MECHANICS

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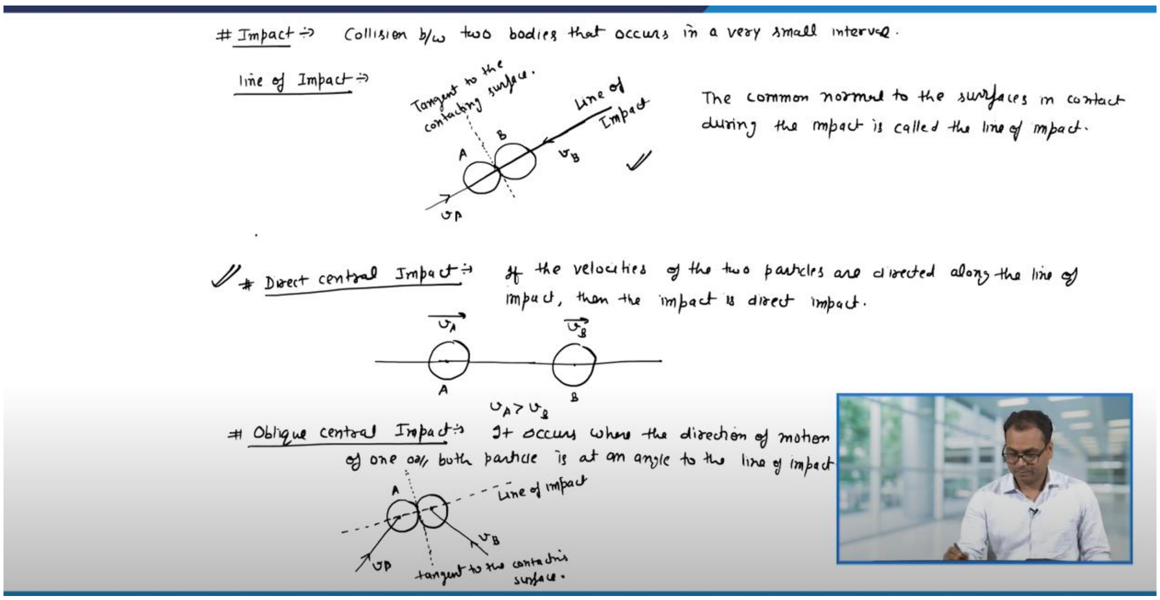
Lecture: 41

Direct central impact

Hello everyone, welcome to the lecture again.

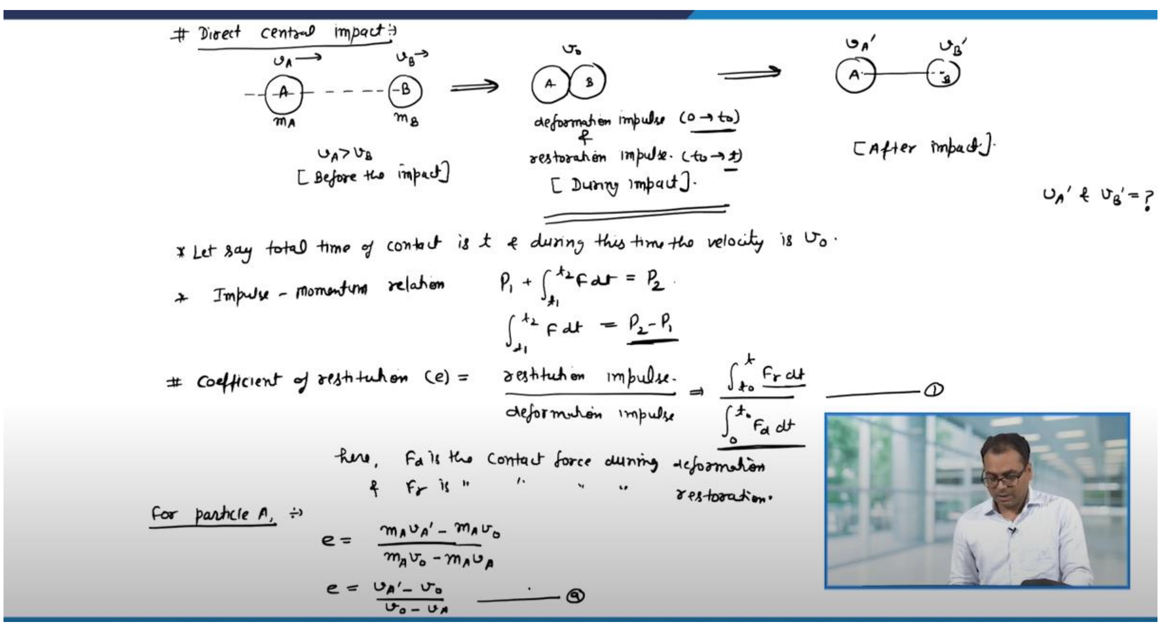


Today, we are going to study impact, particularly the direct central impact. So, let me define what is impact. Impact is basically the collision. So, it is the collision between two bodies that occurs in a very small interval. And to study the impact, we have to define what is the line of impact. So, let us say I have a body, let us say A which is moving in this direction with a velocity v_A, I have another body, let us say B, which is moving in this direction with velocity v_B, then the common normal to the surfaces in contact during the impact is called the line of impact.

