Compliant Mechanisms: Principles and Design
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Lecture – 48
Designing compliant suspensions using kinetoelastic maps

Hello, we were discussing non dimensional measures for quantities other than displacement; we looked at what we called multi dimensional or multi axial stiffness that is stiffness in along the 3 axis and then rotational stiffness about the 3 axis, 3 orthogonal axes also cross axial stiffness. And we discussed how to arrive at non dimensionalized factors for various quantities, multi dimensional stiffness, stress, natural frequency and so forth.

Today will continue that lecture that was 47, now this is 48. But a continuation in the same representation files that you will have with it. So, let us briefly review what we talked about in the last lecture and then move on to today's part.

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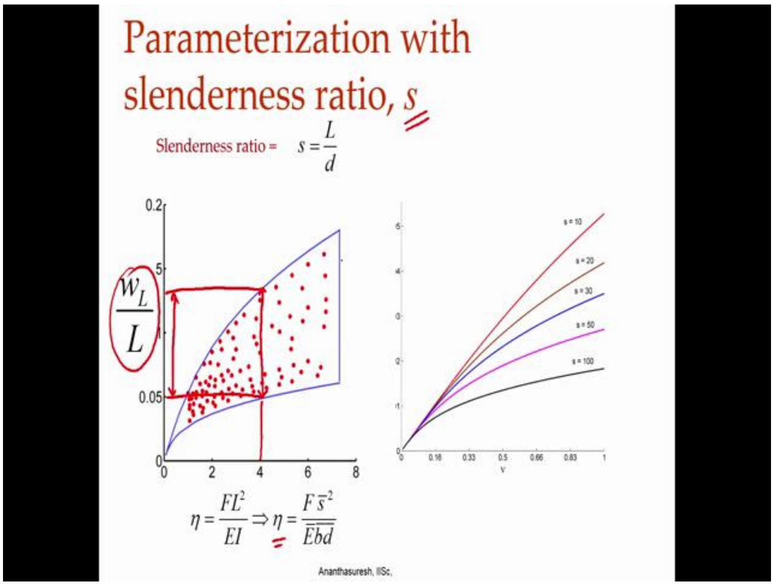

So, our idea is to have this non-dimensionalization for other elastic responses other than displacements and geometric advantage and mechanisms advantage that we had talked about.

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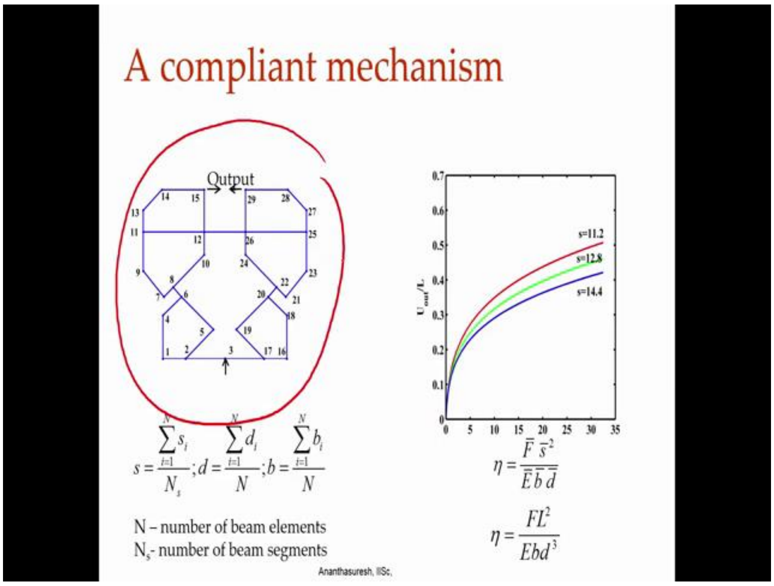

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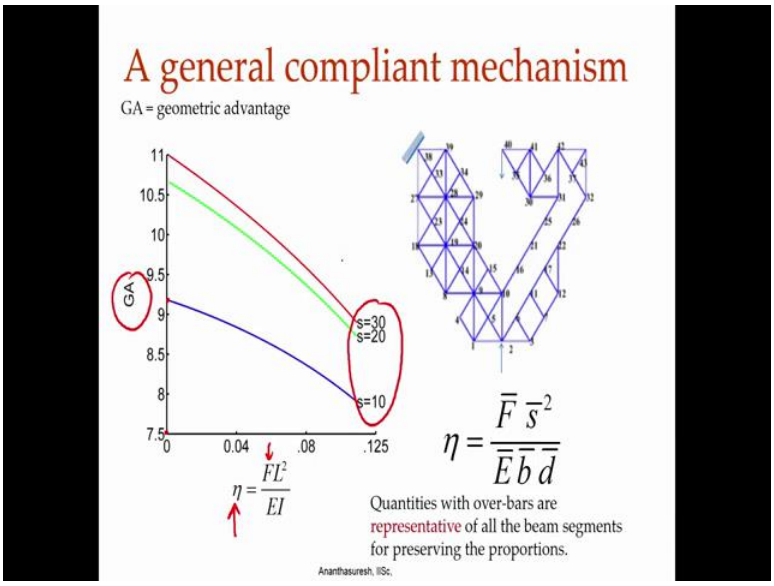

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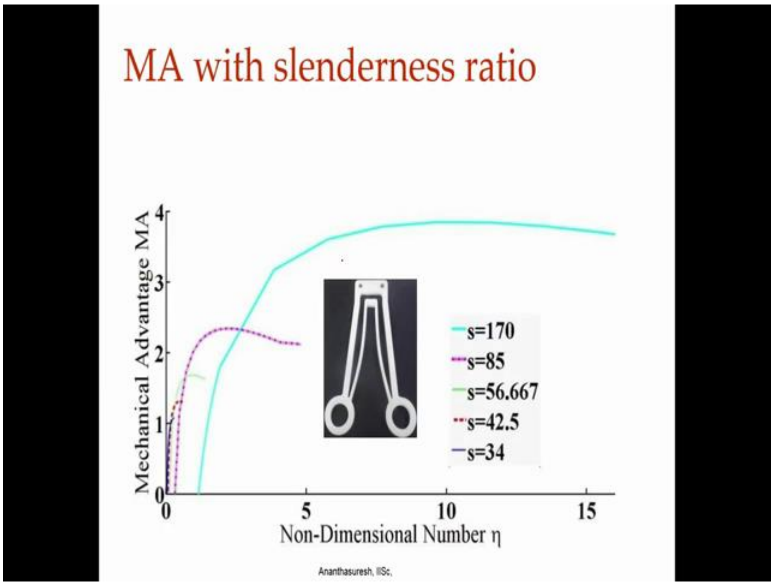

So, this is the math that we are familiar with the Kinetoelasto static map now, becomes Kinetoelasto map because we extended that approach and these are all the slides we discussed in last lecture.

