MECHANICS
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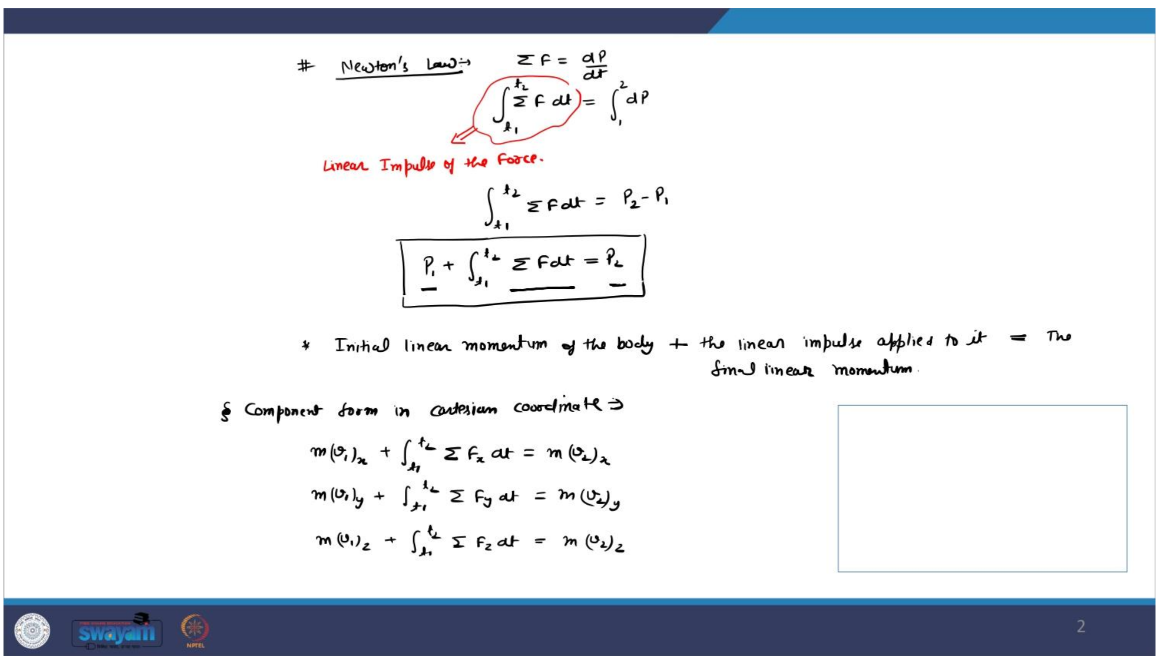
Lecture 39
Impulse momentum relation

Hello everyone, welcome to the lecture again. In the previous lectures, you saw that integrating the equation of motion that is $F = ma$ with respect to displacement gives you the work energy equation. Today, we are going to see that integrating the equation of motion that is $F = ma$ with respect to time will give you the impulse momentum relation.



So, let us see that. Let us start with the Newton's law. $\sum F = \frac{dP}{dt}$. Now, integrate this with respect to dt. So, $\int_{t_1}^{t_2} Fdt = \int_1^2 dP$. The quantity which is on the left hand side is called the linear impulse of the force. So, this is linear impulse of the force. So, we have $\int_{t_1}^{t_2} Fdt = P_2 - P_1$ or let me write it down in the natural sequence.

So, we have $P_1 + \int_{t_1}^{t_2} Fdt = P_2$. So, this is the impulse momentum relation.

