Introduction to Soft Matter
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Lecture 26
Kelvin Meyer Voigt model

Welcome back and last time what we were doing is, we were looking at the Kelvin Voigt body and the response of the Kelvin Voigt body to an applied stress. And, we used very similar methods to what we had originally done for Maxwell's body. And, we derived the constitutive equation that this particular system suggests.

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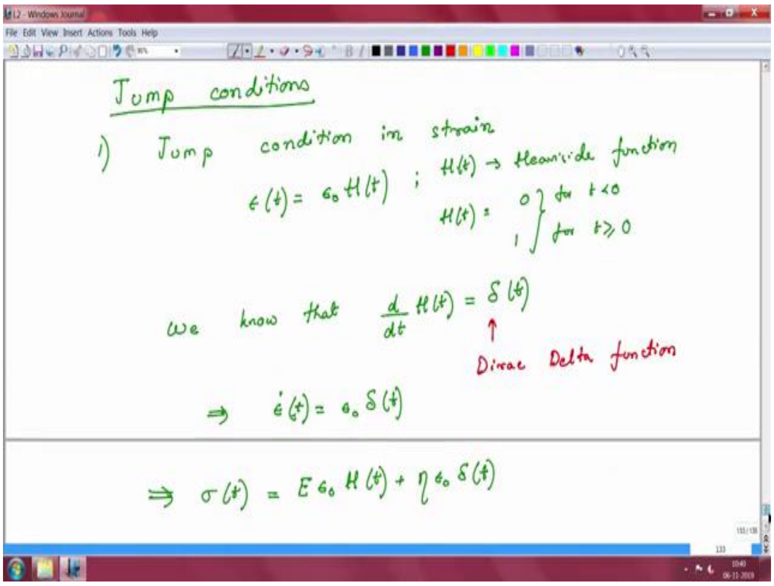

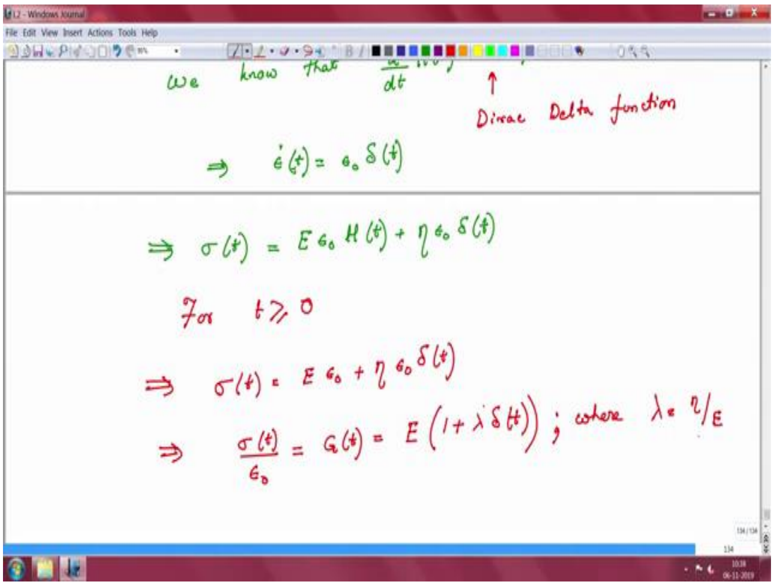

And we ended up with an equation that is linear ODE once again, means the stress and the strain and you have the stress here, as express is the function of the strain plus the strain rate multiplied by suitable factors. So, now, just like we had done for the case of the Maxwell system, we are going to look at jump conditions. So, let us look at jump conditions. So, jump condition, so the let us say the first thing we are looking at is jump condition in strain.

So, I am going to express my strain, some as epsilon not times the Heaviside function just to remind ourselves, what was the Heaviside function? H t is the Heaviside function. And H t is basically given as 0 when t is less than 0, equal to 1 for t greater than 0. So, this is our Heaviside function.

So, if you have a jump in strain, let us say, and you want to find out the effect of that on stress, then what we can do is we can straight away insert this functional form right here. But before we do that, there is one more thing that we have to know. We have a derivative in stress sorry, derivative in strain. So, what happens if you take the derivative of this function?