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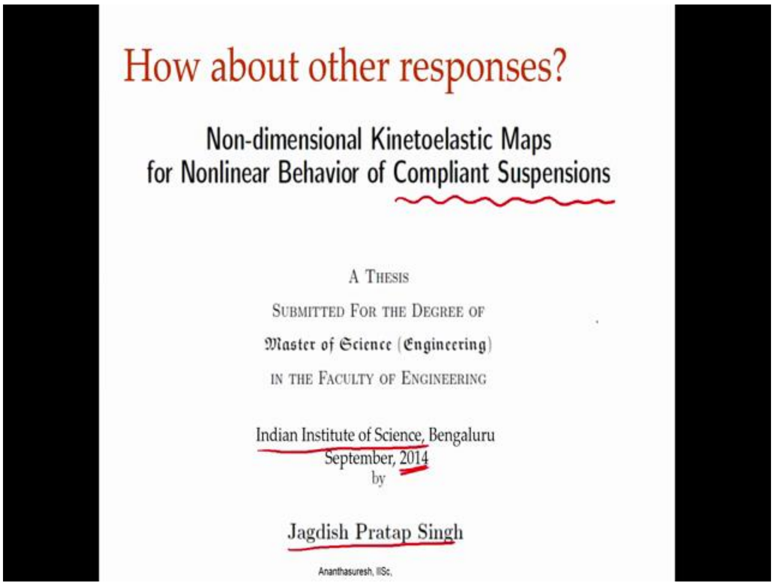

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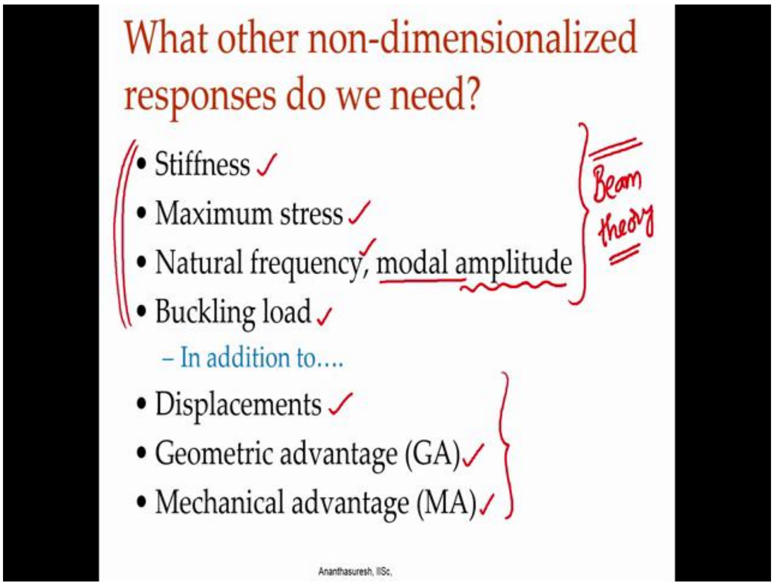

So, it is all from this thesis of a Jagdish Pratap Singh, who worked out a method to non-dimensionalized stiffness, maximum stress, natural frequency, modal amplitude or modal magnitude as we had called Buckling load is not in this lecture, but it can be done for that as well other than Displacements, Geometric advantage, Mechanical advantage which we had discussed in the previous lectures.

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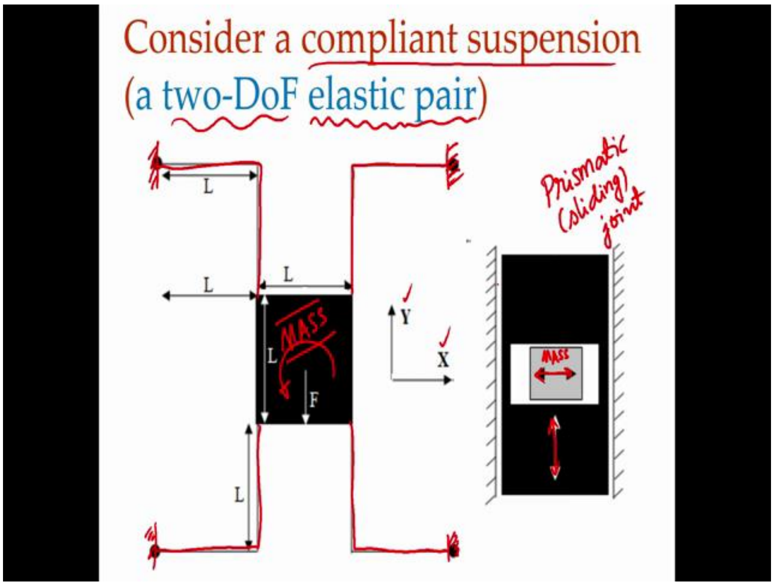

So, we considered this elastic pair and talked about this multi axial stiffness where K x x would be for the stiffness along the x axis.

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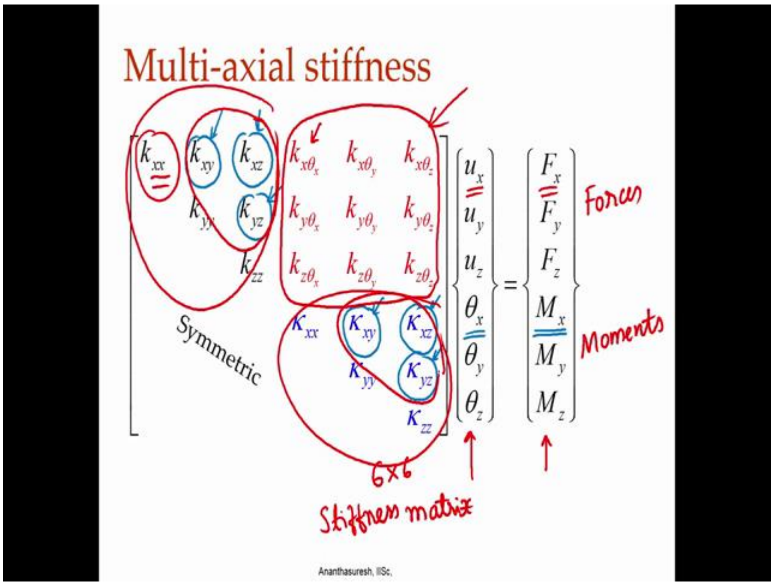

If there is a force applied along the x axis, how much displacement you get and then when you apply force along the x axis, you would also get displacement along the y axis, z axis, probably rotation about x axis, y axis and z axis as well. So, that is why we have all this stiffness, this is the translation part and this is the rotation part and this is the coupling part between rotation and translation within that this subset is the coupling among the 3 translation axis, this will be coupling among the 3 rotational axis and so forth.

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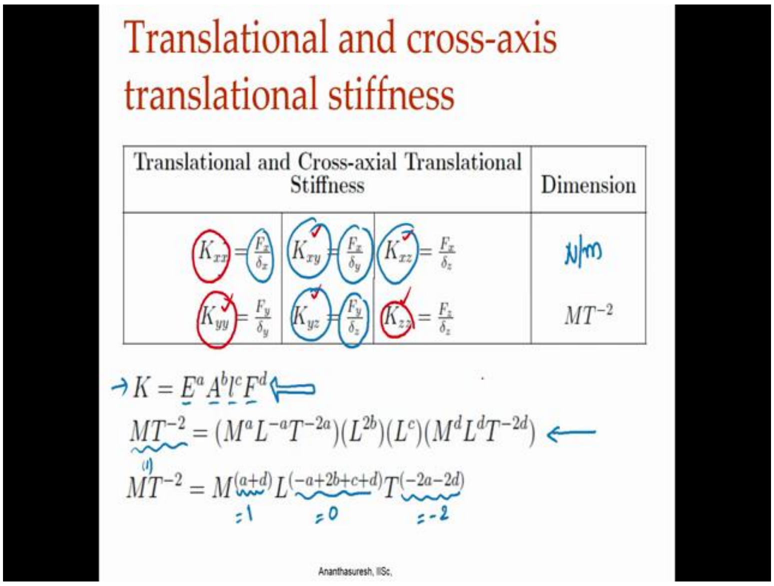

So, we can derive all of this and draw these non dimensional maps. For all of this we want to find the non dimensionalization factor which we discussed a procedure for it.

