

NOISE CONTROL IN MECHANICAL SYSTEMS

Prof. Sneha Singh

Department of Mechanical and Industrial Engineering

IIT Roorkee

Week:10

Lecture:047

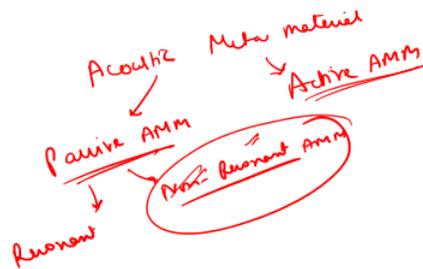
Lecture 047: Space Coiling Acoustic Metamaterial 1

The slide header features a blue and white color scheme. At the top, there are three logos: IIT Roorkee, Swayam (Free Online Education), and NPTEL Online Certification Course. Below the logos, the title 'Noise Control in Mechanical Systems' is written in a large, dark blue font, followed by 'Lecture 47' in a smaller blue font, and 'Space Coiling Acoustic Metamaterial - 1' in a bold blue font. The presenter's name 'Dr. Sneha Singh' and department 'Mechanical and Industrial Engineering Department' are listed below. At the bottom, there is a photograph of the IIT Roorkee main building, a large white structure with a central dome and columns, set against a green lawn and trees. A small number '1' is visible in the bottom right corner of the slide.

Hello and welcome to this lecture course on Noise Control in Mechanical Systems with me, Professor Sneha Singh. We have begun our discussion on acoustic metamaterials, and we saw that there is a passive AMM and an active AMM. Passive means no external energy is supplied; it works by the internal structure itself. And active means that it needs some external supply of energy to manipulate the wave.

Among the passive types, we have the resonant and the non-resonant. And today we will discuss one type of non-resonating AMM, which is the space-coiling metamaterial. We will discuss its working principle, the mechanisms of noise control, and the resonance frequency.

Summary of previous lecture



Outline

- Space coiling Metamaterial \Rightarrow Internal geometry or path modification + Resonance
 - Introduction
 - Working Principle
 - Mechanisms of noise control
 - Resonance frequency or fundamental frequency



Although this metamaterial can work without resonance, most of the time, the space-coiling metamaterial uses both principles.

Its internal geometry allows it to work without resonance. Internal geometry or path modification means we are trying to modify the path of the wave, plus, we are also creating some resonance, and both work together to attenuate sound waves in a space-coiling metamaterial.

Introduction to space coiling metamaterial

- A **space-coiling metamaterial** is a type of artificial material designed to manipulate waves—such as sound or electromagnetic waves—in unconventional ways by controlling the path the waves travel through.

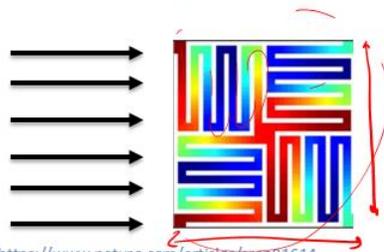


Source: <https://www.nature.com/articles/srep01614>



What is it? It is an artificially designed material to manipulate waves, and what does it do? It controls the path that the wave takes.

- The term "**space-coiling**" refers to the way these metamaterials are engineered: instead of letting waves move in a straight line, they force the waves to take a coiled or twisted path within the material.
- This allows the metamaterial to interact with the waves over a longer distance, within a compact structure.



Source: <https://www.nature.com/articles/srep01614>



Let us see the term space coiling means that whatever the space is available you have coiled it up into some maze-like channels. You are not having a straight away path but rather the

space is now made zigzag or half azad twisted and turned paths. What is happening that these materials they instead of letting the waves move in a straight line they force the wave to take a very coiled up and twisted path within a material.

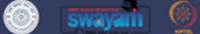
This allows the material to interact more with the sound waves over a longer distance within a compact structure. Now, think about it let us say this example we have this figure here. The dimension of this material is only this much. But within this small dimension, the wave is interacting over a much longer path. It is interacting over a much longer path within this material itself. Within a compact size, we have twisted the pathway in such a way that even in this compact size, we are elongating the path of the wave propagation. Interaction is increasing. The wave is interacting more it looks like the wave is interacting 20 to 50 times more than it would if suppose you had uncoiled it and made it into a straight line.