So, we will, so we know that the derivative of the Heaviside function is the Dirac delta function. This is the Dirac delta function. So, your epsilon dot, so if you are taking the derivative of this quantity, the previous functional form then you will have epsilon delta t. So, if we use our constitutive relationship to find out the resulting stress, then we get stress equal to E epsilon not H t plus eta times of epsilon naught Del t.

So, now if I just want to simplify and I want to write for only t equal to 0 cases, then this implies that the stress is given as E times epsilon naught plus eta times epsilon naught delta t, which implies that we have, if I divide both sides vice epsilon naught then you have this and this quantity is my stress relaxation function.

So, you have sorry, so I have taken E as common, so you have eta by E. So, I am just going to write that as lambda again. So, just remember, where lambda is equal to, so this should remind you that this functional form is something that we also got in the Maxwell case. So, this is a form of this is response time of the system that is the same in both the cases.

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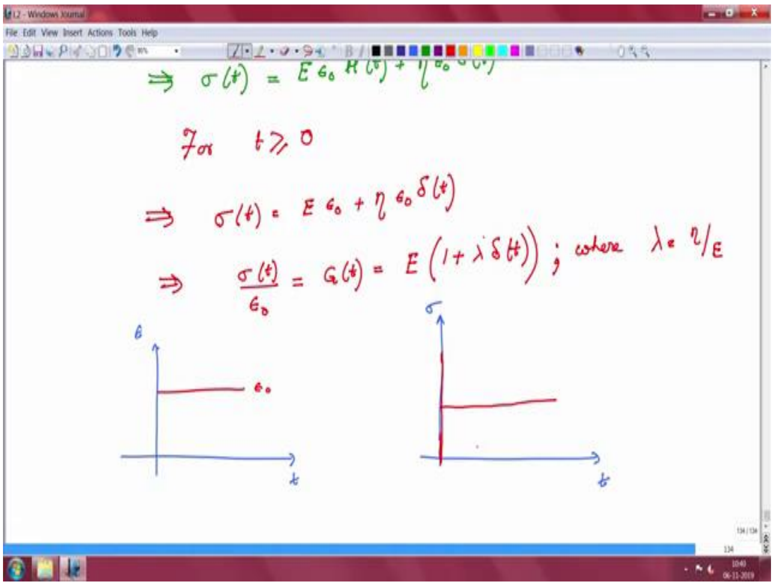

So, we have this functional form, but what does this mean? So, if we are to plot our results, so let us say you have here epsilon verses time, you have, so in this case, what you are doing is you are applying a known strain to the system and then you are studying what happens to the stress history. So, what will happen is because of the delta function, there will be a very sharp jump here, which allows the form for delta function, which goes towards infinity.

And then immediately at any other value of t, this will come down to a constant value. So, we can see that the stress relaxation function is not really very realistic in this case. You have a very-very sudden jump and then it comes back and then it says constant, it does not decrease to 0 which tells you that this system almost behaves like a not almost, it behaves like a solid at times greater than 0. So, this is sort of viscoelastic solid response.

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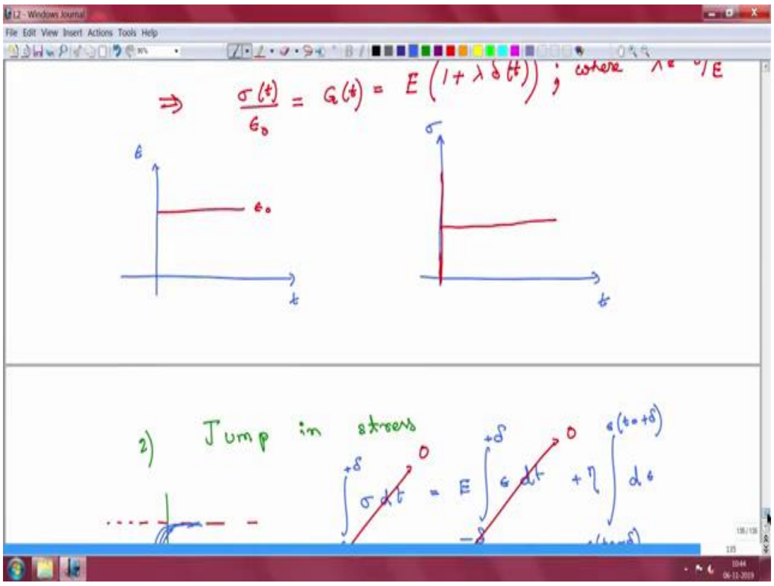

