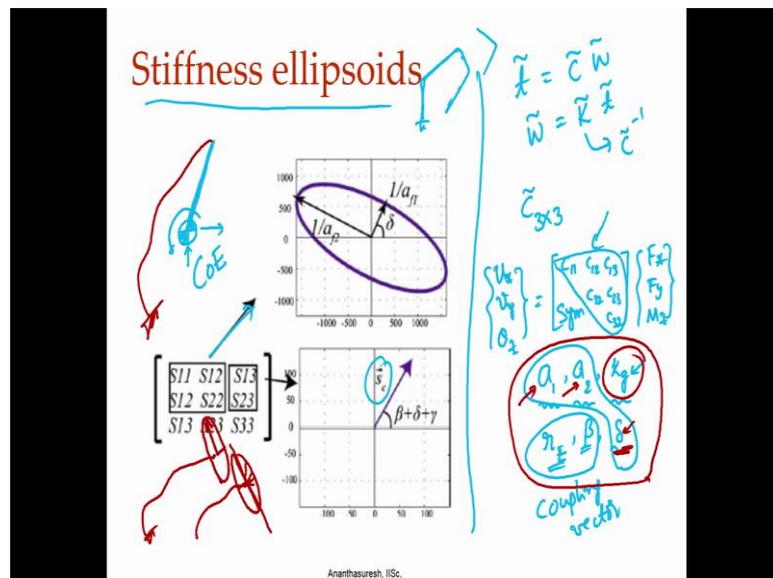


**Compliant Mechanisms: Principles and Design**  
**Prof. G. K. Ananthasuresh**  
**Department of Mechanical Engineering**  
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**Lecture – 51**  
**Building block method of designing compliant mechanisms**

Hello, this is continuation of the previous lecture which was 50, this is lecture number 51. In the previous lecture we talked about compliant ellipsoids where we said given a compliant segment how many our number of segment it has, we can find one end is fixed other end is where you want to imaginary rigid body being there apply forces x force, y force and rotation about the z axis movement about z axis, then we can represent the effect of the entire compliant segment with basically springs. There is x spring, y spring and torsion spring, when you do that you will have matrix relating the twists what we call displacements and ranchers which have a forces.

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So, what we have represented was twist vector is this compliance matrix times ranch factor, other way of writing this is if I want to make it in our usual a stiffness way then the ranch is k times t and k here is the inverse of c or rather c is inverse of k this is called compliant, that is called stiffness. With complaints we said if it is a 2 d problem this c will be a 3 by 3 vector.

So, what we have here is that, we have this  $C_{11}$ ,  $C_{12}$ ,  $C_{13}$ ,  $C_{22}$ ,  $C_{23}$ ,  $C_{33}$  and then here will have  $F_x$ ,  $F_y$  and  $M_z$  and here will have  $V_x$  and I can say  $V_y$  or  $\theta_z$  of a rotation that is what is a compliance is not zero sorry symmetric like so it is a stiffness matrix compliances symmetric. How many quantities do we have here, we have 6 and those 6 are represented in there in variant form when we do Eigen analysis that is what discussed in last lecture.

So, we said that there will be a  $\alpha_1$  or  $\alpha_2$  and then there is  $k_g$ , this is the compliances, minimum compliances, maximum compliances and rotational compliances that is we are represented that like a Eigen spring in 2 directions and Eigen rotation spring about the  $z$  axis and then we have also something that locates the coupling vector magnitude  $r$  is call  $r_E$  locate center of elasticity and then there is an angle  $\beta$  and there is a  $\delta$  related to this. These three will tell you the ellipse or ellipsoids orientation, major axis, and minor axis this is the rotational stiffness; these two indicate the coupling vector.

So, again we have to look at what we had discussed that if I have a segment let me take any segment does not matter we are only took compliant dyad does not matter whatever is the elastic body that is there, it can be even continuum. We represent this whole thing instead of doing all this first imagine that there is a rigid body here; there is a rigid body this rigid body let me use a different color for the rigid body, let us imagine that there is a rigid body which is now suspended by this compliant segment we do not want worry about details the compliant segments we only want to look at that one in terms of a lumped model as if there is a spring here, there is a spring here and there is a torsional spring here, that is how we say lumped model.

But, now what we are saying is that it depend the coordinate system the way you do it, but these things that six quantity that we have here there in variant to the coordinate system, once you have this there is a direction in which there is maximum and minimum compliances and minimum or maximum compliances orientation there, that it the stiffness that is if I take this funny looking thing here there will be in ellipse.

