NOISE CONTROL IN MECHANICAL SYSTEMS

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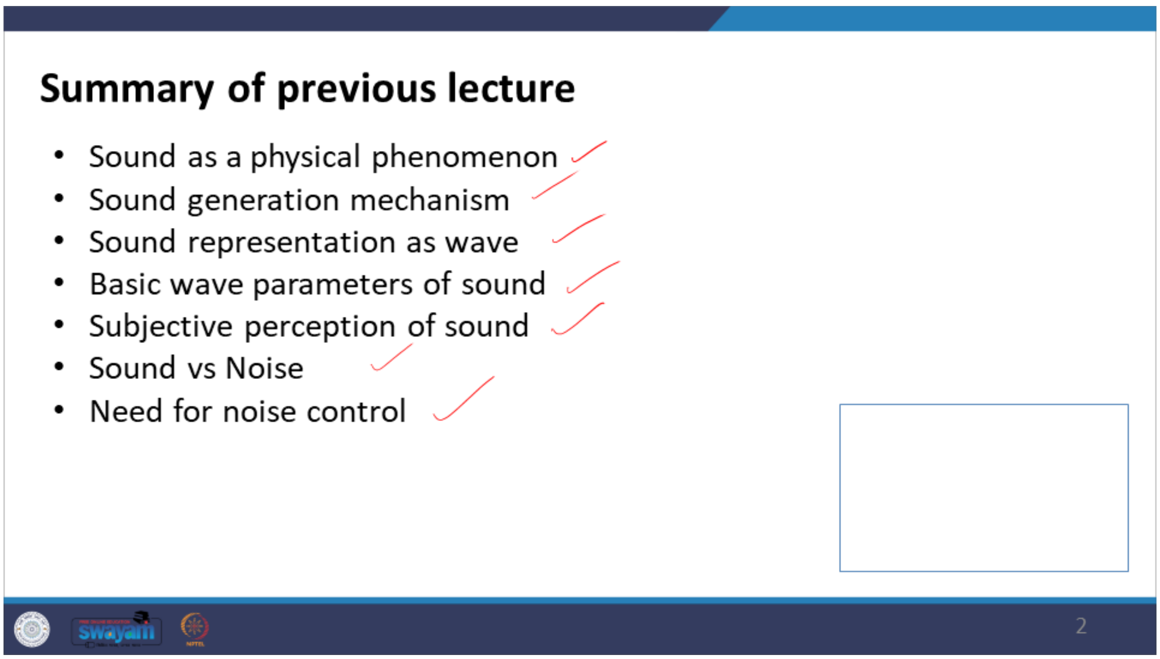
Week:1

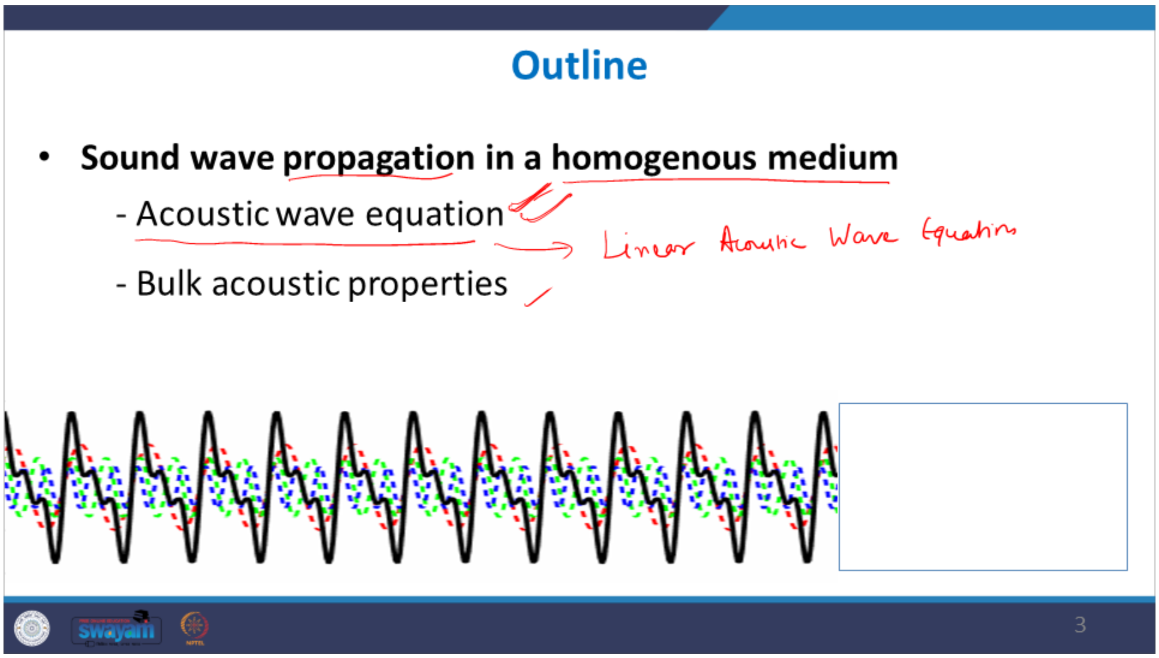
Lecture:3

Lecture 03: Sound Wave Propagation

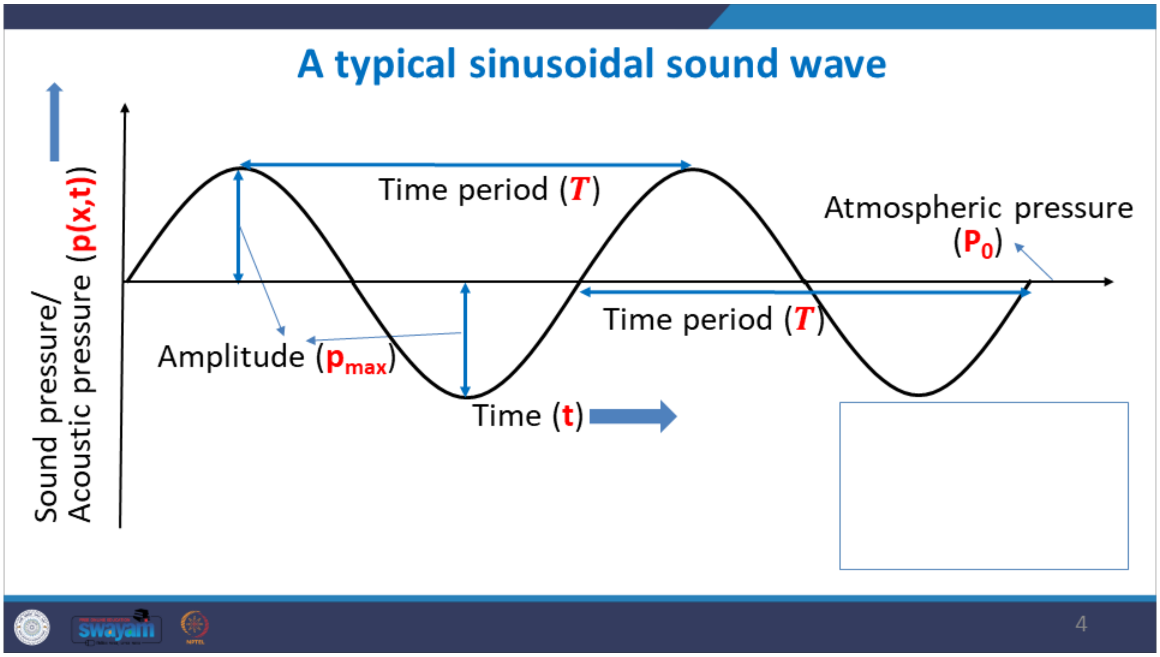
Welcome to the lecture 3 in this course on noise control in mechanical systems. I am Professor Sneha Singh. So, this lecture 3 is on sound wave propagation. So, from here we are beginning our first our module on the acoustics fundamentals and as discussed in the last lecture it is needed to establish the context for noise control. So, to summarize what we have done till now we have the discussed we have I have set an introduction to the various phenomena related to sound and noise, how sound is a physical phenomenon, how it is being generated and represented as a wave, what are the basic wave parameters and their relationships, then how sound perception becomes very subjective, annoyance, presentness of a sound, whether it is a noise or not, all of this becomes subjective to the human. Then noise phenomenon, what is the need for noise control, all of that we have discussed till now. So, today we will discuss about what is the sound wave, how it propagates.



