Heat Transfer And Combustion in Multiphase Systems

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Lecture 14
Interface shapes

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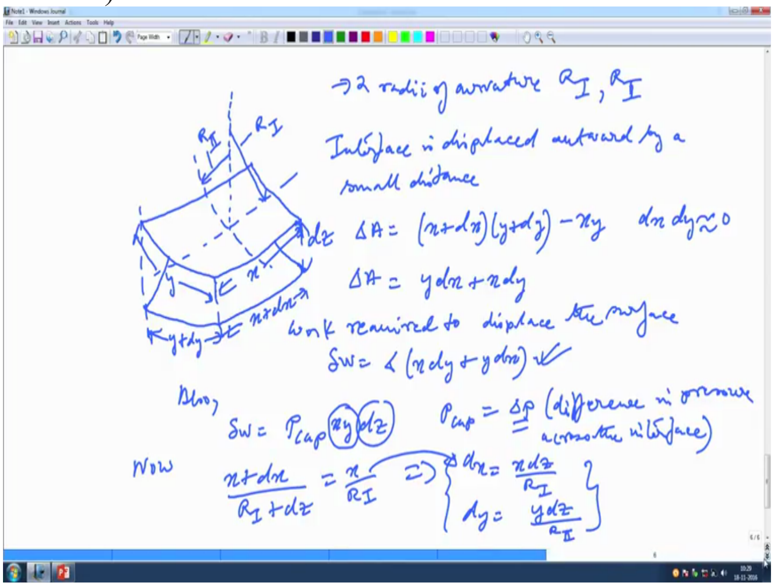

Now let us look at okay, how we can actually interpret this right how we can cause this in a proper way okay. So, any arbitrary interface if I can draw the interface there is a picture also in the book which can be seen okay. So, this interface I am trying to draw an arbitrary interface which has got basically to radius of curvatures okay.

So, this is one radius of curvature this guy okay other guy has got this other radius of curvature okay so this radius of curvature is RI right this radius of curvature is RII okay. This is given by say x, this is given by your y okay. So, these are the two; so, that this particular 2d interface this is an interface or 3d interface there are two radii of curvature okay.

This is RI and RII, R roman 1 and R roman 2 okay. Now this is this interface is now shifted or displaced okay outward by a small distance right. That means it can be compared that there is a liquid vapour interface I am trying to change the interface, the interface is deformed okay. Deform means it is being displaced in some way okay.

So, let us look at that and it is by a small amount. So, all these things we are talking about are minuscule quantities okay. So, it is displaced by this okay and this is how it is when interface looks like, forgive the drawing okay. So, this now becomes x + dx and this becomes y + dy because of the small displacement. This is displaced by dz, right, okay.

So, that interface is changed okay, so, this interface is displayed outward by a small distance okay. So, that also leads to a change area right. You can imagine this to be like a membrane like a balloon right and this balloon is actually expanded a little bit. So, naturally there is a stretching, right, of the balloon okay.

So, this leads to a change in area. So, that change in area is given by x + dx into y + dy correct, that is a change minus the xy, okay. Here we have assumed of course x and y are sufficiently small though these are elemental kind of; it is not a either this entire interface have got one radii; this is radius of curvatures at each point.

So, actually the thing can be very complicated shape right. At each point okay or at each interfacial require I mean, at each delta A or elemental pieces okay. We can define this to radius of curvatures okay. But that can also very spatial okay, what we are not taking that into consideration. So, if you subtract one for the other and if you recognize that quantities like this are basically zero.

That is because it is because it is a product of two very small quantity okay. So, that can be construed as zero. So, delta A is given by ydx + xdy okay. This is what you will get, got it, okay. Now the work that is required to displace the surface okay, now let us look at what work is done because the surface has changed okay.

So, the work done work required to displace the surface is basically given by delta w right. And what it should be it is nothing but Sigma which is nothing but the surface tension x dy + y dx correct, this is the work necessarily that is done against surface tension, right, okay, so, to displace that particular amount okay.

Now also you should recognize that this work is basically done by what? Done by some kind of a; it is done basically by some kind of a capillary pressure we will define the capillary pressure shortly. Capillary pressure is nothing but delta P which is nothing but the pressure difference across the interface, difference in pressure across the interface, got it, clear.

So, the work that is done by this is basically given by the surface tension multiplied by the change in area okay. Similarly the same thing we can also say that this is the work that is done by the capillary pressure right. Capillary pressures which were which actually basically took this area and displaced it by dz, right.