Key Characteristics

Design:

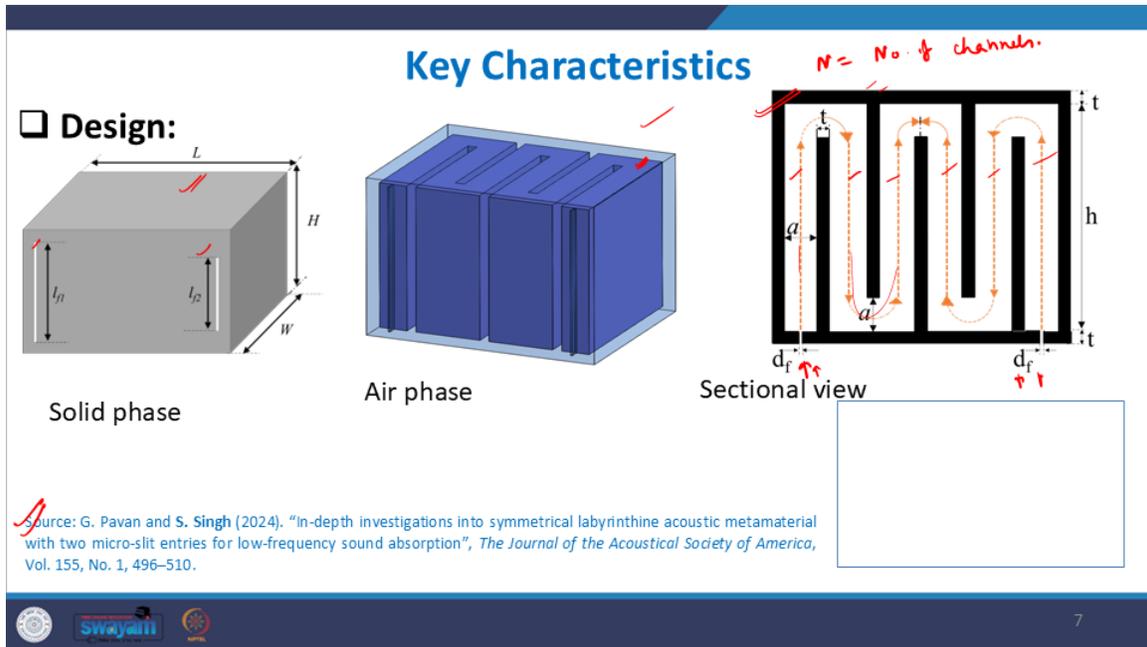
- The structures are often small, intricate, and may resemble twisted or spiral shapes (hence the name "space-coiling").
- These shapes create the illusion of a longer path inside a limited physical space.



6

The design as you see it is often small intricate and twisted or spiral shapes, space coil shapes so in any way if you can twist the space around so that the sound wave have to turn around and go into the twisted and the maze like channels or maze like paths. So that they travel a much longer distance within a short physical space.

a much longer path the sound wave is taking within a limited physical space as seen here. Within this limited size the wave is going through a longer pathway and interacting with a material in a much longer pathway.



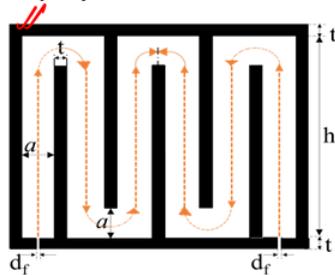
This shows some metamaterial designed by us. You can see our research paper on the space coiling metamaterial. What we had done here is that, a typical design of a metamaterial I am showing you. It looks like a black box from the outside or a rigid box or a rigid cuboidal structure with two slit openings for the sound waves to enter inside the material. And then what happens once it enters inside the material? This shows us the air domain. When it moves inside, it goes through these alternating pathways. This shows the sectional view of the metamaterial.