And most importantly we will try to derive the equation for sound wave propagation which is called as the acoustic wave equation. It is also called as the linear acoustic wave equation. and what are the bulk acoustic properties and the reason we will derive this is because this wave equation is sort of this holds true for any form of media and any form of sound wave propagation okay so while we go further we will discuss about that there are different types of sound waves there are different types of sources that are creating the different wave fronts and the different sound sources the different media in which the sound wave is propagating but whatever be the media whatever be the type of sound this equation is always going to hold true and that's why it's very important so if suppose you're given As a noise control engineer or as a student or a researcher in this field, suppose you are given some new device or some new situation of sound wave propagation and you have to do some fundamental derivations or you have to develop a model of sound wave propagation in an entirely new system. You can always begin with this particular equation because this is going to hold true. So, let us see the derivation of it.



So again this is the typical representation of a sound wave we have the amplitude time period etc.



So we are going to see how the sound wave is propagating in a homogeneous medium in fact this particular equation gives you the propagation in the homogeneous medium so what do you mean by homogeneous medium so a homogeneous medium this is how i have defined it here is one that has constant bulk modulus, a constant mass density you will see this later that if the bulk modulus and the mass density becomes constant then automatically the speed of sound also becomes constant. So, if you just stated constant bulk modulus and constant mass density even then you would have a homogeneous acoustic medium. So, these parameters are constant not just with space but also with time. So, within the duration in which we are observing a medium. and the space over which we are observing the medium. So, within that particular space and within that particular time of observation, if these quantities say constant, we can say that the medium is homogeneous in nature and then the acoustic wave equation is going to hold true for such medium. So, let us see how the sound wave is propagating through this medium. This sound wave is creating pressure variations. So, it will cause both the spatial and temporal variations in the pressure. So, we can represent it let P_0 be the mean atmospheric pressure or the mean pressure of that medium and on top of that we have this pressure which is fluctuating in nature this is the sound pressure. So, the total pressure is given by this equation:

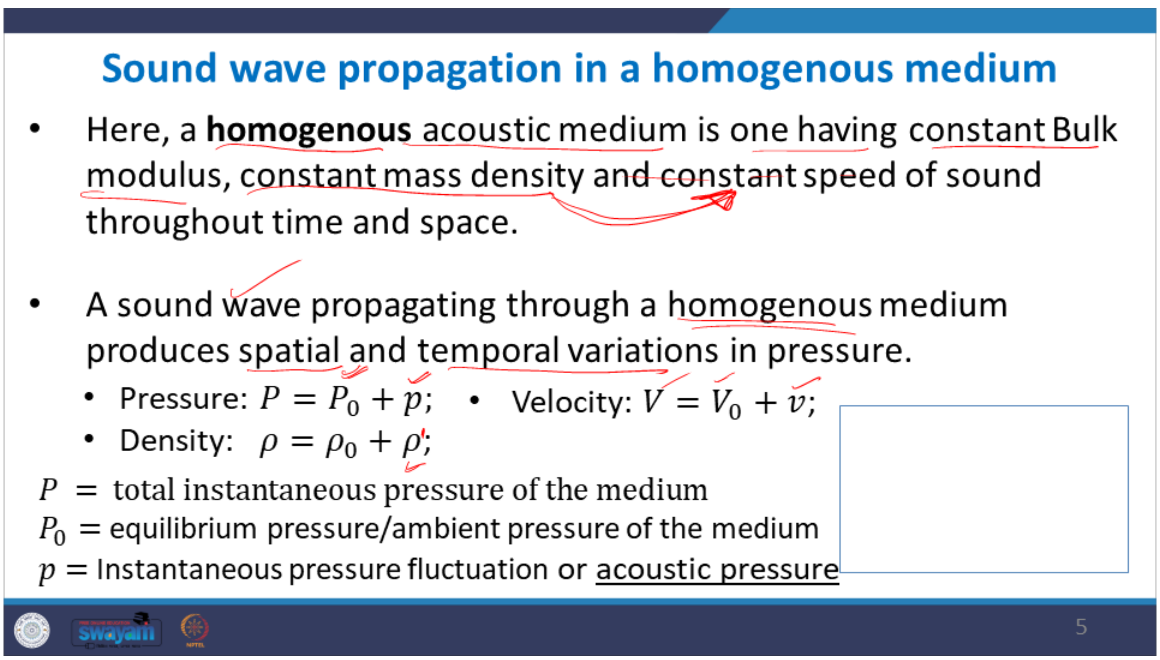
$$\textbf{Pressure: } \mathbf{P = P_0 + p}$$

In the same way suppose the particle already had some velocity and in top of that we have a fluctuating velocity of the particle and it will become the total velocity:

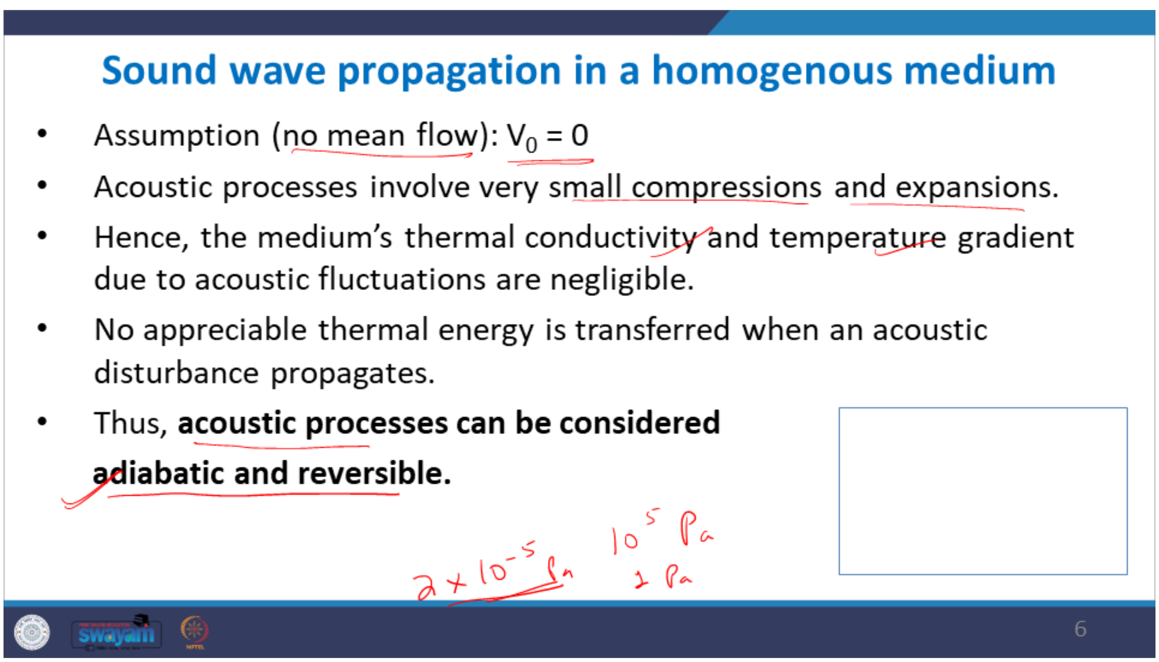
$$\textbf{Velocity: } \boldsymbol{V = V_0 + v}$$

And in the same way suppose the medium already has a mean density but because of the flow of the sound wave or the propagation of the sound wave the density because the pressure is increasing and decreasing momentarily and instantaneously over space and time in the same way even the density will have the fluctuations so This becomes the fluctuation in the density. Okay. So, this I think this we can represent it as rho dash. This fluctuation in the density. So, this is a small fluctuation in the density which is getting added with the mean density of the medium.

$$\textbf{Density: } \boldsymbol{\rho = \rho_0 + \rho}$$



Now, for the deriving the condition, we will take the first condition as no mean flow which means that the V_0 becomes 0. Because for the flow conditions, we have a different set of equations and a different acoustics altogether and that becomes out of course for this particular level. So, the acoustic processes they usually have very small compressions and very small expansions. In fact, in the very first lecture I told you that suppose our atmospheric pressure is of the order of 10^5 even a small variation in pressure let us say 1 Pascal becomes a very loud noise. And we can hear as low noise as 10^{-5} Pascal or rather it is 20 micropascals which is $2 *10^5$ Pascal. This becomes our lowest level. So, in comparison to the mean level you can see that it is so orders of magnitude low. So, all these processes they involve very small compressions, very small expansions. And in such smaller scales or smaller magnitudes, the thermal conductivity, the thermal gradients, they can be neglected. So essentially during these processes, there is no thermal transfer in the medium. So the processes are usually considered as adiabatic and reversible in nature. And hence all our derivations follow this particular assumption you do not call that as an assumption because it holds true that the acoustic processes they are adiabatic and reversible in nature.



