NOISE CONTROL IN MECHANICAL SYSTEMS

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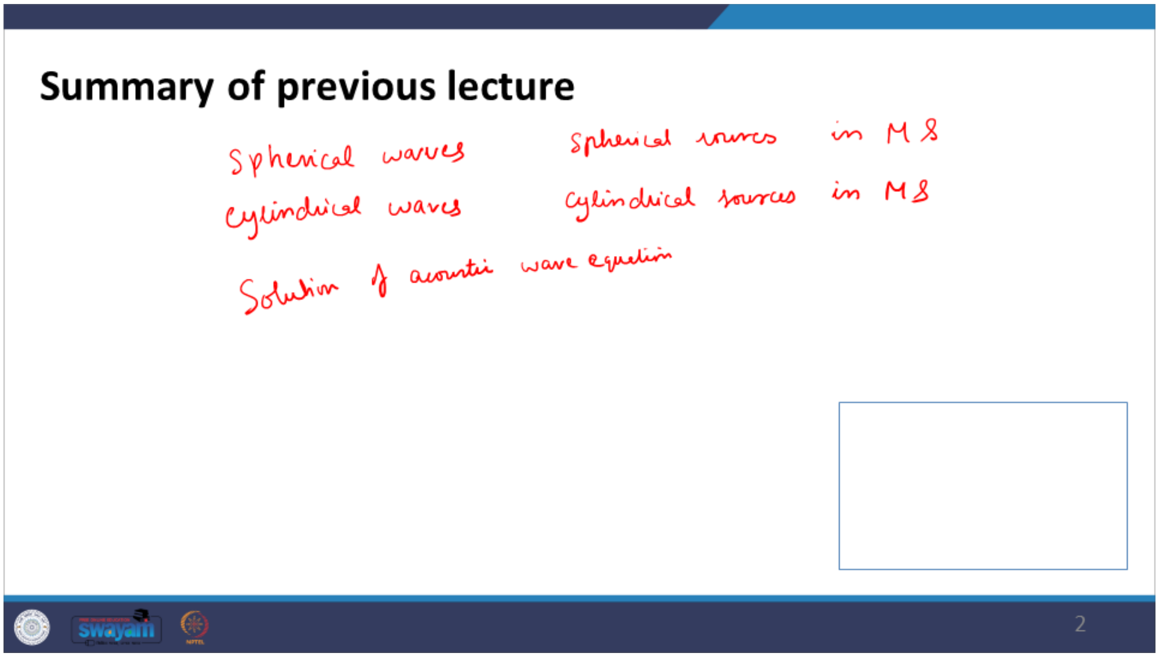
Week:02

Lecture:07

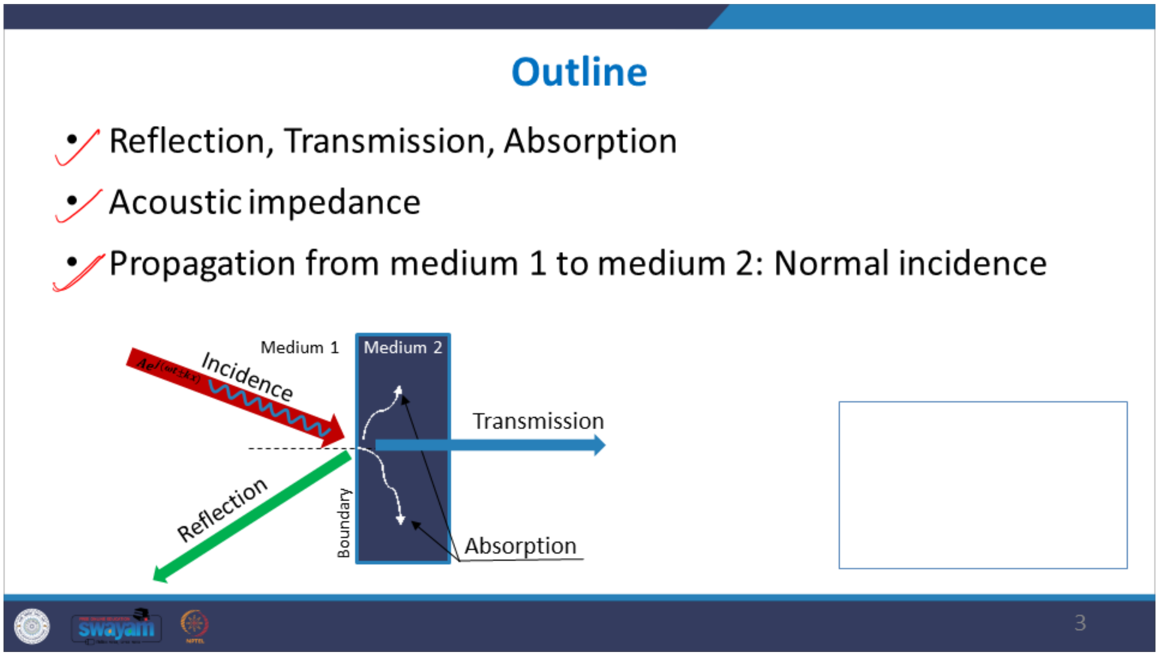
Lecture 07: Sound Propagation Through Medium Boundaries

Hello and welcome to Lecture 7 in this course on noise control in mechanical systems. In this lecture, we will study sound propagation through medium boundaries. This is a very important topic if you are in the field of noise control, especially passive noise control.

So, let's quickly review what we studied in the previous lecture. We studied spherical waves, cylindrical waves, the common spherical sources in mechanical systems, the common cylindrical sources in mechanical systems, and we solved and found the solution to the acoustic wave equation for these sources. The important takeaway was that from the noise control perspective, harmonic plane waves are more dangerous and require priority because the attenuation rate is almost zero in an ideal situation. Whereas, spherical waves have the highest rate of attenuation over space, followed by cylindrical waves.



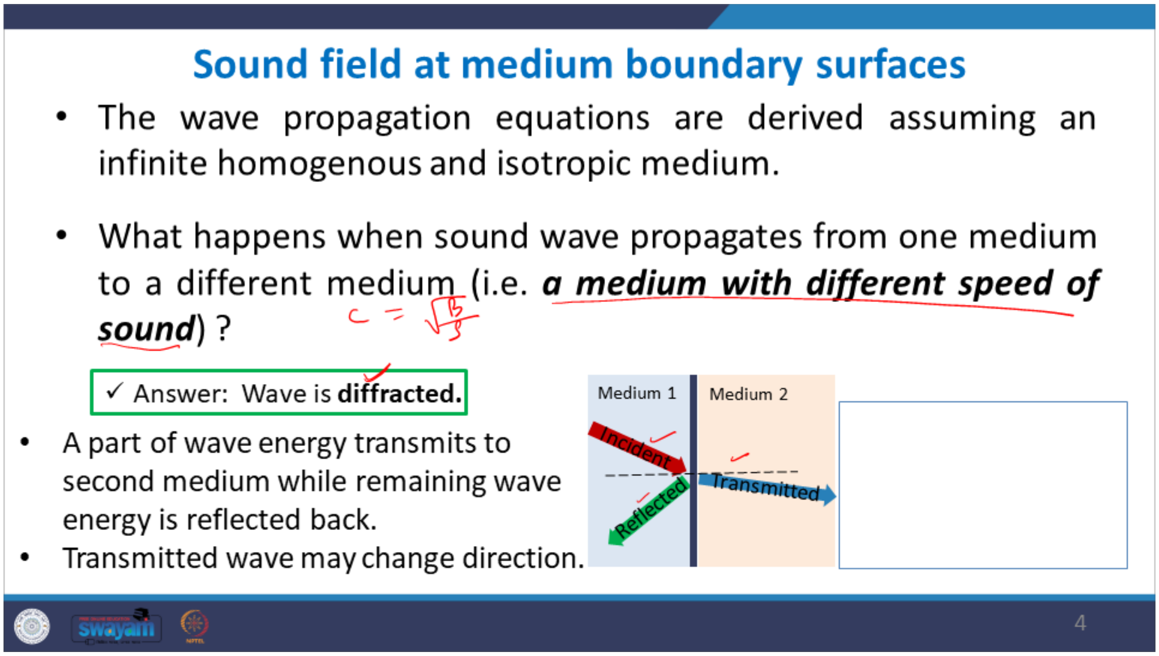
In this lecture, we will study the important concepts of reflection, transmission, absorption, and another terminology called acoustic impedance. Once we have derived some relations, we will proceed to examine what happens when a sound wave propagates from Medium 1 to Medium 2, starting with the case where the wave is incident on a medium boundary at a normal angle.