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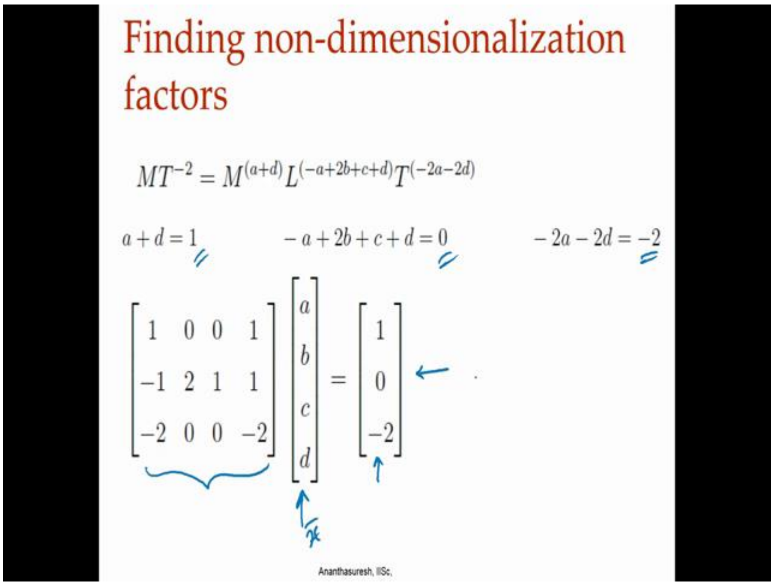

So, we construct this dimensional matrix, taking into account the quantities with which we want to arrive at the non dimensionalization factor or non dimensional, non dimension quantity.

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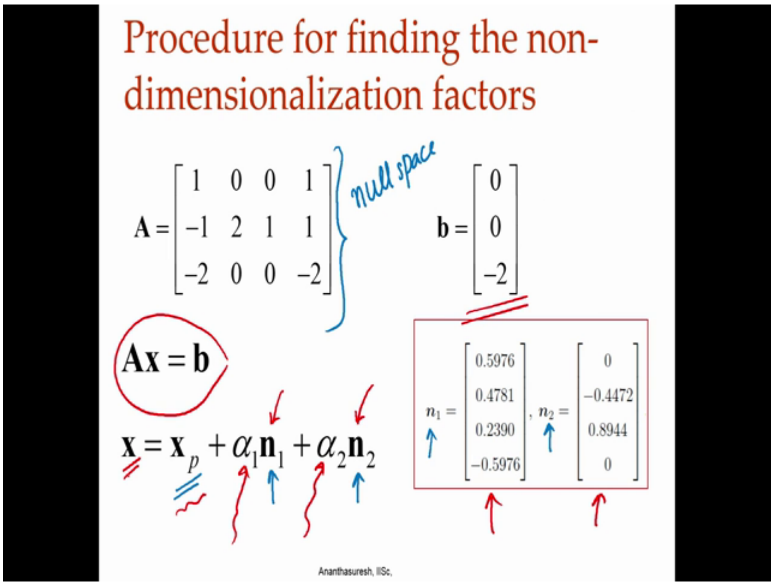

So, we had this thing where x are the exponents in those quantities because you have number of quantities, you have to raise each of them to a power and we had to find that exponent or the power. So, that when you put them in a combination, will get a non dimensional number; this equation that we need to solve with that non dimensional thing A and then b will be the dimensions of the quantity.

So, there is a particular solution which we called x p and then in this particular case it was rank efficient by 2, so there are 2 null space factors, in general there will be as many as the rank deficiency, these are the null space factors that we have. By taking different values of this alpha 1, alpha 2 after finding x p we can generate a number of non dimensional factors, which are of them works that would try and then see which are them leads to a unique curve when you draw this Kinetoelastic map.

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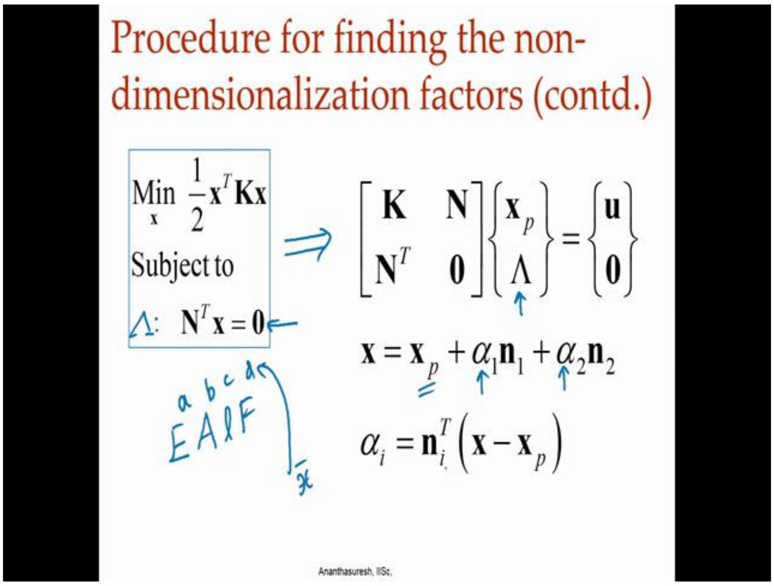

So, for finding x p we had this procedure which is to pre multiplied by a transpose and k equal to a transpose A, u equals to A transpose b, we solve this optimization problem which is one way of solving ax is equal to b in this case and we found that solution for it x p.

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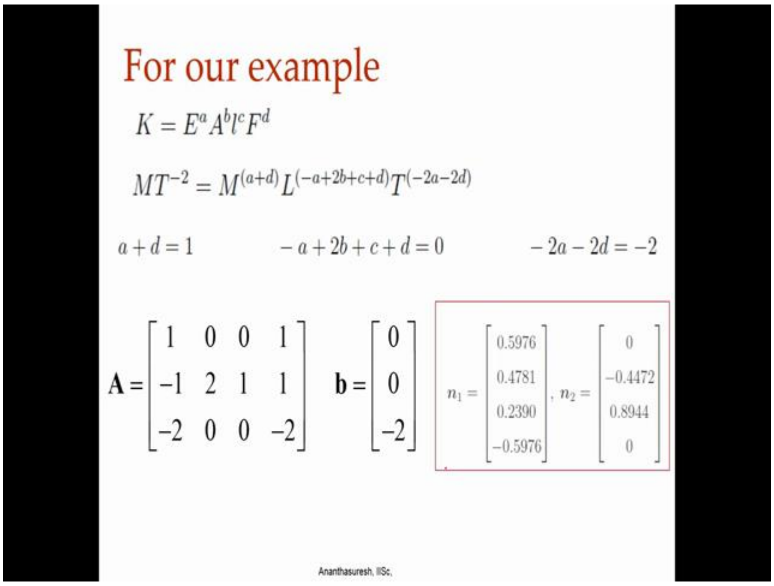

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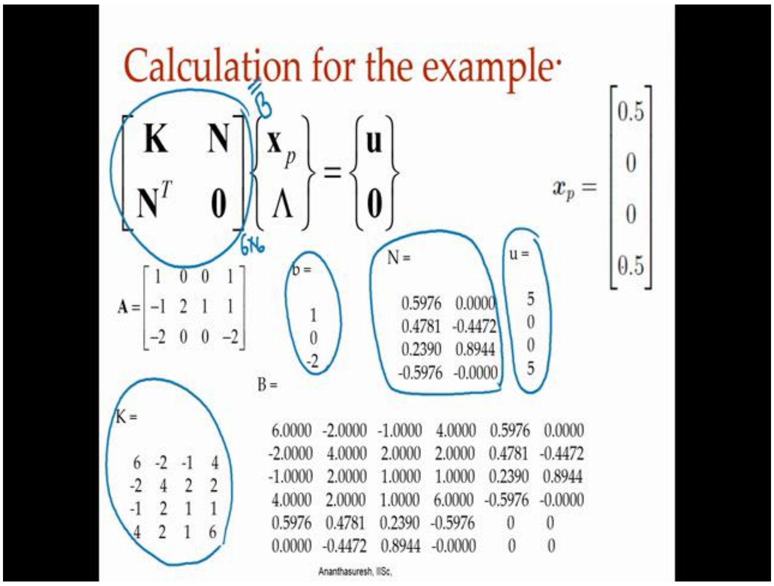

For this particular example we had done and x p turned out to be a value that is given here, it is I think point 5 and point 5. So, x p was 0.5 0 0 0.5.