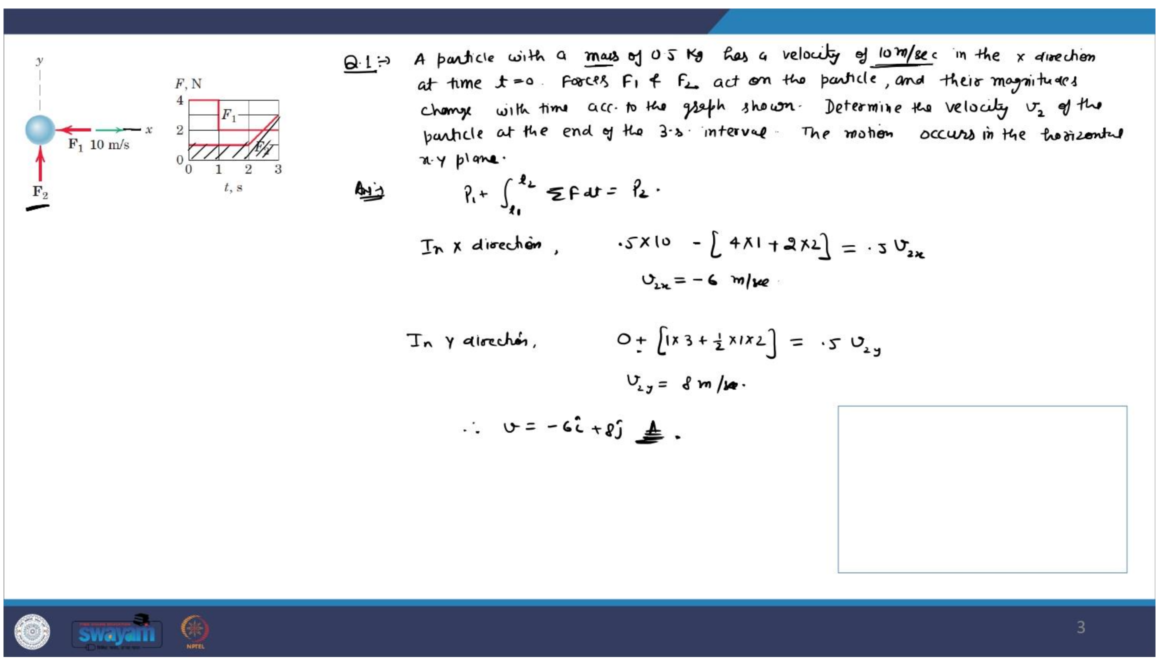
This tells us that if a particle is moving with a momentum P_1 impulse Fdt is applied on it, then its momentum becomes P_2. So, let me write it down in the text. The initial linear momentum of the body which is P_1 plus the linear impulse applied to it will be equals to the final linear momentum. Now, this equation we can write down in Cartesian coordinate component by component. So, let me just write down this component form in Cartesian coordinates. Okay. So, for x component, we have $P = mv$. So,

$$m(v_1)_x + \int_{t_1}^{t_2} \sum F_x dt = m(v_2)_x$$

$$m(v_1)_y + \int_{t_1}^{t_2} \sum F_y dt = m(v_2)_y$$

$$m(v_1)_z + \int_{t_1}^{t_2} \sum F_z dt = m(v_2)_z$$

Now, with this very basic introduction, let us look at some of the examples based on this concept.



