Convective Heat Transfer

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Lecture – 40
Regime IV – Shallow Enclosure Limit II

So, in the last class, we started with the shallow enclosure and we wrote down the basic governing equations, we wrote down the scaling that in the core parameter, in the core, which are the numbers which are the dimensionless variables. So, if you recall uc was given by as u g beta H cube delta T by gamma L.

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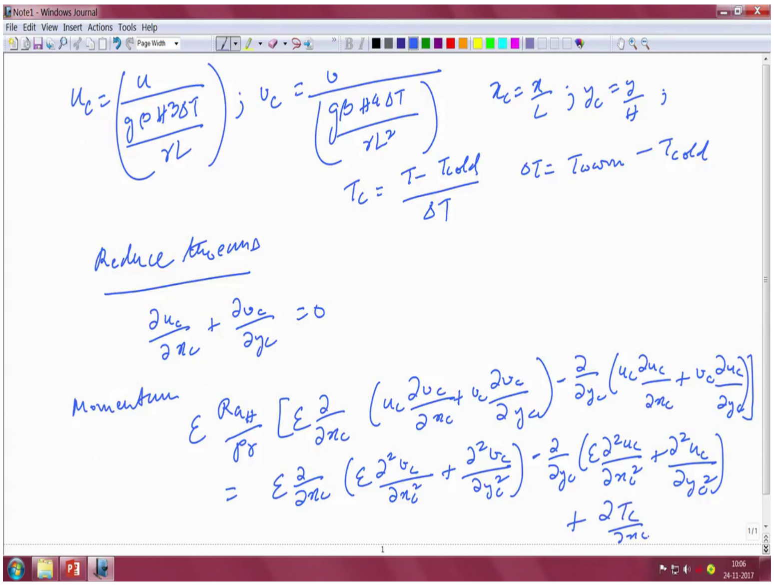

So, that was the first one, then vc was given divided by v divided by g beta H 4 and delta T by gamma L square, where xc, if you recall was x by L y c was y by H and Tc was given as T minus T cold divided by delta T where delta T was basically T warm minus T cold.

So, those were the normalizations that we had. So, you can reduce the equations that we had this also we wrote I am rewriting them once again. So, reduce the equations, so uc by xc plus dvc by yc is equal to 0 . So, that would be the continuity equation and the momentum equation. So, I am rewriting this just for the convenience. So, that one can recap what happened last time, so this is u c. So, that was the momentum equation, similarly, the energy equation.

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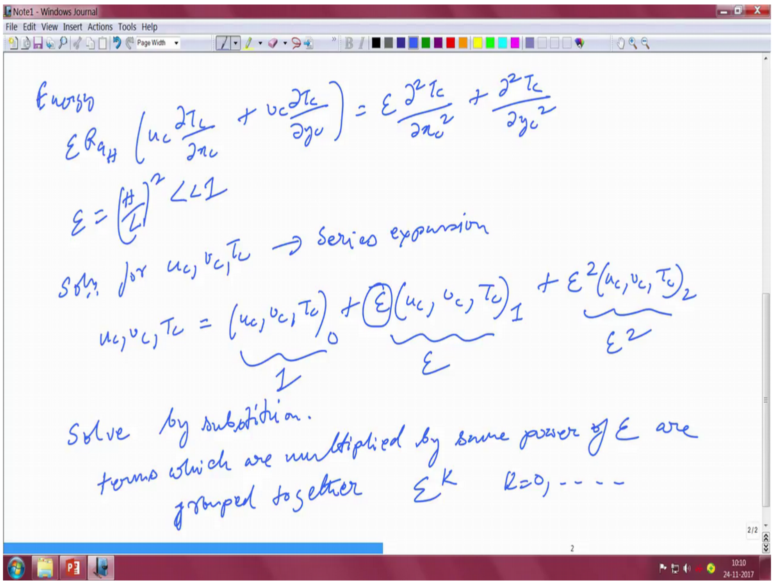