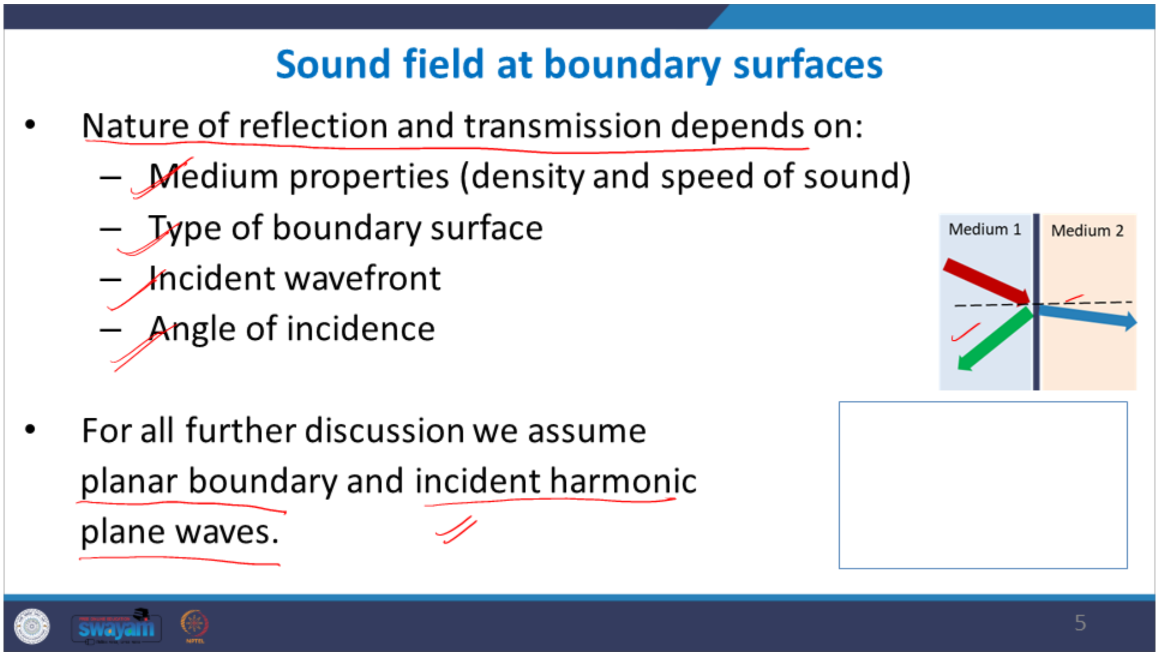
So far, we have dealt with the linear acoustic wave equation and the solutions we obtained for different waves. The main assumption is that the wave propagates in a homogeneous medium, meaning the medium does not change its bulk acoustic properties, i.e., beta and rho remain constant over space and time for the volume of fluid under consideration. But what happens if, suddenly, a wave traveling encounters a new medium while propagating? So, what will happen, and what do we mean by a new medium? A medium, in short, is a medium with a different speed of sound, where the speed of sound is given by:

$$\mathbf{c = \sqrt{(B/\rho)}.}$$

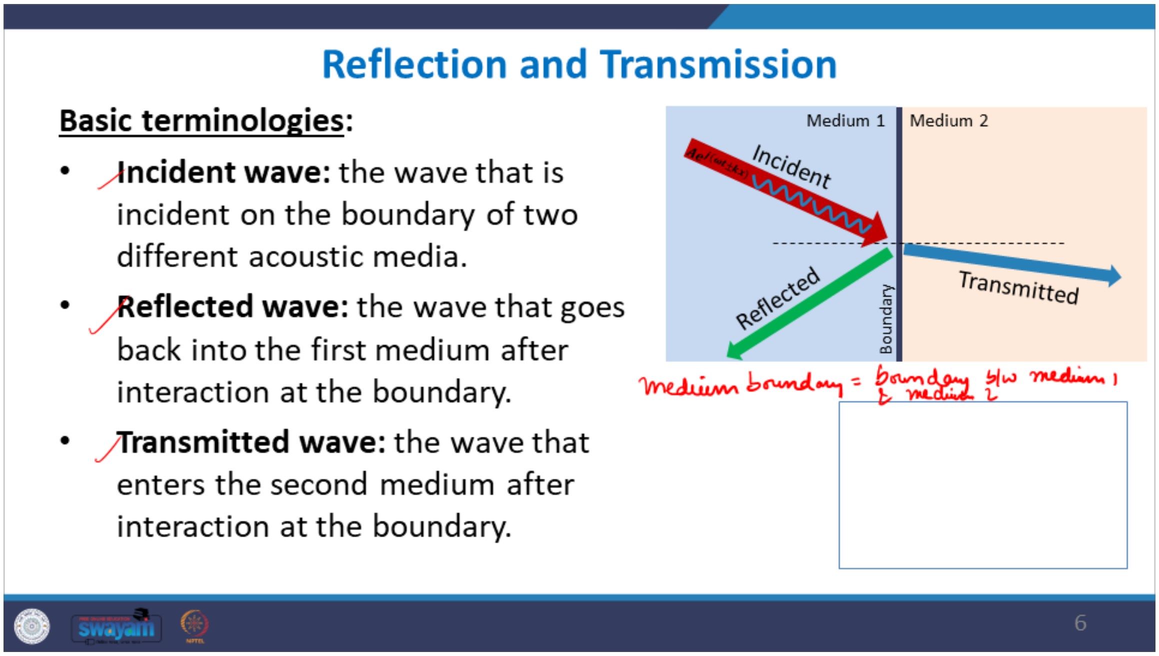
So, it means a medium with different beta and rho. So, what happens? It is very similar to how light waves, as electromagnetic waves, encounter one medium to another with different speeds, leading to refraction. In the same way, when sound waves encounter a different medium, the wave gets diffracted. This is the diffraction phenomenon which constitutes three important phenomena. Which means incidence, reflection, and transmission. So, a part of the wave that's incident gets reflected back into the same medium, and then some part of it propagates into the second medium.



So again, if you think like a noise control engineer, as soon as you come to know that, okay, there is a change in the medium, I see that, you know, some wave is reflecting back, and only a small portion is going to get transmitted. So, the first thing that comes to your mind is that, okay, suppose I have got some machinery and I want to control the noise, let me put some barriers or some different materials in the pathway of propagation, because you are putting some different material in the path of the propagation, then you are sort of installing or introducing some different media in its pathways. And when you are installing different media in the pathways, you would be able to encounter this phenomenon, which means the waves, you know, some part of it reflecting, some part transmitting, and you can use this phenomenon to your advantage and create the kind of acoustics you need for your particular problem or particular space at hand. Okay, So, the nature of reflection and transmission, this is the thing that the noise control engineers want to manipulate to get their desired result. And what does it depend on? It depends on obviously the two mediums that it encounters and their properties. It also depends on, you know, the type of boundary surface, the incident wave front, and the angle of incidence. But in this particular introductory course, we will stick with, you know, the planar boundary and the incident being harmonic plane waves. But, whatever I would like to state here is that whatever results we derive for harmonic plane waves, it can automatically be used for spherical or cylindrical because after all, spherical waves can be thought of as harmonic waves propagating radially outwards, and the amplitude instead of being constant is a function of constant divided by r, it is a constant divided by r. So, if you think of spherical waves as, you know, a special type of harmonic plane then whatever relations you are deriving, it will hold true. And similarly, it will also hold true for the cylindrical waves.



So, the basic terminologies in this is that here a wave is incident on a medium boundary. What do you mean by the medium boundary? Very simple, boundary between medium 1 and 2. Acoustic medium 1 and medium 2, fine. So, the incident wave is the wave that is incident on that boundary. The reflected wave is the one that, after incidence, returns to the first medium after its interaction at the boundary, and the transmitted wave is the one which, after the interaction, enters into the different medium or the second medium, okay. Incident, reflected, transmitted.



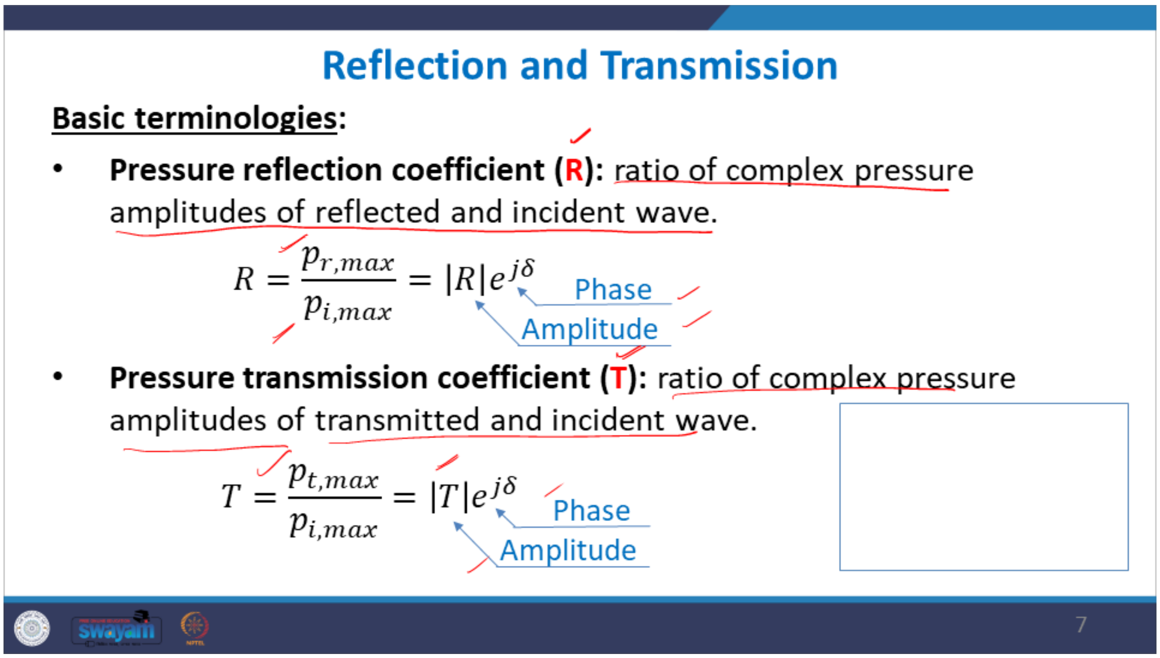
So, then what should be the pressure reflection coefficient? It should be some relation between the reflected and the incident wave. So, here it is: it is the ratio of the complex pressure amplitudes of the reflected and the incident wave, or simply:

$$\mathbf{R} = \frac{\mathbf{p}_{\mathbf{r,max}}}{\mathbf{p}_{\mathbf{i,\ max}}} = |\mathbf{R}|\mathbf{e}^{\mathbf{j\delta}}$$

because it is the ratio of two complex quantities. I already mentioned in some of the earlier lectures that although in reality you see the acoustic pressure, if you think about it, it is actually a scalar quantity and it is real. But why do we take complex quantities into account? Because it makes some of the derivations and computations much easier if we deal with exponential functions, as their differentiation and integration are easier, and we know that the basic wave equation is a partial differential equation. So, we would be doing a lot of integrating and differentiating if, for example, we are dealing with a complex system. So, hence we represent it in the complex form, and then when we do the ratio of these complex amplitudes, we end up with a complex quantity, which should have a phase and an amplitude. In the same way, this is denoted by capital R, the pressure reflection coefficient.