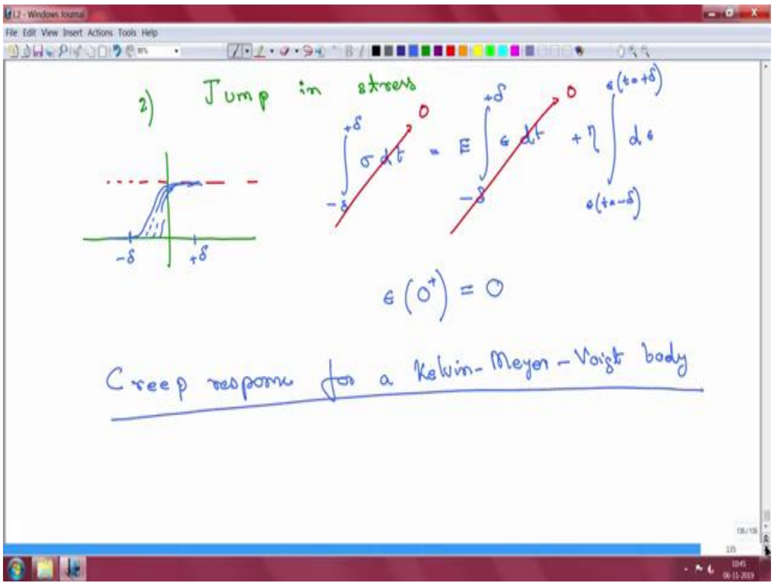

So now, let us take, we have, so we just we just looked at a jump in strain. So, what we want to do now is to look at a jump in stress. So, to look at a jump in stress, what we are going to assume is that with instead of a direct jump, you have the jump as a series of functions that go from 0 to a value of one or sigma not here in a short period of time, this is minus delta, this is plus delta and the biggest deviation occurs between plus and minus delta.

And that you have a series of such functions such that you are basically going to take the limit of delta tending to 0. So, to do that, what we are going to use is we are going to use the previous this constitutive equation and then we are going to integrate over this small timescale of plus minus delta. So, let us do that. So, we are integrating both sides. And here you are integrating is the stream.

So, if the stress is bounded then, so by the way this is once again, this is not very mathematically rigorous, what I am trying to show you is slightly hand waving argument here. A more detailed course, we will go through the mathematical rigor that is required to get to this but the idea is sort of similar, that you consider a set of approximating functions that rise from 0 to value and you take them that they differ from the required functional form in a very short in time interval minus and plus delta.

And in this particular case, what you have is, you have some bounded number multiplied by delta, 2 delta here, similarly here. And as you take the limit of delta tending to 0, both these 2 terms will end up with a 0 and here you will be left with absolute 0. So, this is going to give you the jump in the strain. So, epsilon 0 plus and because these are 0, you will end up with 0.

Now, there is an important issue here, we could not apply this in the previous case and the reason for that is in this particular system, you end up having so just let take a look at this functional form that the stress has a delta function, the delta function is not a bounded function. In fact, it is not even a proper well defined function it is, we use it for its particular properties, but it cannot be used in it is not a regular function in that sense.

So, this functional form here is not bounded and because it is not bounded, you cannot use the idea that this will go to 0 because delta is tending to 0 because you might a 1 term is tending towards 0 but the other term if it becomes unbounded then you not have that situation. So, that is why we had to use 2 different arguments for the system here to get our jump conditions. So, now, let us take a look at, so now that we have the jump conditions with us. Let us take a look at the creep response, Creep response for a Kelvin Voigt body.

