NOISE CONTROL IN MECHANICAL SYSTEMS

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Week:1

Lecture:5

Lecture 05: Sound Wave Propagation Numerical

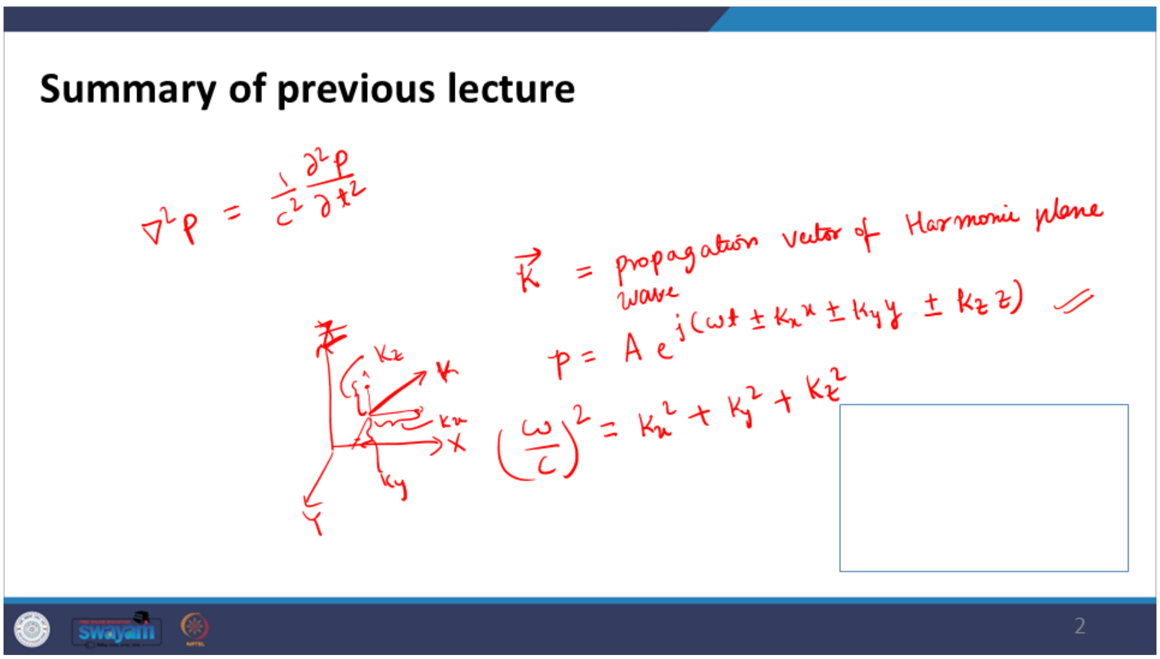
Hello and welcome back to the lecture series on noise control in mechanical systems with me Professor Sneha Singh. So, this is lecture 5 where we will solve some numerical problems with respect to sound wave propagation.

So, to quickly summarize what we have discussed in the topic of sound wave propagation so far, we were able to derive the linear acoustic wave equation which is given by this particular equation here.

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

And we were also able to derive the solution for a harmonic plane wave and the end result of that was that suppose we have a harmonic plane wave travelling in a generic direction where this vector k is the propagation vector of the harmonic plane wave. So, it will have a certain component in the x axis, y axis and the z axis and this is the direction k which is a generic vector which represents the direction of the wave propagation and the magnitude of this represents the wave number k which is equal to $\frac{\omega}{c}$.

So, the general solution which we had found was



$$p(x,\ t) = \boldsymbol{A}\boldsymbol{e}^{\boldsymbol{j}(\boldsymbol{\omega t} \pm \boldsymbol{k_x x} \pm \boldsymbol{k_y y} \pm \boldsymbol{k_z z})}$$

where kx, ky and kz these are sort of the resolution or the these are the components of the k vector in the x axis direction, y axis direction and z axis direction respectively and there was another equation which is

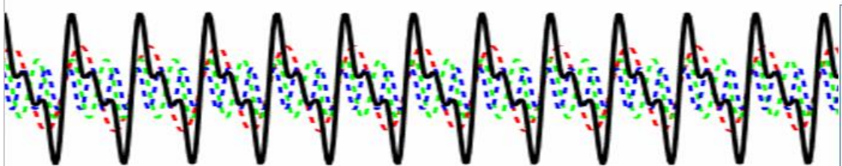
$$\left(\frac{\omega}{c}\right)^2 = k_x^2 + k_y^2 + k_z^2$$

with this let us solve some numerical problems.

the first problem it is seems a rather lengthy problem, but using this would be able to sort out a lot of the fundamentals in sound wave propagation.