Let us say for the sake of discussion it in the direction, here is where it will have maximum displacement or maximum compliances perpendicular to it will be the minimum compliances; you have to understand that when I have a segment when I take this a same

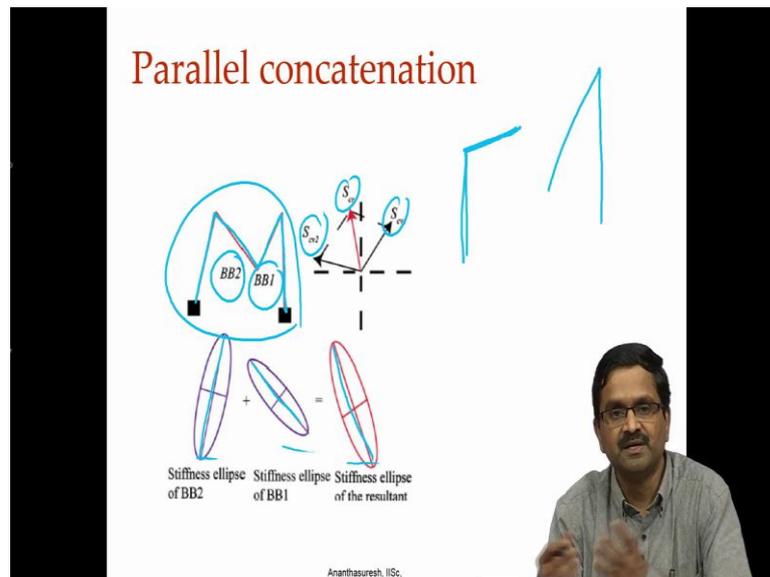
force as I already said in the previous lecture, the same force if applying in this direction am I get only that much displacement that is stiff, another direction it may be little bit more, another it will little bit more, there will be one direction where it will be the maximum and there will be another direction where it will be minimum and likewise others and that is how we get the ellipsoids.

What is the direction of this? That is given by this delta, what is the major axis minor axis are given by  $a_1$  and  $a_2$  and then torsion spring we are talking about is captured by this  $k_g$  and then this coupling vector is important part what we had already said that, said in the previous lecture is that when we have a compliant segment like that, if I were to attach a long this coupling vector there is a magnitude and there is direction if I get the coupling vector located here, this point is called the center of elasticity.

So, at this point if you are to apply a force in x direction, it will purely moving in the x direction why it will purely in the y direction, if I apply a movement it will simply rotate, that is the key aspects here that if you understand then it will be very clear. So, this coupling vector locates the center of elasticity as we showed in the previous lecture, that is the critical part of this, you can do that with compliants as we have done now we can also do with stiffness instead of  $C_{11}$   $C_{12}$  all this we have now stiffness  $S_{11}$ ,  $S_{12}$  and you can do this. The idea is that as we discuss with the compliant ellipsoid if I take two segments, there is one segments and another segment if I add them like this what happens here? What happens to this big segment now? We can simply add these in a particular way what we discussed last lecture; in the case stiffness ellipsoids also you can do the same thing.

So, in the stiffness ellipsoids again there is one over  $f_{n1}$  over  $f_{n2}$  there is delta there is coupling vector again shown here  $S_c$  and there will be a rotational stiffness the  $k_g$  equivalent also will be there first stiffness.

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So, in the case of parallel concatenation that is we have one segment here and another segment if we join them then you actually add these things. In the case of stiffness ellipsoids we can simply add, meaning there will be a direction and direction this will be the resultant direction and there will be measure axis minor axis you add them. So, like would add those vectors or tensors. So you can computationally add we do not have to redo the analysis for the new one.

So you have done for this compliant dyad you have another compliant dyad like this, when you add together the new one that you get you do not have to redo the calculation you can do the building blocks and get them that is the algebra here; you have to work out the details then will understand better, but first try to understand the concept that when you add is building blocks you do not have to redo the analysis, analysis is not needed you can simply take the parameter that exist for BB2 and BB1 that is building block 2 building block 1 simply have a different algebra here to add, we can add them in series are parallel. Now let us look at how to do synthesis with this building blocks, now may be this will become will be clear as to where it is useful.

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## Problem Restatement

Vision Based Force Design Goal:

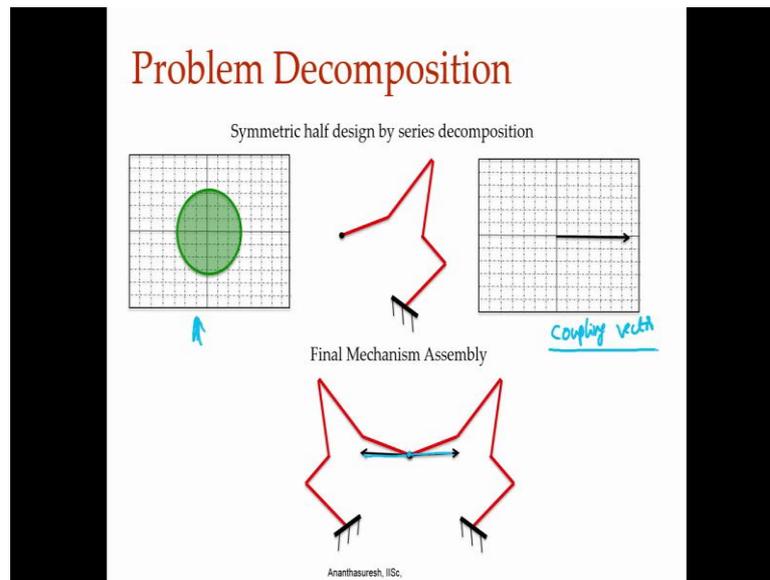
1. Design a compliant suspension with decoupled translations and rotations in the XY-plane
2. Equal stiffness in X and Y directions

Ananthasuresh, IISc.