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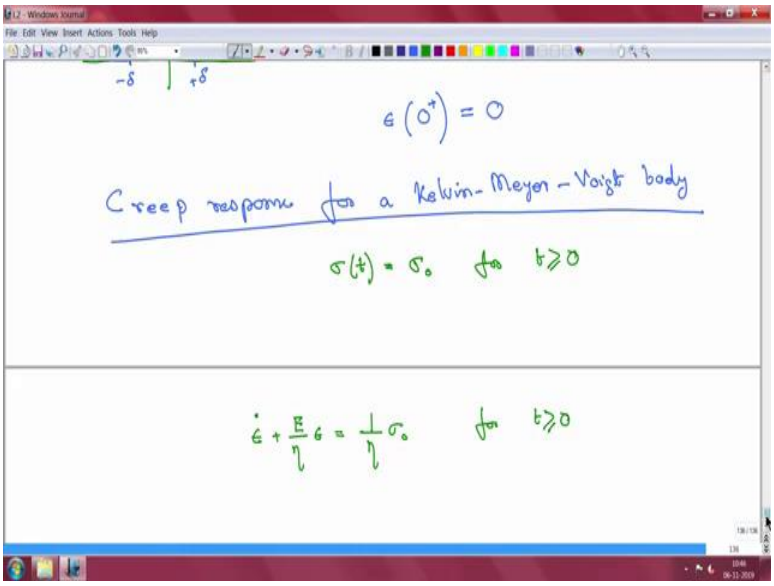