Sound wave is incident here and the entry of the sound waves have been blocked from all the sides. the entry of the sound waves is blocked from other sides. It can only enter into the material and then get strapped inside the materials and dissipated. It does not have any other part to get out of the material.

There is no transmission from the other end, and what happens within this small volume or within this small space is that it is taking this much longer pathway. Within this small space itself, it is taking a much longer pathway and is getting dissipated along the path. How does this dissipation happen?

Working Principle

- The key working principle lies in their **internal design**, which forces waves to take coiled or twisted paths, effectively slowing down their propagation and altering their interaction with the material. The waves, despite travelling through a larger volume, are confined within the material's small physical dimensions.



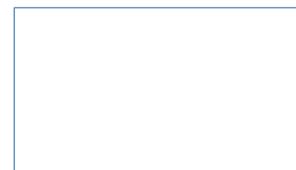
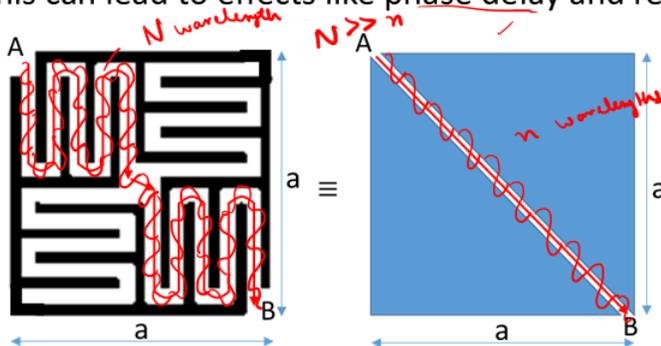
First concept is clear: We have a limited compact-size material, and within that compact-size material, we are making the sound wave go through a much elongated or much longer path, increasing the interaction between the material and the wave.

Whatever this base material is, this black material—and the wave. We are increasing the interaction between them. When the sound wave is passing through, it crosses the boundary of the solid material. The interaction has increased. Because of this, the first phenomenon that happens is the wave compression.

Working Principle

Wave Compression:

- Internal coiled structures create a longer propagation path for waves, which compresses their wavelength relative to the material's thickness. This can lead to effects like phase delay and resonance control.



What it means is that—suppose if this metamaterial was not present, but rather just a homogeneous material was present, and the wave traveled from path A to B within this homogeneous material, what would happen? Suppose the wave has a certain number of wavelengths, so suppose we have a certain number of wavelengths passing like this and reaching from point A to B. But now that we have elongated the path, suppose we were able to cover some ‘n’ wavelengths within this physical space. Now, we have many wavelengths within the same physical space now because the path length is so long that a lot of wavelengths are getting covered within the path—a lot of wave cycles we can observe before coming here?

Basically, we are sort of compressing a lot of wavelengths within a small size. Suppose n wavelengths were there; here, we are capturing capital ‘N’ wavelengths. Where capital N is a much larger number than small n. Basically, within the limited size, more wavelengths are getting accommodated. This phenomenon is also called wave compression because a lot of sound waves are being covered within the same physical dimension, and this leads to things such as phase delay, resonance control, and so on, which we will see subsequently. It feels like the waves are now being compressed within a small volume. The first thing that happens because of this is that there is an effective refractive index control.

Working Principle

□ **Effective Refractive Index Control:** By manipulating the path of waves, these materials can produce extreme values of effective refractive indices, including negative indices. This means that waves can be bent, focused, or redirected in ways that are impossible with conventional materials.

$$\text{Refractive Index for Acoustic Waves} = n = \frac{\text{speed of sound in air}}{\text{speed of sound in Material under consideration}}$$

$$\text{For AMM} \cdot n = \frac{\text{speed of sound in air}}{\text{speed of sound in AMM}}$$