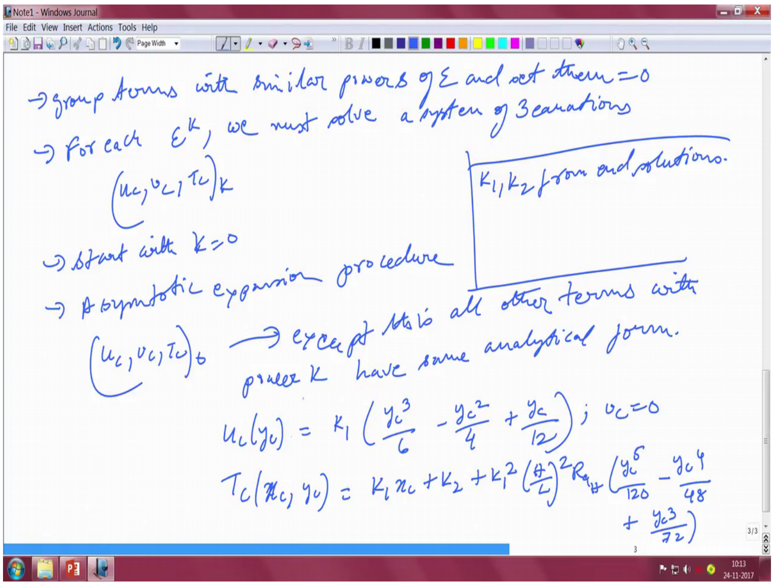
So, were your epsilon was basically H by L square which is much, much less than 1. So, till this point we kind of did earlier also. So, based on this now, what will be the type of solution that we can expect? So, what people did was that, so this is a smaller question. So, epsilon is a smaller quantity, so the solution for uc vc and Tc right. So, these are the 3 variables.

So, we do what we do is basically we do a series expansion. Now, the series expansion is uc vc Tc it is like triplets and then it is uc vc Tc order 0, then uc vc Tc 1 plus uc vc Tc 2. So, this will be of the order of epsilon, this will be the order of epsilon square. So, and this will be of the order 1. So, this is like a series expansion in which because this term is actually very small, we can do a series expansion of this.

Now, what we do is that we take this and we substitute it in our individual momentum energy and energy equation and solve them systematically. So, solve by substitution. Now, so what we do is that we substitute and so, and then what we do? We gather all the terms which has got the same power of epsilon. So, we basically group the terms which has got the same power of epsilon and we set individually those terms as equal to 0. So, this is what we do.

So, basically what we do is that the terms which are multiplied by same power epsilon are grouped together. So, we group the terms which are together. So, for example, if the power is epsilon to the power of K, we group all the terms which has got power of epsilon to the power of K and so we start with K equal to 0 and then we proceed up, this is how we gathered a term sequentially.

So, this procedure is also called the asymptotic procedure or the asymptotic expansion. (Refer Slide Time: 06:45)



So, they basically we group terms with similar powers of epsilon and set them equal to 0. So, for each epsilon K we must solve a system of 3 equations, that means, basically uc vc Tc K. So, we basically start with K equal to 0. So, this is the asymptotic expansion procedure.

So, based on this after we solve, we find that except for uc vc Tc 0 except for this term. So, accepting this all other terms which are of the order x has got the same analytical form. So, except this all other terms with power K have same analytical form, got it. So, uc yc is equal to K 1 this is obviously, 0, is that a core part that we are actually looking at. So, Tc xc yc; K 1 xc plus K 2 plus K 1 square H by L square Ra H yc 5 by 120 minus yc 4 by 48 plus yc cube by 72, got it.

So, this will be the generic form. So, how would you do now? You have to determine this K 1 and K 2 from end solutions; this is remember the core solution. So, from the end solution we have to find out that what will be the form of K 1 and K 2. So, the net heat transfer can also be calculated, so but before doing that we can take a quick look at one of the important graphs.

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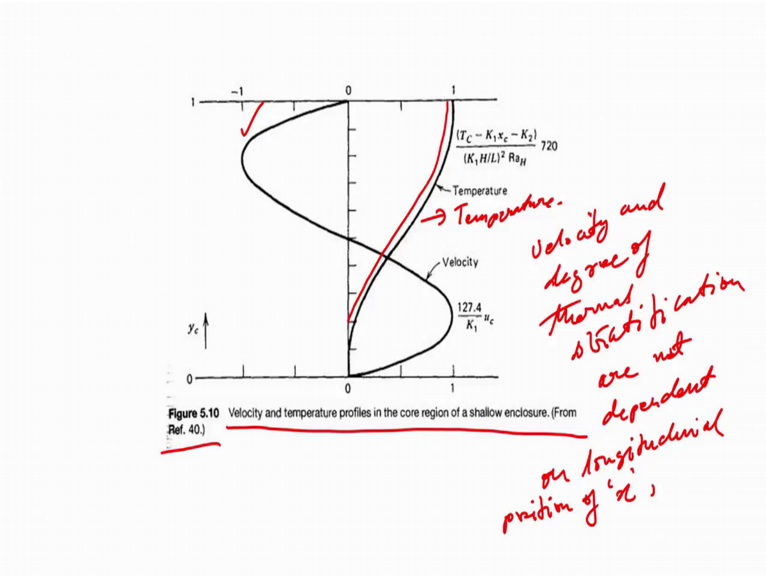

So, this is for example, the graph for the velocity and the temperature profiles in a core region taken from reference 40 from Bejan's book. So, what you can see over here is that this is the velocity profile that you see and this is the corresponding temperature profile. So, the temperature and the velocity profiles are kind of plotted. So, the degree of, so there are 2 things, we can see that the velocity profile and the degree of thermal stratification does not depend on which x you are looking at.