So, let us look at the common normal. So, here this is the common normal tangent and this one is the common normal. So, therefore, this is the line of impact and this is the tangent to the contacting surface. Let us look at what is direct central impact if the velocities of the two particles are directed along the line of impact then the impact is direct impact. For instance, let us say I have a particle A which is moving with velocity v_A, and I have a particle B which is moving with velocity v_B, and they are moving along the line of the impact. So, along this line and of course, for the impact v_A has to be larger than v_B, then this is the direct central impact.

Over here also, it was the example of the direct central impact. Now, let us look at the oblique central impact. So, here or it occurs when the direction of motion of one particle or both particle is at an angle to the line of impact. For example, let us say I have a particle A which is moving with velocity v_A like that and I have a particle B which is moving with velocity v_B like this.

Then the common tangent will be this and the line of impact will be that. So, this is your line of impact and this one is the tangent to the contacting surface. So, you can see here that the direction of motion of the particles are at angle to the line of impact. Therefore, this is oblique central impact.



Now, let us analyze the direct central impact first. So, we are going to analyze the direct central impact. So, let us say, again, as we are keep saying, we have a particle of mass m A, Let us say this is A. It is moving with velocity v_A along the line of impact. And we have another particle B. Its mass is m_B and it is moving with velocity v_B.

And of course, for the impact, v_A should be larger than v_B. This is the case before the impact. Okay. And after that, we have impact. So, this was particle A and that was particle B. During the impact, both of them travel with some velocity.

Let us say that velocity is v_0. And during this time, because this is the impact, there will be impulse. So, first there is a deformation impulse and So, the body will deform and then there will be restoration impulse. Let us say the overall time for the deformation impulse and for the restoration impulse is t. So, from 0 to t_0 time, there is deformation, and then from t_0 to t time, there is restoration. So, this is the case when the impact happens. So, this is during impact and then after the impact, we have particle A and particle B. they travels with a velocity v_A prime and v_B prime. This is the case after the impact.

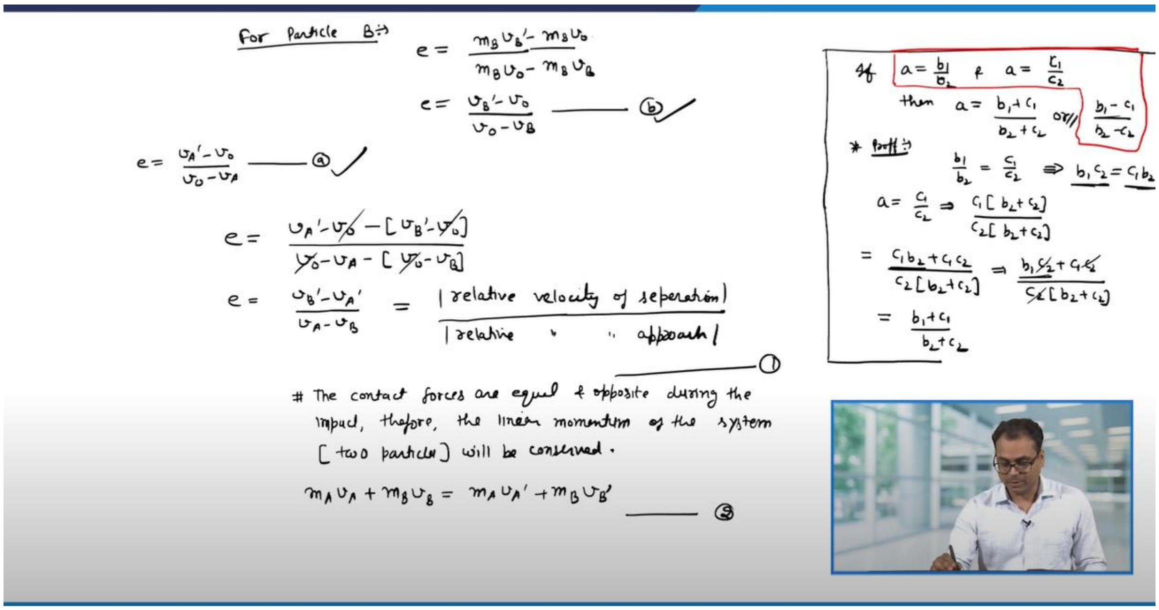
And we want to find out what is the velocity after the impact. That means what is v_A' and what is v_B'. So, since there are two unknown, we need two equations to solve this problem completely. As I said, Let us say total time of contact or impact is t, and during this time, the velocity of A and B is v_0.