Outline

- **Numerical problems on:**
  - Acoustic wave equation
  - Sound as wave
  - Harmonic plane waves



Problem - 1

- A tuning fork is struck on an object and it starts vibrating at frequency of 500 Hz. Assuming a listener is located at 2 metres from the fork, and there is constant sound wave propagation without any attenuation, find the following quantities (Take speed of sound in air as 345 m/s).

a) Wavelength of the sound wave generated
b) Time period of the sound wave generated
c) Time taken by the sound wave to reach the listener
d) Number of sound waves that occupy the spatial distance between fork and listener
e) Number of radians covered by the sound wave between fork to listener
f) Number of radians covered by the sound wave in 2 seconds.

So here we are using a tuning fork. So, we know that the tuning fork you strike it, it starts vibrating, the tongs they vibrate and they create these longitudinal waves. So, we are using a tuning fork. It is being struck to create a sound wave of frequency 500 Hz and a listener is located 2 meters away from the fork and there is a constant sound wave propagation without any attenuation. If nothing is given to you, we are saying that it is a constant sound wave propagation and there is no attenuation. So, by default now you can assume a harmonic plane wave solution because in the harmonic plane wave solution you have the constant amplitude and there is no attenuation with respect to spatial distance. So, this is given in your problem. So, you are assuming a harmonic plane wave solution. Now, with this we take the speed of the sound as 345 meters per second and we try to solve the various things one by one.

First of all, In the first two parts, which is pretty straightforward, we are asking about what is the wavelength of the sound wave that is generated and the time period of the sound wave generated. So, let us solve the first two parts.

$$f = 500\ Hz\ and\ c = 345\frac{m}{s}$$

$$\lambda = \frac{c}{f} --------- f = \frac{1}{T}$$

So, we get this answer as so the this is having the units as meters per second and this is having the units as hertz which is per second. So, when you solve this, you get 0.69 meters. So, this becomes your wavelength.