10

We can control the refractive index of the material. What is the refractive index of a material? It is simply defined as, just like in optics, the refractive index is the speed of light

in air divided by the speed of light in the medium. That is a relative refractive index. In the same way, the refractive index for acoustic waves, just like how it is for electromagnetics, can be defined as some value η . For electromagnetics, we consider the speed of sound in a vacuum, but here, for acoustic waves, they cannot travel in a vacuum. We take air as our reference medium divided by the speed of sound in the material under consideration.

$$\eta = \frac{\text{Speed of sound in AIR}}{\text{Speed of sound in Material}}$$

for the AMM, this will become, for AMM, space-coiling metamaterial

$$\eta = \frac{\text{Speed of sound in AIR}}{\text{Speed of sound in AMM}}$$

Working Principle

Effective Refractive Index Control - continued:

Arrows indicate the wave propagation path

$\eta = \frac{\text{speed in air}}{\text{speed in AMM}}$

in air in absence of AMM: $\text{speed of sound} = \frac{\text{Length } AB}{\text{Time taken}} = \frac{l_{AB}}{t}$; where $t = \frac{l_{AB}}{c_0}$

when AMM is present: time, $t' = \frac{l_{eff}}{c_0}$

speed of sound in AMM $t' \gg t$

$= \frac{l_{AB}}{t'} = \ll \text{speed in air}$

$\eta \gg 1$

Actual AMM Equivalent medium with high refractive index

11

Now, if you once again look at the same schematic as explained in the previous lecture, if you suppose only air was present. The speed of sound in the air in the absence of AMM would be the path length AB because, when there is no obstruction, the sound will directly travel from A to B divided by the time taken.

$$\text{Speed of sound without AMM} = \frac{\text{Length } AB}{\text{Time Taken}} = \frac{l_{ab}}{t}$$

$$(Speed\ of\ sound\ without\ AMM)_{Air} = 343\ m/s$$

However, when AMM is present, when the space-coiling metamaterial is present, then to cover the same distance between A to B, the same displacement from A to B, it takes a much longer path. The length of this elongated path. Let us call this an effective length. Which is the total length traveled by the sound wave within this metamaterial, which is the length of this red line. This is $L_{effective}$. The net length is divided by C_0 . Now, this T' here would be much larger. Why? Because this length has increased. It would be much greater than T . Therefore, speed of sound in AMM is what? It is the net displacement it is doing,

$$speed\ of\ sound\ in\ AMM = \frac{l_{ab,AMM}}{T'}$$

Now, it is taking the same distance in a much longer time. This speed is coming out to be much lower than the speed in air. What is the refractive index?

$$Refractive\ index = \frac{Speed\ in\ Air}{speed\ in\ AMM}$$

And this speed has decreased.

Why? Because it is taking a much longer path to cover the same distance. This refractive index increases in value. We get a very high refractive index material because, to cover the same distance, we are making the sound wave travel a much longer path, and it takes a much longer time to cover. Basically, we are slowing down the waves. What does the high refractive index material do? It slows down the speed, and that is the same thing this metamaterial is doing. It is slowing down the speed of the sound.

This η has become much greater than 1 because the speed in air, in the absence of AMM, is much greater compared to the speed in the presence of AMM due to path elongation, waves have slowed down.

When we have a high refractive index material, it affects how much the sound waves bend. Suppose the sound waves are traveling in the air and they encounter a medium with a higher refractive index; Then they would bend. And the higher the refractive index, the more bending you can achieve. When η becomes much larger, then we have the bending of the waves or the refraction of the waves. The degree of refraction also rises. Compared to conventional material, we can achieve a much larger amount of bending and refraction in the waves than in conventional material, and this is how we can manipulate it.

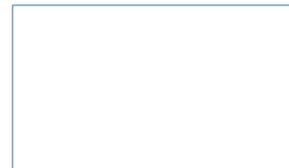
Working Principle

□ Enhanced Wave Interaction:

- The coiled pathways allow for multiple reflections and interferences of waves inside the structure, giving rise to advanced control over wave propagation characteristics, such as selective filtering or absorption of certain frequencies.