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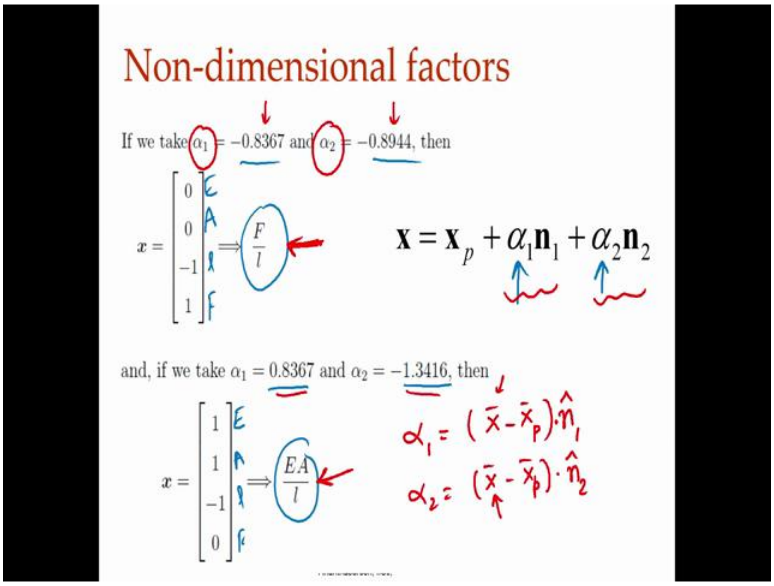

Ones you get x p, if you want to find alpha 1 and alpha 2, in this particular case we said that if you where to take this alpha 1 and this alpha 2, we get a non dimensional quantity of factor which is F by l, if we take different set of values you will get EA by l, how do you get that because you have to take lot of different values, but how do we know this particular values? For that we understand that, the null space factors are orthogonal to one another, here we have 2 they both are orthogonal to each other if I want to find alpha 1, I need to take this x p the other side.

So, I have x minus x p these are the vectors, then I have alpha 1 and of course, we had alpha 1 times n 1, alpha 2 time n 2, if you multiply by n 1 So, if I multiply by n 1 dot product let us say if I do that, then this quantity will become alpha 1 times 1, where are this will become zero because n 1 and n 2 are orthogonal to each other, that is why we getting at alpha similarly alpha 2. If you want to get alpha 2, it will be x minus x p dot at the n 2, n1, n 2 are unit vectors that is how (Refer Time: 06:19) b space vectors come about, then if we make unit vector that becomes one and you get this quantity. But how do you guess what is alpha? What this n 1 this x is? That is probably with your intuition.

So, there are many ways you can combine the quantity that you have into non dimensional factor, there lots of solutions alpha 1, alpha 2 you can take (Refer Time: 06:42) pair of numbers. So we have 2 infinity of solution but you take based on your intuition and then find alphas if you want. So, this procedure only to tell you that there is a systematic procedure, but a systematic nature of the procedure stops at the point, where you need to guess the values of alpha 1 and alpha 2 you have the freedom, you get lot of quantities, but the one that you take may be the best thing is to go by linear analysis, something analytical it should be there to get this non dimensionalization.

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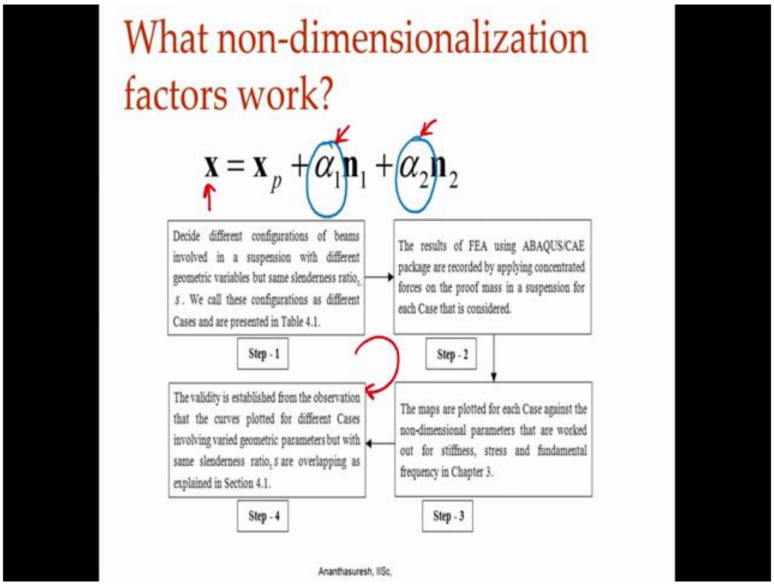

You guess them and find them and then you plot all this point this is a procedure that we had described in the last lecture, not that we vary alpha 1 and alpha 2 and find this, but you use some kind of an analysis could be linear analysis also and based on that you get this non dimensionalization powers, this is x will have the powers for each quantity you have and there is the step 1, step 2, step 3, step 4 procedure here, if we do that if we get a single curve then you can be assure that you would get a non dimensional factor.

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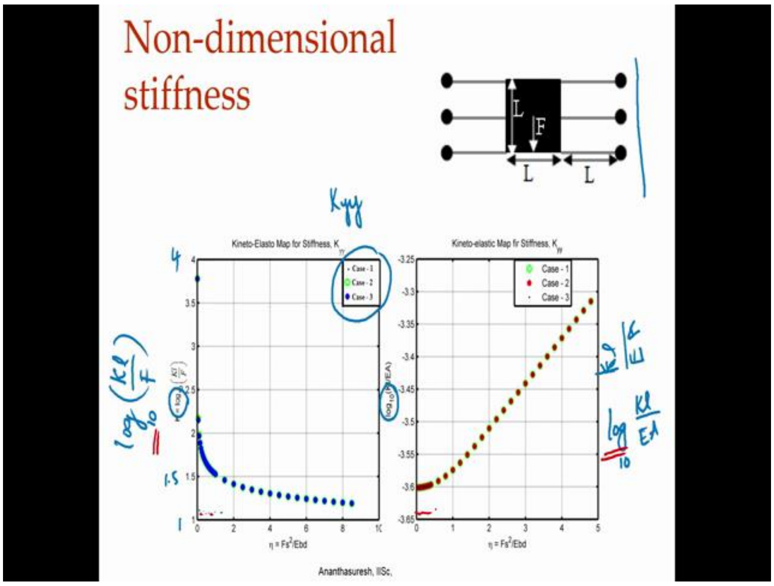

Non-dimensionalzation factor in this particular case we are using L by F as well as L by EA. So both of them because the non-dimensionalized the stiffness here, both look different of course, because this different non dimensionalized factor, but both of them have used log, log to base 10 here. So, that you can cover the range, they do not trust together into one small thing somewhere.

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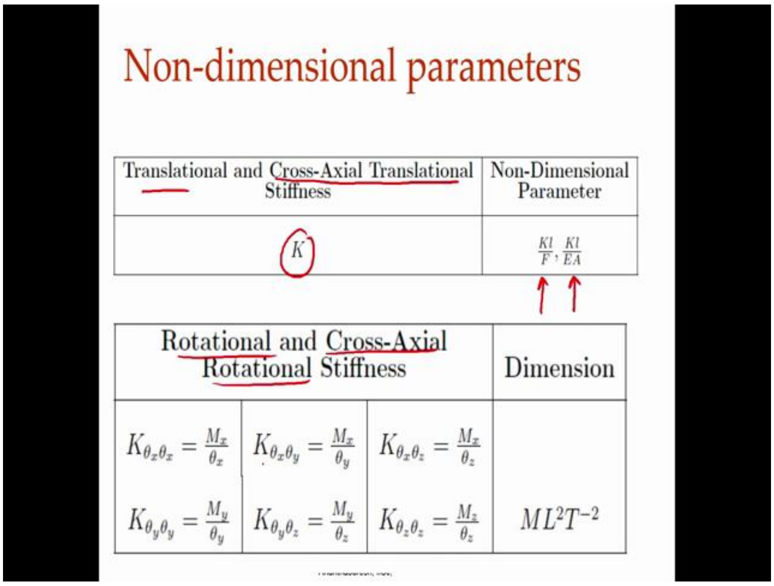

So, ones you have these you can actually design using this, which we what we will do today. So, if you want to take Translational and Cross Axial Translational, then you have

2 factors what we just did K l by F, K l by EA, in terms of units they match, dimensional match and physical quantity also they match; l by F simply (Refer Time: 08:44) some force you get some displacement and displacement can be related to the characteristic length, here we have put length and then EA by l is axial stiffness we can take that if you want you can take bending stiffness also. So, which ever you feel like you can do that and then see that, you get a single curve as you plot that non dimension quantity in this case stiffness versus eta that is f x square by e b t.