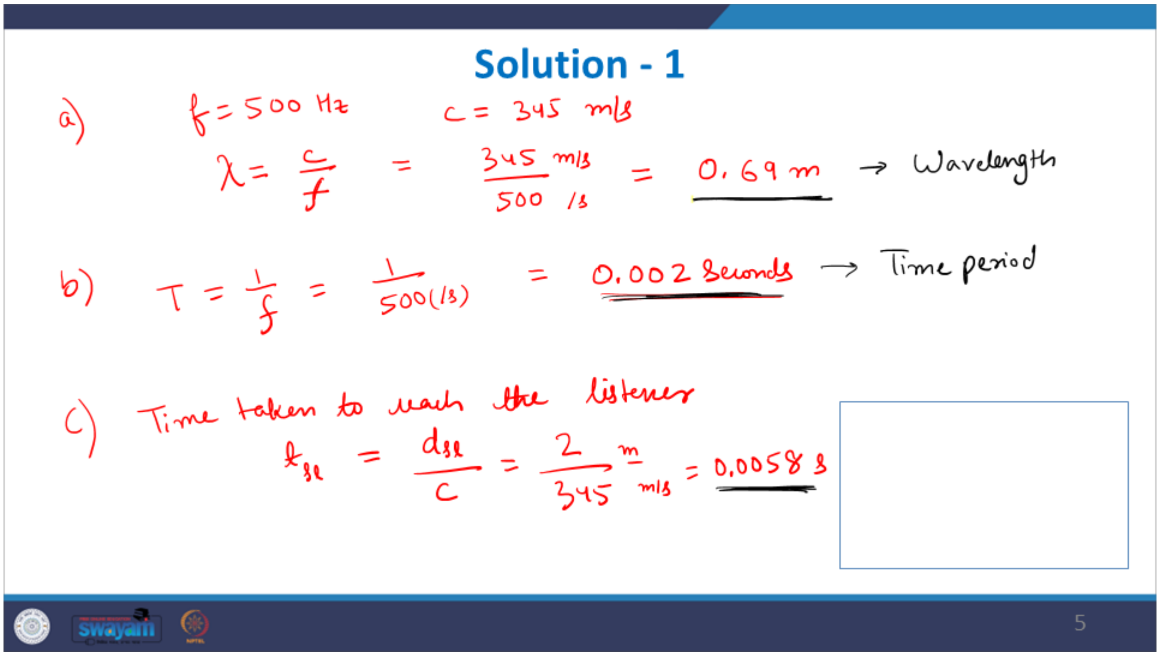
$$f = \frac{1}{T}$$

$$T = \frac{1}{f} = \frac{1}{500} = 0.002\ seconds.$$

This becomes your time period of the wave generated. Now, let us go and solve this part C and D. What is the time taken by the sound wave to reach the listener? Now, it is given that the source or the fork which is the source of the sound is 2 meters away from the listener. So, the distance that the sound wave has to travel is 2 meters to get to the listener. And no other information is mentioned. It's not mentioned that, you know, there are walls, there are reflections or anything. It's just propagating forward. So, we assume that there is no reflection and the sound wave is propagating.

So, it has to reach the listener, which means it propagates and covers a distance of 2 meters. And what's the speed? 345 meters per second, very straightforward. For the part C, the time taken to reach the listener. let us indicate it as TSL that is the time taken from source to listener. TSL is given as the distance between the source to listener divided by the speed with which it is travelling.

$$t_{sl} = \frac{d_{sl}}{c} = \frac{2}{345} = 0.0058\ seconds$$



Let us go and see what is part d. What is the number of sound waves that occupy the spatial distance between fork and listener? Now, if you look at it It will look complicated, but it is rather simple. It is asking how many waves are occupying the space between the source and the listener. And what is the distance between them? It is 2 meters. Essentially, how many waves are occupying a distance of 2 meters? For that, let us find out number of waves in a distance of 1 meter first. What would that be?

Each wave takes a distance of lambda meters in the space. So, the number of waves in 1 meter would be $\frac{1}{\lambda}$ by definition of the wavelength, okay? Each wave taking lambda meters, okay? So, the number of waves between source to listener is what?

$$= \frac{d_{sl}}{\lambda} = \frac{2}{0.69} = 2.899$$

or approximately 2.9 waves just short of 3 waves. Now, let us see part e number of radians covered by the sound wave between fork to listener. So, just like we had how many waves in between fork to listener we want to find how many radians it is covering. Now, we have already found that between source to listener we have got approximately 2.9 waves and each full wave is equal to a complete revolution or a complete full complete circle which is 2π radians. Every wave is a 2π radian. So, the number of radians would be between the source and the listener.

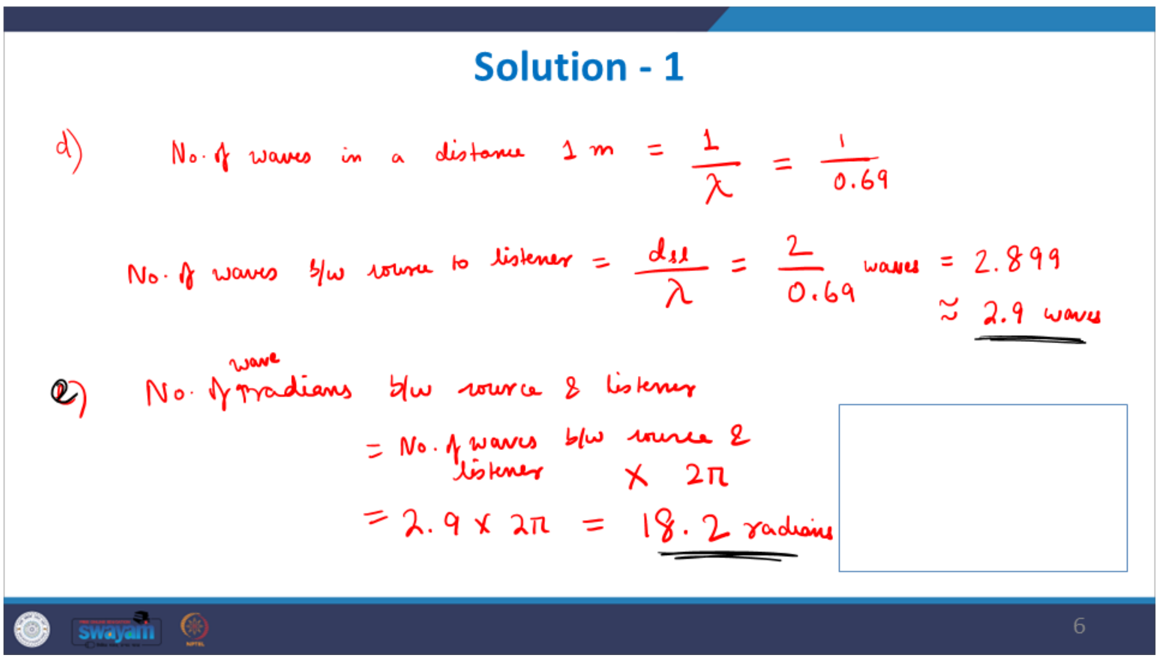
The number of wave radians between the source and the listener would be