We have another quantity, the pressure transmission coefficient, denoted by capital T, which is the ratio of the complex amplitude of the transmitted wave to the incident wave. It is denoted by capital T and has an amplitude and phase:

$$\mathbf{T} = \frac{\mathbf{p_{t,max}}}{\mathbf{p_{i,\,max}}} = |\mathbf{T}|\mathbf{e^{j\delta}}$$



Then we have the intensity reflection coefficient, just like the pressure reflection coefficient was a ratio of the respective pressures. Intensity is the ratio of the respective time-averaged intensities of these two waves.

The RMS intensity of the reflected wave and the RMS intensity of the incident wave:

$$\mathbf{R_I} = \frac{\mathbf{I_{r,rms}}}{\mathbf{I_{i,rms}}}$$

Let us derive it in terms of the pressure reflection coefficient. So, we know that I_{RMS}, which we have already studied in the previous lecture, can be written as:

$$\mathbf{I_{rms}} = \frac{\mathbf{p_{max}}^2}{\mathbf{2\rho c}}$$

So, now this R_I can be written as:

$$\mathbf{R_I} = \frac{\mathbf{p_{r,max}}^2}{2\boldsymbol{\rho_1}\mathbf{c_1}}\frac{2\boldsymbol{\rho_1}\mathbf{c_1}}{\mathbf{p_{i,max}}^2}$$

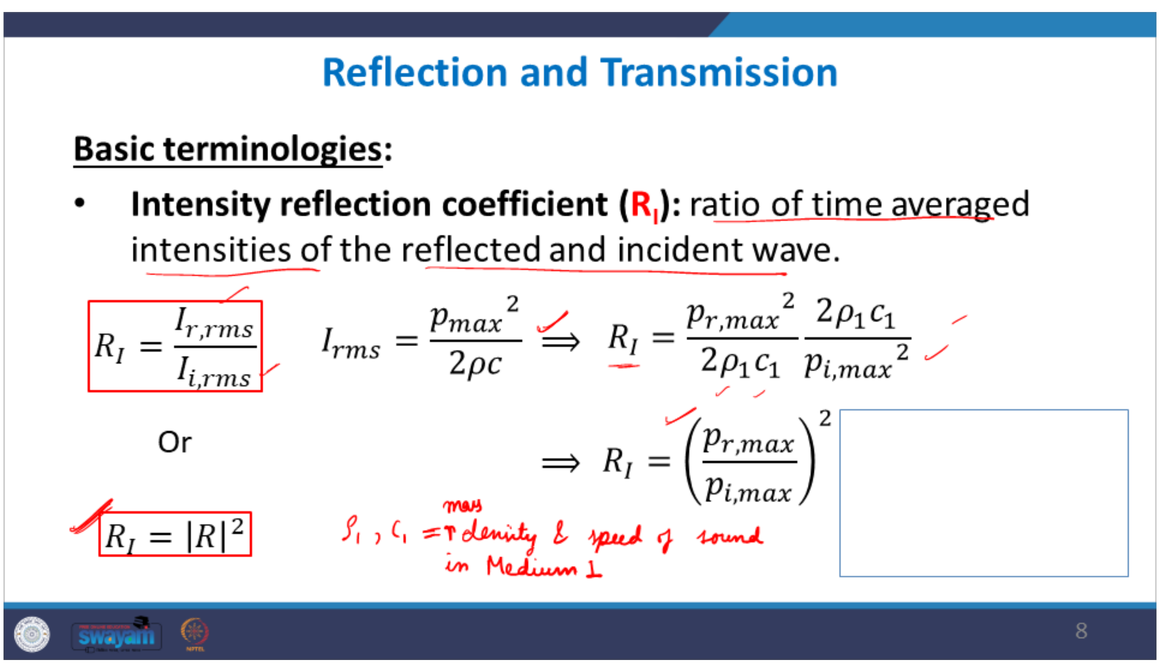
because here the incident and the reflected waves they are both in medium 1. So, it is $\boldsymbol{\rho_1}\mathbf{c_1}$, which means the density and the speed of sound of medium 1. Okay, this is something which I will make clear here, and then this is how it is: this gets cancelled out, and ultimately we end up with:

$$\mathbf{R_I} = \left(\frac{\mathbf{p_{r,max}}}{\mathbf{p_{i,\,max}}}\right)^2$$

and what is this inside the square? It is already defined as the pressure reflection coefficient.

$$\mathbf{R_I} = |\mathbf{R}|^2$$

So, this is the thing we get. So, here $\boldsymbol{\rho_1}\mathbf{c_1}$ is the density and the speed of sound. Again, density I will mention is mass density because as you go into higher sciences, you have different types of density. So, mass density and speed of sound in medium one. So, this becomes the relation.



The intensity transmission coefficient, again the same way, is the ratio of the time-averaged intensities of the transmitted and the incident wave. It is represented by this τ symbol.

$$\boldsymbol{\tau = \frac{I_{t,rms}}{I_{i,rms}}}$$

Let us derive a relation. So, again using the same format. We have τ as this whole thing:

$$\boldsymbol{I_{rms} = \frac{p_{max}{}^2}{2\rho c}}$$

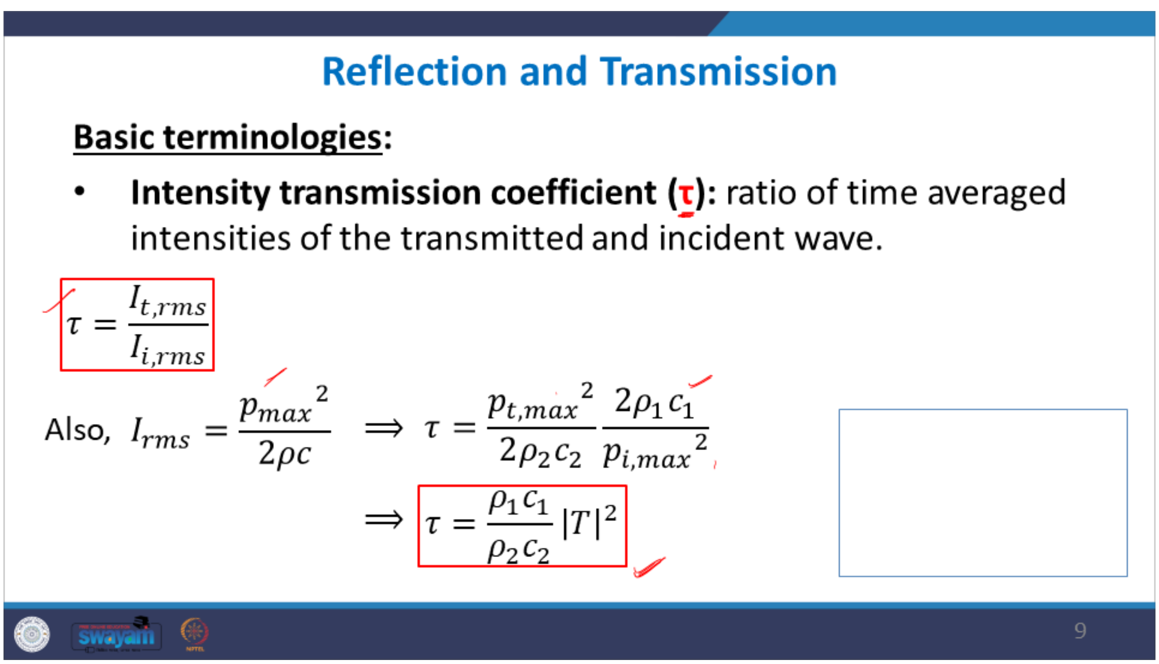
Here, the transmitted wave is in medium 2. So:

$$\boldsymbol{\tau = \frac{p_{t,max}{}^2}{2\rho_2 c_2}\frac{2\rho_1 c_1}{p_{i,max}{}^2}}$$

and the incident wave is in medium 1. So, here we have $\boldsymbol{\rho_1 c_1}$. So, we club it together, we get $\boldsymbol{\rho_1 c_1}$ by $\boldsymbol{\rho_2 c_2}$ and $\mathbf{P_{Tmax}}$ by $\mathbf{P_{Imax}}$ whole square is what? It is the square of this because T is a complex function.

$$\boldsymbol{\tau = \frac{\rho_1 c_1}{\rho_2 c_2}|T|^2}$$

So, directly the magnitude square we are doing. For the pressure transmission coefficient, Okay.



Now, what is the absorption coefficient? So, we know that when the wave is incident, here again I would like to make a distinction. When we study porous materials further down in this lecture series, we will deal with the absorption coefficient and we will come to know that when the wave is incident, what happens? Suppose you have medium 1, medium 2, and you have a layer of material in between. So, essentially, it is a 3-medium interaction.