So, long as you are into the core region. So, it is largely independent of x, so you can understand that if you are looking in the core region it is not sensitive to which longitudinal location you are looking at, it is kind of consistent across all the, both the velocity as well as a degree of thermal stratification this is like the degree of thermal stratification both are independent of the longitudinal position x.

So, velocity and degree of thermal stratification are not dependent on the longitudinal position of x, got it. So, you can see that a velocity profile shows that typical form that there is a recirculation. So, you have the typical form it is going back and forth the temperature profile shows this gradual kind of a stratification level from top to bottom which is what you would normally expect, but it is independent which is an important thing to note over here.

So, going back; now, if we look at the heat transfer argument now, so the net heat transfer from T warm to T cold.

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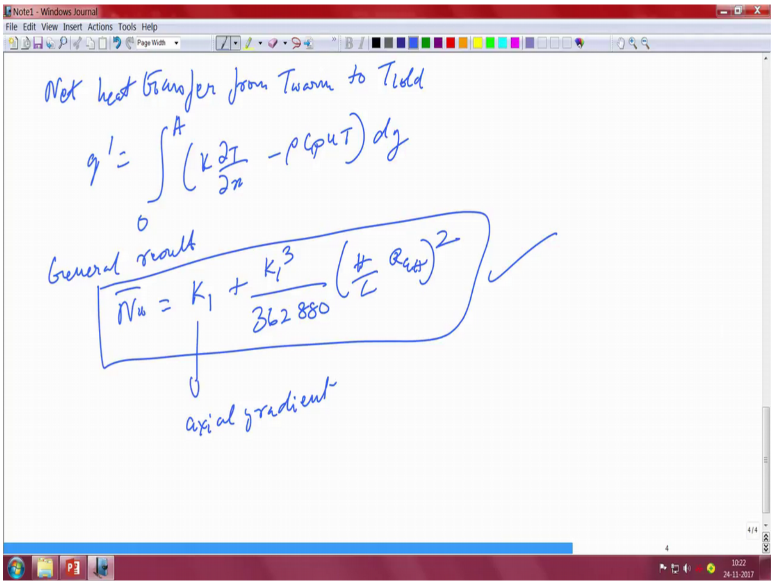

So, that is q prime again this is not really difficult. So, it is the same form as what we had in our earlier discussion that is the form that we have. So, generally this should have a general result will be nusselt number bar will be K 1 plus K 1 cube 362880; H by L Ra H square. So, the general solution, so this is the nusselt number, so you can see that the heat transfer rate depends on the core axial gradient, so this is the axial gradient. So, this generalized formulation for the nusselt number is actually shown in another result which I will pull up right now.

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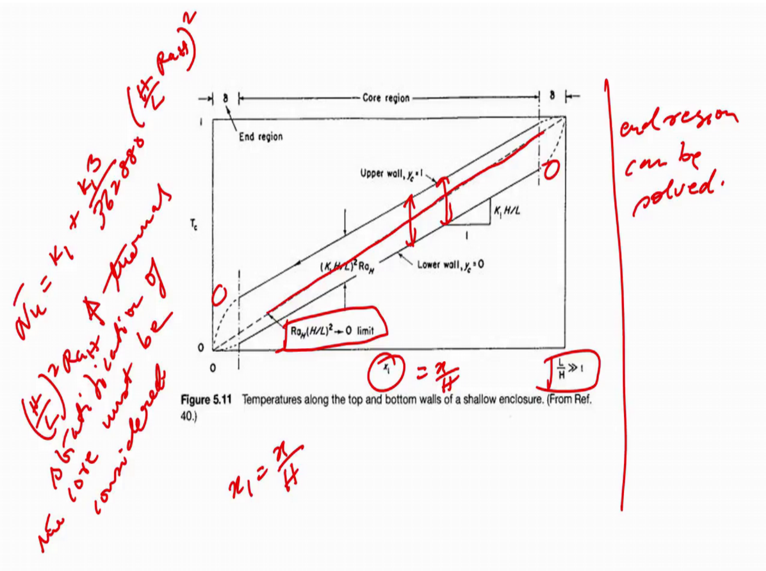

Figure 5.11 Temperatures along the top and bottom walls of a shallow enclosure. (From Ref. 40.)