$$= number\ of\ waves\ between\ the\ source\ and\ the\ listener * 2\pi$$

each wave has a 2 π revolution so

$$= 2.9 * 2\pi$$

$$= 18.2\, radians$$



so, the last part is the number of radians covered by the sound wave in 2 seconds ok so how many radians does it cover in 2 seconds So, here see each wave means 2π radians. So, to find out how many radians it is covering we need to find out how many waves do we have in 2 seconds.

So, I will start like this. So, we will follow the similar approach as we did in part D and E and we will get the answer. So, here one wave takes T time in seconds. because T is the time period. So, number of waves in 1 second would be

$$= \frac{1}{T}$$

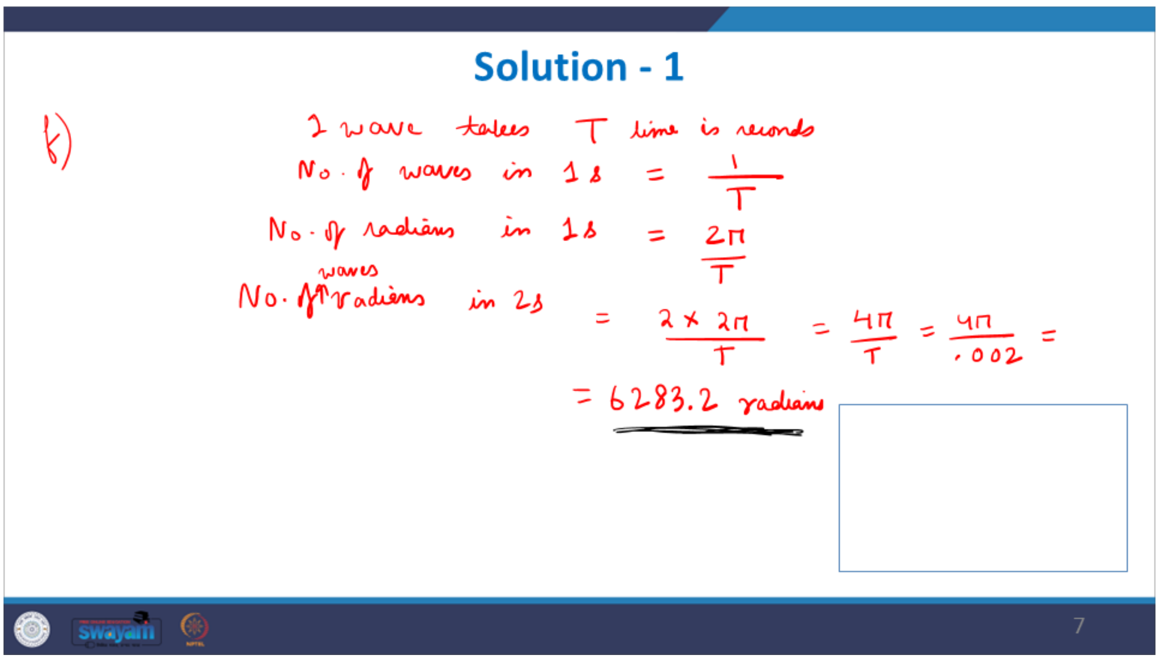
So, the number of radians in 1 second would be