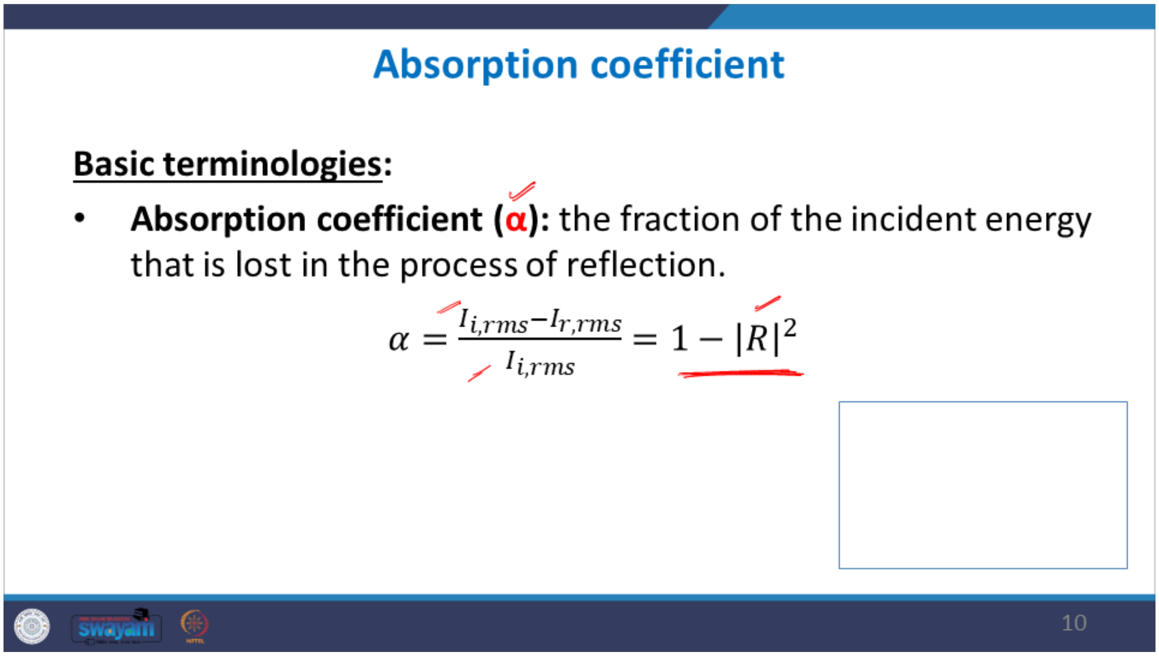
So, in that three-medium interaction, what happens at each boundary? You have reflection, incidence, reflection, transmission, incidence, reflection, transmission, and in that layer of material that is there in between, some energy gets lost. That becomes, you know, the energy that is being absorbed. So, we use the term absorbed, but Physically, what is the definition of absorption? It means that whenever there is an interaction at a medium boundary, some energy is lost. What is the loss in the reflected energy? Because essentially, you just see that suppose this is one layer of material. So, you view it like this.

A wave came, hit it like this, and came back. So, it is hit and come back. But the hitting wave and the coming back wave have some loss. In the reflected wave compared to the incident wave. So, just with this, you are just using the reference of medium 1.

So, it is going, hitting, and then there is some loss in the energy while returning back. So, what is that loss as a fraction of the incident energy that gives you the absorption at the surface? So, then it becomes: what is the difference in the intensity of the incident and the reflected wave divided by the incident wave in relative terms? So, if you do this:

$$\boldsymbol{\alpha} = \frac{\mathbf{I_{i,rms}} - \mathbf{I_{r,rms}}}{\mathbf{I_{i,rms}}} = \mathbf{1} - |\mathbf{R}|^{\mathbf{2}}$$

So, this becomes another important relationship.



Now, there is one other important terminology which we need to know whenever we are discussing, you know, sound propagation through different media, that is called the impedance. So, we have already, if you are familiar, if you are a science student, you are familiar with the concepts of, you know, electricity, electric current flowing, and then the concept of force and the energy flow. Then you would know that, in general, in these kind of situations, impedance refers to the resistance to the flow of whatever quantity you are measuring, and most of the time, because acoustics is newer than the field of electricity.

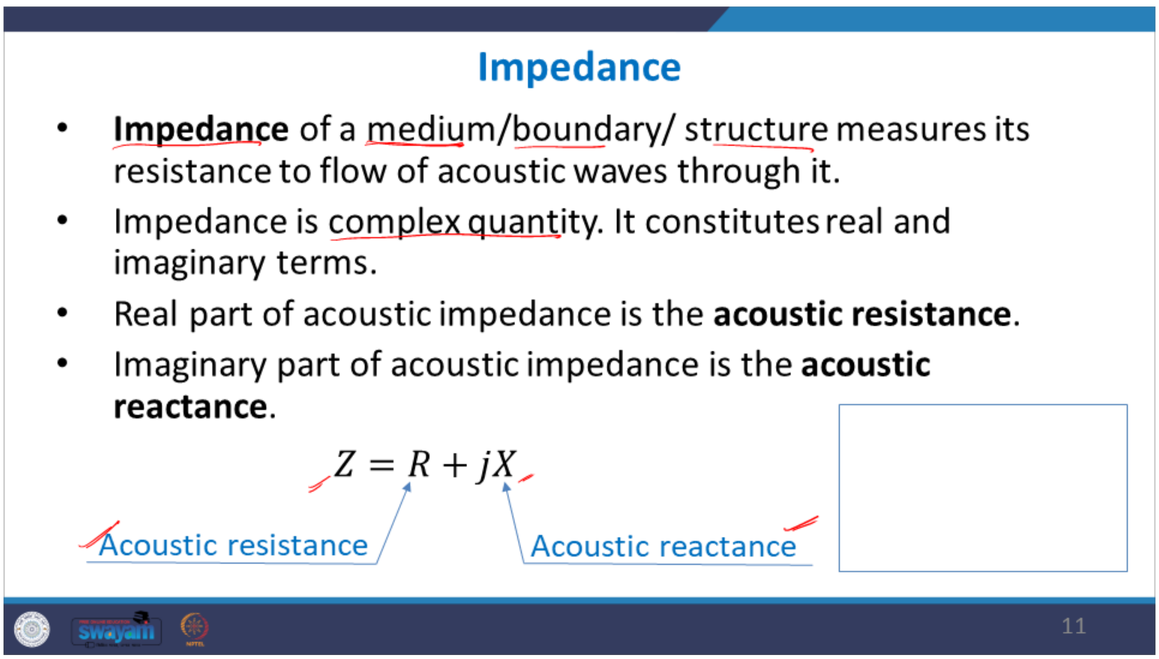
Let me put it that way. So, most of the time, we derive various analogies from the field of electricity and apply them to the field of acoustics. So, in the field of electricity, what is the resistance of any element? It is the resistance to the flow of current through that element. And impedance is the complex quantity of that resistance.

In the same way, impedance in the field of acoustics can be thought of as the resistance to the flow of sound waves. This impedance could be offered by either a medium, or it could be offered as resistance to anything, you know, whatever you are trying to measure. Suppose you just want to know, in general, what resistance this medium is offering to the flow of sound waves, or you can measure a particular boundary to see what resistance it is offering to the entering of sound waves. Or some kind of lumped structure, you can have, like some kind of a black box device where the sound wave is incident and then comes out.

So, it is offering some kind of resistance to the flow of sound waves. What is the impedance it is offering? Okay.

It is a complex quantity; the real part is called the acoustic resistance, and the imaginary part is called the acoustic reactance, denoted by the letter Z.

$$\mathbf{Z = R + jX}$$



Now, there are various types of impedance, as you would have seen in the definition itself; it could be for anything. So, depending on whose resistance you are trying to measure, you can classify your acoustic impedance in that way. So, the first type of impedance is the specific acoustic impedance. So, you can denote it as capital Z_{SA}, or you can simply write it as the characteristic impedance or small z.

So, in many books, you see that sometimes people use this term, and in some books, you see this one; they are both the same. So, what do you mean by that? It is the ratio of the acoustic pressure at a point divided by the particle speed at that point. So, remember, we are using the term speed here, which means it is not velocity; it is the magnitude of the velocity. So, pressure itself is scalar, the speed also becomes scalar, and we end up with a scalar quantity.

Okay.