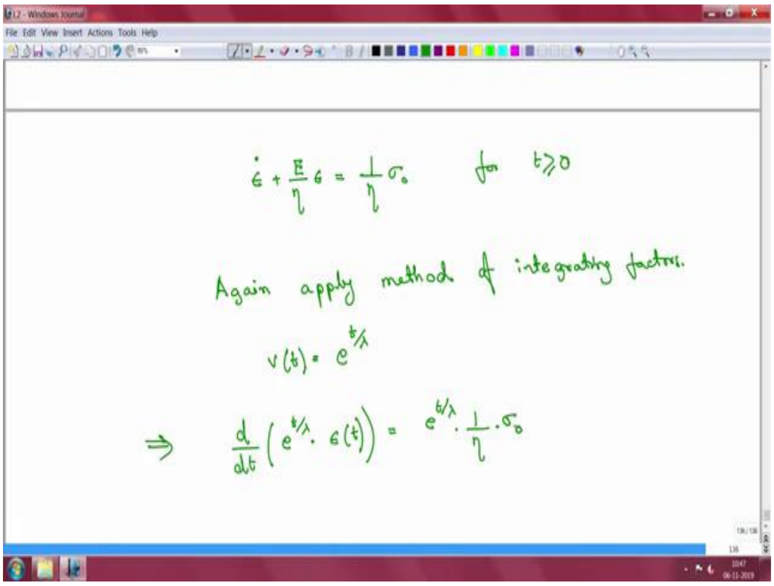

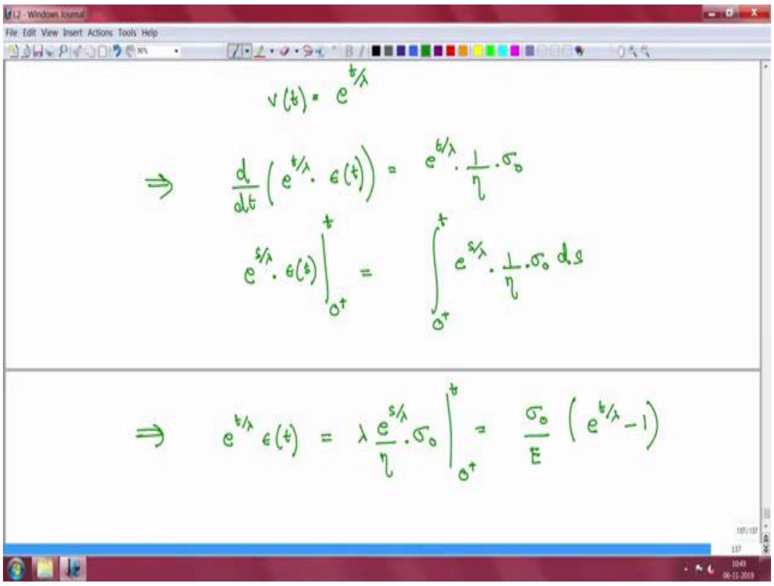

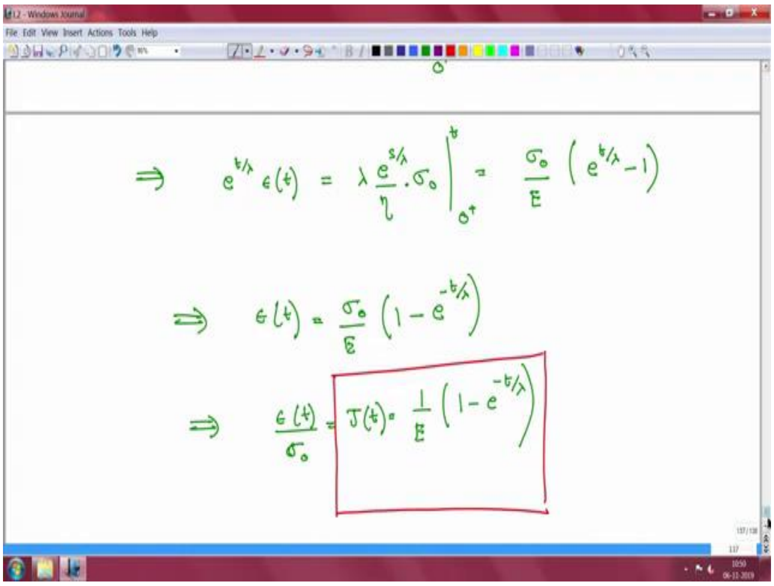

So, for Creep response, we have to what is provided to you is that sigma t is equal to sigma naught for t greater than 0 and now we have to solve this particular equation epsilon dot plus e by eta is equal to 1 minus eta naught sigma naught. So, this is your governing ODE for t greater than 0. So, this value is constant here. So, again we have to apply the method of integrating factors, so again apply method of integrating factors.

So your v t in this case, once again, you will see that it ends up as the same quantity e to the power t by lambda. So, we are left with, so this implies that we have sigma naught. Now, we will integrate both sides from 0 plus to t. So, you have e to the power t by lambda, I just use some other variable because you already have e to the power s by lambda into epsilon s. This we have to consider between 0 plus to sometime t.

And on this other side, we are just going to integrate this particular form. So, you have 0 plus to t, e to the power t by, so again we have to apply some other dummy variable here into sigma naught ds. So, you end up with an exponential e to the power t by lambda epsilon t and epsilon 0 plus is 0 that we just did. So, this is the entire left hand side. And on the right hand side you will have, so this is e the power s by lambda into lambda by eta sigma naught, this goes from 0 plus to t.

So, this will become 0. So, lambda by eta is 1 by e this is, so your epsilon t is equal to sigma naught by e, so this is says 1 minus, I am taking the exponential onto the right hand side. So, now, my final objective is to find the Creep function. So, what I do is I take the sigma naught onto this other side. So, my epsilon t by sigma naught which gives me the Creep function is equal to 1 by e, 1 minus e to the power minus e to the power minus t by lambda. So, this is now your Creep function for the Kelvin Voigt body.