So, this is the adiabatic empirically determined adiabatic relationship in a real gas:

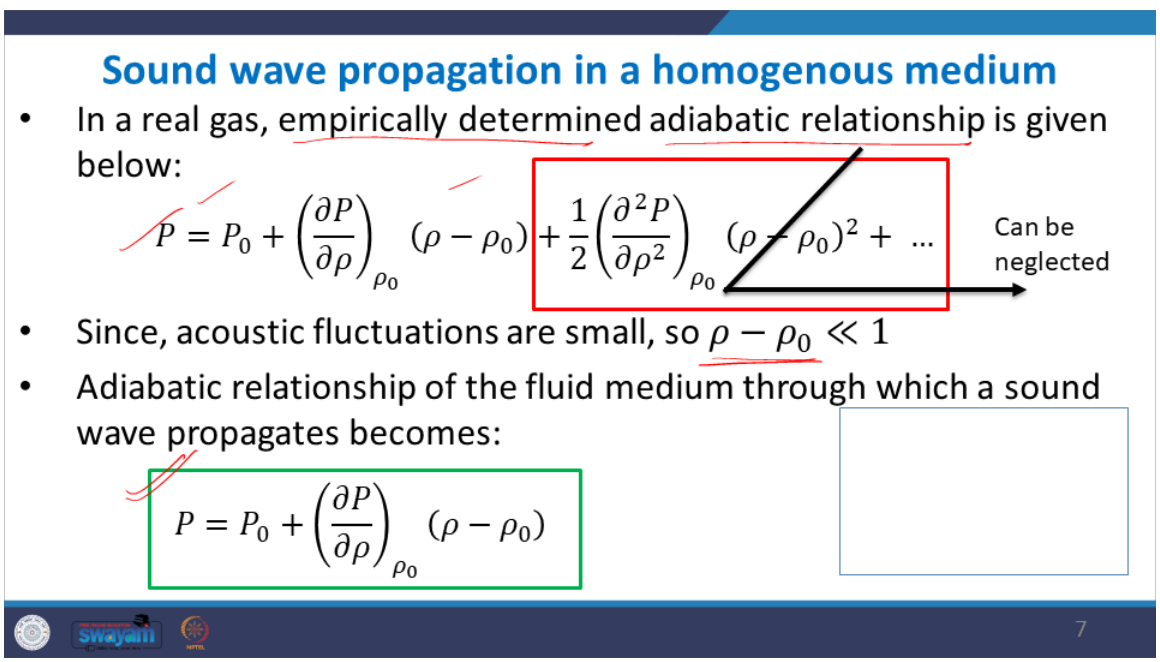
$$\mathbf{P} = \mathbf{P_0} + \left(\frac{\partial \mathbf{P}}{\partial \boldsymbol{\rho}}\right)_{\boldsymbol{\rho_0}} (\boldsymbol{\rho} - \boldsymbol{\rho_0}) + \frac{1}{2}\left(\frac{\partial^2 \mathbf{P}}{\partial \boldsymbol{\rho}^2}\right)_{\boldsymbol{\rho_0}} (\boldsymbol{\rho} - \boldsymbol{\rho_0})^2 + \ \ldots$$

and because the fluctuations are very small in magnitude. So, the higher powers of this term:

$$\boldsymbol{\rho} - \boldsymbol{\rho_0} \ll \mathbf{1}$$

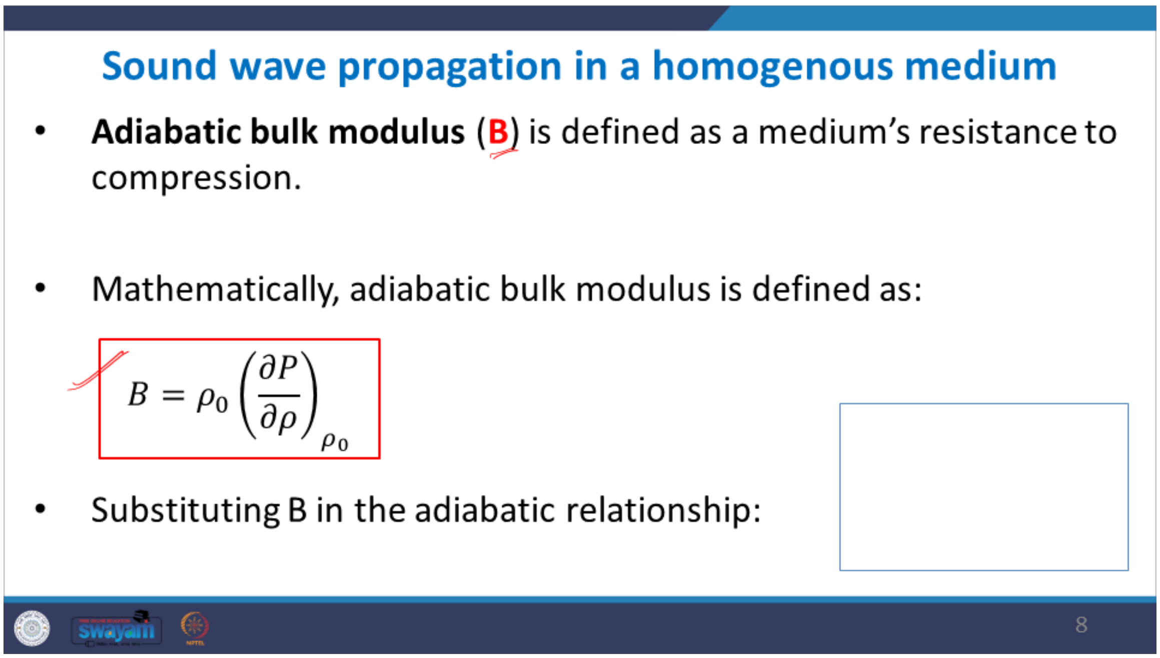
can be neglected. So, till this we can consider and this higher powers and further on can be neglected and hence the relationship can be shortened into only having the first order of this smaller term which is this one.

$$\mathbf{P} = \mathbf{P_0} + \left(\frac{\partial \mathbf{P}}{\partial \boldsymbol{\rho}}\right)_{\boldsymbol{\rho_0}} (\boldsymbol{\rho} - \boldsymbol{\rho_0})$$



Let us define a term called as the adiabatic bulk modulus B which defines the property of an acoustic medium it is the resistance to compression of the medium and mathematically it is represented using this particular formula which is the mean density of the medium times the rate of change of the pressure with respect to density.

$$\mathbf{B = \rho_0 \left(\frac{\partial P}{\partial \rho}\right)_{\rho_0}}$$



So, if you see this and see this particular equation:

$$\mathbf{P = P_0 + \left(\frac{\partial P}{\partial \rho}\right)_{\rho_0} (\rho - \rho_0)}$$

let us say we divided it by ρ_0 and multiplied by ρ_0. So, we will get this particular thing represented here the same equation becomes this:

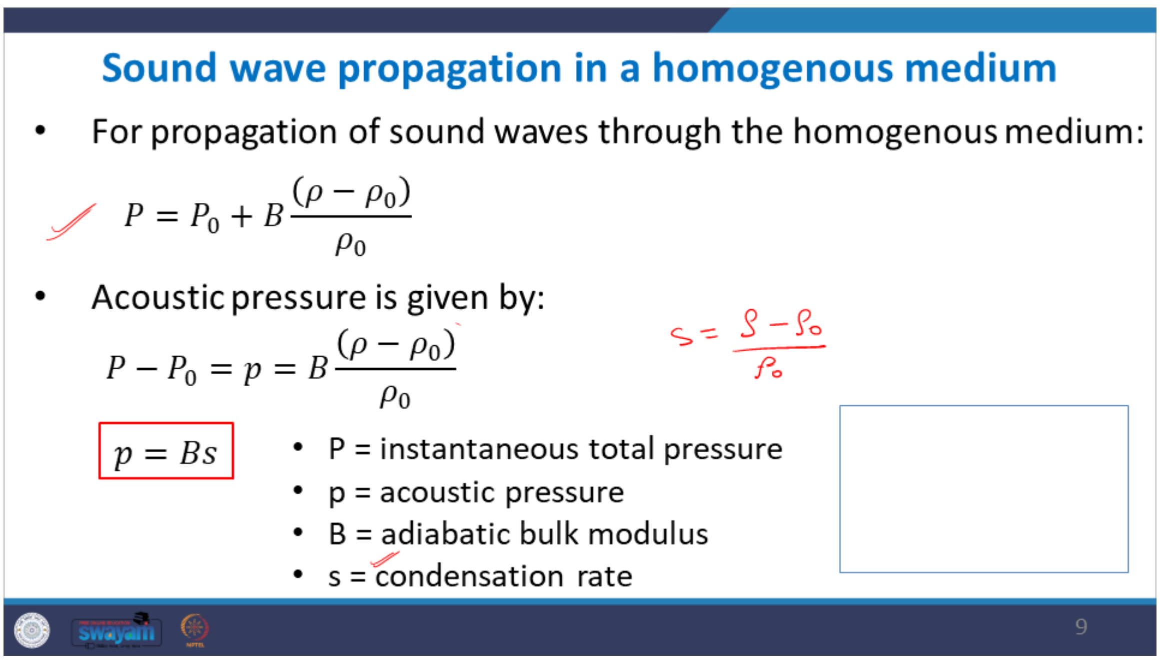
$$\mathbf{P = P_0 + B\frac{(\rho - \rho_0)}{\rho_0}}$$

We are dividing and multiplying by ρ_0 and we are able to represent the term left side of this by the bulk modulus. So, the acoustic pressure is what? It is $P - P_0 = p$. It is that fluctuation that we are representing. So, this can be represented as:

$$\mathbf{B}\frac{(\boldsymbol{\rho} - \boldsymbol{\rho_0})}{\boldsymbol{\rho_0}}$$

Let us denote this entire term which is by a new terminology called as the condensation rate(s) of the medium which is the relative rate of change of density.

$$\mathbf{p = Bs}$$



Then let us take a small volume of that medium we are discussing and within that small infinite small volume, we will apply the principle of conservation of mass. So, based on the principle of conservation of mass what we see is that the rate at which the mass is flowing into the system and going out of the system. So, whatever is the difference between the mass flowing into the system and the mass going out of the system or out of the element that is essentially the increase in the mass of the system. So, let us see how much mass is flowing into this volume. The net mass flowing into the volume would be the difference in the mass flowing from this particular surface and going out of it. So, the difference between the mass flowing into the volume and the mass going out of the volume and we are discussing them all with respect to the change in time. So, it is the. So, what is this? So, the mass let us discuss here. Let us take along the positive x direction to see the mass that is going into the volume becomes the velocity multiplied by the density, it will give you the rate of mass flow. So, velocity multiplied by the density of the medium multiplied by the surface area of this volume will give you the net volume of mass that is flowing per unit time. Okay So, that is the representation here with the first term ρu_x multiplied by dy dz. Then the second term here is the rate at which the mass is going out of this infinite small volume. So, this is the net mass flowing plus the change in this multiplied by dy dx. So, this becomes your overall when you solve it this becomes your overall mass that has flown into the volume in the x direction where this is what this is the change in the density. that is happening because of mass is entering from one end to the other end of the infinite small element:

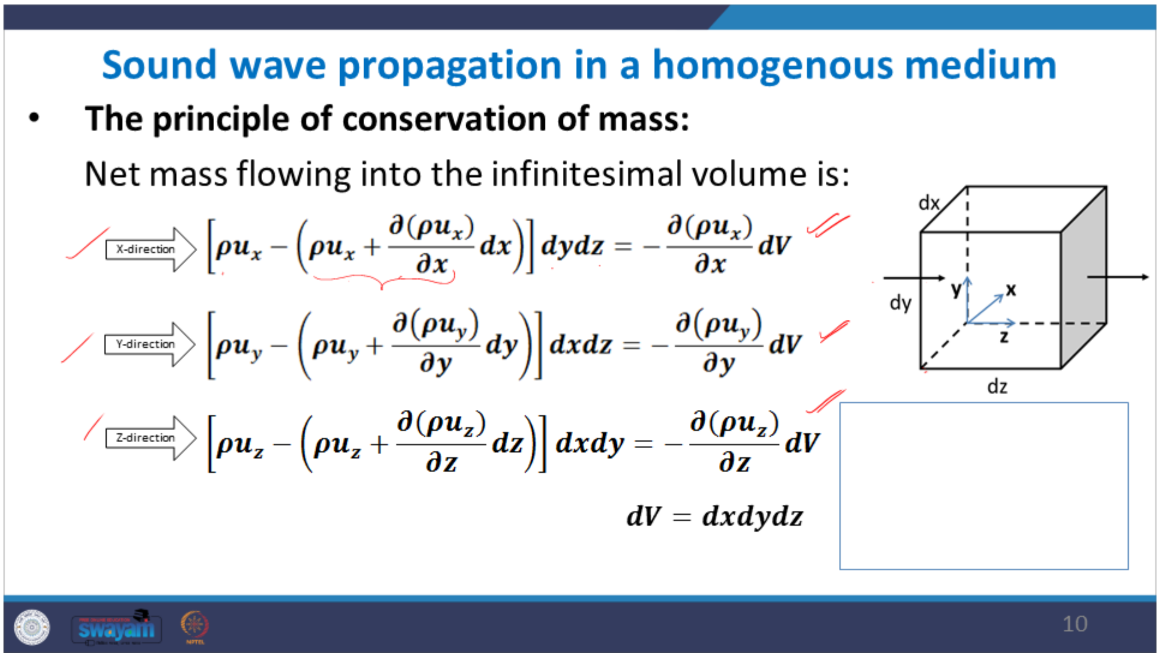
$$\left[\rho u_x - \left(\rho u_x + \frac{\partial(\rho u_x)}{\partial x} dx\right)\right] dydz = -\frac{\partial(\rho u_x)}{\partial x} dV$$

In the same way you can represent in the y and the z direction where dv is what it is dx into dy into dz which is the infinite small volume of this element:

$$\left[\rho u_y - \left(\rho u_y + \frac{\partial(\rho u_y)}{\partial y} dy\right)\right] dxdz = -\frac{\partial(\rho u_y)}{\partial y} dV$$

$$\left[\rho u_z - \left(\rho u_z + \frac{\partial(\rho u_z)}{\partial z} dz\right)\right] dxdy = -\frac{\partial(\rho u_z)}{\partial z} dV$$

So, this is the net mass flow in the x, y and z directions. So, the net mass flow or the net mass increase in the volume will be the addition of these mass increase in each of the directions.