So, Z_{SA} is:

$$\mathbf{Z_{sa}} = \frac{\mathbf{p}}{|\vec{\mathbf{v}}|}$$

and we already know that V is the relation between pressure and velocity. So,

$$|\vec{\mathbf{v}}| = \frac{\mathbf{p}}{\boldsymbol{\rho_0}\mathbf{c}}$$

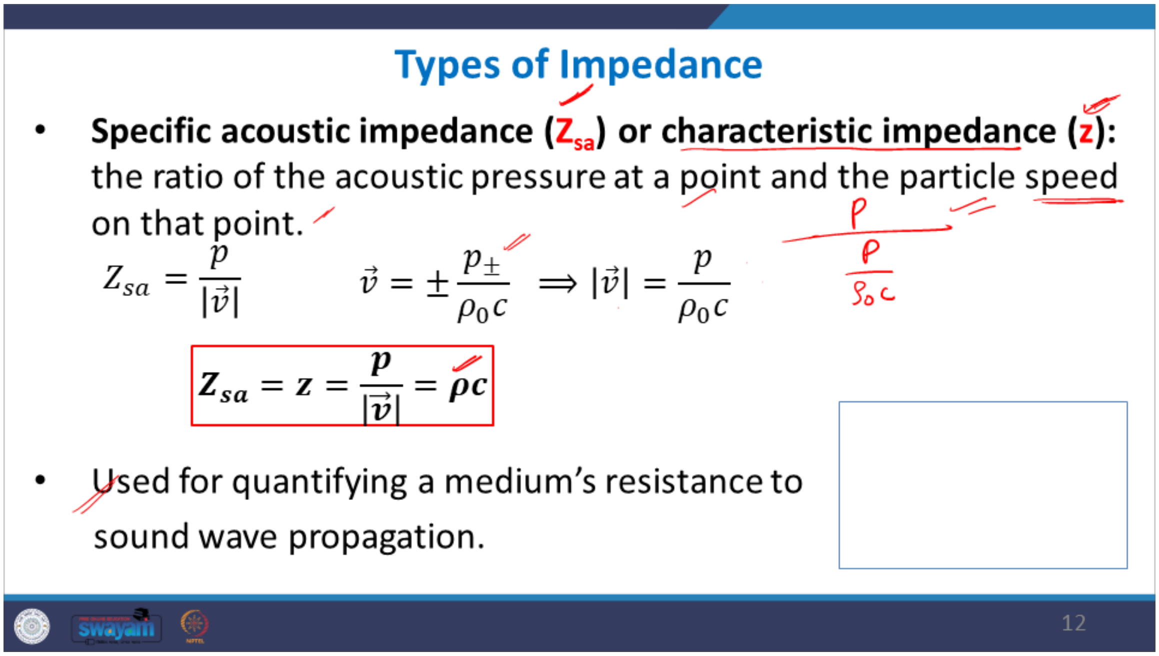
So,

$$\mathbf{Z_{sa}} = \mathbf{z} = \frac{\mathbf{p}}{\frac{\mathbf{p}}{\boldsymbol{\rho_0}\mathbf{c}}}$$

$$\mathbf{Z_{sa}} = \mathbf{z} = \frac{\mathbf{p}}{|\vec{\mathbf{v}}|} = \boldsymbol{\rho}\mathbf{c}$$

whatever way you want to use it. So, you will come up with this term.

This is the mass density multiplied by the speed of sound. Now, you do not see any surface area or anything coming over here. So, this means that this is for any point in space, and it is usually used to characterize a medium. So, let us say we can, if we just want to compare the medium air and the medium water, to find out which is better for sound wave propagation. So, here there is no structure in mind, that we have got a cylindrical tube of wave and a cylindrical tube of water, or we have this much area of water or this much area of air, no structure in mind, just the medium in general. Then we can use this term to quantify the medium's resistance to sound wave propagation.



But if, suppose, we have got a certain element, So, let us say we are having the sound waves propagating, and suddenly they encounter this kind of Helmholtz cavity, some kind of cavity they are encountering, and this also has the medium air; this also has the medium air; both of them are having the same medium, right? Air and air, but this air is enclosed into this kind of a neck-shaped, narrow neck-shaped, enclosed cavity, right? So, now in this, suppose you have something like this, you know, you have got a resonating element, and the sound waves are incident on that. So, you cannot just use this to quantify the difference, right? Because this is used to just measure for different media, like air versus water versus some kind of steel or some kind of solid structure.

So, here both the media are the same. So, how will you, so effectively, there is no impedance? That is not the case, right? So, if some resistance is being offered by this element by the virtue of how the medium is being enclosed or how the medium is structured around, So, we need some new type of impedance just to measure what is the resistance offered by this particular structure combining different media. So, there we use acoustic impedance.

What is the definition of it? It is the ratio of the complex acoustic pressure on a medium surface. So, suppose a sound wave is incident here. So, what is the pressure at which it is incident on this particular element? Here, this thing is the entire element we are taking into consideration, and this is the surface over which it is being incident, that small neck surface.

So, the ratio of the acoustic pressure on the medium surface divided by the complex volume velocity on that surface. So, what is the volume velocity of the waves propagating from that surface? So:

$$\mathbf{Z} = \frac{\mathbf{p}}{\mathbf{U}}$$

which is the volume velocity. The volume velocity is what? It is the velocity multiplied by the surface area.

$$\mathbf{U} = |\vec{\mathbf{v}}|\boldsymbol{S}$$

Here, I have considered that you know the surface area and the velocity incident is normal, and I have directly taken the product, but if it is some complex situation where only a component of the velocity is incident, automatically you will have to take that component of the velocity which is normal to the surface area because to measure volume velocity, you want to see how much volume of air, basically you want to see what is the rate of the volume of air that that particular, you know, wave is pushing forward, okay. So, the velocity should be able to push it along that surface area. So, automatically, this you have to take the component of your V vector that is normal to your surface area under consideration.

So, when they are aligned, you get:

$$\frac{\mathbf{p}}{|\vec{\mathbf{v}}|}$$

if, suppose, they are both in the same direction, in that case only when this holds true, that is V when V is perpendicular to the surface area under consideration, then we will have this relationship:

$$\frac{\mathbf{p}}{|\vec{\mathbf{v}}|}$$

becomes your small z,

$$\mathbf{Z} = \mathbf{z}/\mathbf{S}$$

So, this is one way to, you know, make this capital:

$$\mathbf{Z} = \mathbf{z}/\mathbf{S}$$

that is acoustic impedance equals to characteristic by small by the surface area. Another way to write this is you can simply write:

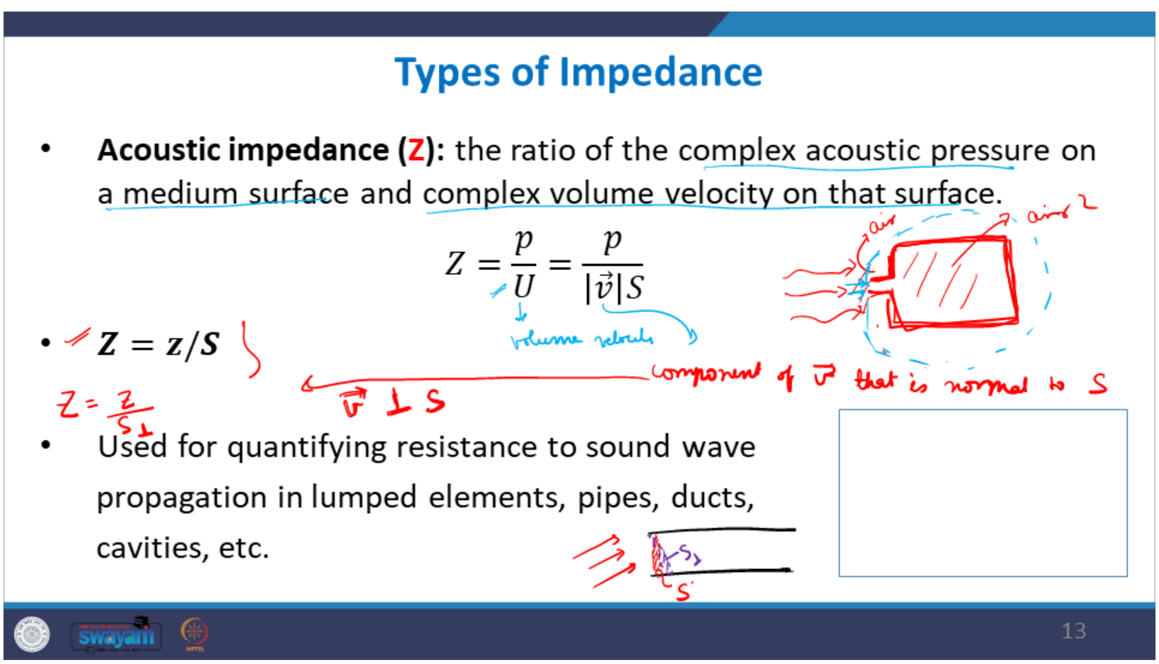
$$\mathbf{Z = z/Sp}$$

So, suppose this is some structure through which it has to flow, some kind of structure, and the sound wave is incident, let us say, along this direction. So, you take the equivalent surface area in the direction of the wave propagation.

So, S perpendicular means the surface area where this V becomes perpendicular. So, instead of taking this surface area, you would take something which is like this in this particular tube, ok. Let me illustrate it slightly better. Suppose you had this as a structure, some kind of structure. Okay, and the sound wave is incident on that, but instead of incident normally, it is incident in this particular direction.

Okay, so maybe the surface area, this is, let us say, the surface area S of this particular element, but the wave is not incident normally. So, what would you do? You would take the S perpendicular, the effective surface area normal to the V vector. So, you would take something like this. And this becomes your S perpendicular.

So, this way you can represent it, and it is used, as I told you, for quantifying the resistance of various elements, pipes, ducts, cavities, etc.