So, the problem statement here is that, we want to design compliant mechanism such that it will have decoupled translations and rotations that is one thing, it should have equal stiffness in x and y directions; that means, that the ellipse that we saw there will be one major axis one minor axis, meaning that one direction has lot of compliants other direction has little compliants, but if you want have same stiffness in both directions, the ellipse becomes a circles, say what target is to have a circle of compliants and the coupling vector is actually shown like a point meaning that they should be complete decoupled one right to where the point is, that is center of elasticity should co inside with the thing itself that can also happens sometimes, whenever coupling vector actually becomes a null vector it will be just a point not a thing here.

So, in order do this thing, what is shown here is that instead of taking only one of these you takes two of them add them up, two circles add them up. Let us say one of them as a coupling vector like this other one has opposite, you add them you get you are null coupling vector, that is the point that you are taking that itself becomes the center of elasticity that is what is asked here. So that means they are asking for a compliant mechanism, where you apply the forces or moments it will be completely decoupled rather than up going per some other point.

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So, let us see how it is done, If I take this segment that is shown here it is compliant element ellipsoid is shown over here coupling vector is shown here, let me write coupling vector that is there will be a center of elasticity if you take this direction over here.

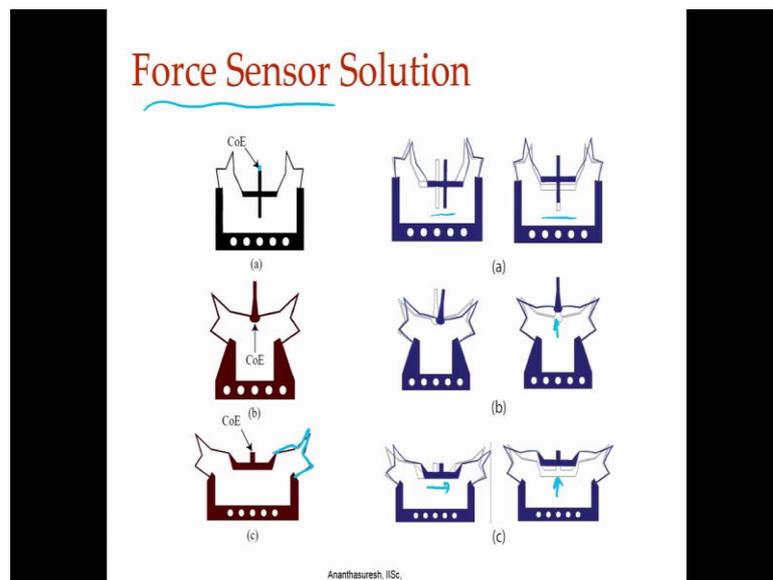
Now, you add one other one, so then coupling vector changes, so let me do it one more time coupling vector is like that, compliant ellipse at keep watching both figures. Now if we add another segment there is building blocks you know add another segment, then compliant ellipsoid changed coupling vector also changed, add another segment ellipsoid has changed coupling vector has changed our aim is to make this a circle, make this one be in this direction then you add another one becomes like that.

So, now imagine that if we keep on adding things change graphically it becomes like a video game right, in the sense that you can synthesis with building blocks you draw that building blocks being that here it is shown as a you are adding these things one by one by one, later you joint two of them here that is coupling vector here and here that becomes zero, now basically adding symmetry.

So, in the symmetry one if you take this one this will have the property that when apply force purely moving the x direction, y direction and rotation that is idea. So, basically what the in the previous slide was we said there are two of these bunch together be gives you the zero coupling vector, that is what is written there it is basically using symmetry.

But the idea is how do you combine these things? So you have to go step by step imagine that there are these little building blocks, we can drag and drop like electrical circuit that I set at the beginning of the last lecture, like resistors capacity inductors are there now we have compliant segments you grab one grab bean assemble them like a high school kid can design compliant mechanism of course, then it will be have some intuition here. So, you do that you get that thing.

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So, based on this few of them or done here like a force sensor solution they call it. So, if you have center of elasticity over there, you apply force there purely move in the x direction y direction and likewise this is like a different center velocity here where applying force it is actually rotating also little bit; maybe there are two of them are done or y is simply going up and same thing is x and y.

So, different things that you are adding here, so that is this design method. If you have this graphical visualization behind that all these compliant as ellipsoids being added coupling vector added in a particular way whether there is series or parallel combination of the building blocks is how? This is synthesis; of course in order to do what we say in is to be done here one it is intuition. So, you are seeing, you are you have to interpret what this means and what this means and if you know what do you want, it is possible to construct this. So, this is the method for building blocks.

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**Main points**

- A mathematical representation of lumped stiffness of a compliant dyad. *segment*
- Computational concatenation of compliant dyads.
- Synthesis (intuition-based)

Ananthasuresh, IISc.