So, we simply add them up because mass is a scalar quantity we can directly add up these orthogonal components:

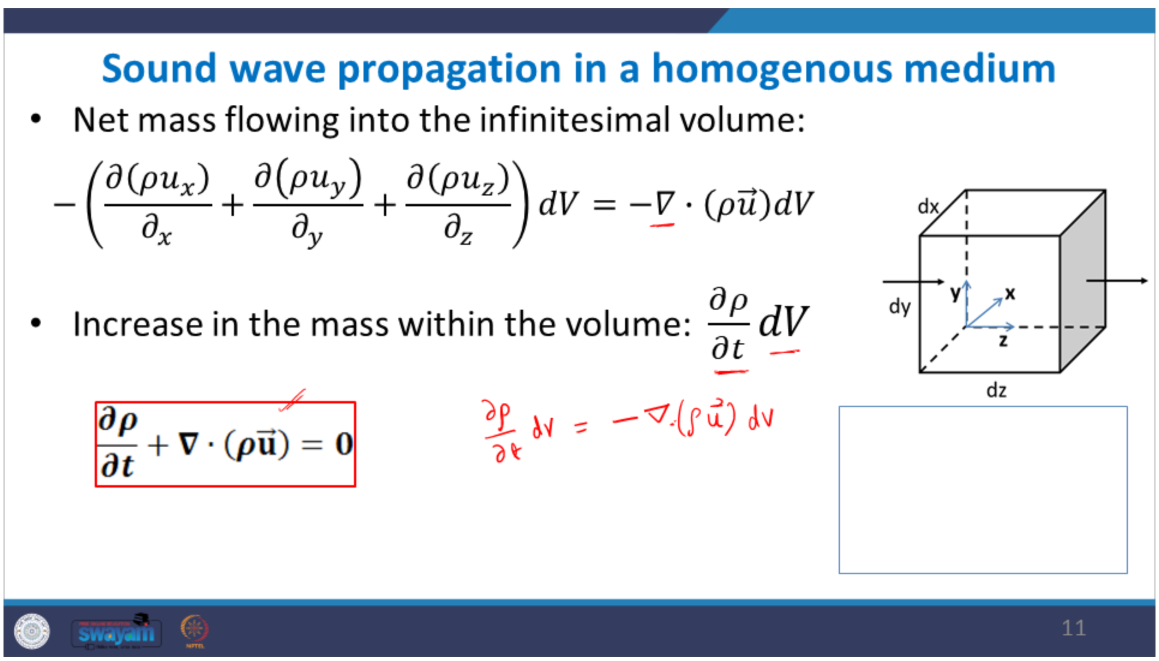
$$-\left(\frac{\partial(\rho u_x)}{\partial_x} + \frac{\partial(\rho u_y)}{\partial_y} + \frac{\partial(\rho u_z)}{\partial_z}\right)dV = -\nabla \cdot (\rho\vec{u})dV$$

When we add them, we represent this using the ∇ operator. So, it is the $\nabla \cdot (\rho\vec{u})dV$ and this becomes your net mass flowing into the volume. And because of the conservation of the mass, this should increase the mass of the volume by this amount. So, mass is what? The density multiplied by the volume. So, the small infinite if you are considering this infinite small element this becomes the volume of the element and this becomes a net increase in the density which shows you the net increase in the mass.

$$\frac{\partial \rho}{\partial t} dV$$

So, this $\frac{\partial \rho}{\partial t} dV$ should be equal to the net increase that we have calculated across every direction and summing them up. So, this equation comes in.

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{u}) = 0$$



In the same way, this particular infinitesimal may experience some net force because of the fluid flow. And how do you find out the force due to a fluid flow? You find out the pressure gradient and you multiply it with the surface area. So, here the net gradient of the pressure or the net increase in the pressure due to this fluctuating motion that is passing through the fluid element is the pressure that is total pressure that is there and the net increase in the pressure multiplied by the area over which it is acting. So, first we are seeing in the positive x direction what is the net infinite small force that is acting due to the fluid flow. So, it is given by this which becomes the pressure gradient into the volume:

$$df_x = \left[P - \left(P + \frac{\partial P}{\partial x} dx\right)\right] dydz \ = -\frac{\partial P}{\partial x} dV$$

Similarly, this one:

$$df_y = -\frac{\partial P}{\partial y} dV$$

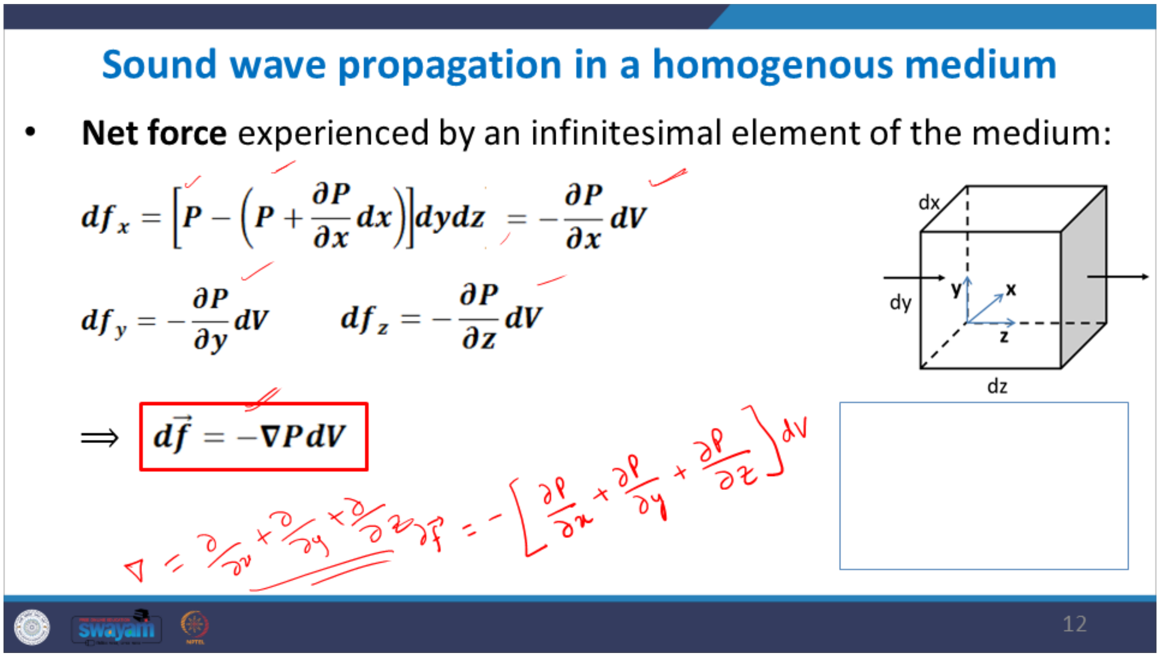
and this one:

$$df_z = -\frac{\partial P}{\partial z} dV$$

So, all of this gives you the net force. If you sum them up again you will have total df would be what? It would be if we take dV as common as well. Okay This is the ∇ operator very well-known mathematical operator in calculus.

So, we are doing essentially a ∇ operator over the pressure. So, overall this is the equation:

$$d\vec{f} = -\nabla P dV$$



So, the net acceleration experienced by the infinite small element of this medium can also be calculated using the Taylor's theorem. So, by the Taylor's theorem, when the dt tends to 0, in that limit, you calculate the difference in the velocity divided by dt. So, you calculate essentially, suppose the velocity at the time t is in this infinite small element at x, y, z and t is given by this.

This is the velocity at time instance t. And this is the velocity at time instance of t plus dt. So, in that instance the x, y and z coordinates have also changed. The change is calculated by u_x into dt, u_y into dt and u_z into dt.

$$\vec{a} = \lim_{dt\to 0}\frac{\vec{u}(x + u_x dt, y + u_y dt, z + u_z dt, t + dt) - \vec{u}(x,y,z,t)}{dt}$$

So, in that particular instance you get it and once you solve this you will see that it comes out to be this operator here.

$$\vec{a} = \frac{\partial\vec{u}}{\partial t} + u_x\frac{\partial\vec{u}}{\partial x} + u_y\frac{\partial\vec{u}}{\partial y} + u_z\frac{\partial\vec{u}}{\partial z}$$