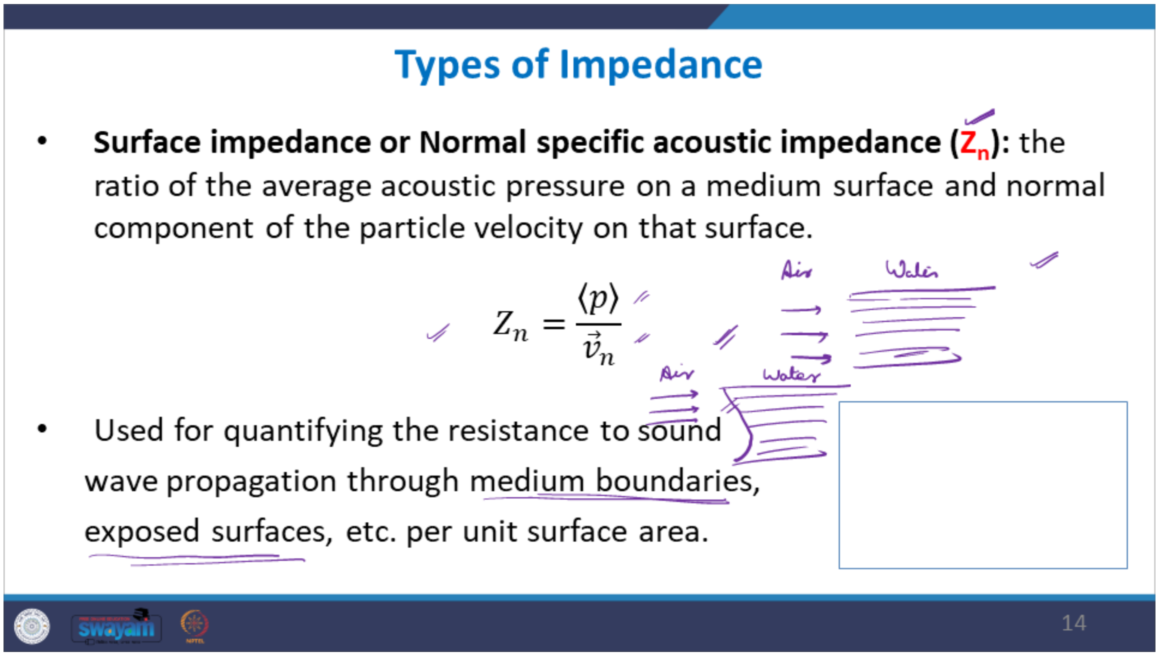
Then there is another thing called surface impedance or normal specific acoustic impedance Z_n. What is Z_n? It is the average acoustic pressure on a medium surface and the normal component of the particle velocity on that surface.

$$\mathbf{Z_n = \frac{\langle p \rangle}{\vec{v}_n}}$$

Now, this is the term which is used for quantifying what is the resistance offered by a medium boundary.

Okay So, various medium boundaries of various kinds of exposed surfaces, just we want to know what is the impedance because of this exposed surface, we can use this terminology. Okay So, let us say again, take let us take the example: we have an air medium and the wave and it is incident on a water medium, okay, in one way it is incident like this, and this is the, you know, water medium, and then there is a case too when again we have the air medium and the water medium, and the wave is coming like this, and it is incident on a water medium whose boundary might be like this, again we have the same media

We cannot use just 'we have' in both cases. We have got the same set of media, okay? So, we cannot here differentiate between the two. Things using just the characteristic impedance because both the media or the media pair are the same. The difference is in the type of boundary and the exposure, okay, in the incident wave. So here, in that case, to characterize this difference on how it is being exposed, we use this particular normal specific acoustic impedance. So, this is a per-unit area kind of quantity.



Okay. Then, the last type of impedance is the radiation impedance. It is the ratio of the force on a vibrating surface and the resultant particle speed of the sound waves it generates.

$$\mathbf{Z_r} = \frac{\mathbf{F}}{|\vec{\mathbf{v}}|}$$

$$\mathbf{Z_r} = \frac{\mathbf{pS}}{|\vec{\mathbf{v}}|}$$

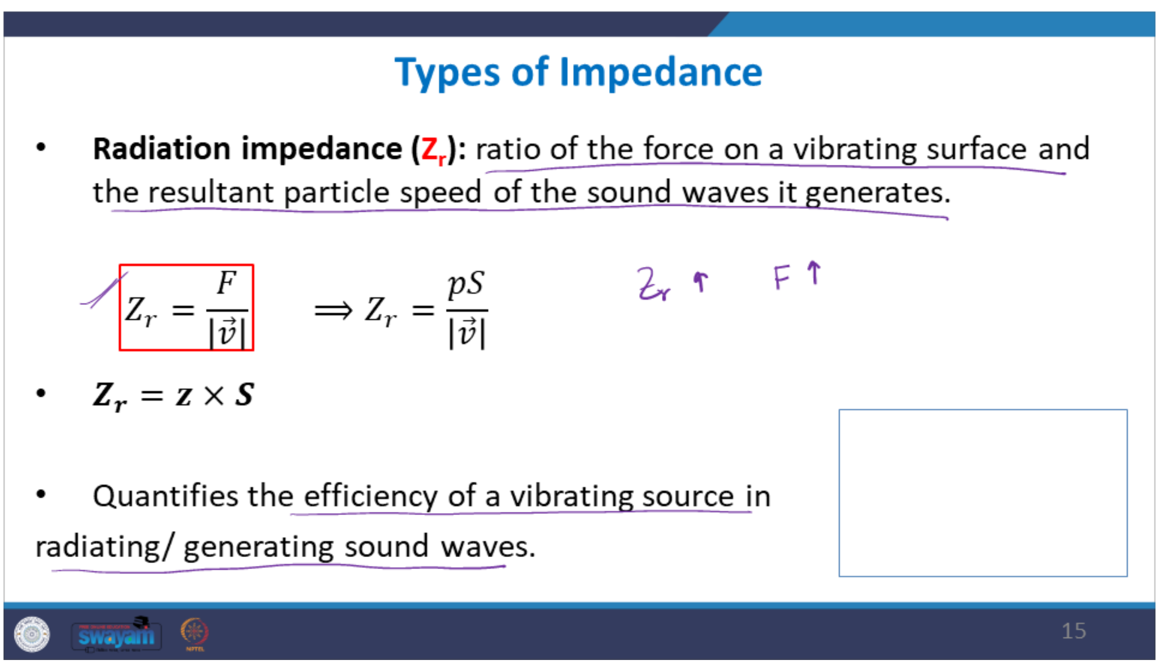
$$\boldsymbol{Z_r = z \times S}$$

So, this actually measures what? It measures the efficiency with which a vibrating source is generating or it is creating the sound waves. So, here what happens? Let us take the example of the sphere which is contracting and then which is sort of contracting and expanding like this, and it is creating the spherical waves all around. There could be one wave that is sort of moving back and forth with the same kind of volume velocity, and then there is another wave. Suppose we have two spheres; both of them are pulsating, or they are expanding and contracting with the same force.

So, the force measured on the surface because the surface is sort of vibrating because of the expansion and contraction, we can represent it as the vibration of the surface. So, here both the surfaces they are in. So, the structure that is, you know, vibrating is with the same force. So, both the spheres they are sort of pulsating with the same force, but with the one sphere we are getting a much larger amplitude sound waves, whereas with the other sphere we are getting smaller amplitude sound waves, okay? So, they are not both generating an equal amount of sound, which means that inherent to that solid structure which is creating.

There is some form of impedance element there itself. So, in that situation, how to quantify this? We use the radiation impedance, which says that you know how efficiently a vibrating source can create sound waves. So, it would be the ratio of the force acting on that vibrating surface or the force of the vibrating surface and the resultant speed of the sound wave it generates, and the speed is directly proportional to the acoustic pressure, okay? So, if you know this definition, then you can think about suppose some surface has got a higher radiation impedance, what should that mean? Suppose some structure has got a higher radiation impedance, so what does it mean? Is it more efficient at creating sound waves or less efficient?

So, let us see. Suppose something has got higher Z_r. Suppose Z_r is high; this means for the same particle speed to create the same kind of waves; you need a higher force. Okay, so that means that surface efficiency of creating sound waves is less. So, the more impedance inherent within it for radiating sound waves, the lesser obviously it's going to generate the sound waves.



Okay, so with all these definitions, let's do one classical, you know, phenomenon where a plane wave is incident on a planar boundary. So, we already know that, you know, it depends on the type of boundary as well. So, we will take into account a simple case: a plane wave incident on a planar boundary at the normal angle, okay.

So, here, how do we, you know, represent this? We can represent this $\mathbf{P_i}$ as:

$$\mathbf{p_i = p_{i,max} e^{j(\omega t - k_1 x)}}$$

the plane wave representation. And the reflected wave, in the same way, it should also have a certain amplitude.

$$\mathbf{p_r = p_{r,max} e^{j(\omega t + k_1 x)} = R p_{i,max} e^{j(\omega t + k_1 x)}}$$

But here, the incident wave, if suppose we are taking the boundary at x=0, and this is our positive x direction, then the $\mathbf{p_i}$ is moving in the positive x direction, and the $\mathbf{p_t}$ is also moving in the positive x direction, but this is moving in the other way. So, here, this would be a plus sign. Now, this is what this is: $\mathbf{p_{r,max}}$, and we know that R is what it is:

$$\mathbf{R = \frac{p_{r,max}}{p_{i,\,max}}}$$

So, $\mathbf{p_{r,max}}$can be represented as R Times $\mathbf{p_{i,\,max}}$

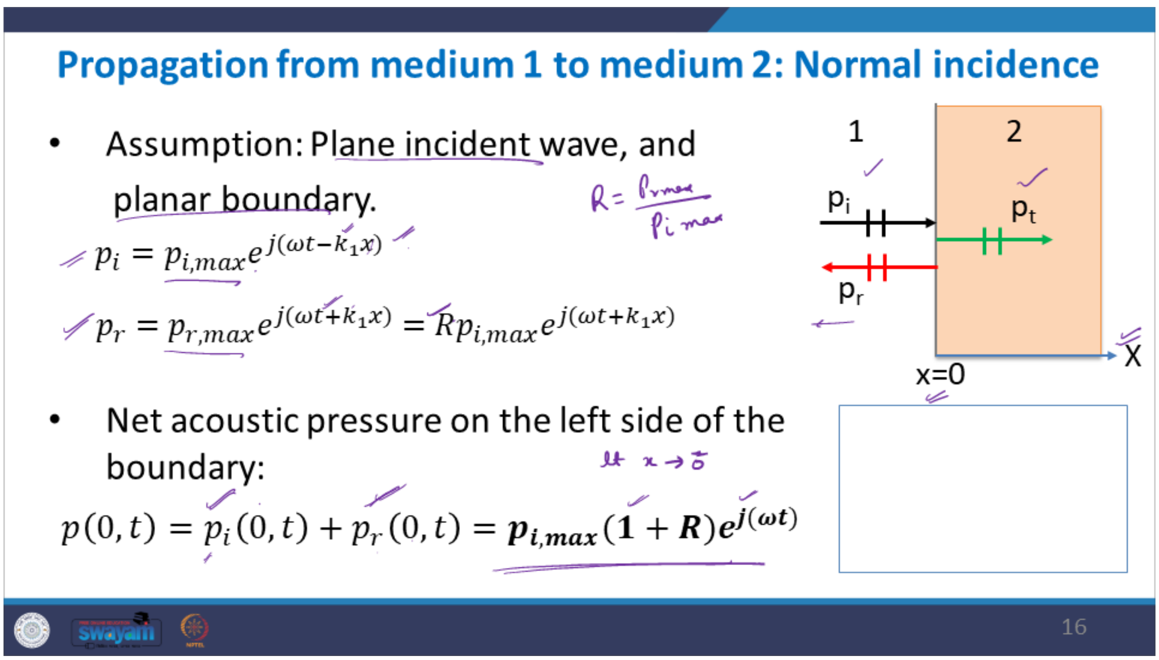
So, that is how we are writing it. Now, let us see what is the net acoustic pressure exactly just left side of the boundary in the limit x tends to 0 negative. You can talk in this term of mathematics as this x tends to 0 negative, so just left side of the boundary, what is the total acoustic pressure which is present? So, you will simply sum up because this is acoustic pressure, so it is independent of the direction of the wave. It is a scalar quantity; we simply add up the pressure. So, we add up the pressure due to the incident wave at x equals to 0, and we add up the pressure due to the reflected wave at x equals to 0.

$$\mathbf{p(0,t) = p_i(0,t) + p_r(0,t)}$$

And when you sum up these two equations, what you get is $\mathbf{P_{Imax}}$ into, because its x is 0 here.

So, this quantity automatically becomes 0. So, you get:

$$\mathbf{p_{i,max}(1 + R)e^{j(\omega t)}}$$



So, from the first two equations. Now, we find out what is the net velocity exactly on the left side, just on the left side, or limit x tends to 0 negative. So, v_i, let us see the form. It would be the pressure of the incident wave, which is

$$\mathbf{v_i = \frac{p_{i,max}e^{j(\omega t-k_1x)}}{\rho_1 c_1}}$$

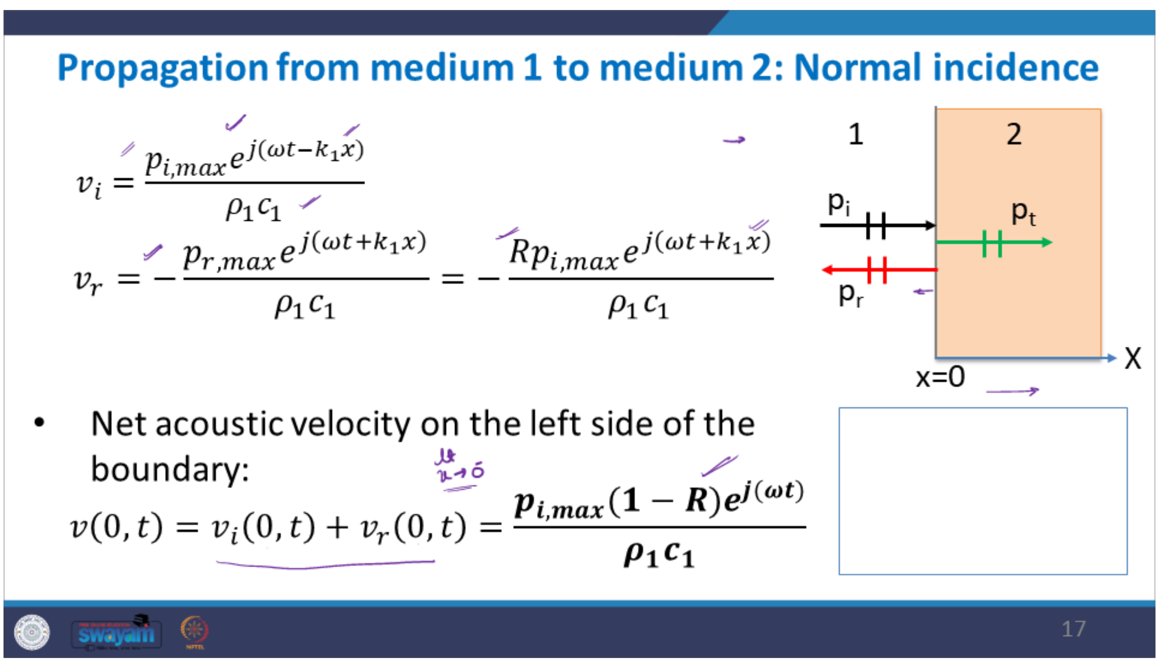
So, here, you see here why I am here. The plus and the minus sign depends on the velocity.

So, you find the magnitude and you see which is the velocity of the particle wave. Now, because sound is a longitudinal wave, so overall the medium particles are moving forward like this. It is along the P_I direction, so positive. So, we have a positive P by $\boldsymbol{\rho_1 c_1}$, but here the reflected wave is moving backwards. So, because this is our positive direction, right? Positive x. So it is moving backwards or away.

So here the velocity of one is forward whereas the velocity of the other wave, or the velocity is the same as the direction of the wave propagation, is backwards. So here we have a minus sign. The velocity is what? It is a vector quantity. So, we ultimately do not just add them up like pressures.

We need to have the sign also sorted out. So, we have sorted out the sign and did the same kind of computations. And what do we end up with? Suppose we want to find out the boundary, we put x is equal to 0 on both these equations here and here. And we add them up to find out the total velocity. We are directly adding them because both the velocities' direction is the same, and the plus minus takes care of whether it is forwards or backwards, so directly we are adding them. So this is the term we end up with.

$$\mathbf{v(0,t) = v_i(0,t) + v_r(0,t) = \frac{p_{i,max}(1-R)e^{j(\omega t)}}{\rho_1 c_1}}$$



By definition, what is the normal specific acoustic impedance? It is the

$$\mathbf{Z_{n,boundary}} = \frac{\mathbf{p(0,t)}}{\mathbf{v(0,t)}},$$

but here let me quickly tell you this is the specific acoustic impedance. If suppose the normal component of the velocity was to be the same as the actual velocity, which means that the entire velocity is sort of normal to the surface, then the Z_{SA} would be similar to Z_N because here the normal component is the same as the velocity itself. So, the impedance we are calculating.

So, here in this particular case, when it is normal incidence, it is:

$$\mathbf{Z_{sa}} = \frac{\mathbf{p}}{|\vec{\mathbf{v}}|}$$

They are the same, so we simply do **P by V** at x equals 0 to find out for this surface what is the impedance, and the equations we have derived, we put them.

$$\mathbf{Z_{n,\ boundary}} = \frac{\mathbf{p(0,t)}}{\mathbf{v(0,t)}} = \frac{\mathbf{p_{i,max}(1+R)e^{i(\omega t)}}}{\frac{\mathbf{p_{i,max}(1-R)e^{j(\omega t)}}}{\boldsymbol{\rho_1 c_1}}}$$

and this is by definition all same for the medium 2 because it is a medium 2 we are entering. So, this becomes by definition the Z of the medium 2. So, the Z of the medium 2 is this by this into this quantity.

$$\mathbf{z_2} = \frac{\mathbf{1+R}}{\mathbf{1-R}}\boldsymbol{\rho_1 c_1}$$

Now, we know that $\boldsymbol{\rho c}$ is what? It is the characteristic impedance of that medium. So, here it is $\boldsymbol{\rho_1 c_1}$. So, which means it is the characteristic impedance of the first medium z_1.

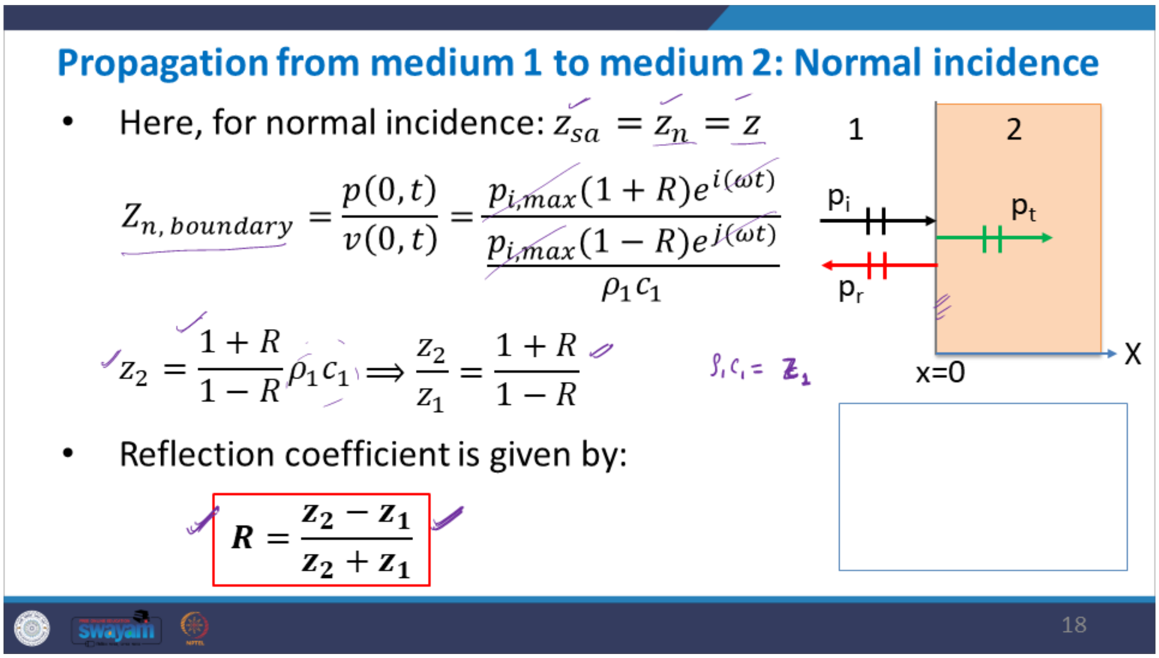
So,

$$\frac{\mathbf{z_2}}{\mathbf{z_1}} = \frac{\mathbf{1+R}}{\mathbf{1-R}}$$

If you solve this equation, you can solve it like the traditional way, or you can use, you know, the componendo and dividendo property of the proportions, and you end up with this equation. So, this is very important.