So, this is figure 5.11 again from reference 40, where we show that these are the 2 these are the temperature profiles, where here of course, x 1 is the same as x by H please note, this x 1 is nothing but x by H this has been done just for the purpose of normalization nothing else and we normalize just by H, L by H because of the shallowness of the enclosure to make the results kind of tractable, so that is what we have done.

So, in this particular thing and I would write down the nusselt number here, just in case we forget because we will spend just a little bit of time discussing this H by L RaH square, so that was the nusselt number that we had. Now, if this value of nusselt number that is H by L square into Ra H, this particular factor as it increases the thermal stratification of the core must be taken into consideration; that means, the 2 temperature profiles that you see we will actually have a divergence, they will actually have there will be a little bit of a gap between the 2, they do not they would not actually overlap with each other.

So, let me write this down also as H by L square Ra H goes up the thermal stratification of the core must be considered. So, the thermal stratification of the core must be considered. So, in other words it essentially means that you have a degree of thermal stratification between the upper wall and a lower wall, that is essentially what we mean and these are obviously, the end effects close to the wall. So, you can see that there is a degree of thermal stratification between the lower and the upper wall, which is quite evident from this plot.

Now, in the limit when this goes to 0, the 2 walls are basically one and the same. So, there is no thermal stratification at all, between the top and the bottom wall there is no thermal stratification the core, but as it increases as the limit of the Rayleigh number this H by L into Rayleigh number square increases you do get a thermal stratification of the cord that becomes very important. So, you cannot make them visible like by a single quantity.

So, however, but when we actually this happens; that means, Ra H into HL square approaches 0, what we can have is that the nusselt number can actually be represented we know much more, I mean by a unique function. So, and not only that the K 1 and K 2 kind of merges in such a way that there is no gradient as such between them. So, this is the main point that we wanted to kind of emphasize and of course, now what you can do parallelly which I am not going to do over here, you can solve for the end region now, end region can be solved.

So, that is rather complicated and it does not really give you any more insights and what we already have. So, what we do is basically that you have to match the solution at the core coming from the end region also. So, that end region has to be now solved categorically. So, for that there is the discussion in Bejan, which you I urge you to take a quick look, but, however, it is not that important to do it over here and it can be left as a homework assignment, but in essence we have covered the core region and he has shown that thermal stratification is important provided you have a high Rayleigh number and HL square at a higher value. So, that is the most important part of this particular argument.

So, now let us look at, so this is the form of nusselt number we already established that. So, there is a discussion in Bejan, where you actually have a summary of all the results of the different types like for example, tall enclosures, shallow enclosures, what nusselt number to use, basically what form of the nusselt number to use and figure 5.14 of Bejan actually shows, some of those nusstle's numbers as well.

So, let us see if I have it here, then I do not have it here, but anyway. So, that is kind of left as an exercise, but that is more like a handbook approach. So, you can just if anybody needs to consult and handbook any practicing engineer among the audience if they want to use an handbook where they want to read out what nusselt number correlations to use say for example, for square enclosures, shallow enclosures, tall enclosures and in the different limits what you are supposed to use then I think that is a good starting point, that graph is a good starting point it has got lots of data it has been compiled based on a lot of data.

So, now that we have done it, we have done this let us look at some of the problem of partially divided enclosures.

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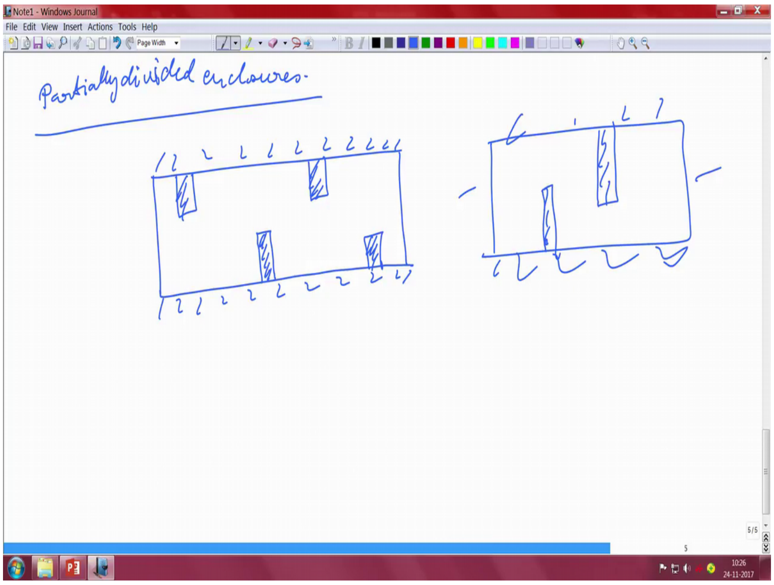