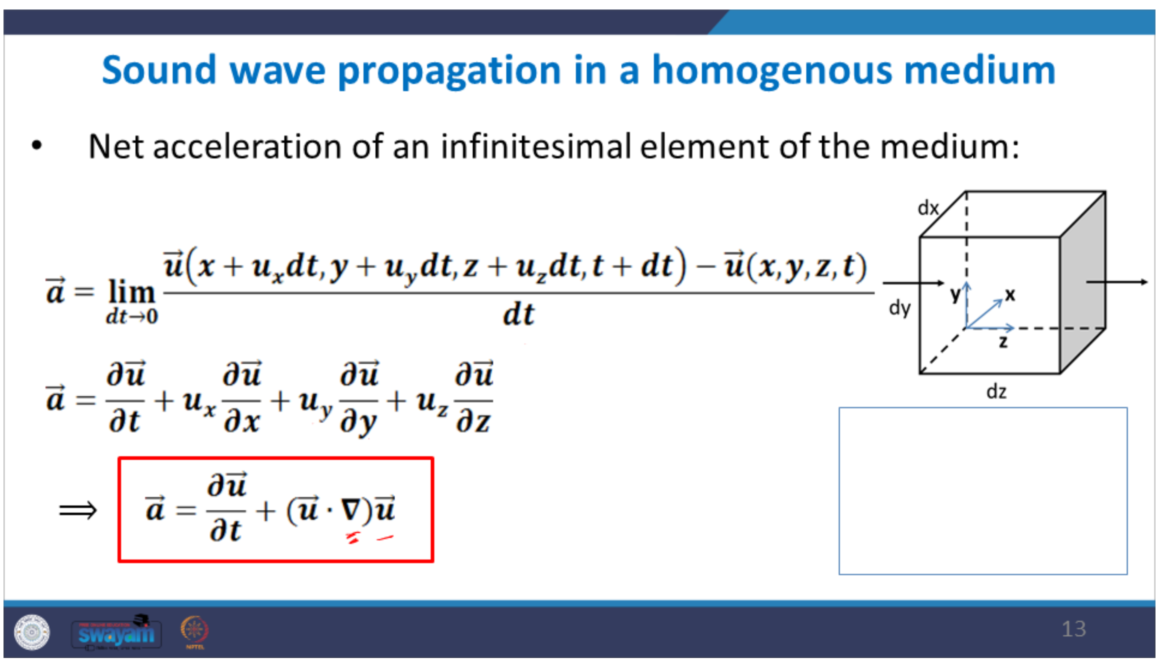
So, overall if you see this $\frac{du}{dt}$ is common and what is this?

This is again:

$$\vec{a} = \frac{\partial \vec{u}}{\partial t} + u_x \frac{\partial \vec{u}}{\partial x} + u_y \frac{\partial \vec{u}}{\partial y} + u_z \frac{\partial \vec{u}}{\partial z}$$

So, it is the ∇ of this u vector. And it is being multiplied with u_x, u_y and u_z components. So, it is essentially represented as a dot product because you are multiplying the x component of the u vector with the x component of the ∇ vector, the y component of the u vector with the y component of the ∇ vector and the z component of the u vector with the z component of the ∇ vector. And you are getting this formulation:

$$\vec{a} = \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}$$



And the law of momentum for the fluid or the Newton's second law we apply.

We already have an equation for the force. We have an equation for the acceleration. So,

$$\mathbf{d\vec{f}} = \mathbf{\vec{a}dm}$$

We put the equations already found for the force and the acceleration:

$$\boldsymbol{d\vec{f}} = -\boldsymbol{\nabla P dV}$$

$$\boldsymbol{\vec{a}} = \frac{\boldsymbol{\partial \vec{u}}}{\boldsymbol{\partial t}} + (\boldsymbol{\vec{u}} \cdot \boldsymbol{\nabla})\boldsymbol{\vec{u}}$$

$$\mathbf{dm} = \boldsymbol{\rho}\mathbf{dV}$$

and this is the assumption that:

$$|(\mathbf{\vec{u}} \cdot \boldsymbol{\nabla})\mathbf{\vec{u}}| \ll \frac{\boldsymbol{\partial}\mathbf{\vec{u}}}{\boldsymbol{\partial}\mathbf{t}}$$

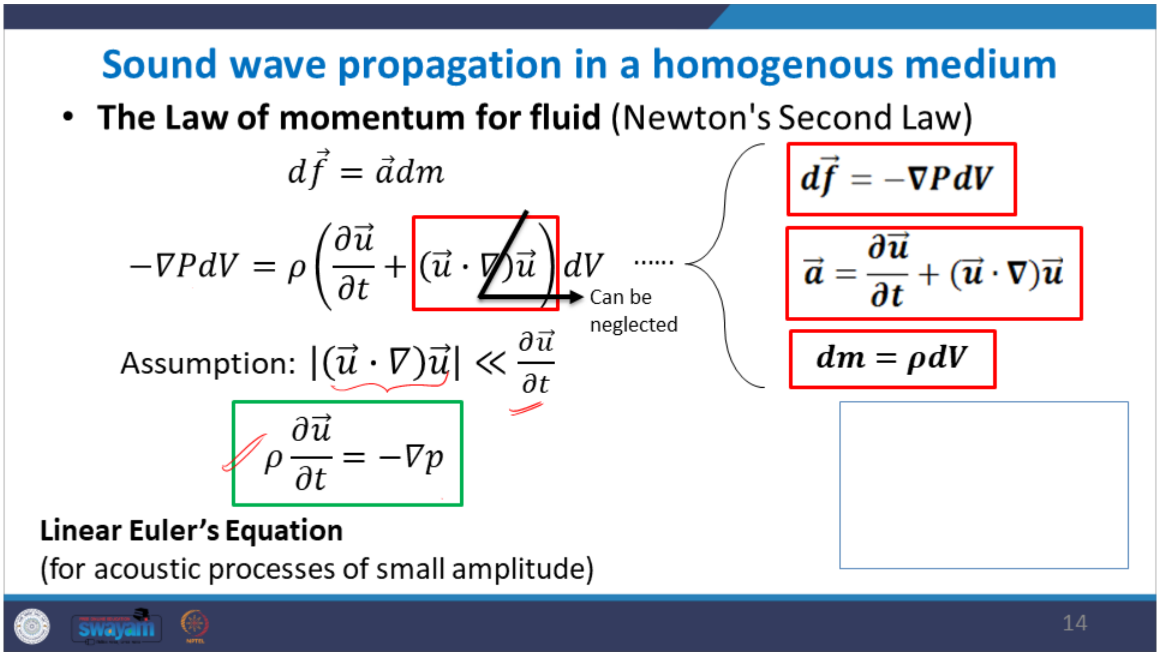
this particular part can be neglected.

$$(\mathbf{\vec{u}} \cdot \boldsymbol{\nabla})\mathbf{\vec{u}}$$

Now, why this is there? This is because of the property of the ∇ operator itself and the assumption we can simply write it like this that the spatial variation of the velocity is much less compared to the temporal variation in the velocity. Okay So, that is what we are representing here. So, we are neglecting this operator and finally, this particular equation then becomes this:

$$\boldsymbol{\rho}\frac{\boldsymbol{\partial}\mathbf{\vec{u}}}{\boldsymbol{\partial}\mathbf{t}} = -\boldsymbol{\nabla}\mathbf{p}$$

This is called as the linear Euler's equation.



So, these are the equations obtained for the sound wave propagation. The first equation was obtained using the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{u}) = 0$$