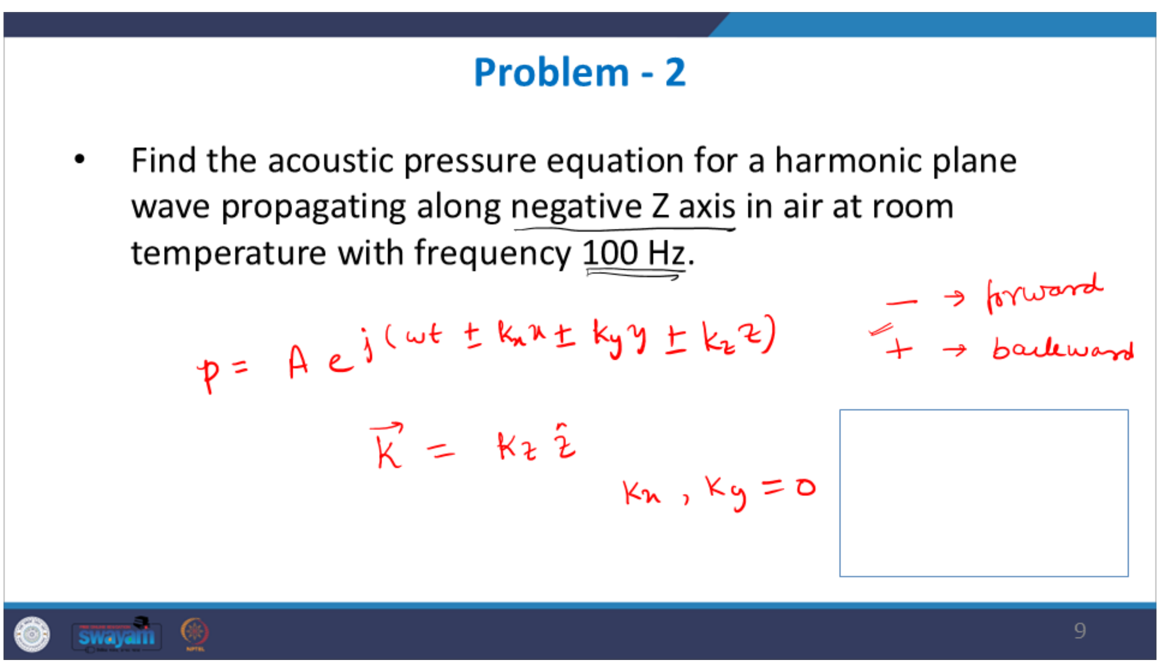
$$= \frac{2\pi}{T}$$

So, we are asked how many radians in 2 seconds. How many radians of waves created in 2 seconds? So, it should be

$$= \frac{2 * 2\pi}{T} = \frac{4\pi}{T} = \frac{4\pi}{0.002} = 6283.2\ radians$$

of waves that are being created. So, we solve this particular problem.





Now, let us move to the next problem. Find the acoustic pressure equation for a harmonic plane wave propagating along the negative z axis in air at the room temperature and the frequency is 100 hertz. Now, we already know that the generic solution is what? The generic solution for a harmonic plane wave is given by.

$$p(x,\ t) = \boldsymbol{A}\boldsymbol{e}^{\boldsymbol{j}(\boldsymbol{\omega t} \pm \boldsymbol{k_x x} \pm \boldsymbol{k_y y} \pm \boldsymbol{k_z z})}$$

So, here the negative sign is for forward propagating or propagating along the positive x axis and the positive sign is for backwards propagating or propagating along the negative axis. So here it is along the negative axis which means we will be using the plus sign and it is solely propagating in the z direction. So, which means that in the k vector is completely along this z direction. the kx and the ky component are 0. So, the solution should be something of the form

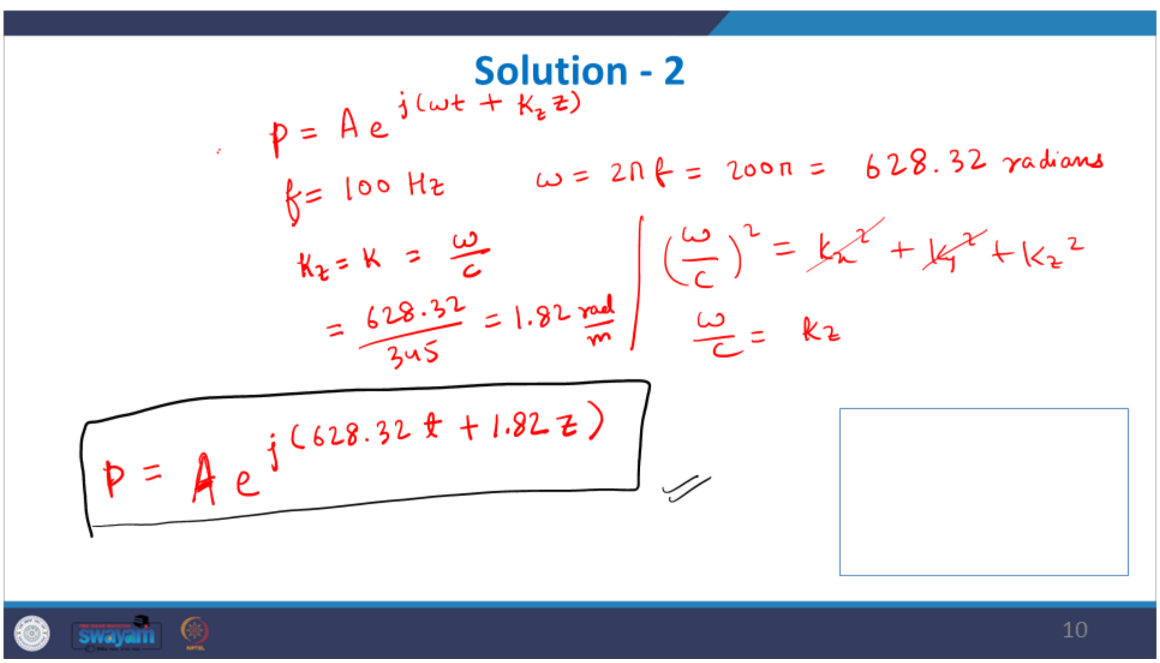
$$p(z,\ t) = \boldsymbol{A}\boldsymbol{e}^{\boldsymbol{j}(\boldsymbol{\omega t} + \boldsymbol{k_z z})}$$