And the deformation time is t_0 and the restitution time is from t_0 to t. So, let us use the impulse-momentum relation. So, this is something that we can use to find out what is the momentum after the impact. So, impulse momentum relation, it tells that if the initial momentum is P_1 and the impact is Fdt then the final moment will be P_2 or I can find out what is the impact integral Fdt that is $P_2 - P_1$. Now, let me define the coefficient of restitution.

It is denoted by e, and this is the ratio of the restitution impulse divided by the deformation impulse. So, we have a restitution impulse divided by the deformation impulse. Let us look at the deformation impulse. So, the deformation is from 0 to t_0. So, 0 to t_0.

Let us say the deformation force is F_d. So, it is $F_d dt$ and the restitution is from t_0 to t. $F_r dt$. Here, F_d is the contact force during deformation and F_r is the contact force during restoration. Now, let us use the definition of e, let us call it equation number 1 for particle A. So, for particle A, e will be, let us first look at this term 0 to t_0 . $F_d dt$. $F_d dt$ Will be $P_2 - P_1$. So, P_2 will be the momentum after the deformation. So, P_2 will be $m_A v_0$ minus the P_1 which is the initial momentum. So $m_A v_A$. And the $F_r dt$ will be again $P_2 - P_1$, wherein P_2 will be the momentum after the restoration has happened.

So, that is $m_A v_A'$ minus the P_1, which is the initial momentum before the restoration. So, it is $m_A v_0$. From here, m_A will get cancelled and we get $e = v_A' - v_0/(v_0 - v_A)$. Let us call it equation number (a). Now, let us use the same definition for particle B. So, for particle B, we have e equal to 0 to t_0, $F_d dt$.



So, $F_d dt$ will be $P_2 - P_1$ and P_2 is the momentum after the deformation. So, that is $m_B v_0$ minus the initial momentum m_B, it was travelling with velocity v_B and here I have $F_r dt$. So, $F_r dt$ will be again $P_2 - P_1$, and P_2 is the momentum. After the restoration has happened, so that is $m_B v_B' - m_B v_0$. So, again, the m_B will get cancelled, and we have $e = v_B' - v_0/(v_0 - v_B)$.

Let us call it equation number (b). And remember our equation number (a) was$e = (v_A' - v_0)/(v_0 - v_A)$. This was our equation number (a). Now, here, the unknown is v_0 because that is something that we do not have access to. Therefore, we have to eliminate v_0, and we can eliminate v_0 from equation numbers A and B if we use the identity A. that if suppose you have a quantity a which is b_1/b_2 and the same quantity a is equal to let us say c_1/c_2, then I can write down a as $(b_1 + c_1)/(b_2 + c_2)$ or I can write down $(b_1 - c_1)/(b_2 - c_2)$.