So, main points for this part for the lecture is that, we have mathematical representation of lumped stiffness of a compliant dyad, for that any compliant segments it is not may not be just a dyad. And you can add these computationally, that is concatenation can be done computationally and if you some intuition you can actually synthesis.

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**Further reading**

ASME Journal of Mechanisms and Robotics, February 2011.

**An Intrinsic Geometric Framework for the Building Block Synthesis of Single Point Compliant Mechanisms**

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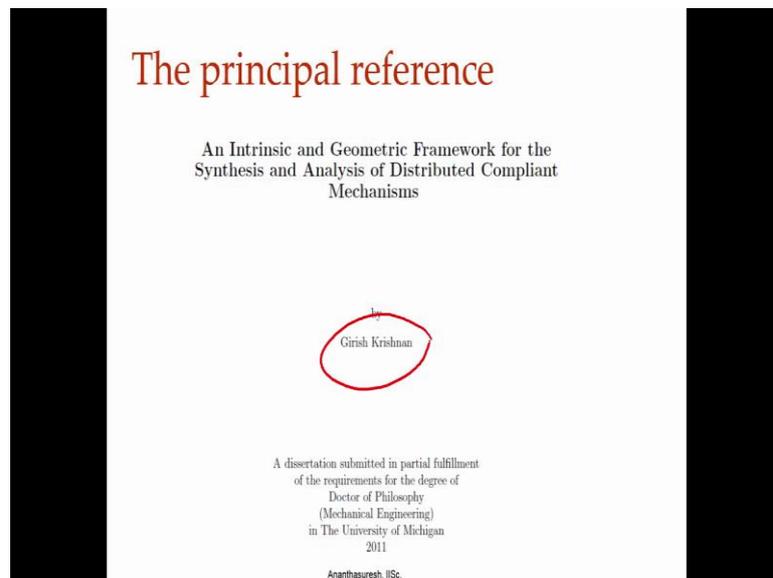
*In this paper, we implement a characterization based on eigenwists and eigensprings for the synthesis of a compliant mechanism at a given point. For 2D mechanisms, this involves characterizing the compliance matrix at a unique point called the compliance center of elasticity, where translational and rotational compliances are decoupled. For 3D mechanisms, the translational compliance may be represented graphically as an ellipse and the rotational compliance as a vector. These representations facilitate geometric insight into the operations of serial and parallel concatenations. Parametric trends are ascertained for the compliant dyad building blocks. The synthesis technique is also extended to combination of series and parallel concatenations to achieve any compliance requirements. [DOI: 10.1115/1.4002511]*

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As already said for further reading, this paper will give you compactly and if you read the PhD thesis you will know lot more, this is partly intuition partly math representation one needs to look at them carefully in order to understand.

Now, we will go the next part which is slightly different, but along the same lines which will do now. So, in this slides you will get there will be different lecture numbers, but anyway this is we will take up the 51. Now we are already into 51st lecture. So, we just continuing the 50 first lectures, but it has a new name here, but anyways the same thing, we are going to talk about load path method which is again in intuition method there is a little concept that we need to understand, how to design very interesting compliant mechanisms using this method?

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So, this is based on the PhD thesis of Girish Krishnan, which was there in the previous method also, here there a concept of transferred load meaning that if I have one point and apply  $f$  force.

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## A Kineto-Static Representation

Definition of Transferred Load

$$\begin{bmatrix} u_i \\ u_j \end{bmatrix} = \begin{bmatrix} C_{ii} & C_{ij} \\ C_{ij}^T & C_{jj} \end{bmatrix} \begin{bmatrix} f_i \\ f_j \end{bmatrix}$$

$$\underline{f_{jtr}} = \underline{C_{jj}^{-1} C_{ij}^T f_i}$$

Relationship between any two points in a continuum can be represented by Transferred Forces

Ananthasuresh, IISc.

Then let us say point has moved somewhere, now I do not apply the original force but I want to get the same displacement what force (Refer Time: 15:45) be that is the transferred force.

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## Transferred force

$$\begin{bmatrix} u_{in} \\ u_{out} \end{bmatrix} = \begin{bmatrix} C_{in} & C_{in-out} \\ C_{in-out}^T & C_{out} \end{bmatrix} \begin{bmatrix} f_{in} \\ f_{out} \end{bmatrix}$$

$$u_{out} = C_{in-out}^T f_{in} \quad f_{out} = 0$$

$$u_{out} = C_{out} \tilde{f}_{out}$$

$$\tilde{f}_{out} = C_{out}^{-1} u_{out}$$

$$\tilde{f}_{out} = C_{out}^{-1} C_{in-out}^T f_{in}$$

$$\underline{T_L} = C_{out}^{-1} C_{in-out}^T$$

Ananthasuresh, IISc.

So, what we mean here is that, if I have some compliant segment or whatever it is now it some it can actually add another some also let us say have a compliant mechanism or segment like that, when I apply force at one point the other point would let us say has

gone to that point. Now I do not want to apply the force there, you want to instead apply a force here itself, how much force would that be is called the Transferred load.