plus we will be using because it is a negative z axis propagation. So, some kz times of z this should be a generic solution. Now, what is given to us we are given the frequency is 100 Hz. Okay. So, 100 Hz frequency. So, what is omega?

$$\omega = 2\pi f$$

$$= 200\pi = 628.32\ radians$$

And now we need to find kz or simply k we can represent.

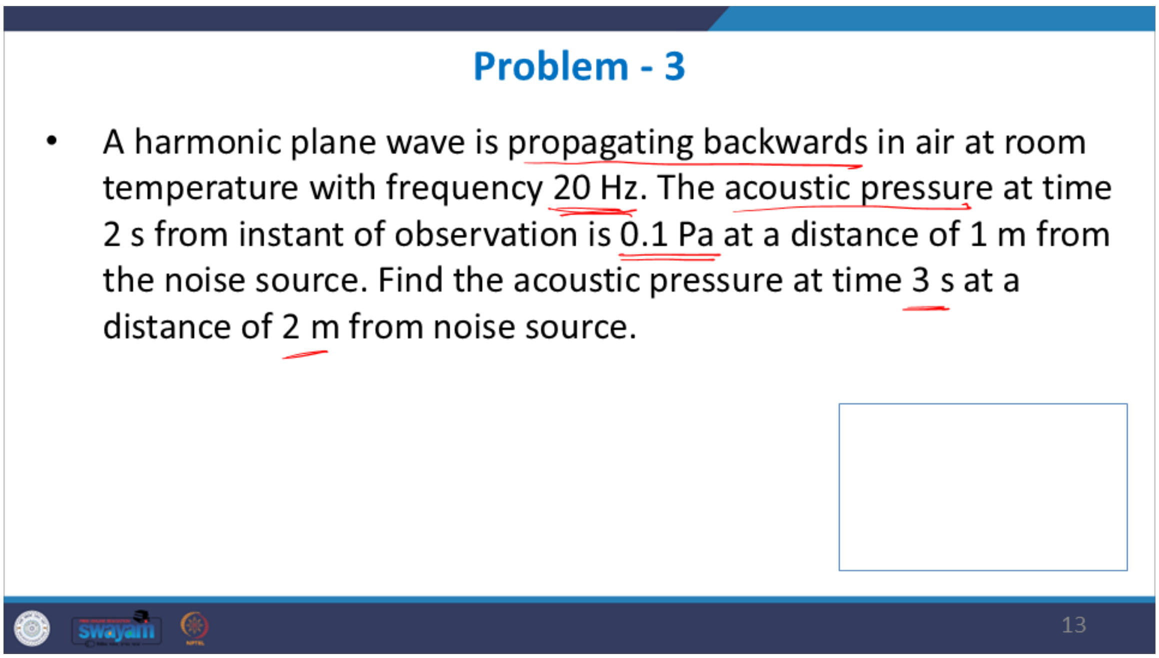


$$k = k_z = \frac{\omega}{c} = \frac{628.32}{345} = 1.82\, rad/m$$

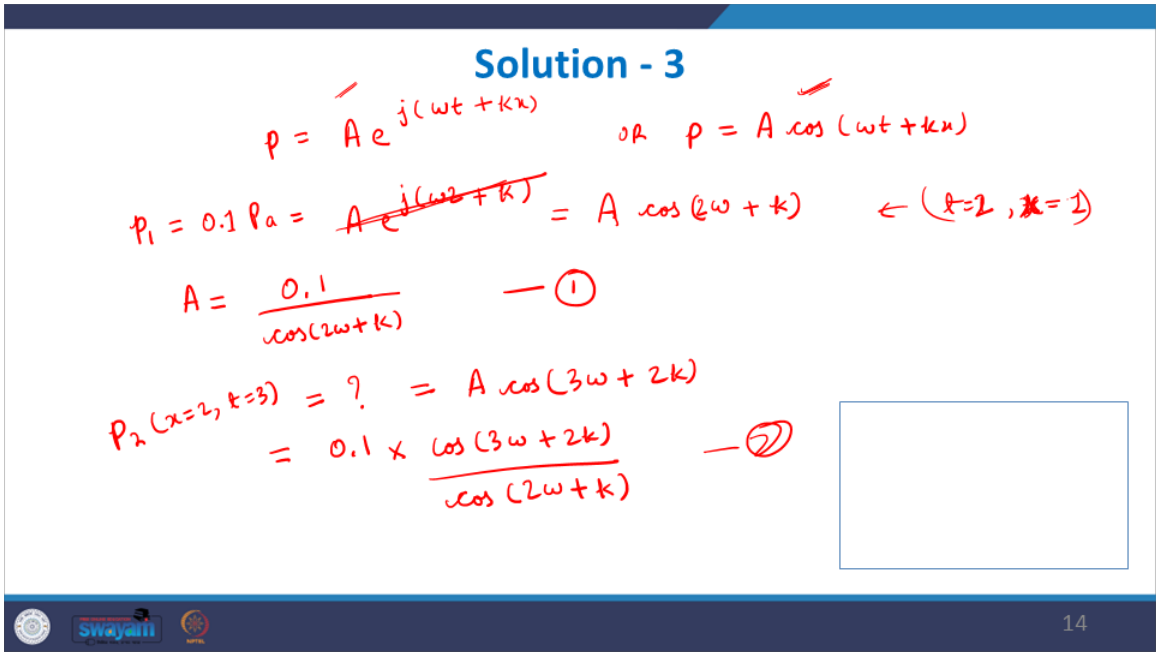
So, we substitute these values into this particular equation.

$$p(x,\ t) = \boldsymbol{Ae}^{\boldsymbol{j(628.32t+1.82z)}}$$

So, this becomes our answer. Are we given any further information? No, we are not given any more information. So, we cannot find out what is the amplitude. So, we leave the answer here.



So, now the third problem which is probably the last problem of this lecture. So, here it is given to us that a harmonic plane wave is propagating backwards. Once again just like in the previous case we had this negative z axis here also it is propagating backwards. So, we will take the plus sign for the solution in the air at the room temperature. So, we can take the c as 345 and the frequency is 20 Hertz. So, we have a generic solution ready. Now, the acoustic pressure at time T equals to 2 seconds. From the instant of observation is 0.1 pascals at a distance of 1 meter. And let us say what you have to find out the same acoustic pressure at the time T equals to 3 seconds at a distance of 2 meters from the noise source. So, let us see. So, over here the generic solution will become it is travelling along some negative axis. Let us say it is negative x axis.



So, it would be

$$p(x,\ t) = Ae^{j(\omega t + kx)}$$

Now, over here I would before we solve it I would like to point out that we can take a solution like,

$$or \qquad p = A\cos(\omega t + kx)$$

because this is the real solution and this is the exponential solution. If you would continue and solve it using either of them, you will get different answer. So, if you solve it using a cosine wave, you will get a different answer and using an exponential wave, you will get a different answer. And in the previous, from my prior experience, I have seen that the students struggle. They use the exponential form and they come up with the wrong answer. So here I would like to clarify that you have these two options and you get different answers from them, which one to choose. So over here, what is given? It is given that the acoustic pressure at a certain time and certain distance is given to be 0.1 pascals.

Now, tell me is this 0.1 pascals a real quantity or a complex quantity? It is going to be a real quantity. We are not giving the pressure as a complex quantity. We are giving it as a real part. So, this means that when you do or solve it, it is a real part you are equating to because it is 0.1 pascals which is a fully real quantity with no imaginary part.

