will have a set of rectangles that can approximate any particular signal.

So, basically you are going to go from a discrete case to an integral case, that is what we are trying to do here, you are trying to take a system says that the delta sigma keeps going to 00 keeps becoming infinitely smaller, then what happens. So then, this generalization will get you to so this generalization will get you here. Some people write it as minus infinity to t, some will write it as 0 minus it depends on how you are.

So this is now the generalization of the this. And you can show that this form of the conservative equation is actually it is So, here you can replace minus infinity by minus 0 also, because, what you if we assume that there is no signal t equal to 0 and t equal to zero there is a jump then you can replace it with that. So, we see that this particular format of this equation, this is a consequence of a deeper principle that just comes simply from the idea of the step responses and the generalization of that.

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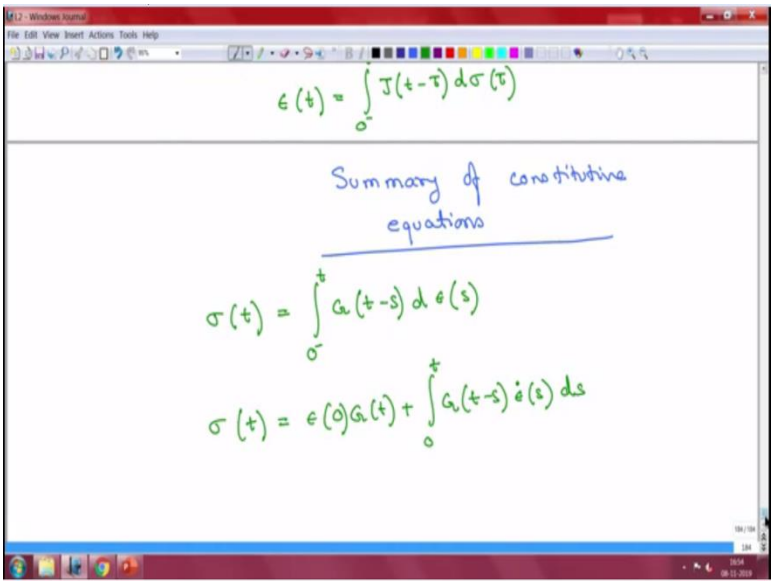

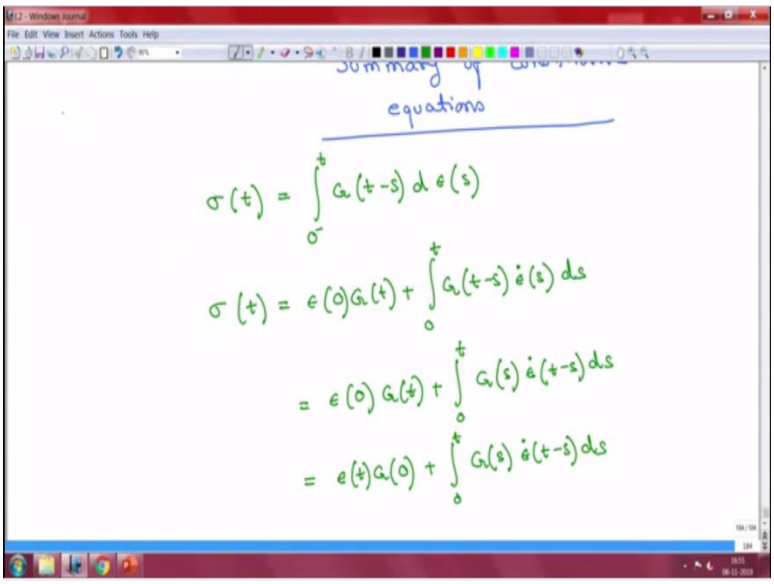

So, now, so, what we are trying to do is we are trying to sort of conclude what we have in general learned and we are now in a position to write the summary of the various constitutive equations. So in our case, this is this goes actually from 0 minus, so I will just put 0 here, minus 0 minus 0 minus.

So, what we saw is that this, this current state of stress can be written as an integral or tau, this s is a dummy variable inside. And this should be such that it is to be interpreted as, you can rewrite this entire thing as epsilon 0 Gt plus 0 to t and you can just change using chain rule, you can change this to this form. Now, this is a convolution equation.

And here you can actually just interchange the order of these and it will still remain unaffected the integral, so you can rewrite this entire thing as, and then you can also switch these two here. So, instead of epsilon 0 multiplied by Gt, you can make it G 0 multiplied by epsilon t, and it will still remain same. So I will just write it here you could just switch the order of this. So, similarly, you will write it for the Creeps, Creep compliance function. So, maybe I just write one of them and then you can.

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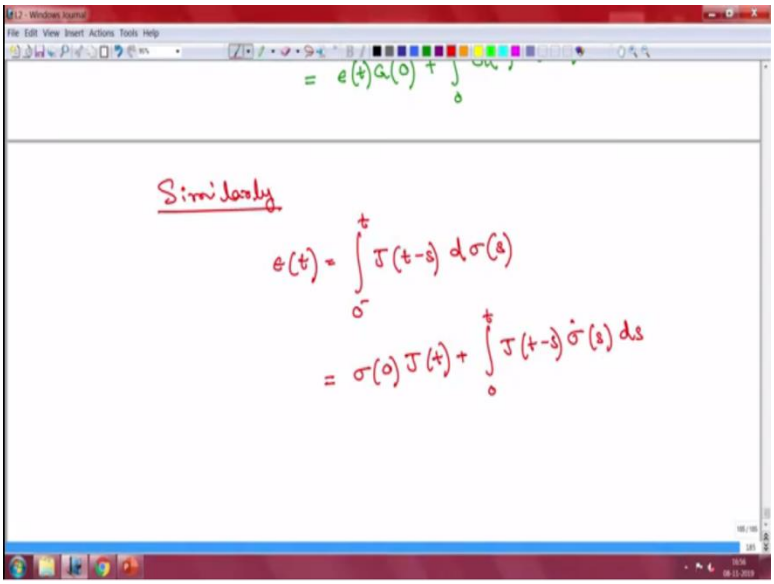

So, similarly, you can write J t minus s, d sigma s and you can derive this from instead of the Superposition of responses. You can either take first the input can either be the stress or the strain. So, the way you argue you can get this. And this is going to be equal to sigma 0 Jt plus 0 to t, J t minus s sigma dot s ds. Yeah. And you can again switch the order of these are these just like the previous case.

So, we started off with a very simple case, the case of a very straightforward one single spring and one single dashpot, either in series or in parallel and we found two ways straightforward, or some of the simpler the two of the simplest viscoelastic models. And then we built on and generalized, it finally do get insights into viscoelastic behavior in a more general sense, so we will stop here for today's lecture. And we will see you next time. Thank you.