And this capillary pressure is nothing but the pressure difference across the interface right okay. Now that we so these are the two basically the expression okay. Now from similar triangles okay from similar triangles we can say x + dx divided by RI + dz right, must be equal to x by RI, got it, okay, for infinitesimal displacement okay.

You can look at it up from the similar triangle point of view okay. So, in this way you can show that dx is nothing but x dz by RI similarly you can do the same thing for RII okay not all right it is RI and this is RII. You can show that y dz is divided by RII okay. You can show these two expressions.

The first expression comes from this, the second expression you can write an equivalent expression for RII as well right involving y, okay.

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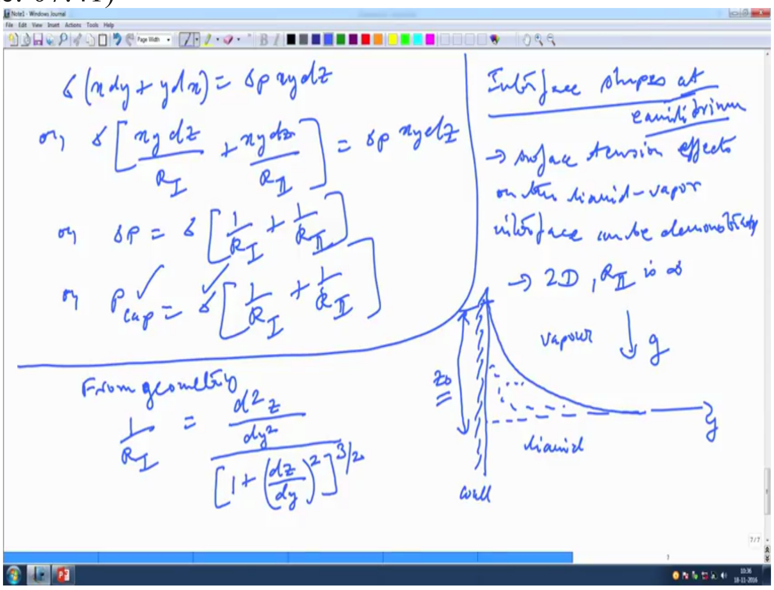

So, based on this now we can say that Sigma x dy + y dx right must be equal to delta P or basically the capillary pressure into xy into dz right. That has to be true, so that has to be true because the two works have to match okay. So, now we can substitute this xdy these expressions. So, it can be represented by x dyz by RI + xy dz by RII.

I am substituting the quantities this is delta P xy dz right. So, xy dz cancels from all three sides right, all three expressions. So, you will basically get delta P = sigma into 1 by RI + 1 by RII okay. So, this is or you can write it as the capillary pressure is given by 1 by RI + 1 by RII right. So, that is the expression that is the Young Laplace equation, okay.

We know this and of course in spaces where there is only one single radius of curvature. Basically the two radius of curvatures of the same as it happens in a sphere for example, okay. So, basically these two will be the same, so you will get 2 Sigma by R okay. So, and in some cases like for example in the case of a cylinder one radius of curvature is infinity, okay.

In that case we will get Sigma by R okay, so you are, so, this is the expression which links a capillary pressure with Sigma right and that is the Young Laplace equation which we will profusely use throughout the course of this of this course okay. Now, so, once that we; now that we have done that okay.

Let us look that what are the interface shape at equilibrium, right, okay. So, this is what we have said that there is an interface and we can cast that how that inter; what will be the pressure difference and Sigma. And we also know that why the interface should have the minimum area. We also have a point of view from the intermolecular forces perspective that how the liquid vapour interaction actually works, right.

Now let us look at the interface shape that is important okay. So, interface shapes okay and well we can demonstrate it in multiple ways but the interface shapes need to take into account basically the surface tension effect like, what surface tension does to the interface shape okay. So, surface tension affects or effects okay on the liquid vapour will choose liquid vapour that is more convenient for us.

Liquid vapour interface can be demonstrated by this example demonstrated okay. By considering this particular example this is a wall okay and there is a liquid which basically wets the wall right, partially wets the wall. So, this is given as z naught this is the y axis this is the z axis okay.

So, there is vapour here this is liquid here okay liquid all over right, okay. So, what happens is that this liquid okay a free liquid surface of a completely waiting liquid okay meets a planar vertical wall right, and it wipes a part of that wall okay. So, if this is basically a typical example in which the interface is 2d in nature okay.