□ Wave Manipulation:

- Path the wave travels gets lengthened without increasing the overall size of the material. This changes properties like wave speed and phase.
- This manipulation allows for effects like wave redirection, negative refraction, etc.



What are the other principles? The enhanced wave interaction. The coiled-up pathways allow for multiple reflections happening, multiple interferences, which gives rise to advanced control. Such as selective filtering and absorption. More interactions are happening. More internal refractions and more interferences happen. Wave manipulation, the wave travel, gets lengthened without increasing the overall size, this changes the wave speed, as already discussed. The wave speed, or the assumed wave speed, becomes much lower because now, we have slowed down the waves. And this also leads to a phase delay. The delay in the phase happens, and then you can redirect the waves; negative refraction can also be achieved. These are the various phenomena that can happen in a space-coiling metamaterial.

But what is the chief mechanism of controlling the noise, or the chief mechanism by which is the sound level going to go down? The first is that the wave-structure interaction has increased because now the wave is traveling a much longer pathway, and it is interacting much more with the material in a smaller size. Because of that, the dissipation of the acoustic energy also rises due to increased internal air friction and the increased scattering that happens. I have already explained that when the wave is traveling, it is taking a much longer path. There is internal friction in the pathway, and the longer the path, the more friction the waves have to face, and the more decay will happen.

Mechanisms of Noise Control with Space-Coiling AMM

Increased wave – structure interaction:

- The coiled paths increase the interaction between the sound waves and the material. This can result in dissipation of acoustic energy as heat due to increased internal friction and increased scattering.



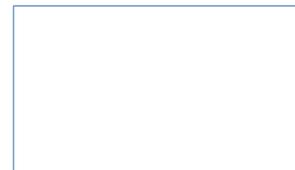
Obviously, a sound wave that has to travel a shorter distance or a shorter path will have a higher magnitude than the sound waves that must travel a much longer pathway because of the air resistance and the various other frictions, such as air friction and other frictions that take place in the path. And friction is a quantity directly dependent on the length of the path. It increases and keeps attenuating the wave; more scattering happens at all the turns and twists, and more dissipation occurs. Basically, dissipation rises due to this.

Mechanisms of Noise Control with Space-Coiling AMM

Acoustic Impedance Matching:

at Resonance frequency (freq. of internal Absorp. is matching with the Incident freq.)

- The metamaterial can be designed such that surface impedance of the AMM structure matches i.e. is nearly equal to the surface impedance of the air medium. This phenomenon is called: impedance matching. It leads to **high absorption** with SAC \rightarrow 1, because nearly all sound waves can freely enter the AMM without any resistance.
- Now, if the other end of the structure is closed by rigid boundaries, then all entered sound waves are trapped in the structure.



Then there is another phenomenon called acoustic impedance matching. What happens is that here, when, at certain specific frequencies, the way this metamaterial is designed, there is a specific phenomenon called acoustic impedance matching. From our experiments, what we have observed is that this impedance matching happens at the resonance. When the fundamental frequency of the internal structure matches with the incident frequency. The frequency of the internal structure matches with incident frequency. What happens is that the surface impedance of the metamaterial, so suppose you have sound waves that are coming and hitting some material, so the surface impedance matches with the impedance of the air. To the sound wave, it means that when you have the same impedance, there is no reflection, and all the transmission happens, and the sound waves just go through. To the sound waves, it looks like there is almost no change in the acoustic medium, and they just pass through. That phenomenon is called impedance matching because when the impedance of the air and the metamaterial match, then all the sound waves simply pass through. The absorption coefficient is going to approach unity. Why? Because no reflections will take place. For example, we have a closed room and an open window in the closed room. The sound waves can directly pass through the open window. The absorption coefficient is going to be one there. Why? Because it is like a medium continuity.