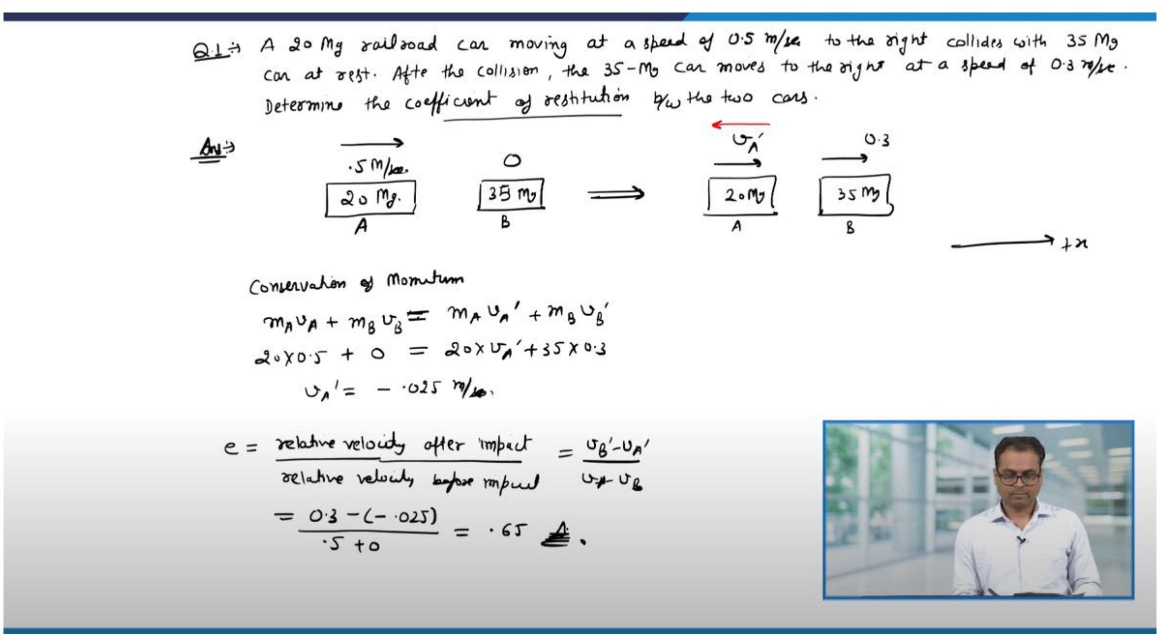
This we can prove very easy. So, let me just Prove this. So, from here, because a is equal to b_1/b_2 and the same a is equal to c_1/c_2. Therefore $\frac{b_1}{b_2} = c_1/c_2$, or I can write down $b_1 c_2 = c_1 b_2$.

Now, let us start with a. a Was c_1/c_2. We can multiply it by $b_2 + c_2$. So, $(c_1 b_2 + c_2)/(c_2 b_2 + c_2)$, and this I can write as $(c_1 b_2 + c_1 c_2)/(c_2 b_2 + 2)$ or $b_1 c_2 + c_1 c_2$. Herein, i have used $c_1 b_2 = b_1 c_2$.

So, this divided by $c_2 b_2 + c_2$. Now, c_2 will get cancelled, and you will have a equal to $(b_1 + c_1)/(b_2 + c_2)$. So, therefore, let us use this identity. And therefore, e can be written as, because e is this and e is that, therefore e will be $(v_A' - v_0 - (v_B' - v_0))/(v_0 - v_A - (v_0 - v_B))$. And this v_0 will get cancelled with that and this v_0 will get cancelled with this.

So, we get rid of v_0. We get $e = (v_B' - v_A')/(v_A - v_B)$. And note that what was v_B' and v_A'? Well, the difference gives you the relative velocity of the separation. So, this is the relative velocity of separation, and at the bottom, you have the relative velocity of the approach.

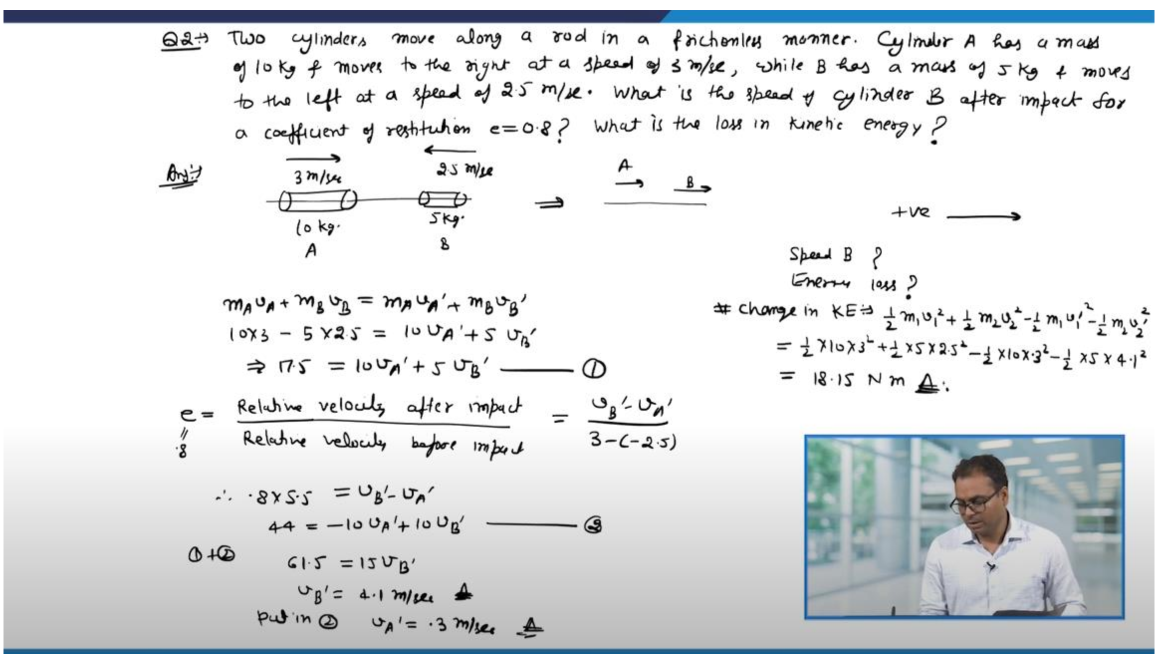
So, this is the velocity with which the particles are coming to each other. And $v_B' - v_A'$ is the velocity of the separation. Let us call it equation number 1. Now, we need one more equation and for that we can use the fact that the contact forces are equal and opposite during the impact. Therefore, the linear momentum of the system, so here system means the system of two particles will be conserved. So, therefore, we have $m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$. Let us call it equation number 2. Now, we have two equations and two unknown.