And implying that RII is infinite okay so one part of the interface that is one that is going inside the board right that radius of curvature is basically infinite okay, so you can almost consider that this particular problem extends infinitely into the board. So, is in finite wall is an infinite water source like water or whatever liquid that you can think of.

So, this liquid which partially this particular wall is waiting liquid type okay. And it shows this kind of a curvature and it goes up the wall for about for the distance of approximately z0 and there is gravity that is acting downloads sense and okay the vertical wall. And the liquid is wetting the wall and one radius of curvature is basically infinite, got it, okay.

Now in order to analyze this particular problem, I think you have got a clear idea of what the problem is right. So, we have to study what this interface actually looks like what is the shape of this interface right. How we can describe this interface understood okay. So, this is one of the problems, there can be many other kinds of problems that we can think of.

But this particular problem show that what the interface shape should be okay and this is the interface shape at equilibrium remember at equilibrium. So, there can be cases as we will see shortly where the interface itself will be changing because of the motion okay. So, there can be motion of the interface okay because the liquid and the vapour mighty exhibit relative motion.

But this is the interface shape at equilibrium that we are concerned with in this particular example okay. So, let us look at that from geometry 1 by RI that is the radius of curvature that the single radius of curvature or RI sorry that is given by this is standard you can actually if you have forgotten it please look it up okay.

In this is written in terms of that gradient right the zy surface is basically a zy surface right. It is a geometric surface okay. So, that is the expression that is that should be the expression for R but we have to find out what these terms exactly are, what is the functional form of these terms, right. So that is the whole purpose of doing this particular exercise, okay. Let us move to the next one.

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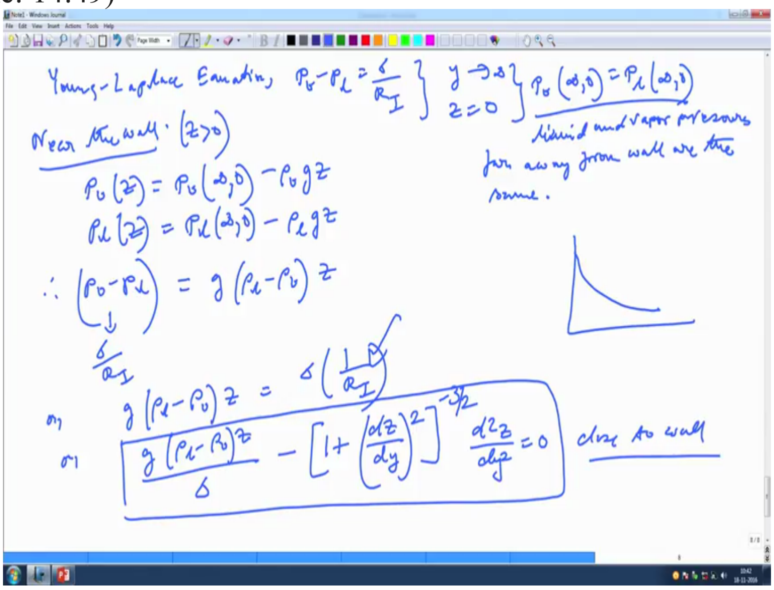

Now for this particular condition whatever is the interface shape we can apply the Young Laplace equation okay which is basically Pv - Pl because here the two phases are the liquid and the vapour and this is given by Sigma by RI, right, okay. Now the interesting part is okay as y goes to infinity okay and z goes to 0 and z is basically 0, okay.

What we will have, you will have Pv at infinity, 0 must be equal to Pl at infinity,0 why is that, just let us look at that. So, what is y equal to infinity and $z = 0$ not be here right, must be here. So, here what happens is that the interface is basically flat right, okay. So, they are the two pressure of the liquid and the vapour phase must be the same, okay.

There is no interfacial curvature so even if you apply young Laplace equation there okay the interfacial curvature is basically infinite, RI is also infinite. So, therefore your P capillary pressure is zero, okay. So, that implies that the pressure in the liquid phase and the vapour phase are the same, right. So, that is why we have written this okay.

So, that means liquid and vapour pressures far away from the wall, from wall are the same right, okay. Now let us look at near the wall. So, that is far away from the wall. So, near the wall what happens, near the wall means z greater than 0, okay, got it, z just greater than 0 that means along the wall okay.