So, it is not complex. So, let us go ahead and take the real solution here. let us say that the first pressure which is 0.1 pascals is when the time is equal to 2 seconds and the distance is equal to 1 meter. So, it is

$$p_1 = 0.1 \ = Acos(2\omega + k)$$

T is given to be 2 and x is given to be 1. So, this becomes your first equation. So, with this what would be the amplitude? So, the amplitude can be solved as

$$A = \frac{0.1}{cos(2\omega + k)}$$

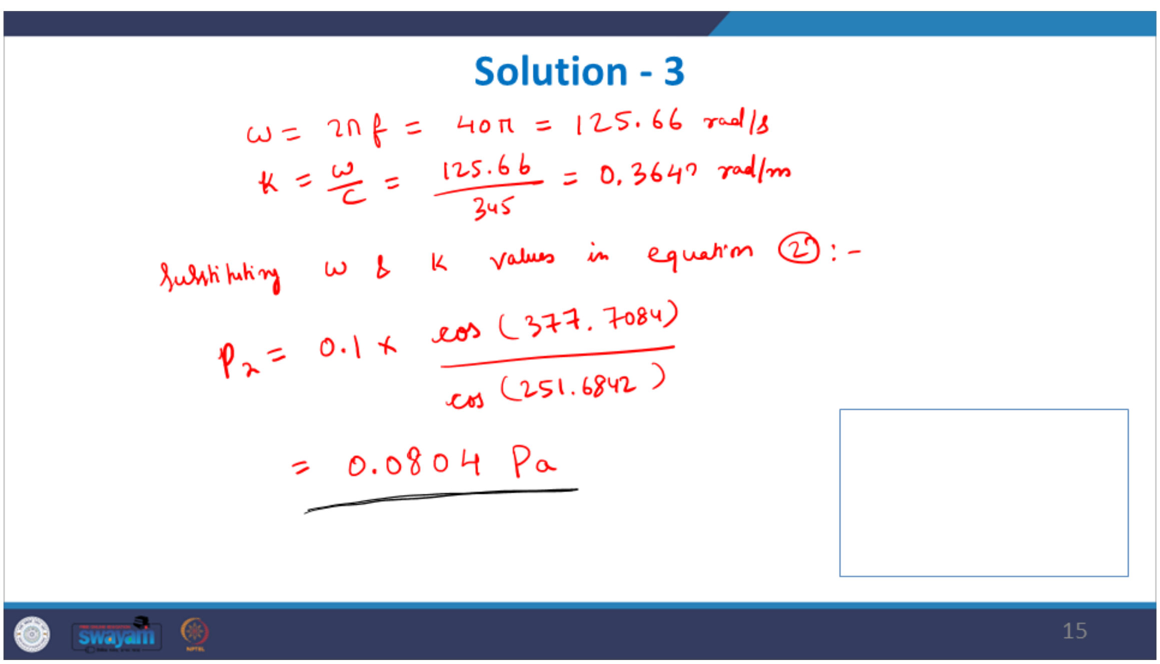
We will put the values later. So, this is our first equation. Now, we have to find out P2 which is the acoustic pressure at times 3 seconds and distance of 2 meters from the source. So, this,

$$p_2(x = 2, t = 3) = Acos(3\omega + 2k)$$

okay. and a we have already found out from the previously given pressure which is 0.1 so 0.1 multiplied by if you put the A substitute it from the equation 1 you get this

$$p_2 = 0.1 \frac{cos(3\omega + 2k)}{cos(2\omega + k)}$$

so this is what you have gotten. Now we already are given the frequency (f) which is 20 Hz. So, we can find out what is k and what is ω and we can substitute into this final equation.



First of all, let us find out what is ω and then we can use it to find k.

$$\omega = 2\pi f = 40\pi = 125.66 \quad \frac{rad}{s}.$$

$$k = \frac{\omega}{c} = \frac{125.66}{345} \quad \frac{rad}{m}$$

So, omega and k we have obtained. now we will simply substitute it here in equation 2. Substituting omega and k values in equation 2 What you would get is, so I will directly jump to the answer you can try it at home and see for yourself. So, it would directly be

$$p_2 = 0.1 * \frac{\cos(377.7084)}{\cos(251.6842)} = 0.0804\ Pa$$

(Consider quantities are in radians). So, when you solve it further you end up with this pressure and this is the pressure that was asked for.

So, this is how you solve it and this being the last numerical for this lecture. I hope you enjoyed it and you would be able to solve further lectures which refer to the sound wave propagation part and,

thank you for listening.