Transferred load or force is nothing but, what force there are two points we are chosen one here, another point here, two points when apply some force  $f$  one here it would have moved by that distance, but now I want to know how to get that displacements by applying force over there, that is the idea that is the transferred force, this is what is used to represent the flow of force from this point to that point. Similarly I can choose this point this point, this point, this point, this point everywhere or even these points I can see the force that are applied how does it get transferred to different parts of the mechanism that is why it is shown in the previous that I can continuum, where the force it is applied it actually feels like that much force over there, other part it may be less than that and so forth.

So, different points we have do this calculation. How is it done? If you look at the degrees of the freedom  $u_i$  and  $u_j$ ,  $u_i$  is over there  $u_j$  is over there, there will be corresponding forces  $f_i$  and  $f_j$ , there will be a compliant matrix. Again displacement and forces are related to here, these compliant matrix  $c_{ii}$ ,  $c_{ij}$ ,  $c_{ij}$  transpose and  $c_{jj}$ . Now this transferred force is given by  $c_{jj}^{-1}$  and  $c_{ij}^T$ , times  $f_i$ , that is a transferred force from this point to that point. That is what we need to understand by this transferred load concept that is an important concept of this method.

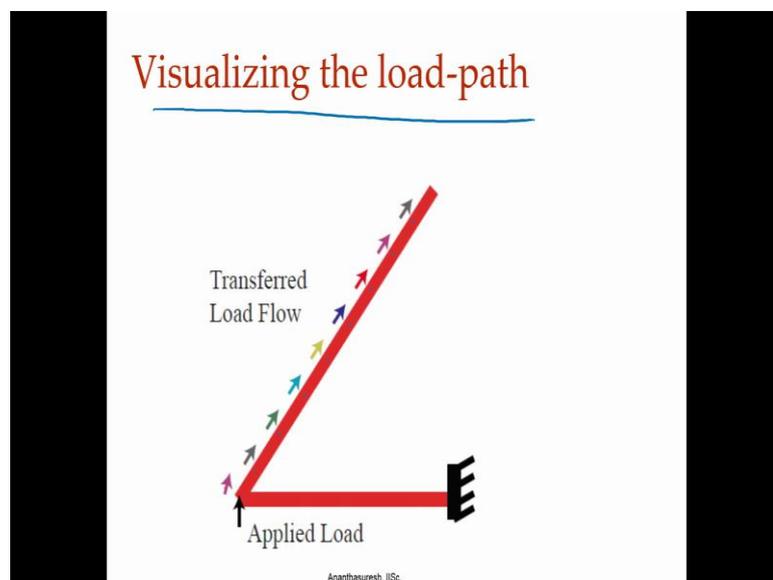
How does it come over? We have this  $u_{in}$   $u_{out}$ ,  $f_{in}$   $f_{out}$ , in the first case we would make  $f_{in}$  equal to zero sorry,  $f_{out}$  equal to zero, we are applying only  $f_{in}$  in which case  $u_{out}$  is  $C^T u_{in}$  times  $f_{in}$ , that is what we have  $f_{out}$  is zero here, first case  $f_{out}$  is zero only  $f_{in}$  is there so we got you out. Now we do the other way that is we remove  $f_{in}$  and get  $f_{out}$  in that case we have to will be  $C u_{out}$  multi divided by  $f_{out}$ , it is a different  $f_{out}$   $\tilde{f}$  a different force that is the force that you need to apply there, but what we want to know is what is that  $f_{out}$ ? That  $f_{out}$  will be  $C^{-1} u_{out}$ , what is  $u_{out}$  we have here.

So, what we get is this  $f_{out}$  here is  $C^{-1} u_{out}$ , which is we have  $C u_{out}$  that again comes from  $u_{out}$  if you take,  $C^T f_{in}$  overall you get this. So, you can talk about this part to be the load transferred matrix, that is if apply  $f_{in}$  at one point elastic body apply force at point, how much get felt here there will be T L between this

point this point, that is how we can get the transferred load at different points that is the concept here.

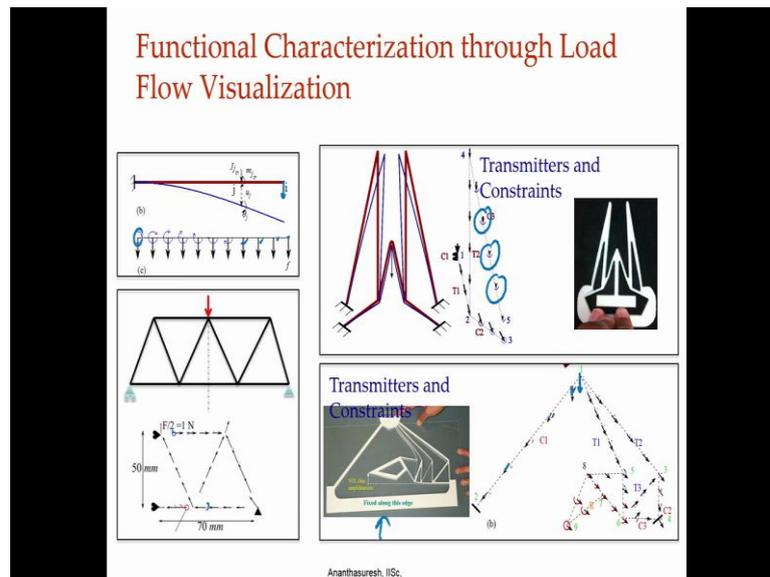
How much force can transferred different points can be obtained computationally in this manner, this is as it was shown in the previous slide it is  $C_{j j}^{-1}$  times  $C_{i j}^T$ , that is how the force course it is a inclusive concept there is a computational basis now, that is what shown here, if applied a force here at different points how the force gets transmitted we can actually compute.