So, the topic is partially divided enclosures. So, partially divided enclosures there will be a few of these kind of things which are kind of rather interesting and comes from a design engineer perspective that what are these enclosures and how they can be actually used, but in essence we will look at some of the interesting problems in such and we will learn that just by using engineering analysis, just by using heuristic analysis how you can actually treat and kind of draw, what will be the kind of design for these enclosures.

So, there are some interesting issues which we will start here and we will take it to the next class also. Like for example, if I give you an enclosure like this. So, whatever I will mark as hatched those sides would be kind of insulated at which they would have that adiabatic condition. So, now, if this is a room, so and if I have suddenly a prop and this is the adiabatic, this particular thing, this may be adiabatic, this may be adiabatic and these are the walls which are basically positive plus delta T and minus delta T.

Now, if somebody asks you that what will be the recirculation pattern in this? Now, you have to, you if you start doing the math out then it is rather complex, but can we through the scaling argument. So, many things that we have learned can we do something to answer this kind and what about if we have another enclosure sitting here? I can have a third enclosure which can sit here; I can have another thing that can prop over here. So, actually rooms, halls that you design can have lots of these kind of enclosures, it would be many things, it could be ceiling tops that are kind of coming down, it could be like kind of partitions that you have created in the room inadvertently or just by design.

Now, you need to know that whether those kind of designs are helpful in maintaining a proper recirculation in the room. So, otherwise we will see what happens if you are unable to maintain such recirculation. So, what happens for example here, this is another typical problem that you have 1 enclosure here, 1 enclosure here like that; so, what will happen to those kind of rooms? Once again this wall and this wall are open the rest are all insulated.

So, if somebody ask you to analyze these kind of problems, then I knew first and foremost should know which design is good. So, the design that design is good which gives you a lot of recirculation, which gives you that kind of natural circulation pattern, but if some redesigns may not be actually good at all, it might get slit to what we call thermal stratification. So, we have to look into some of this design.

So, in the next class what we are going to do? We are going to start evolving some of these designs and we will try to see which enclosures are good, which enclosures are bad. So, in the next class we will just pick it up from here and just do a little bit more systematic study on this, without going into a whole lot of math. So, that should be the whole idea to kind of from an engineering perspective, how can we actually evolve such designs.

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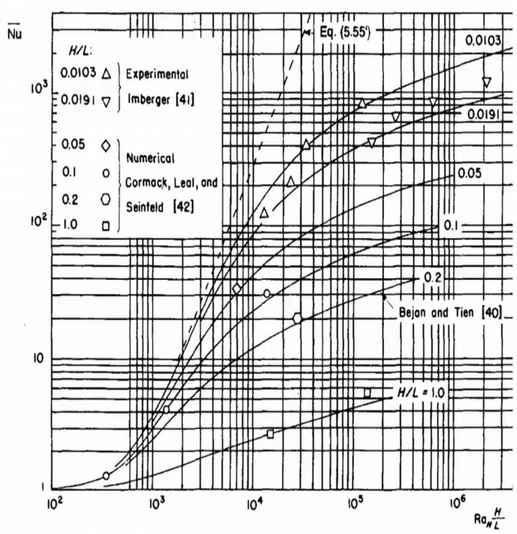

Figure 5.13 Natural convection in a shallow enclosure heated from the side. (From Ref. 40.)

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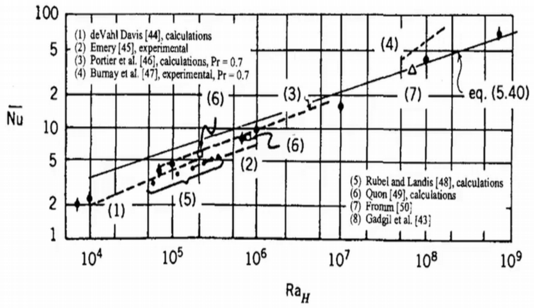

Figure 5.14 Experimental and numerical natural convection in a square enclosure. (From Ref. 43.)

Thank you.