So, your z is upward axis right, okay. So, in this particular case Pv z should be equal to Pv infinity , 0 - Rho v into hydrostatic z, right, got it, okay. Similarly Pl at any z should be equal to Pl infinity , 0 – Rhol into gz, got it, because this is just we are adding whatever is the height of the liquid of the vapour, column, right, over here. Now in this particular; therefore Pv - Pl which is basically nothing but the pressure difference across the interface right.

That should be equal to g Rho l – Rho v into z okay. So, this is nothing but your sigma by RI right or in other words now g Rho l - Rho v into z minus should be equal to Sigma divided by 1 by RI and 1 by RI we have already wrote the expression what is going to be 1 by RI. So, therefore or g Rho l - Rho z divided by sigma - 1 + dz by dy square - 3 by 2 d squared z by dy squared must be equal to 0, got it.

If you do not understand just look at it so what we have done is that we have to get this expression right this is already there okay and we have applied it here right it is a straightforward application of the same right. And we have brought things to the same side okay. So, this is the expression that that is, that we get by considering close to the wall.

Away from the wall we are not bothered the pressures are the same right. The curvature is non-existent right. So, close to the wall is what we are really interested in because there it shows that curvature, right. Close to the wall is the where it shows that curvature, correct. So, this is close to wall not far away from the wall, okay.

And this happens for example if you put a liquid in a capillary tube or through if it tries to rise through a narrow channel all right or so, that is the kind of curvature that you would expect the liquid to show, right, okay after this part it should be clear. Now we take this expression and try to see how we can get a more simpler form.

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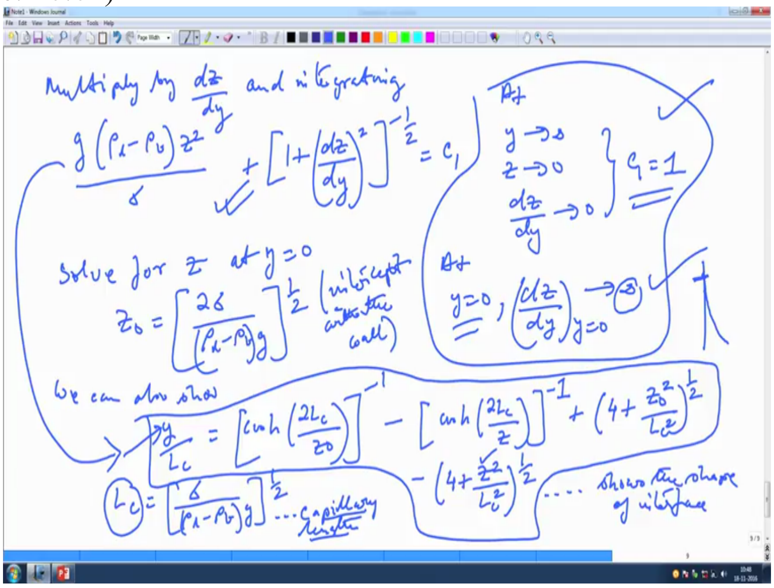

So, we multiply by dz by dy this expression and integrating, integrate the same what we get is g Rho l - Rho v z square you can work out the math individually I would just show the expressions, right, okay. So, this comes with the boundary conditions R at y goes to infinity okay once again away far away and z going to 0 okay, dz by dy should also go to 0 correct because the curvature becomes non-existent.

There is no relative slope between z and y okay that interface anymore. So, this should lead to C1 going to 1 okay. So, in other words, so, that is one boundary condition at y = 0 okay that means absolutely at the wall right at the wall y = 0, dz by dy at y = 0 approaches infinity, right, okay.

Because it meets, how it meets the surface okay at it approaches actually infinity. So, therefore using these two boundary condition and this expression okay. We solve for z right at y = 0 that means what is the rise or basically z naught okay. So, z naught is basically therefore given by 2 sigma by Rho l - Rho v into g raised to the power of half okay.

It is raised to the power of half that is z naught that is the height through which the liquid actually moves okay. We can also show that y by Lc, we will define Lc shortly do not worry. So, this is if you solve for the whole thing more complicated okay. This particular expression basically shows the shape of the interface.

We will see that what Lc is do not worry the Lc is basically given by just an expression and okay, got it, okay. This has also got a formal name it is called the capillary length is a characteristic capillary length basically okay. So, if you look at this expression what does this expression actually give you.

This expression actually tells you how y is varying right, y are the interface is varying okay as a function of z right at different z's what is the value of y, right. So, that how; that is a functional form right and it is we have cast it we have normalized it in terms of the capillary length, got it, okay.