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So you can visualize the load path, when apply the force and segment is there it is as if the force is going like that that is a “Transferred Load Flow”.

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Now, if we actually want to understand it, we can just think of them as bending movements and shear force diagram. So, if I have let us say cantilever beam where I am applying a force, so we have this arrows know little arrows here this bent arrows, the bending movement you know the bending movements is zero here little more little more little more maximum over here, that we know bending movements this is the shear force, if I have a force here then everywhere the same force will be acting you can think of in that way.

In the case of cantilever, when thing is getting complicated let us say truss system, truss is can take only extension and contraction basically axial forces load flow will be like this. Somewhere this little movement is also shown, but trust members do not really take but it is may be frame member not trust because there are no joints there. So, there will little arrows there to indicate the bending movement also.

So, if you take a three dimensional or more complicated compliant mechanism, these not three dimensional two dimensional still, how the force and movement will be there at every point you can actually compute in the manner that we disguised. So, it basically the force flow are load path.

So, in a complicated mechanism like this, again you have every point the force that you applied here a force and little movement little movement under force how it aligns when it goes to this, so you can compute all this. So, given a compliant mechanism by taking

from the input point to every other point, we can compute this transferred load when I say load, load forces and movements both that is how this force flow they talk about.

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### Decomposition into Transmitter-Constraints

Output  
Constraint ( $C_o$ )

---

Transmitter  
( $C_t$ )

---

Input  
Constraint  
( $C_i$ )

$f_{tr} = [C_t + Q^T C_i Q]^{-1} Q^T C_i f_i$

Independent of  $C_o$

Dyads are fundamental building blocks for distributed compliant mechanisms

Ananthasuresh, IISc,

And this force flow they say for everything any compliant can be split like the decomposition which is a inverse a assembly we talked about earlier, you can take this little pieces all are them will have a certain way of flow of force, that needs little bit of imagination, some intuition to say to see how otherwise you can compute the way we have just discussed.

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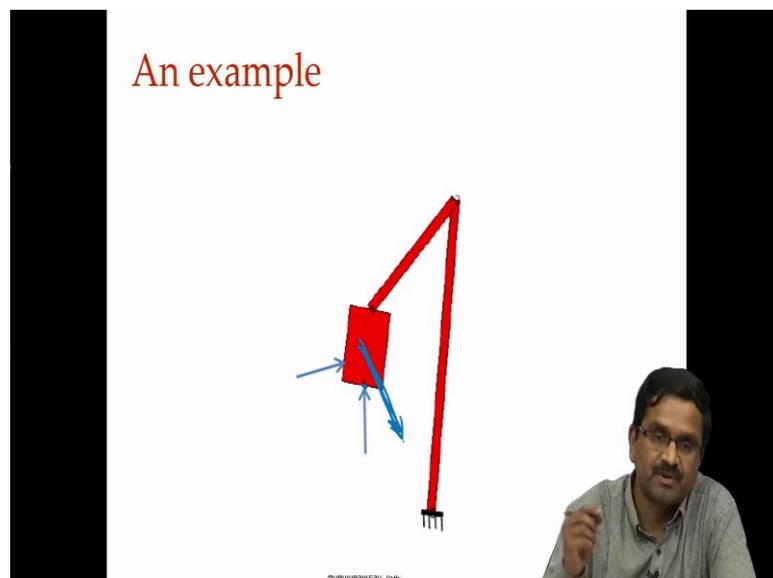
### Constraints on load-transmission

Ananthasuresh, IISc,

These other thing also the constraint that is apply a force here, this point is not likely to move any other directions it only move moves in this semicircle (Refer Time: 22:31) only goes up you do not expect it to move down. So, at every point you can think of these things you want to constraint it all the possible directions are shown here, if you want to constraint it this way then you add this thing here like that, then it will be constraint to move that way.

So, once you know the force flow direction, then if you want to constraint to one of those many directions you add this constraints you get this. This gives what we had called in one of the previous lecture compliant 4 bar.