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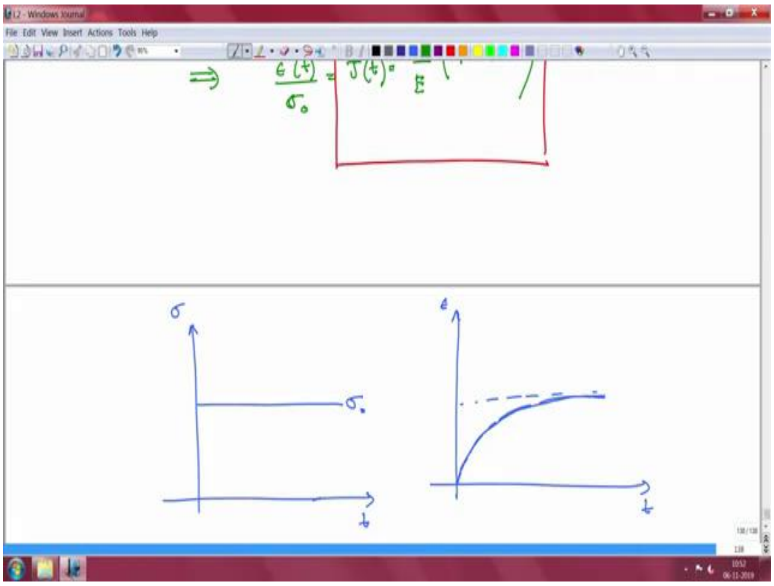

Let us plot this, so that we get a feel of what this response really looks like. So, you are applying a straight stress, a constant stress. So, you will be applying sigma naught here. And then you are going, you have to measure for the creep or the strain value is. And, what you will get is this particular curve? You will get a curve like this, which asymptotically reaches some value. This is your exponential curve right here.

And this represents your Creep response function for a Kelvin Voigt Meyer body. Now, you can ask yourself whether this is physically realistic or not. And the answer is that it does not look very satisfactory, because it starts from a value of 0. If you recall that the intuitive diagram that we had drawn that there should have been some quick elastic response to an applied stress and then there should have been increased with time.

But that initial elastic response is totally missing in this particular case. So, even though when you looked at the stress relaxation function, that behaved as if the Kelvin Voigt, this particular system has a very strong solid body like response, the Creep function does not reflect all of that, it does saturate to a value. But here you do not have that. So, this is acceptable probably in some cases, but it is not a very satisfactory case.

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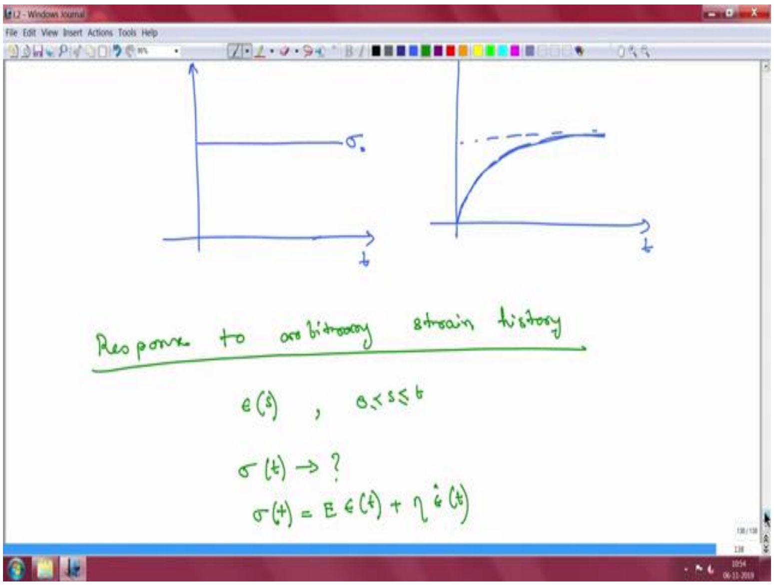

So, we have had that Creep response in this and the stress relaxation function. So, we will quickly look at response to an arbitrary strain that is what we had done for the Maxwell model. So, the first thing that we had looked at there was response to arbitrary strain history. So, basically you have provided a strain history of epsilon s is provided to you for all values of time leading up to t. And the question is what is sigma t?

Now, if we go back to the equation, the governing equation for this particular body, then we have a very simple form. So, we have sigma t is equal to e epsilon plus eta epsilon dot t. So, given arbitrary strain history, all you need to do to find out the current stress, value of stress is simply you have to compute this. So, you can just directly find it. So, if you are given this particular functional form and all you need to get this result is to just plug it in here and compute it directly. So, this is really easy that is it.