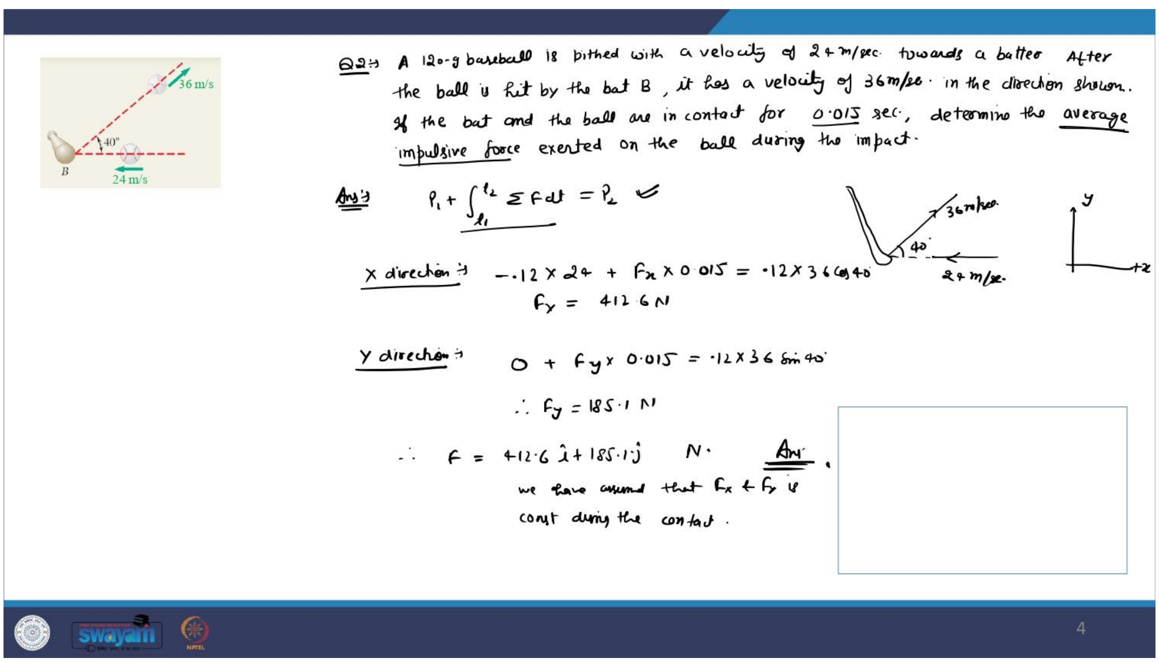
Question number 1, the problem statement is a particle with a mass of $0.5\ kg$ has a velocity of $10\ m/s$ in the x direction at time $t = 0$. Forces F_1 and F_2 act on the particle and their magnitude change with time according to the graph shown, determine the velocity v_2 of the particle at the end of the $3\ s$ interval and it is given that the motion occurs in the horizontal xy plane. So, here note that the x-axis and the y-axis are given. Two forces F_1 and F_2 are acting on it. F_1 is acting in $-x$ direction and F_2 is acting in plus y direction and their magnitudes are given here. So, we can use the impulse momentum equation to find out what is v_2. So, we have $P_1 + \int_{t_1}^{t_2} Fdt = P_2$. This equation I can write down in the x direction. So, in the x direction, we have momentum $P_1 = mv$, mass is given. So, 0.5 into its velocity v, it is given that it has a velocity of 10 m/s in the x direction. So, multiplied by 10 and then plus $\int_{t_1}^{t_2} Fdt$. Now, F is acting in $-x$ direction. So, therefore, minus the force from 0 to 1 s, it is 4 N. So, 4 into 1 Fdt plus then from 1 to 3 s, that is for 2 s, the force is 2. $P_2 = mv_2$. So, from here, I can find out what is v_{2x}.