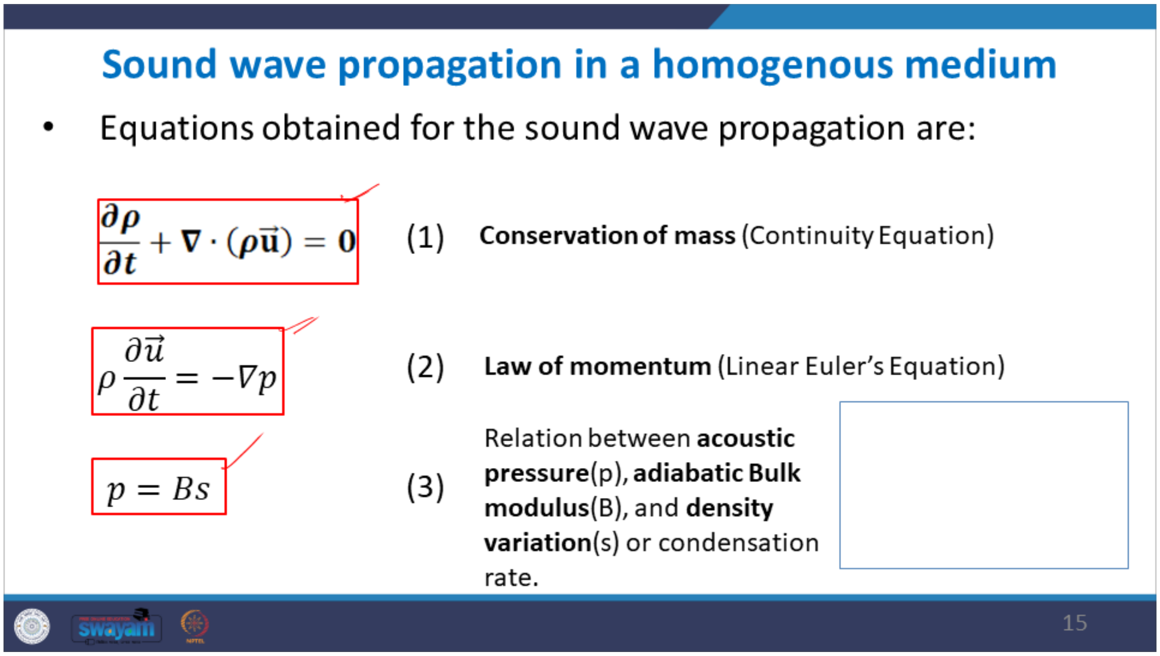
This one using the law of momentum:

$$\rho\frac{\partial \vec{u}}{\partial t} = -\nabla p$$

And this one was based on the relationship we derived for the pressure in terms of the bulk modulus and the condensation rate.

$$p = Bs$$

So, we will use these three equations.



So, let us solve the first equation and substitute the condensation rate in that. Now, we know that s is what? It is:

$$s = \frac{(\rho - \rho_0)}{\rho_0}$$

This is the definition we have taken for the variable s. okay so ρ can be represented as what it can be represented as if you see it here this becomes:

$$\frac{\rho}{\rho_0} - 1$$

so this

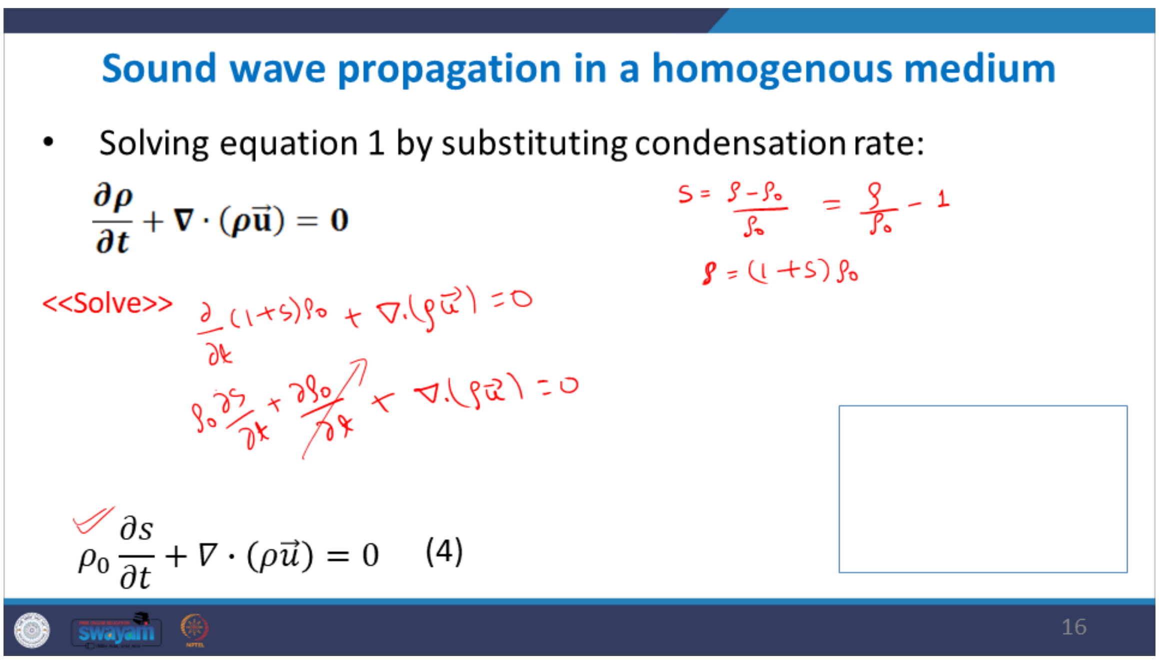
$$\rho = (1+s)\,\rho_0$$

this is what ρ is in terms of s so let us represent it here in terms of s and see what happens So, what you will see here is that obviously that becomes that

$$\frac{\partial}{\partial t}(1+s)\,\rho_0 + \nabla\cdot(\rho\vec{u}) = 0$$

but we are assuming a homogeneous medium. So, homogeneous medium means the mean velocity, the mean pressure and all this are constant for the medium. So, this does not change. So, overall this is the equation we are getting.

$$\rho_0 \frac{\partial s}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$$



We differentiate this equation that we have obtained with respect to time, we get this double derivative:

$$\rho_0 \frac{\partial^2 s}{\partial t^2} + \nabla\cdot\left(\rho\frac{\partial\vec{u}}{\partial t}\right) = 0$$

Okay.

And

$$\nabla \cdot \left(\rho \frac{\partial \vec{u}}{\partial t}\right)$$

it becomes. Now, differentiating the equation 2 with respect to space let us go and quickly see what the equation 2 was. This is our equation 2.

$$\rho \frac{\partial \vec{u}}{\partial t} = -\nabla p$$

So, when we differentiate it with respect to space we will get.

So, here we have the in this term we have the ∇ operator with respect to p and here we have got

$$\rho \frac{\partial \vec{u}}{\partial t}$$

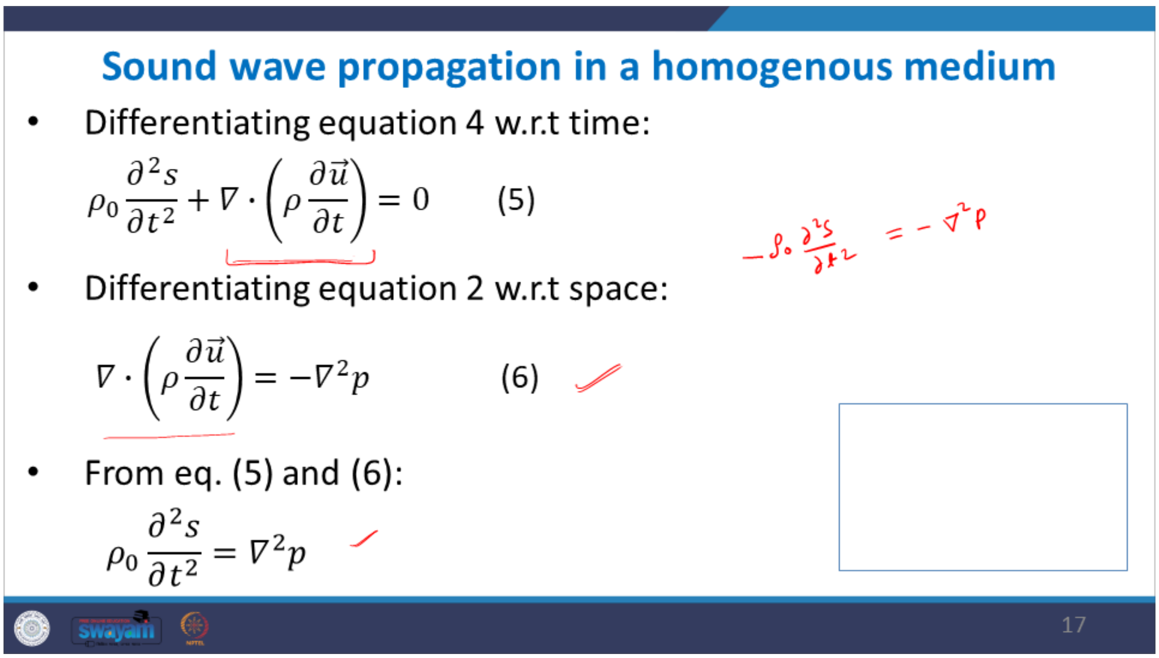
So, this becomes our equation

$$\nabla \cdot \left(\rho \frac{\partial \vec{u}}{\partial t}\right) = -\nabla^2 p$$

the ∇ operator because we are differentiating with respect to space and here are double ∇ square p for that particular equation.