Similarly, you can do this for a cross axial rotational to rotational. So, we have k theta x theta x theta, theta y and so forth. Here (Refer Time: 09:25) have covered all k x x, x y, x z all of that here you have theta x, theta y and all those 6 combinations that we have here. 3 linear that is that rotation to itself not linear, rotation to itself the cross axial rotations one rotation to another rotation.

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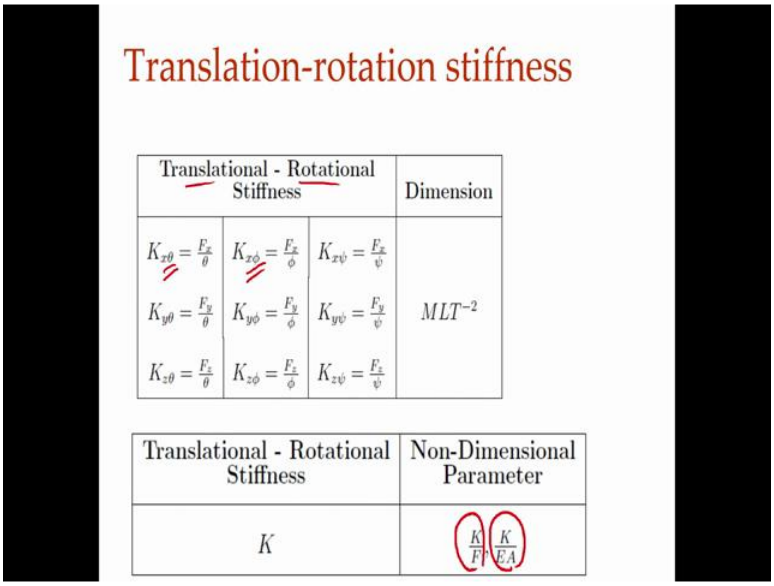

So, here also the non dimensional factor if you work it out you will get a quantity. In this particular case it will be again, this one is rotation to rotation we had there, now we have translational to rotation that is x and theta, x and phi so forth, for such a thing the non dimensional factors are K by F and K by EA, basically l is missing because now in this particular case, this would have units which will be involving Newton meter because there is a torque or rotation involved.

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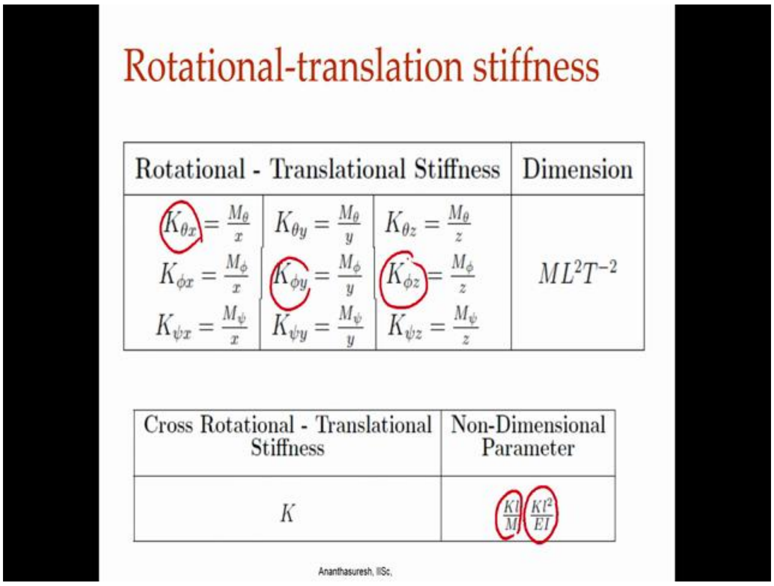

And you can also look at this rotational translational in the case of movement applied. So, here you can put it has K l by M or K l square by EI. So depending on the units or the quantity you are looking at, it is cross rotational and translational that is theta and x and phi and y, phi and z, that is one rotation another translation which was in the top right curl of the matrix that we had looked at. Let us look at that matrix one more time, so it is multi axial, this one which was these are the quantities we are looking at now. So, for all of those there are non dimensionalization factors, which are flipping through the slides (Refer Time: 11:17) already gone through.

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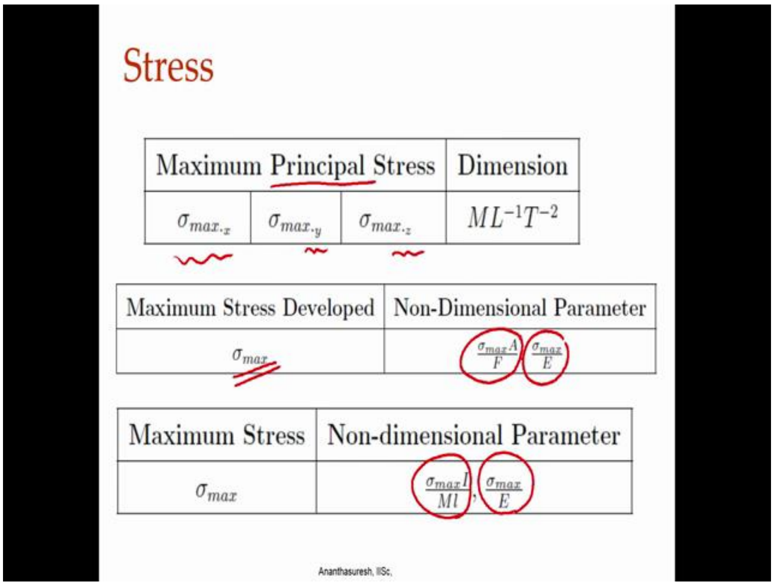

So we have stiffness, multi axis stiffness we know how to non-dimensionalise. Once we have this we move on to another one let us say stress, if we take stress there are lot of measures of stress that you can take, but most of 10 in design conductional material, we use von misses stress and we want to know in a given structure where the boundless stress will be maximum, more than where it is we want to know how much that is. So, you can have maximum stress when I put subscript x it means that when force is applied in the x direction.

When force applied in the y direction for some z direction and so forth or we can just go for just one maximum stress, here it could be principle stress or it could be von misses stress. So, which ever you like we can take for them if you do a similar procedure you would find that, you can have maximum stress sigma max A by F, sigma max by E or you can have sigma max by E and then you can multiplied by S also, that is (Refer Time: 12:32) ratio that we have discussed. So, which ever works you would take that.

So, when there is a movement load, then it will be sigma max I by Ml and sigma max x by E. So you can choose the appropriate non dimensionalization fact of stress.