There is no difference in the acoustic impedance. Reflections and changes in the pathway only happen when there is a difference in the acoustic impedance. Here in this case, what happens is the surface impedance of the AMM matches with the air, and at that point, the absorption coefficient tends to unity. Now, if the other end of the structure is closed by a rigid boundary, what happens now is we are allowing all the sound waves to enter the material. Now, if we close all the other ends with rigid boundaries. Let me go back to this slide where we have this metamaterial (Slide7). We are allowing the sound waves to enter this structure, but we are closing it from all sides. At resonance, almost all the sound waves would be able to enter the material because now the impedance has matched. It behaves like a medium continuity, and all the sound waves are entering. And the absorption coefficient is reaching unity. They get inside easily, but there is no means to go outside. They get trapped inside the material. They get trapped in the structure itself. What should be the condition for this acoustic impedance matching? Simply, the impedance must match, which means that the surface impedance of the AMM should be almost the same as the impedance of the air.

Mechanisms of Noise Control with Space-Coiling AMM

□ Condition for Acoustic Impedance Matching:

$$\checkmark Z_{AMM} \approx \checkmark Z_{air} \approx \rho_0 c \checkmark \quad \text{for normal incidence} \quad Z_{surface} = Z_{characteristic} = \rho c$$

or

$$\frac{Z_{AMM}}{Z_{air}} = Z_{AMM,rel} \approx 1$$

$$Z = R + jX \rightarrow 1$$

Acoustic resistance

Acoustic reactance

- Relative acoustic resistance should reach unity, and relative reactance should become zero. ✓

And we know that for normal incidence, the surface impedance, Z , is the same as the small which is the characteristic impedance, and this is ρc of the medium.

$$Z_{surface} = Z_{characteristic} = \rho c$$

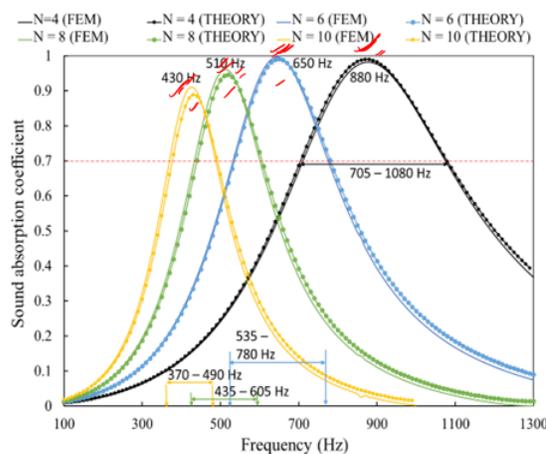
This should be the condition. It should almost be the same. I can say that it will be the same because theoretically it may be possible, but practically it is never achieved in reality—we never get a perfect impedance match with exact equality. The more it approaches this value, the better this phenomenon would be. Which means that the relative impedance has to approach 1.

$$\frac{Z_{AMM}}{Z_{air}} \rightarrow 1$$

and we know that the relative impedance is an impedance term after all. It will have a real part and an imaginary part. In acoustic resistance and in acoustic reactance, this has to approach unity. If this has to approach unity, it means that the real part of the relative acoustic resistance should reach unity, and the imaginary part of the relative acoustic impedance or the relative acoustic resistance has to reach unity, while the relative reactance should become 0. So that we can achieve a perfect impedance. So here is just a simple derivation.

Mechanisms of Noise Control with Space-Coiling AMM

Acoustic Impedance Matching at Resonance Frequency:



Closer AMM is to Impedance Match
SAT at peak \rightarrow closer to Unity

Source: G. Pavan and S. Singh (2024). "In-depth investigations into symmetrical labyrinthine acoustic metamaterial with two micro-slit entries for low-frequency sound absorption", *The Journal of the Acoustical Society of America*, Vol. 155, No. 1, 496-510.