So, this is the intercept that it makes with the wall okay and this is the dynamic shape of the interface okay under steady state conditions obviously okay. So, there is no interfacial movement as such okay. So, what we have done we have just used to geometry, we have used the definition, the Young Laplace equation.

And we have been a; and we have integrated the same and we have shown that this is the total expression that; of course you need to work out the math that how did you get from here to here okay. So, that is a lot of algebraic steps okay you can show that this is the expression that you get using these boundary conditions.

These boundary conditions are pretty simple away from the wall there is no pressure the interface is flat basically okay. And at y = 0 that means at the wall right at the wall okay you have your gradient of dy, dz by dy because it is completely wetting liquid that is we that is what we told okay that should actually go to infinity completely wetting means which the contact angle becomes actually infinite right.

I mean the contact angle becomes actually 0 okay. So, here you can see the meeting is like this. So, the meeting was something like that right okay. So, here we are talking about that okay. So, naturally that curvature we have taken into consideration right, okay. So, this shows that this is the total expression.
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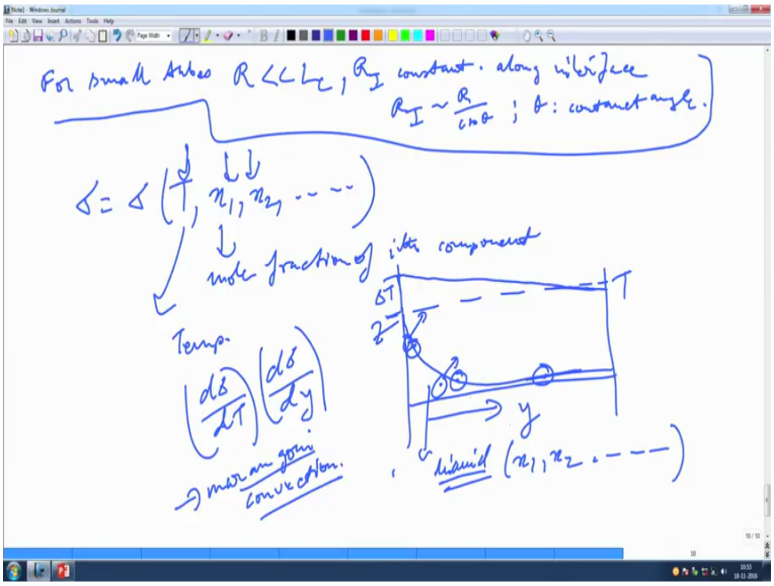

Now we can do some simplification say for example for small tubes here we have solved it without mentioning what is the physical scale of the problem right. For small tubes what happens, the radius of the small cube is much, much less than Lc. LC is basically the capillary length right okay and RI is basically constant, got it, okay.

So, constant RI is constant along the interface and you know its value is basically given as R cos theta okay where cos theta is basically whatever is the contract angle, okay that it makes. So, for small tubes we can write a slightly simplified version. There can be other things also say for example your Sigma can be a function of temperature it can be a function of different molecular species okay.

Where x is basically the mole fraction of ith component you know the standard and T is basically the temperature. So, because surface tension can vary with temperature right, so, even if you have this particular problem, the same problem that we took okay, right, of course this flattens out.

So, here for example there is a temperature gradient say for example, so here the temperature near the interface may be high, here it may be low, okay. So, if I plot the temperature on the other axis of the interface. Let us say this is a hypothetical wholly that the; this is so this is the V axis is the temperature axis right.

So, you are plotting it across the interface is still y okay for you so across the interface say the temperature which assume there is some initial value. So, you can get going like this say for example it can be a linear temperature like that right. So, there is a temperature differential of delta T that is created along the interface right.

We are not concerned about the depth at this particular point it is along the interface there is a temperature gradient that is created okay. Because of this temperature gradient what will happen is that if you plot the corresponding surface tension now, that surface tension will also show a gradient, is not that so.

So, surface tension here will not be the same as surface tension here right and surface tension here will be different from that correct. So, there is a variation in the surface tension because surface tension is a function of temperature like all properties which is the function of the temperature also right.

There can be also one other thing we have assumed that this liquid is of pure liquid right okay. It can be a mixture okay, so for example we can take a ready example, say for example you take a pipe and you just poured some diesel into it. Diesel is inherently multi-component in nature it has won multiple components right.