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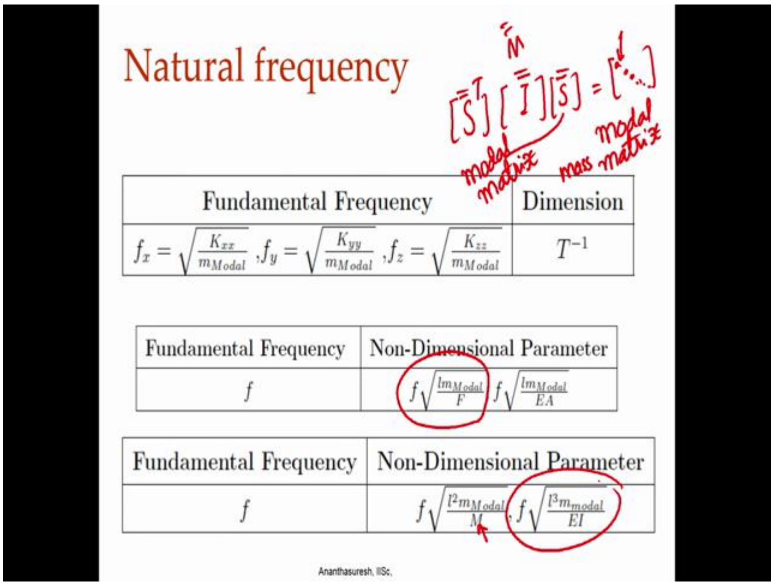

Similarly for frequency if you follow this procedure, then would find how to non dimensionalise that, in this particular case it is shown as f times square route of l and then m modal that is modal mass because they are connecting with each natural frequency there will be a mode shape, for that mode shape we have calculate what we can call Modal mass for it. So, when you take mode shape matrix multiplied by inertia matrix and then post multi pre multiplied with the transpose of the modal matrix, post multiplied by the modal matrix itself of the inertia matrix, something like if we have inertia matrix we would do S transpose then s.

So, S is the mode matrix, I is the inertia matrix, then you would get this as a diagonal matrix which will call a modal matrix and this s is the modal mass matrix, let us called that modal mass matrix, s is the modal matrix itself that will its columns will be mode spaces of the structure.

So, you do s transpose I s are sometimes we denote it also m for initial matrix, that will be the modal mass matrix you take the corresponding mass for that frequency, because there will be first one, second one, third one and so forth because the first frequency you are taking take the first modal mass, divided by F or could be lm by ea square route and when there is a movement load it will be l square m modal mass by m square route of that times frequency and so forth, l cube by EI and there are different non dimensionalized quantity that will compute by natural frequency also.

So, for any quantity that is given it seems possible that we can get a proper non dimensional factor. So, that we get a single curve for that response, when you plot it against the eta that is our primary non dimensional factor.

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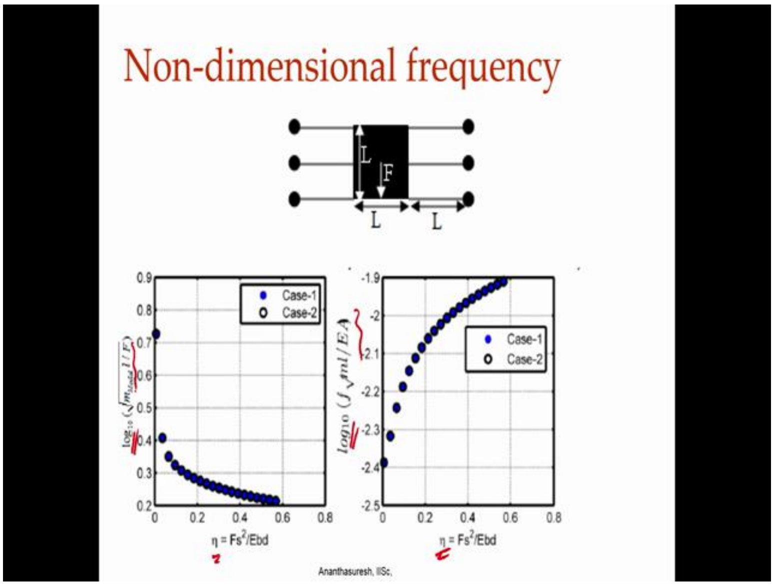

Here it is done for frequency that is square route of m modal L by F and something with square route of ml by EA times F here, again there is a logs scale here how frequency changes, we looking at them we can estimate what happens when increase forces, that the frequency increased or decrease. We can find all that here, but it again depends on what kind of a non dimensionalization factor you have used.

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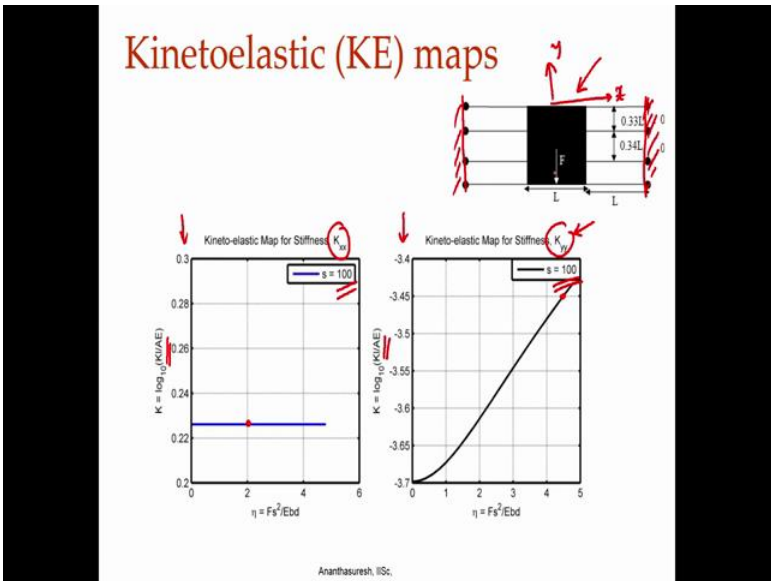

And it will be convenient for us, for every compliant mechanism in this case in elastic pair or compliant suspension we can plot with whatever non-dimensional fact that we want here there is K x x then K y y, if you want cross axis stiffness if you plot it for the same non dimensional number, we can see how the things change and the how the magnitude also change changes. If you see all these are 0.3 0.28 positive value, these are negative values remember they are log negative means that it 10 power minus that.

So, for example, if I take a point here if it is minus 3.45 it is actually, 10 raise to minus 3.45 where as if I take a point here, which is actually constant here its point 2 2 10 raise point 2 2 clearly, this in this case is much lower than this the non dimensional stiffness. So, it will be very flexible in the y direction, very stiff in the x direction. If I call this y and I call this x axis, x is very stiff as you can see because it is fixed here, all this points are fixed. So, it will very stiff to move in the x direction, but very flexible to move in the y direction, we can access the magnitudes also using this maps and here the maximum stress non dimensionalized versus K y y is given.

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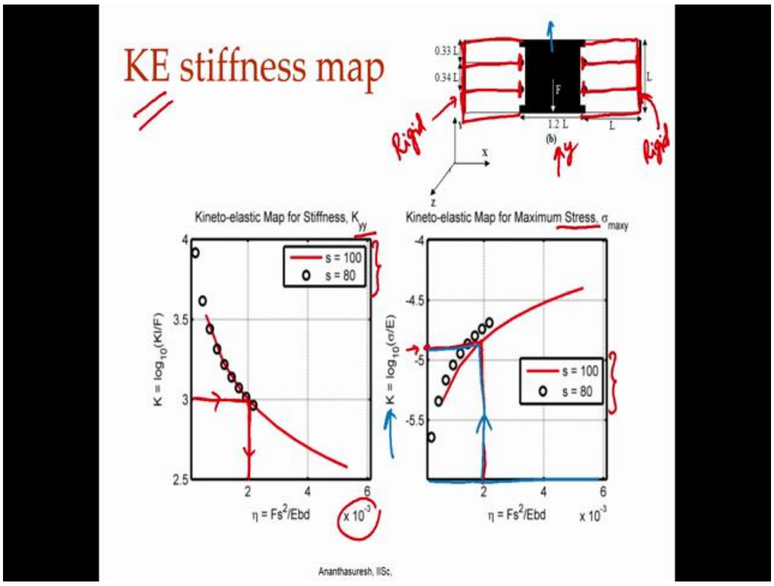

So, if you want some stiffness in y direction that is our y direction, want to know what is the corresponding stress? They are done for 2 slenderness ratios 80 and 100. So, if you want let us say again number this log to base 10 K l by F, if you want 3 then you would know what this eta is, once you know eta let us say true you go there and depending on s is equal to 100 or 80 let us say if we assume is 100, then you can see what will be the non dimensional stress which is K log 10 sigma by E, if I know this value here I can actually estimate sigma.