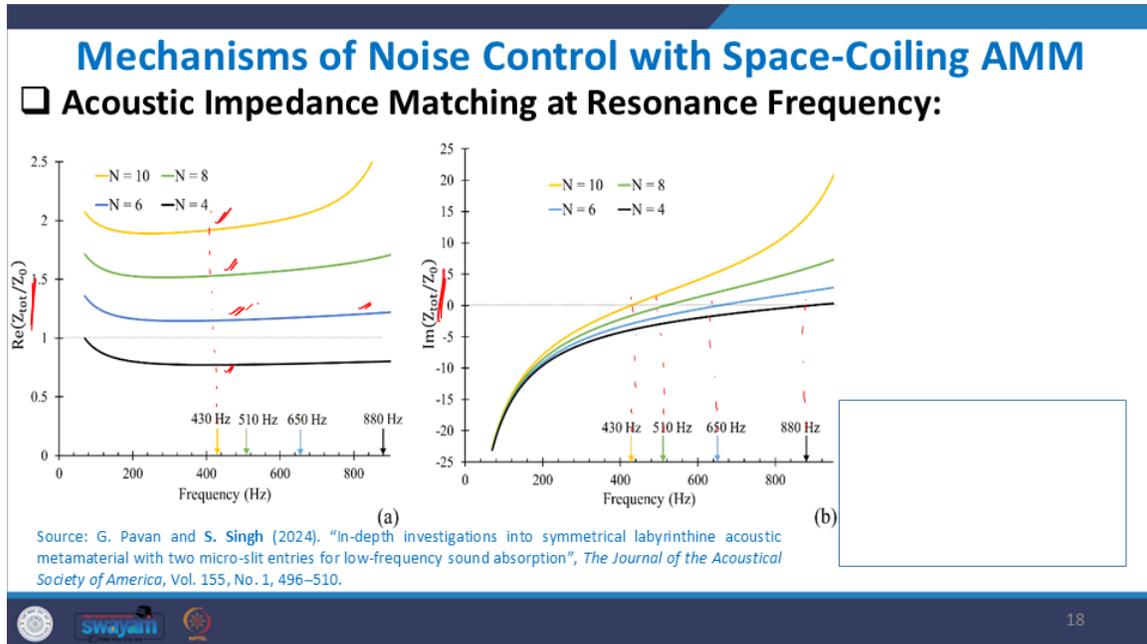
This is again from one of our papers. What we did was we made that material—I have already shown the design—this is the material that we had made, and then we observed its frequency response. What is the absorption coefficient over a range of frequencies? And what we observed was that, if suppose N is the number of channels—so in this case, let us go back (slide 7). Here we have 1, 2, 3, 4, 5, 6—6 channels are there. capital N becomes the number of channels. What we observed is that we modeled the same metamaterial, but we increased its number of channels and observed its behavior.

What we observed was that as you increase the number of channels, the frequency shifts toward the lower value. Then we also observed what the real part of its relative impedance and what the imaginary part of it. Now, at resonance, where the where, you are getting this absorption peak, there should be an impedance match.

The impedance will match, which means all the sound waves are entering the material. They are able to freely enter into the material at that frequency peak. At this peak frequency, they are able to freely enter the material due to the matching of the impedance. At these frequencies, this should be the condition: the resistance should become unity, and the reactance should become zero. That is what we observed when we observed the resistance and the reactance at the peak frequencies—at all the peak frequencies.

What we observed is that the reactance was reaching 0, and the real part did not reach exactly unity, but it came very, very close. And for this particular model, this was the

closest to unity, and that is why this blue curve is here. This shows the highest sound absorption. Why? Because there was the highest impedance matching here.



And then the next nearest curve is this black curve. This had a much closer impedance match. This had a higher absorption coefficient at the peak. Then we had this black curve. This had a slightly lower peak.

Then we have the green curve, which had an even slightly lower peak. And then the furthest away was this yellow curve, or the yellow metamaterial, whose peak was much lower compared to the rest. What we can say is that the closer the AMM is to impedance matching, the SAC at peak would be closer to unity. Closer to the maximum value, which is unity. The higher the impedance match, the more the absorption coefficient observed.

And how do you determine these frequencies at which this happens. Now we know that this is the condition to increase the absorption, but at what frequencies? here the resonant behavior comes into effect. These space-coiling metamaterials show resonance effects. that means that where the fundamental frequency of the coil matches with the incoming wave frequency, it approaches the frequency of the coiled structure. That is the phenomenon of resonance: The incident frequency has the same frequency as the frequency of the structure. And in that case, you have the maximum sound absorption. And that is where the impedance matching also happens.