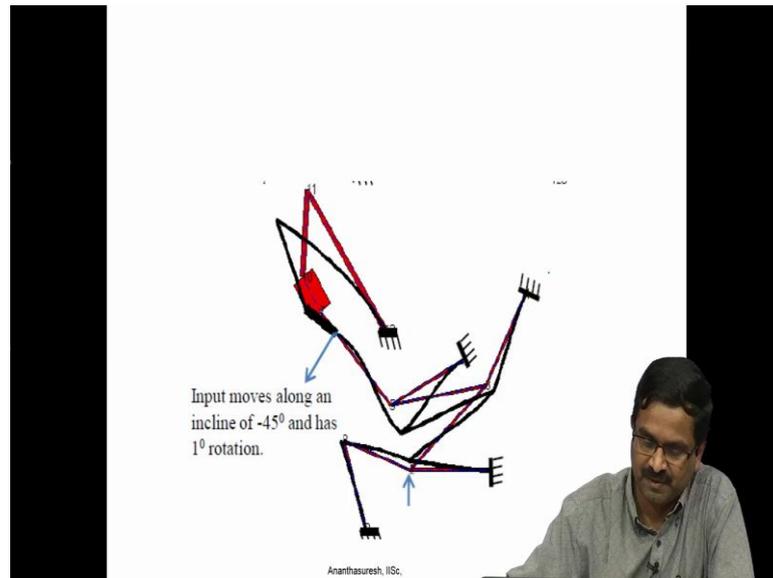
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Let us take an example. So, let us say that we have a rigid box here and a compliant segment, now you want to add something to be so that these rigid blocks moves in a particular direction, they can point here moves in a particular direction. For that what you do is, construct this load paths how you come up this segment are left to your imagination.

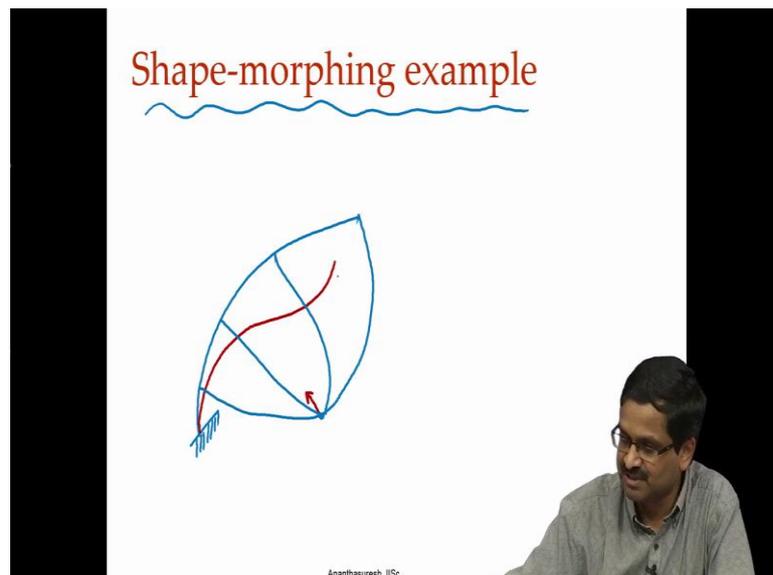


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If we applied force over here this body from there to here it moves. So, everything is deforming in their own way and you can get. So, basically this load path method ask you to have some intuition has to how force gets transmitted, there is a little systematic concept that we discussed about the transferred load between two points in an elastic body that is say very sound concept, beyond that you have to have intuition has to how to assemble.

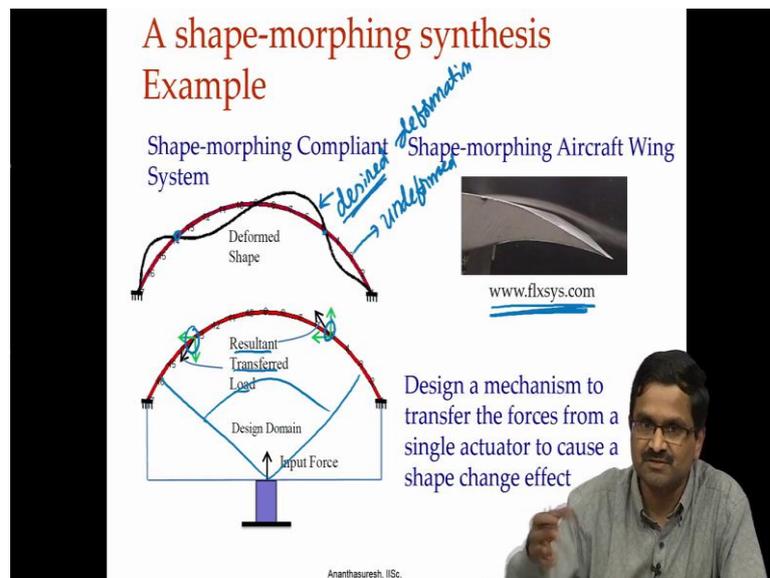
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So, let us look at another example which is quiet complicated, which is the Shape morphing example that is if I have let us say some shape like that is useful for morphing aircraft wing and so forth, let us say I want this to be morph to a shape like that, by applying force somewhere let us I apply the force like that, somehow I want to connect to this blue body, which is the unreformed one.