So, let us use equation 5 and 6 here. So, you have this common term here. So, what does it mean?

It means that $\boldsymbol{\rho_0 \frac{\partial^2 s}{\partial t^2}}$ okay. So, this minus of this because this would be equal to minus of ρ. So, this particular terminology is equal to $\boldsymbol{-\rho_0 \frac{\partial^2 s}{\partial t^2}}$ that should be same as $\boldsymbol{-\nabla^2 p}$ okay. So, you remove the negative term. So, this is what happens. That is one equality you get.



So, this is our equality that we have obtained which we want to solve further to get this particular equation.

So, what we do is now S is what? In the very first equation we found that

$$\mathbf{p = Bs}$$

Okay So, let us solve this if P is given by B times of S then if we have to represent this S in terms of P and remember for a homogeneous medium this B is constant with respect to both space and time. Okay

S is what? It is P by B. okay. B comes in the denominator. So

$$\rho_0 \frac{\partial 2}{\partial t2} \left(\frac{P}{B}\right) = \nabla^2 P$$

where P is the acoustic pressure. So we want to put both in the terms of the same variable P both left hand side and the right hand side. Now, this B is constant with respect to space and time.

So, ρ_0 and B both are constant which we take out. This is the equation we are getting.

$$\frac{\rho 0}{B} (\partial^2 P / (\partial t^2) = \nabla^2 P$$

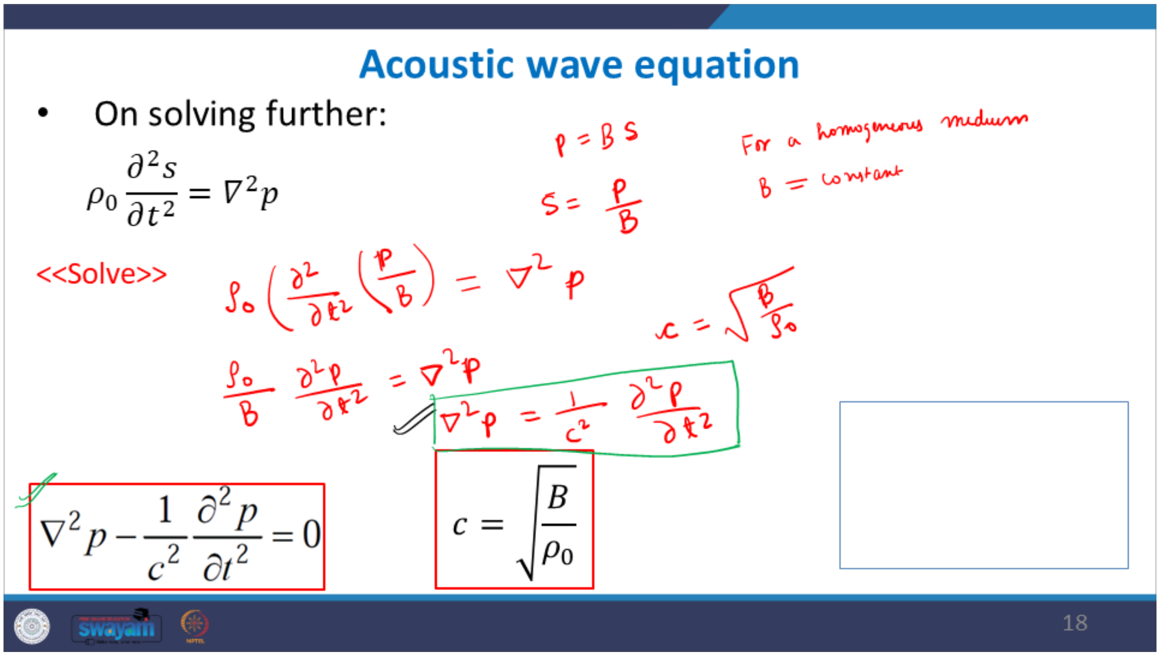
Let us say we have a term called C which is $\sqrt{\frac{B}{\rho 0}}$ Then what could this be represented as? This

$$\nabla^2 P = 1/C^2 (\partial^2 P / (\partial t^2)$$

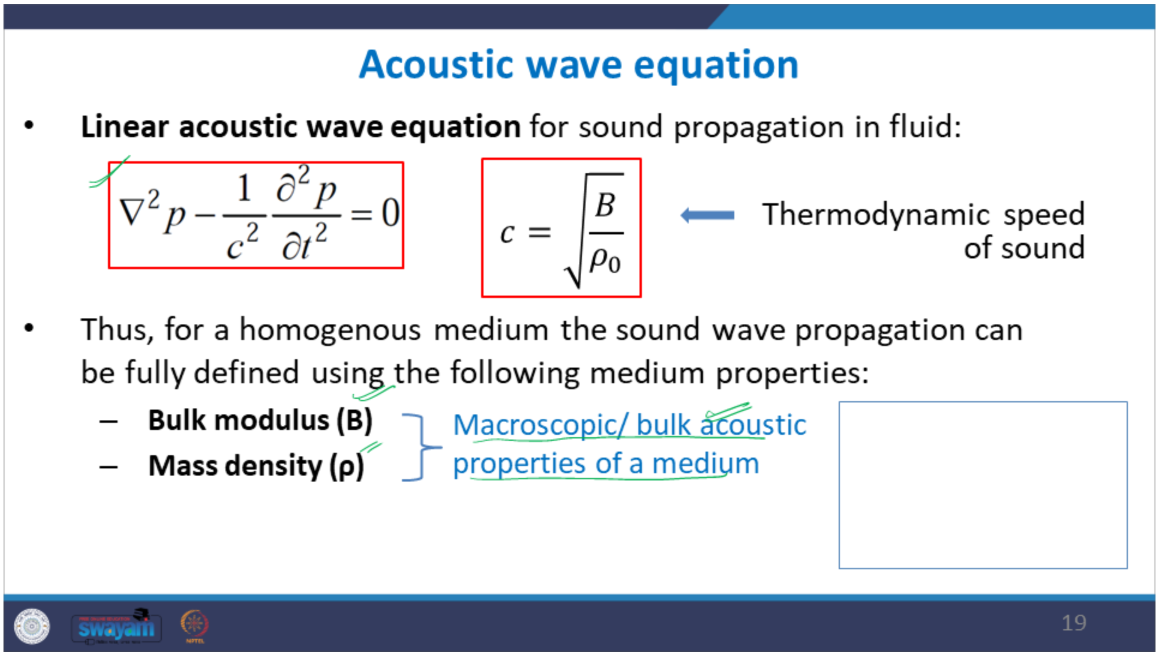
And this is essentially your linear acoustic wave equation:

$$\boxed{\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0}$$

I have put this term here. So, either you can represent it using this particular format here, either in this way or in this particular way both and c is $\sqrt{\frac{B}{\rho 0}}$



which is the thermodynamic speed of sound and so what you see here is that the thermodynamic speed of sound is actually $\sqrt{\frac{B}{\rho 0}}$ this is the speed at which the sound is moving and in this particular linear acoustic wave equation what we see is that the entire equation depends only on the property c which is itself dependent on B and ρ. So, it depends on the bulk modulus and the mass density, which become the macroscopic bulk acoustic properties of a medium. And the entire linear acoustic wave equation can then be derived or dependent on B and ρ. So, if a sound, so essentially the explanation or the gist of this exercise is that, we are getting an equation where a sound wave is propagating in a homogeneous medium with no mean flow and in that medium it only depends the propagation the nature at which the sound wave is propagating depends only on two characteristics which is the bulk modulus of the medium and the mass density of the medium and that is why we call them as the macroscopic bulk acoustic properties. okay.



So, we will continue with this sound wave propagation and we will solve some numerical, we will see some various kind of waves and what are their solutions to this linear acoustic wave equation in the next lecture. So, thank you for listening.

Thank You