$$0.5 \times 10 - (4 \times 1 + 2 \times 2) = 0.5\, v_{2x}$$

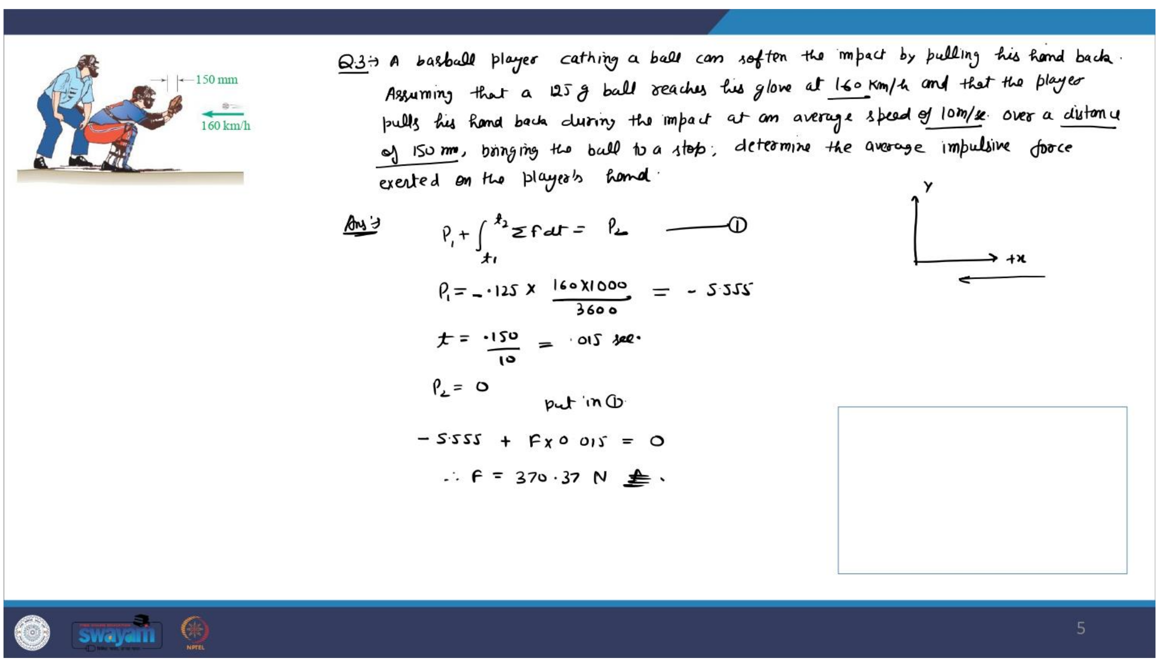
$$v_{2x} = -6\ m/s.$$

Now, let us apply the impulse momentum equation along the y direction. So, in the y direction, only force F_2 is acting and it is given that the mass was initially moving in the x direction only. Therefore, there is no momentum in the y direction. So, in y direction, we have $P_1 = 0$. So, that will be the area inside this force. So, $0 + \left(1 \times 3 + \frac{1}{2} \times 1 \times 2\right) = 0.5 v_{2y}$.

$v_{2y} = 8\ m/s$. Therefore, the final velocity $v = -6\hat{\imath} + 8\hat{\jmath}$.



Now, let us look at another problem statement. A 120 g baseball is pitched with a velocity of 24 m/s towards a batter. After the ball is hit, by the bat B, it has a velocity of 36 m/s in the direction shown. If the bat and the ball are in contact for 0.015 s, determine the average impulsive force exerted on the ball during the impact. In this question, we have been asked to find out the average impulsive force. For that, we can use the impulse momentum equation. So, we have $P_1 + \int_{t_1}^{t_2} F dt = P_2$. Now, the situation is following. We have this bat and a ball is travelling in the horizontal direction with a velocity of $24\ m/s$ and it is hit by the bat and then the ball goes at $36\ m/s$ at an angle of $40°$. So, we can write down this equation in the x and in the y direction. Let us look at this equation in the x direction first. So, in the x direction, the initial momentum $P_1 = mv$, mass is 0.12 because it is $120\ g$ multiplied by the velocity and it will be minus because the ball is travelling in $-x$ direction. So, $-0.12 \times 24 + F_x \times 0.015 = 0.12 \times 36 cos 40°$. So, from here I can find out what is $F_x = 412.6\ N$. Now, let us use it in y direction. So, initially since the ball is moving in the horizontal direction, therefore, there is no moment in the y direction. So, it is $0 + F_y \times 0.015 = 0.12 \times 36 sin 40°$. This gives you $F_y = 185.1\ N$. Therefore, force $F = 412.6\hat{\imath} + 185.1\hat{\jmath}\ N$. Here, note that when we calculate integral Fdt, I have taken F as constant because that is independent of t and we are looking at the average impulsive force. So, here we have assumed that both F_x and F_y is constant during the contact. So, when the ball is in contact with the bat. And it is given here because we have been asked to find out what is the average impulsive force.



Now let us look at another problem statement. A baseball player catching a ball can soften the impact by pulling his hand back. Assuming that a $125\ g$ ball reaches his gloves at $160\ km/hr$ and that the player pulls his hand back during the impact at an average speed of $10\ m/s$ over a distance of $150\ mm$, bringing the ball to a stop.

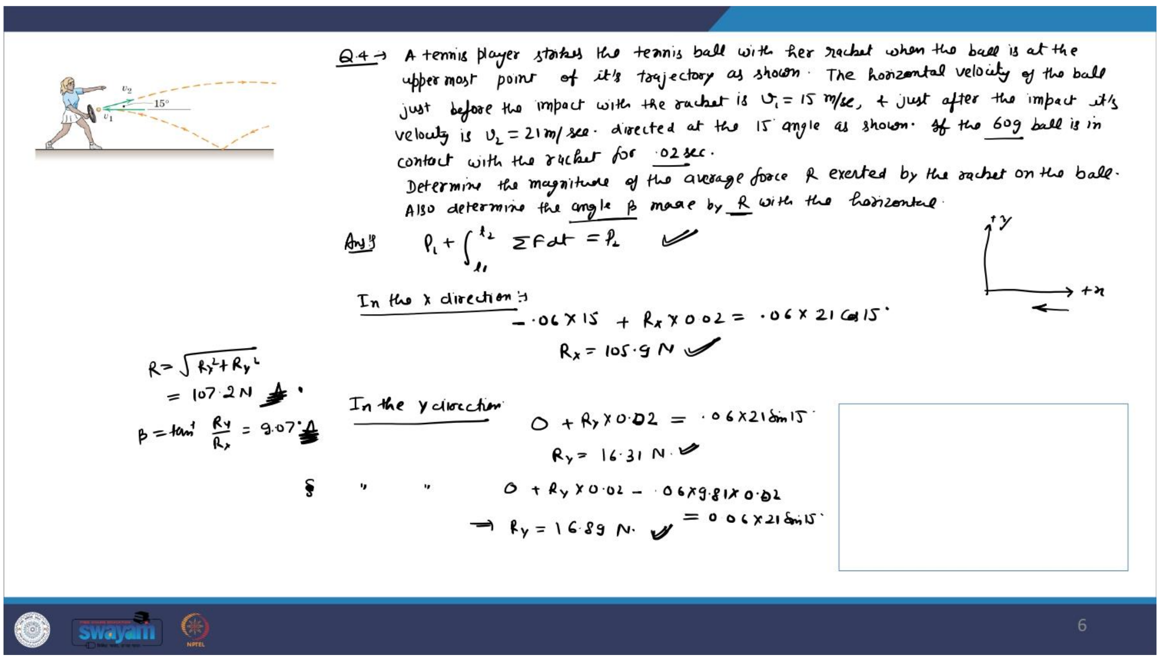
Determine the average impulsive force exerted on the player's hand. So, again to find out the impulsive force, we can use the impulse momentum relation.