So, this liquid can have x1, x2 lot of component right. And these components need not be the same across the interface right. There can be more component there can be more for example x1 in this region compared to this region right, close to the interface right. So, that leads to one that also leads to a change in the surface tension.

Surface tension of each liquid is different. So, when you determine surface tension of a mixture that will be all so different because it is a mixture of different species right. So, moment you actually have this kind of a multi-component system along the interface there will be a lot of variation in surface tension as well right.

So, there is a variation there can be a variation in temperature there can be a variation in species concentration. All these will lead to there is a surface tension gradient basically. So, there is what we call a d sigma by dt or d sigma by dx dy whatever you call it okay. There are terms like this. Now in this previous analysis what did we assume we assumed that the surface tension was basically constant, right.

Is not that what we did we assume that the surface tension was basically constant everywhere right. When the first extension is not constant everywhere like for example in this particular case that we mentioned right okay there can be actually flows or velocities that can be introduced within the liquid okay from high surface tension and low surface tension.

You have a difference in surface tension so basically you have difference in pull right. So, this can actually drive a liquid flow okay. There is a formal name and we will come to it a little later when we do the sessile droplet. This is called Marangoni flow or Marangoni convection. It can happen due to concentration gradient which is basically the species gradient or it can happen due to temperature gradient.

I mean temperature gradient of surface tension dependence on temperature and surface tensions dependence component okay. So, what we have done we have determined the shape of the interface we have understood what the interface should look like right. We have established a Young Laplace equation and we know that how the molecules actually behave inside the liquid in and close to the interface right.

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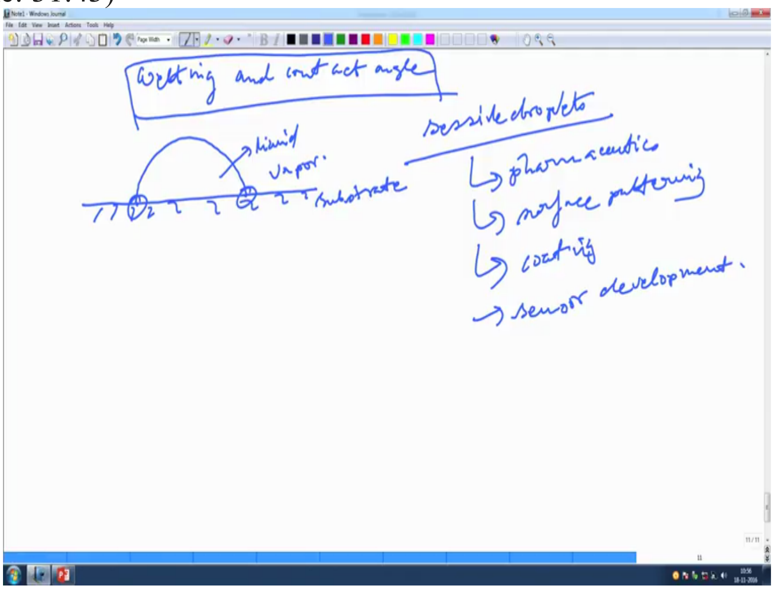

Now we will start with will introduce the problem here and complete it in the next lecture that wetting and contact angle. This is a problem this goes into a very standardized problem that we need to understand and we will do Marangoni and all these other flows in here that for example this is a very common phenomena that you have seen right.

You spill coffee, you spill lot of you know water on the surface sometimes you see that this forms this nice spherical cap type of a droplet correct does it not form. It forms that spherical cap. This is a substrate right okay and this is the liquid and this is a vapour right, okay. Now this is a very typical scenario, now you have all the three phases now, existing at this point, this point basically across the circumference, right, okay.

How to analyze problems like this okay because this is a very common problem this happens not only it, every application will have this sessile droplet this is called the sessile droplet. So, this has not applications from pharmaceutics right, pharmaceutics to surface patterning to coating right got it, okay, sensor the development. You name any application it will have a sessile droplet, okay one way or the other.

So, it is important to know how we can analyze this sessile droplet what are the flows that are introduced within this sessile droplet and how we can actually state. So, why do we actually do this we actually will look into the effect of surface tension and other things on the sessile droplet evaporation phenomena.

If evaporation and the dynamics of the sessile droplet okay. So, this is one of the first specialized topics that we are going to enter immediately. So, you I think you understood what we did in this particular hour. So, we covered all these interfacial physics as much as we could okay.

And now we are going to analyze this particular problem the wetting problem and how you do determine the contact angle okay. So I will see you next class.