$$R = \frac{z_2 - z_1}{z_2 + z_1}$$

So, we have got the first equation for the reflection coefficient in terms of the impedances of the two media.



So, in the beginning, I told you that the reflection and the transmission depend on the medium properties, and that is the dependence that we have derived just now. And for this one, it is now let us take the format of the $\mathbf{P_t}$, which is the acoustic pressure of the transmitted wave; it is this amplitude.

$$\mathbf{p_t = p_{t,max} e^{j(\omega t - k_2 x)}}$$

And the transmitted wave is also traveling in the positive x direction. So, we have a minus sign. And then some component $\mathbf{K_2}$.

So, why are we taking different $\mathbf{K_s}$ for the two media? So, see for the first one, for $\mathbf{P_i}$ and $\mathbf{P_r}$, we have taken some K, $\mathbf{K_1}$. And for the second medium, we are taking $\mathbf{K_2}$. Why? Because these are two different media.

The transmitted wave is in medium 2, right? So,

$$\mathbf{K_1 = \omega / c_1.}$$

So, in any interaction, I want to clarify here that the frequency remains the same; it is the difference in the speed of sound that changes K and $\mathbf{K_2}$.

$\mathbf{K_2}$ = **ω / C2** in both mediums because they are different; they have a different speed of sound. So, hence **K_1 and K_2** are different.

So, this is the $\mathbf{K_2}$ for the transmitted wave. We apply the continuity of pressure, which means that the pressure present just at the left side of the boundary Because the pressure is the same, the pressure is not, you know, vanishing at that point. So, the pressure at 0 negative should be the same as the pressure at 0 positive. It is not suddenly vanishing at the point.

So, we have already solved for the left side. We equate it to the pressure on the positive side, which is $\mathbf{P_t}$. So, P_t at 0_t should be the same as P_i 0_t plus P_r 0_t.

$$\mathbf{p_i(0,t) + p_r(0,t) = p_t(0,t)}$$

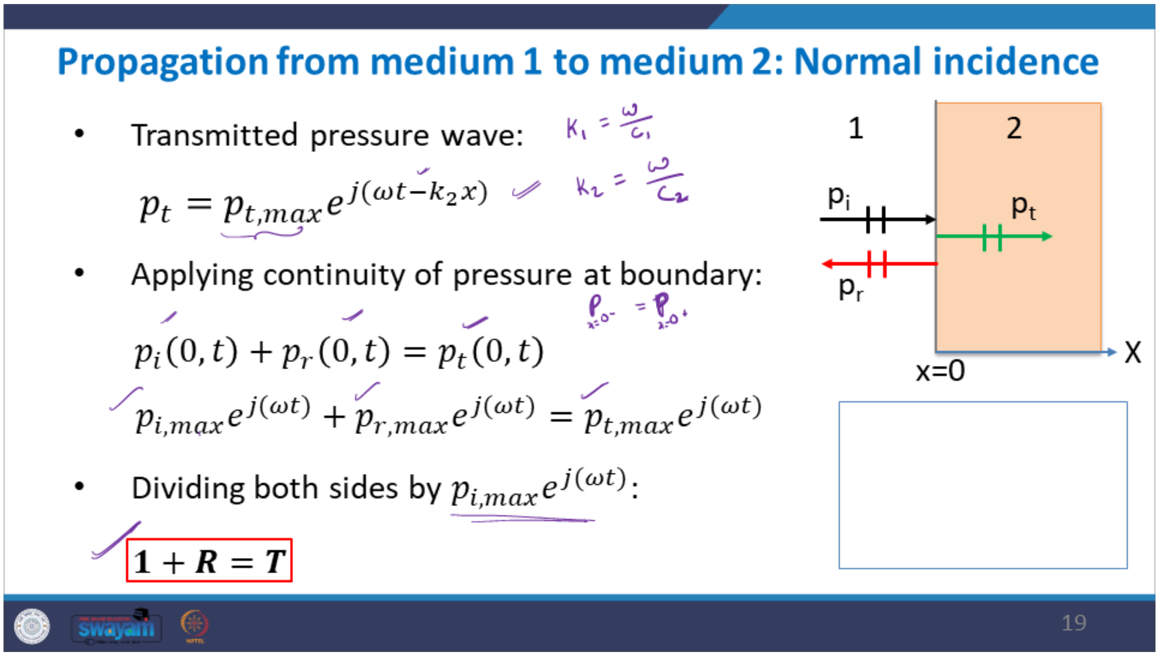
Okay, so let's put them in the respective equations, you know, x equals to 0, so this is the equation we are getting.

$$\mathbf{p_{i,max}e^{j(\omega t)} + p_{r,max}e^{j(\omega t)} = p_{t,max}e^{j(\omega t)}}$$

Okay, and fine. So, when you divide the entire thing by $\mathbf{p_{i,max}e^{j(\omega t)}}$

so this by P_{imax}, becomes 1, this becomes the reflection coefficient, and this term here becomes the transmission coefficient. You end up with this important relationship:

$$\mathbf{1 + \mathit{R} = \mathit{T}}$$



Now you already have found R:

$$\mathbf{R} = \frac{z_2 - z_1}{z_2 + z_1}.$$

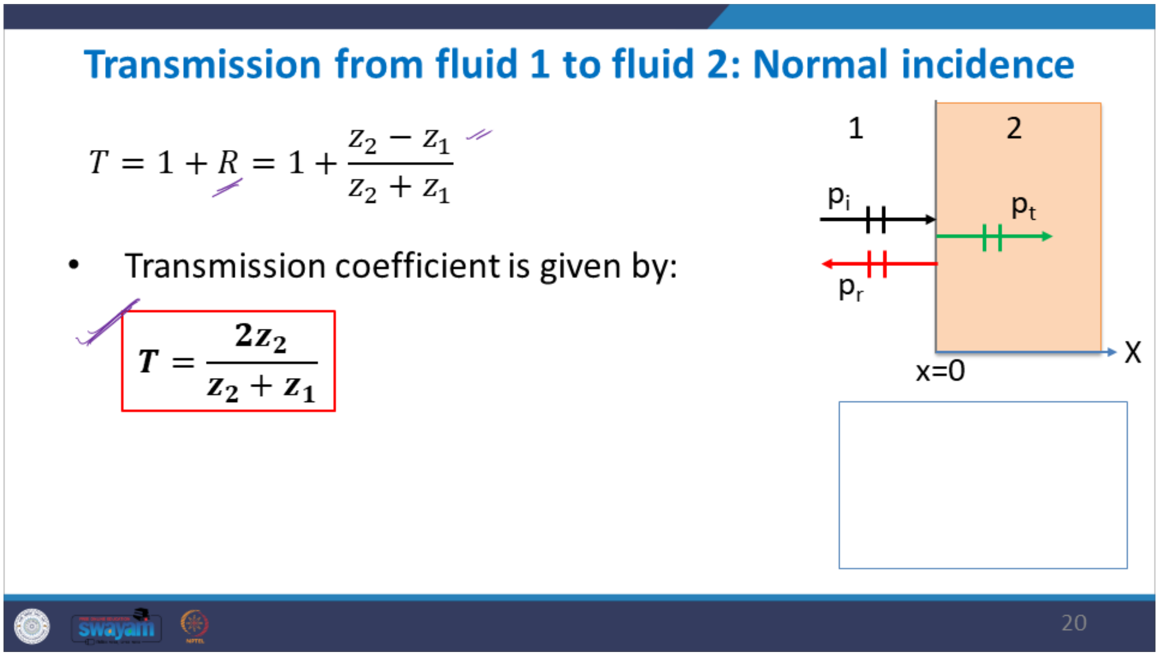
You solve it for T,

$$\mathbf{T = 1 + R = 1 +}\frac{z_2 - z_1}{z_2 + z_1}$$

and this is what your T is:

$$\boldsymbol{T} = \frac{2z_2}{z_2 + z_1}$$

So, the amount of reflection you will get or the amount of transmission you will get essentially depends upon what the characteristic impedance of medium 1 and 2 is. And here, it is only dependent on the characteristic impedance because we have fixed the per unit area thing. We have made it that this is the per unit area, and we have made the planar boundary. Suppose it was some different kind of structure, then obviously it will depend upon the design of the structure itself.



Okay. So, with this, I would like to end this lecture and hopefully see you next time for some special cases of this particular interaction.