Mechanisms of Noise Control with Space-Coiling AMM

Resonant Behavior at Specific Frequencies:

- Space-coiling metamaterials can exhibit resonance effects at specific frequencies, where the geometry of the coils resonates with incoming sound waves, trapping and absorbing them more effectively.
- At Resonance: where incoming wave frequency approaches the coiled structure frequency, maximum sound absorption takes place due to impedance matching.



The same way, resonance happens, and the sound wave is used up in driving the particles to high amplitude. let us find out the resonance frequency. Suppose we have got this space-coiled structure, and we are closing its end.

Resonance frequency of Space Coiling AMM

- For space coiling AMM with one end open for sound wave entry and other end of the maze like pathway closed.

$$f_{SCM} = \frac{(2n-1)c}{4L_{eff}}; n = 1, 2, 3, \dots$$

$$f_{SCM} = \frac{c}{4L_{eff}}, \frac{3c}{4L_{eff}}, \frac{5c}{4L_{eff}} \dots$$

L_{eff} = total path length the wave takes within the material



open-closed rigid pipe



We have some structure like this: a sound wave is entering this pathway—whatever your pathway is—it could be any form, spiral, rectangular, like that, and then at the end, it is

closed to stop the transmission. what does it behave like? It behaves like a pipe with one end open and one end closed, ok. An open-closed rigid pipe, and the fundamental frequency for this case is given by this value. For this particular AMM, it is the odd multiple of C. This is the speed of the sound.

$$f_{scm} = \frac{(2n - 1)c}{4L_{eff}}$$

Where L effective is what? It is the total path length the wave takes within the material. The first fundamental frequency will come when n is equal to 1 which is

$$f_{scm1} = \frac{c}{4L_{eff}}$$

The second will come,

$$f_{scm2} = \frac{3c}{4L_{eff}}$$

this and so on.

Resonance frequency of Space Coiling AMM

- For space coiling AMM with one end open for sound wave entry and other end also open for sound wave exit.

$$f_{SCM} = \frac{nc}{2L_{eff}} ; n = 1, 2, 3, \dots$$

$f_{SCM} = \frac{c}{2L_{eff}}, \frac{2c}{2L_{eff}}, \frac{3c}{2L_{eff}}, \dots$



Open - Open pipe




21

Suppose both ends are open something like this. Both ends are open here and transmission is happening. It behaves as an open pipe and the fundamental frequency for this is given by

$$f_{SCM} = \frac{nc}{2L_{eff}}$$

which means that for this particular case, the first would be at

$$f_{SCM} = \frac{c}{2L_{eff}}$$

followed by

$$f_{SCM} = \frac{2c}{2L_{eff}}$$

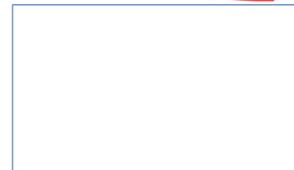
and so on.

This knowledge of this frequency is important because then you can predict that at what frequency you will observe these peaks. These peaks should come at these frequencies, these frequencies. So, this was some of the phenomenon.

Mechanisms of Noise Control with Space-Coiling AMM

❑ Wave Path Elongation and Phase Manipulation:

- The coiling structure creates a longer effective path for sound waves within the material, despite the material's small overall dimensions. This lengthened travel path causes the sound waves to slow down and undergo phase shifts.
- The slowed waves are **delayed**, causing them to interfere destructively with incoming sound waves leading to cancellation of certain frequencies.



Then the other phenomenon is obviously the path wave elongation is happening, and manipulation is happening. We have already discussed how elongated path slows down the wave or delay them and due to that, the phase delay also happens, and the waves coming out from the other end of the material will destructively interfere with the other waves, and

they can cancel each other out. With this, I would like to close the lecture on space-coiling metamaterial and thank you for listening.

Thank You