So, when I apply the force it should morph to the shape, the shape should change to this not (Refer Time: 25:14) morph. So, I have connected somehow. So, this point when I applying the force are to connect this with some segments we do not know what, some compliant mechanism when I apply the force it should actually become like that, that is what we did a possible with these load path method.

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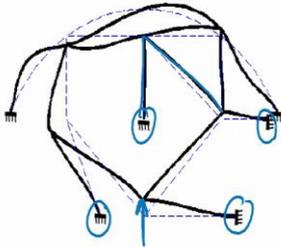


So, that is what is here, I have this arch and that should go to the black one. So, this is the unreformed and deformed now this is the desired deformation, we do not know how it would deform, but we want it deformed to that shape that is a target that desired shape.

So, here resultant transferred load is kind of visualized at these points, why only these two points why not this points? We do not know that is taken and this is the input force we have to apply this we have to connect this to this in some fashion. So, that it actually does that. This is used for this flxsys company uses for morphing wings, there is this where this method was developed by Sridhar Kotas group. So, first visualize the load flow here.

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## Guidelines for Synthesis



Step 1: Visualize load flow  
Step 2: Analyze feasibility  
Step 3: Replace load paths with building blocks  
Step 4: Verify solution

Ananthasuresh, IISc.

So, for this red arch to deform to this you see you think that if I apply of force here, here it will happened, then you imagine some flow how you actually get this way why not some other way we do not know you need to have your imagination. So, you analyze the feasibility and put these load paths and then for these load paths, you come up with an equivalent compliant dyad or segments.

First you say load path, with this nothing here in this space nothing, but you are just imagining are coming up of the intuition how the load path should be? Once you have that, then you can actually imagine adding this compliants segments each one of them you add finally, you get this. So, you apply the force here that dash line and you add them like this it actually going, not exactly the way it is asked for, but it indeed going to a shape like that by having connections here and here.

Why not here? May be that is also possible, but why this segments? So, all that requires little bit of intuition as to how to come up with these load paths, if you have that institution this method is suitable to solve a complicated problems such as this one, that is how to make a beam which is your like an arch beam, applying force single force somewhere how do you do this? Of course you have gotten many new supports, here 4 new supports also get added as you go in the procedure, but you need to have very good intuition to imagine this load flow or force flow.

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**Main points**

- Load-path visualization
- Intuition-based synthesis

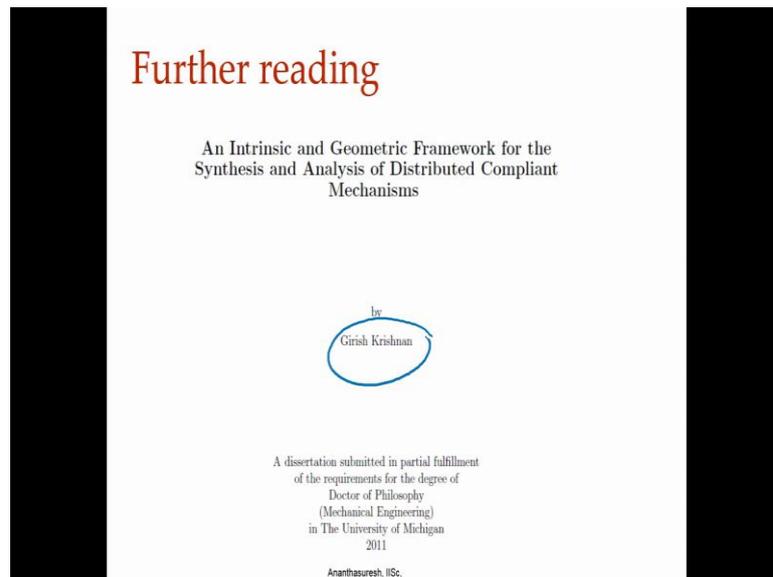
Ananthasuresh, IISc.

So, the main points this is that load path can be visualized if you have intuition, so intuition is very important for this method, then you can do the synthesis. So, given some input output relationship that you are interested is the complete shape morphing we have to see where.

So, let us say I like a that arch one let us say I have a segment like that, my want this to deform like that, it cannot be image should be sharp here, then I have to apply forces to this may be at this point something at this point something, but these are forces I want apply only single forces let us I have apply a force like this. Now to this point how do I connect to my unreformed one? So, that if deformed like this.

So maybe I need to add something like this, add another attachment and for this point also I made add another attachment maybe connect this to this in some fashions, I am just making up. We have intuition of this how the force this force gets the transferred to this that is the transferred force we talked about, if you have that intuition you can do this. So, these a very interesting innovative technique, but it depends a lot on once intuition, if want to know more about it you can read the PhD thesis of Girish Krishnan and some other papers that he has written.

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So, now we have discussed six different design methods for the last few weeks. In the next lecture will summarize all of those and do sort of a comparative analysis of all these designing methods so that we understand the pros and cons of which method. You may like more than the other, you can feel comfortable with that and that is what we will do in the next lecture.

Thank you.