So, $P_1 + \int_{t_1}^{t_2} Fdt = P_2 - - - - - - (1)$. Here, t_1 is given. It is mass into velocity. So, mass is $125\ g$. So, 0.125 into the velocity $160\ km/hr$. So, we can convert it into m/s. So, it is 160 into 1000 divided by 3600 and the direction of P_1, so, let's say this is the y-axis, this one is the x-axis, the ball is moving in $-x$ direction. Therefore, it will be negative. So, that is -5.555.

$P_1 = -0.125 \times 160 \times \frac{1000}{3600} = -5.555$. Now, the time t will be the distance which is given here $150\ mm$ divided by the velocity which is $10\ m/s$. So $t = \frac{0.150}{10} = 0.015\ s$.

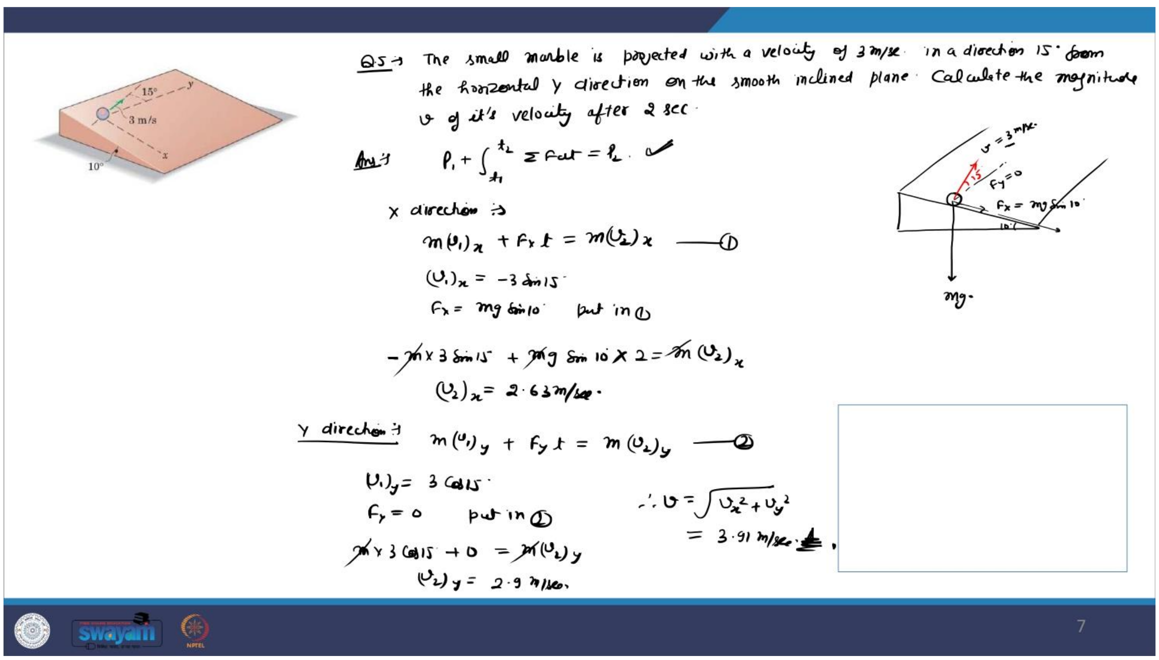
Now, the final momentum P_2 is given. It is 0 because the ball comes to the rest.