So, once you have this you can actually design a suspension for strength as well as stiffness, by just looking at this Kinetoelastic maps.

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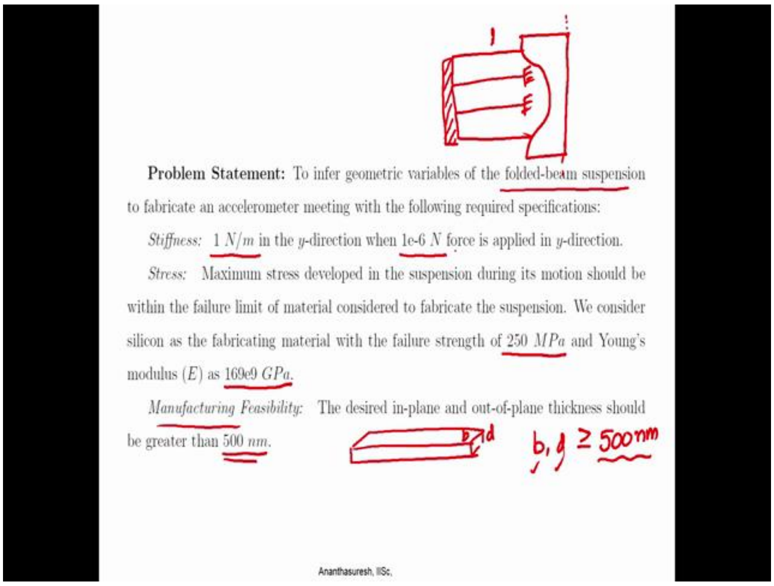

To illustrate that let us take a simple example. So, it is a problem statement, which says we want to find the geometric variables of the folded-beam suspension, full in suspension is something which is a very good approximation for a sliding joint without actually the joint it is a compliant joint or what we called elastic pairs at the beginning of this course, will see a picture of that in the next slide let us actually look at that if it is not here. So, will just draw one here it is. So, the full in suspension it is fixed here and here, here and here.

So, its folded meaning and this portion is rigid but it can move it can slide as this everything slides; it is fixed here where I am putting this red one fixed at those 4 points. That is a folded beam because the beam is folded like that one beam, another beam and then we join them with a rigid one and the likewise here folded back folded back and then join them, there is a folded suspension where you have K y y as well as stress. So, let us use these maps to solve the problem that is post here.

So the stiffness required is 1 Newton per meter, it is a very small stiffness if you thing about it because by applying let us say 1 Newton which is equalent just 100 grams of a gravitational force, gravitation force into 100 grams mass if you apply that much like there is an apple a spring deflects by one meter, you can imagine how flexible a spring that is and the force applied is one micro (Refer Time: 20:33) clearly this for a micro application, usually for biological cells if you are trying to measure the mechanical response your tools should have very low stiffness, something like 1 Newton per meter all the way down to 1 mille Newton per meter, imaging applying1 mille Newton we generate displacement of 1 meter.

So, that is what is required here, maximum stress should be less than 250 Mega Pascal’s with Young’s modulus of 169 Giga Pascal the correspond set of silicon, also desired in plane and out of plane of thickness should be greater than 500 nanometer. So, in terms of the demands if I have a beam somewhere, that will have a thickness and that in plane of width both of those if I call this b, I call this d both b and d should be greater than 500 Nano meters, that is half a macron. Those are the things are given from the manufacturing view point lithography. In fact, that itself is quite small to do, but the problem statement is taken in this fashion, let us see if we can design this. The first thing to do is when you have the suspension; we already know its geometry which we already sketch. So, its goes like this goes like this and here we connect these too with a rigid one.

So, here when it I need to say all of them are the same length and then there is this is fixed here, this is fixed here, in there is a mass here let this is the axis of symmetric, we need to find this l, we need to find b and d and a is given we need to design basically. So, size we need decide cross section size because material is already specified and the force is already specified and if this is what we have, then what will do is will just do a small calculation to see how we can design such a suspension using in the kind of charts that we have here.

So, will beginning by since we have the chart let say somebody has drawn it for s is equal to 100, let us take s is equal to 100 first.

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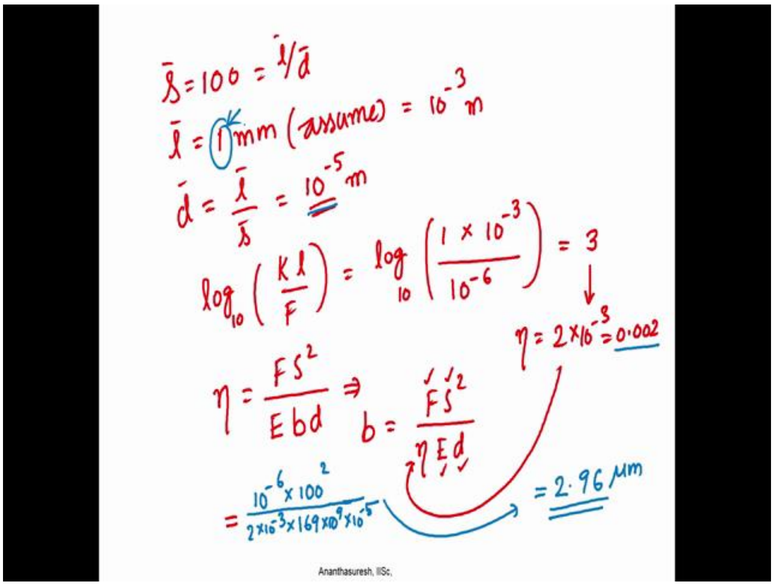

So, we start with s equal to hundred because there is a chart for it, otherwise you will find a chart that is drawn for this (Refer Time: 23:11) that you want l equal to let us first take this as 1 mille meter, it is a micro thing the overall suspension will be only 1 millimeter, this we are just assuming first because of you are retraction the manufacturing, let say we assume l then the d will be because s is equal to l by d when I say s or l these are all the averages. So, there are 4 beams and 4 beams 8 beams in the suspension, we are talking about and they are all the same in this case l bar is equal to l. So, that is does not matter. So, d will be l by s, if I do that I get d this is 1 millimeter. So, I will get that is 100, it will become 10 power minus 5 meters of 10 macrons, 1 millimeter 100 times 100th of that become 10 power minus 5 meter.

So, now we have this and the stiffness that we want is said to be we had 1 Newton per meter. So, what we could do is and also force is also 1 macron Newton is given. So, we want to find this log raise to base 10 stiffness l and F, since we have assume l here we know k already we can compute this. So, we have log to base 10, k is 1 Newton per meter and l is 1 millimeter that is 10 power minus 3 meters 10 power minus 3 meters by force 1 Newton, then that will be 3 because 10 power minus 6 goes to (Refer Time: 25:05) becomes 10000, 1000 log thousand to base 10 is 3 now correspondent to 3 we have to find the eta value.

So, we go back to this thing the 3 I happen to take 3, already correspond to 3 if you take this is 10 power minus 3 is there, 2 into 10 power minus 3. So, eta value that we have is correspondent to this we get n eta, which is 2 into 10 power minus 3 equals 0.002 So, once we have this sense we already know the force F here, we can actually calculate the eta is FS square by E b d is what we have in this particular case, that d we already have because we assume s and l. So, I can get from here b is equal to FS square by eta E and d. We know everything now because F has given, E is given and s we assumed, and hence sense we assumed l we got d also, eta we got from the chart if you put this, then b turns out to be if we do the calculation here, which I will just write and you can work it out.