Therefore, the problem can be solved mathematically. Let us look at some of the examples based on this concept. So, this is the first problem statement. A 20 $million\ gram$ railroad car moving at a speed of 0.5 m/s to the right collides with 35 $million\ gram$ car at rest. After the collision, the 35 $million\ gram$ car moves to the right at a speed of 0.3 m/s determine the coefficient of restitution between the two cars. Now, in this question, it is given that the railroad car has a mass of 20 $million\ gram$. Let us call it A and it is moving with 0.5 m/s to the right. Car B, its weight is 35 $million\ gram$.

It is at rest and after the collision, Let us say car A moves to the right with a velocity of v_A' and it is given that car B moves to the right with a velocity of 0.3 m/s. Let us take this as positive x direction. To find out v_A', we can use the conservation of momentum of the whole system. So, we have $m_A v_A + m_B v_B$, this is the momentum before the collision equal to $m_A v_A' + m_B v_B'$. So, m_A and v_A is given $20 \times 0.5 + v_B$ is 0 equal to $m_A v_A' + m_B v_B'$, and this gives you $v_A' = -0.025\ m/s$.

Therefore, we have to change the direction of the v_A'. Now, we can find out the coefficient of restitution e which is equal to the relative velocity after impact divide by the relative velocity before impact or we can write down $(v_B' - v_A')/(v_A - v_B)$. So, this comes to be $(0.3 - 0.025)/(0.5 + 0)$ which comes out to be 0.65.



Now, let us look at another problem statement. Question number two. Two cylinders move along a road in a frictionless manner. Cylinder A has a mass of 10 kg and moves to the right at a speed of 3 m/s has a mass of 5 kg and moves to the left at a speed of 2.5 m/s. What is the speed of cylinder B after impact for a coefficient of restitution e equal to 0.8, and it is also asked to find out what is the loss in kinetic energy.

So, here it is given that you have a cylinder of mass 10 kg, let us call it A, it is moving with 3 m/s to the right, collides with 5 kg cylinder, let us call it B, which is moving at a velocity of 2.5 m/s to the left. And after the collision, let us say cylinder A moves in this direction and B moves also in the positive direction. And in the question statement, we have been asked to find out. What is the speed of B and the energy loss, the kinetic energy loss?

So, we can use the conservation of momentum for the whole system. $(m_A v_A + m_B v_B) = m_A v_A' + m_B v_B'$. So, m_A is $10 \times 3 - 5 \times 2.5 = 10v_A' + 5v_B'$. We can rewrite this as $17.5 = 10v_A' + 5v_B'$.

Let us call it equation number 1 and now use the definition of the coefficient of restitution e, which is the relative velocity after impact divided by the relative velocity before impact. So, this is $(v_B' - v_A')/(3 - 2.5)$. So, e is given, it is 0.8. So, therefore, $0.8 \times 5.5 = v_B' - v_A'$ or we can write down $44 = -10v_A' + 10v_B'$.

I just multiplied both side by 10. Now, we can Add 1 and 2, so we get $61.5 = 15v_B'$. This gives $v_B' = 4.1\ m/s$. And if we put this in equation number 2, then we get v_A' which is 0.3 m/s.

Let us now look at the change in kinetic energy. So, kinetic energy is $\frac{1}{2}mv^2$. Therefore, the change will be $\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}m_1 {v_1'}^2 - \frac{1}{2}m_2 {v_2'}^2$. Let us put the values. So, we have $\frac{1}{2} \times 10 \times 3^2 + \frac{1}{2} \times 5 \times 2.5^2 - \frac{1}{2} \times 10 \times 0.3^2 - \frac{1}{2} \times 5 \times 4.1^2$. And this gives you 18.15 $N.m$. So, this is the change in kinetic energy. With this let me stop here. See you in the next class.

Thank you.