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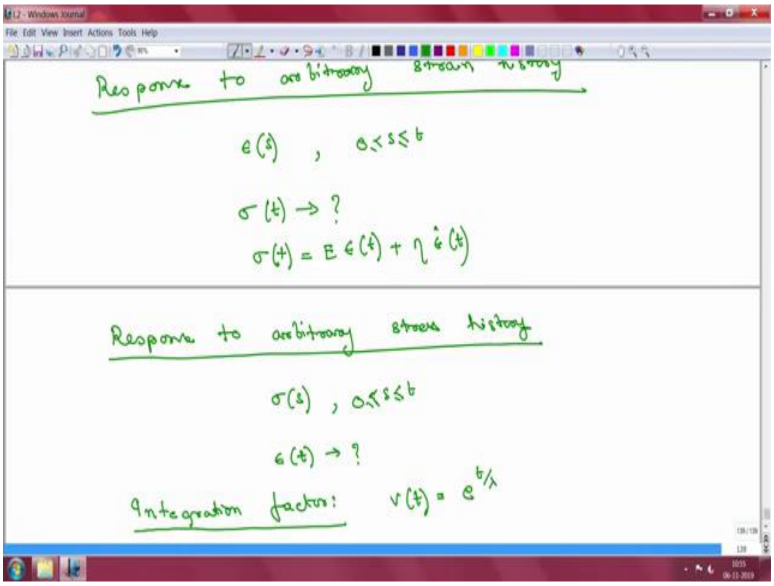