So, F is 10 power minus 6 Newton's, S is 100 square divided by eta which we got 2 into 10 power minus 3, 0.002, E is 169 10 power 9 that is Giga Pascals. So, Pascals 10 power 9 Pascals and then we have d which we have already determined which is 10 powers minus 5. If you do all this you should get a number if my arithmetic is right, will get a number that is 2.96 microns, which is a reasonable thing. If 2.9 say about 3 microns, one can make this using lithography we get the d. So, let us say now we make sure that d and b according to the problem statement there should be more than have a micron their indeed mode, but if you want we can change this let us say 2.96 is too small for your process.

So, fabrication sometimes micro abbreviation some labs want to be more conservative and say I want to make 5 macron as the width. So, in which case this does not work, then you should change either the size a little bit or go for a different case. In this particular thing we had drawn the curves for s equal to ad in 100. You can try a (Refer Time: 28:06) and then see what happens or changes this sizes and find out.

Before that we should also look at what happens to the stress. So, we go back to this chart that we have, now that we know this eta. So, we go to this eta then find out what this is. In this case minus 5 something is there, because again this similar log. So, we have log to base 10 sigma by E is I will say minus 4.5 may be let us say 4.9 minus 4.9.

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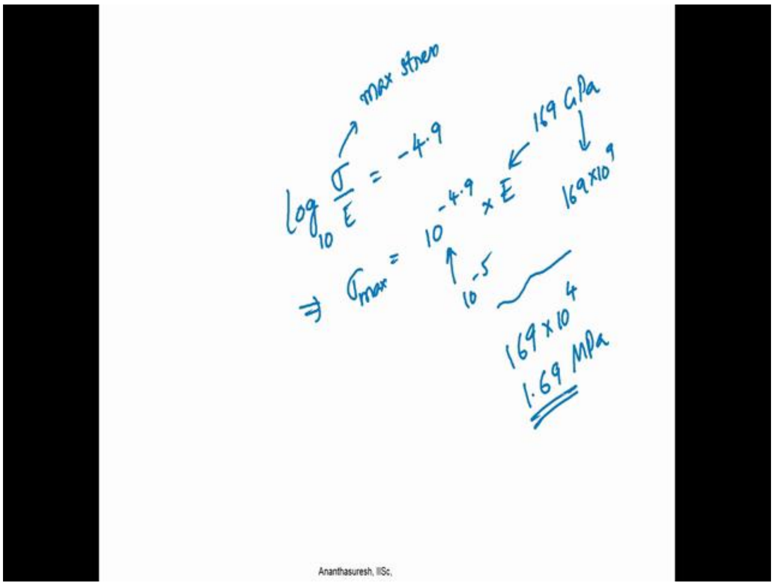

So, base on this I can say log to base 10 sigma I think sigma by where did you put it was just sigma by e in this case sigma by e is minus 4 point 9. So, once I get this from here I can get sigma here is maximum stress m that gives you already. So, you can find out sigma max as 10 raise to minus 0.49 into e which is 169 Giga Pascals. So, you will get some stress and you have to make sure that that is less than what was given which were 267 something is 10 raise to minus 5 we can take this will have 10 power 9.

So, which seems to be here because this is 10 raise to minus 5 let us take this is 169 10 power 9, over all you get if you do this we will get 169 into 10 power 4 or rather 1.69 a mega pascals much less than what was taken. So, very quickly we have the charts you can actually design suspension. So, if you do not want to make it smaller bigger you can see what will be the stress you do not have to (Refer Time: 30:09) analysis everything is here also note that the maximum stress that we are showing here is a function of eta meaning that as you apply force on the max it is a stress is changing. So, when you initially when you do it when applies force will get something, but if you put more force the stress is going to change how it changes is also given here if it is natural frequency the loaded structure natural frequency is shown to you. So, that actually is a lot of calculation you have to first apply the load and then on that you have to do modal analysis is to be get the frequency.

If we do this for stress it is not. So, much, but for the case of natural frequency you have to do redo modal analysis and get that when you have this maps it is easy to calculate that is how this method works.

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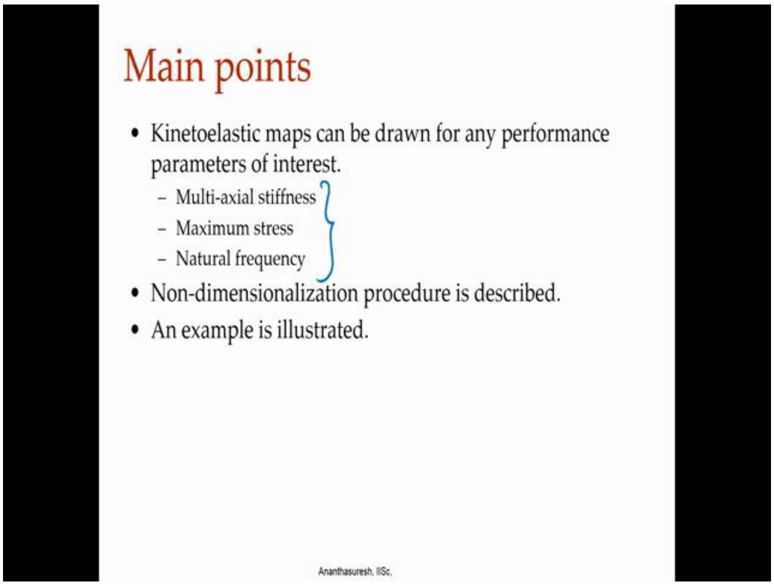

So, just to summarize we have Kineoelastic maps now they are not static they are also done for dynamic cases meaning frequency and other measures modal mass they can be drawn for any performance measure will discuss a systematic procedure and trying to get this.

And then we discuss this procedure and we also illustrated an example to show how this method works. It is very simple because, when somebody creates this maps, creating maps itself is not a whole lot of works because just you have to do it for a few parameters that is for some designs that you have to get and then draw these thinks you have to (Refer Time: 31:50) s that is the only thing for different standardness ratios once you have it you can do it for stiffness and strength that is maximum stress and any other measure and you can easily design the mechanism

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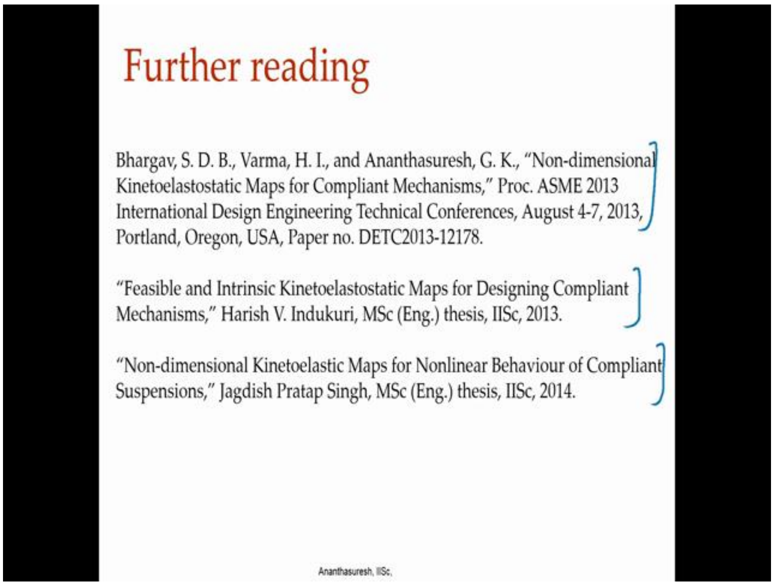

So, for further reading there is this mass stress thesis another mass stress thesis, and there is a paper to read about the details. In the next lectures will look at different design technique. These are the four design technique that we are including which is using these non dimensional maps. We look at few more design techniques in the next week's lectures.

Thank you.