Let us put it in equation number 1. So, we $-5.555 + F \times 0.015 = 0$. And from here, I can find out what is F. So, that comes out to be $370.37\ N$.



Now, let us look at another problem on the same concept. A tennis player strikes the tennis ball with her racket when the ball is at the uppermost point of its trajectory as shown. The horizontal velocity of the ball just before the impact with the racket is $v_1 = 15\ m/s$ and just after the impact, its velocity is $v_2 = 31\ m/s$ directed at the $15°$ angle as shown. If the $60\ g$ ball is in contact with the racket for $0.02\ s$, then determine the magnitude of the average force R exerted by the racket on the ball. Also determine the angle β made by R with the horizontal. So, again we have to use the impulse momentum relation and for that let's say this is our $+x$-axis and this one is the $+y$-axis. So, we can use $P_1 + \int_{t_1}^{t_2} Fdt = P_2$. We can use this equation in the x direction. It is given that the mass of the ball is $60\ g$. So, therefore, $-0.06 \times 15 + R_x \times 0.02 = 0.06 \times 21 cos 15°$

and from here I can find out what is R_x. It is $105.9\ N$. Now, let us use this equation in the y direction. So, in the y direction, initially, there is no moment, so that is $0 + R_y \times 0.00.06 \times 21 sin 15°$ and from here, you can find out what is R_y. It comes out to be $16.31\ N$. Now, let us consider that the gravity also act in the y direction for that small time. In that case, in the y direction, we can write down the equation $0 + R_y \times 0.02 - 0.6 \times 9.81 \times 0.02 = 0.06 \times 21 sin 15°$. In that case, you see that R_y comes out to be $16.89\ N$. So, you can see that, you know, if we do not consider gravity even in that case, the difference is very small because the impulsive forces are acting for very small duration. Now, let us calculate the R. So, $R = \sqrt{R_x^2 + R_y^2}$. Let me take this as R_x and this as R_y. So, it comes out to be $107.2\ N$. And in the question, we have been asked to determine the angle β that R makes with the horizontal. So, $\beta = \tan^{-1} R_y / R_x$. Now, R_y and R_x is known. So, it comes out to be $9.07\ °$.



Now, let us look at one more problem. The small marble is projected with a velocity of $3\ m/s$ in a direction $15\ °$ from the horizontal y direction on the smooth inclined plane. Calculate the magnitude v of its velocity after $2\ s$. To find out the velocity of the marble, we can use the impulse momentum equation, which is $P_1 + \int_{t_1}^{t_2} F dt = P_2$. Let us look at the free body diagram of this. So, we have this inclination and we have this marble. Its weight is going to act downwards. There is no force along the y direction. So, $F_y = 0$ and the component of the mg will act along the x direction. So, since this angle is $10\ °$, therefore, $F_x = mg sin 10°$ and the ball is launched at an angle of $15°$. Now, let us use this equation both in the x and y direction. So, in the x direction, we have $m(v_1)_x + F_x t = m(v_2)_x - - - - - (1)$

Now, $(v_1)_x = -3sin15°$ because the component of this v along the x direction is $-vsin15°$ and v is given, it is 3 m/s. So, let us put this in equation number 1. So, we have $-m \times 3sin15° + mgsin10° \times 2 = m(v_2)_x$. So, m will get cancelled and we have $(v_2)_x = 2.63\ m/s$. Now, let us use this equation in the y direction. So, in the y direction, we have $m(v_1)_y + F_y t = m(v_2)_y - - - - - (2)$. Now, $(v_1)_y = 3cos15°$ because the component of 3 along the y direction. $F_y = 0$ and let us put this in equation number 2. So, we have $m \times 3cos15° + 0 = m(v_2)_y$, m will get cancelled and from here $(v_2)_y = 2.9\ m/s$. Therefore, total velocity of the ball of the marble will be $v = \sqrt{(v_x)^2 + (v_y)^2} = 3.91\ m/s$.

With this let me stop here. See you in the next class. Thank you.