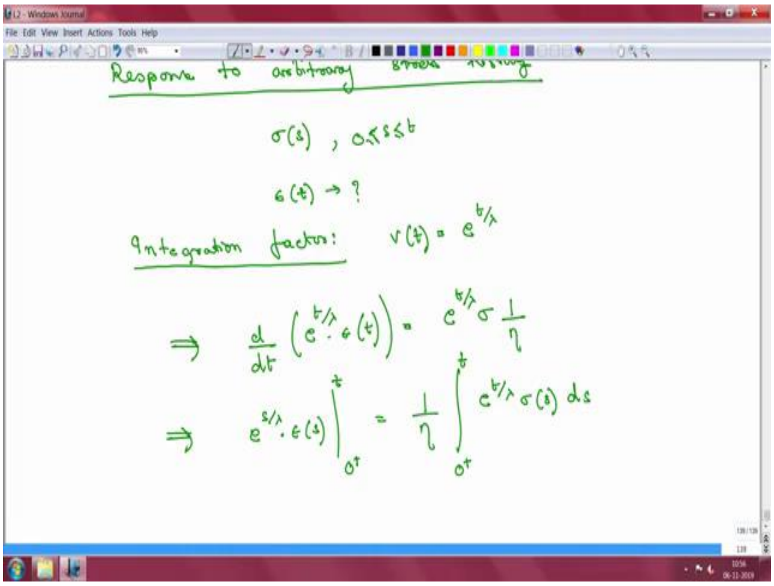

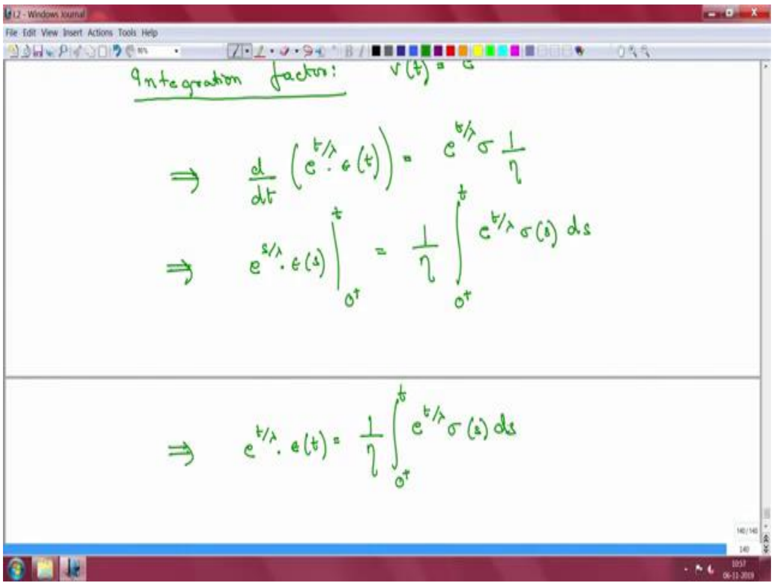

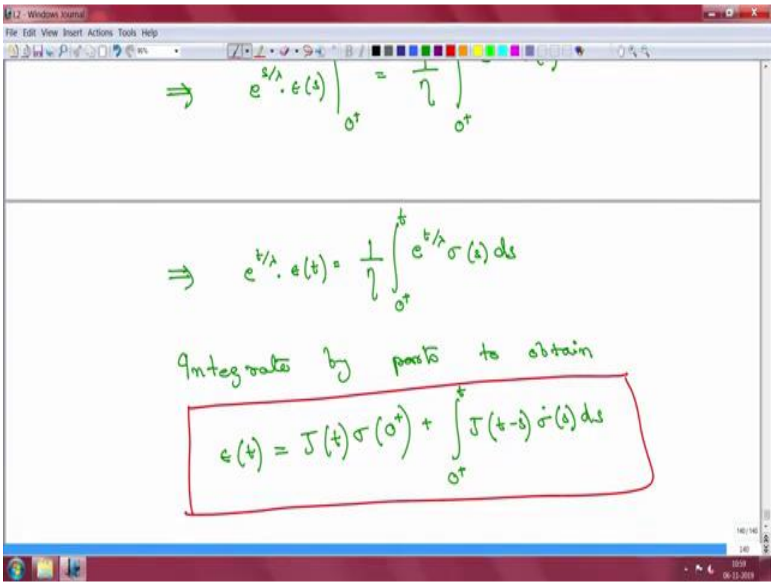

So, now, let us look at the other case, which is, so, this becomes more of us, very simple. So, the response to arbitrary stress history. So, if you want to do that, then basically your stress is provided to you as a function of time up to leading up to all values till t is greater than, and the question is what is epsilon current value of strain?

To do that we realize that we have what we have to solve is basically the same thing, linear ordinary differential equation, which is non-homogeneous and that has to be solved. So, for that once again we need an integrating factor and integrating factor in this case and appropriate integration factor you will see again is the same form t by lambda.

So, in this particular case, what you will get is if you apply this integrating factor and you will end up with e to the power t by lambda epsilon t equal to e to the power t by lambda sigma into 1 by eta which is, so now we have to integrate both sides from 0 plus to t and you have the exponential t by lambda.

So, I am just going to use a different variable here for a second. So, say the exponential s by lambda into epsilon s, this has to be found out from 0 to t. And you have on this side, you have by eta zero plus t, e to the power t by lambda, you have sigma function of s, so we are just going to use the dummy variable s for time. And, we know that epsilon 0 plus is 0.

So, if you apply that, then you will end up with epsilon t by sorry, this is exponential function. So, the exponential t by lambda multiplied by epsilon t is equal to this particular 1 by eta, integral 0 plus to t. Now, I am going to not complete this and I am going to sort of leave this for you as a homework problem.

So, you have to integrate this by parts. So, once again, you see that you have the functional form sigma s, whereas, what you to want is a sigma dot, so here what you can do is integrate by parts because I have done it for the previous case, so leave it here, integrate by parts to obtain and what you will find is, so a surprise, the strain is still given by the same functional form that we had gotten for the Maxwell model.

So, this is and this is rather interesting because the two Creep functions are very-very different yet when you try to find out the strain history, the strain from the stress history, you end up with the functional form that is exactly the same as the Maxwell model. So, this indicates that this particular functional form that we are seeing that I have boxed in red is a more generic equation that governs such models.

So, for today, we will leave it here. And the next class, we will look at once again, the quickly a comparison of the Maxwell and the Kelvin Voigt model and then we will go on to make more complicated models for ourselves. So, we will stop here today.