

Principles of Industrial Engineering
Professor D K Dwivedi
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee
Lecture 50
Forecasting: Methods 3

Hello, I welcome you all in this presentation related with the subject Principles of Industrial Engineering and you know we are talking about the Forecasting Methods. We have talked about the Forecasting Methods like the Simple Moving Average or Weighted Moving Average, Naive Methods. In this presentation I will be talking about the different aspects related with the few forecasting methods.

(Refer Slide Time: 01:02)



And we will be starting with the like Exponential, Exponential Smoothing Method, and it comes with the, lot of variations also like Simple Exponential Smoothing Method, then Exponential Smoothing Method with the consideration of the trend if it is in, if it is present in the demand pattern.

So how to forecast when the trend component is present which is indicating as a function of time, how the demand is increasing or decreasing or it remains constant. Thereafter, we will be talking about the forecasting considering the seasonality factor.

So, considering the seasonality, how to do the forecasting and then we will be also try to talk about as per the availability of time, that is about the Forecasting Error, like how accurate a forecasting is and what are the methods to calculate the forecasting errors about that we will be talking about, and how good forecasting is being done and whether we should continue with the forecasting method which is being used.

There is a, an approach which is called Tracking Signal. So, how to calculate the tracking signal and determine if we should continue the use of the existing or the current forecasting method or we should go for something else.

(Refer Slide Time: 03:17)

$$F_t = \alpha A_{t-1} + (1-\alpha) F_{t-1}$$
 Forecast (F_{t+1})
 Actual demand A_{t+1}
 Smoothing Coeff (α)

$$F_{t+1} = F_t + \alpha (A_t - F_t)$$

0 - 1	down
0.1 - 0.3	stable

$$F_{t+1} = F_t + \alpha (A_t - F_t)$$

$$F_{t+1} = \alpha A_t + (1-\alpha) F_t$$

IF 800008 APJ ONLINE CERTIFICATION COURSE

Exponential Smoothing

New forecast = Last period's forecast +
 α (Last period's actual demand - Last period's forecast)

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1}) \quad F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1}$$

Where F_t = new forecast

F_{t-1} = previous forecast

α = smoothing (or weighting)
 constant ($0 \leq \alpha \leq 1$)

Forecast current (F_{t-1}) = 100 units
 Actual current demand (A_{t-1}) = 120 units
 Smoothing constant (α) = 0.20



Handwritten derivation of the exponential smoothing formula:

$$F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1}$$

$$= \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 F_{t-2}$$

$$= \alpha A_{t-1} + \alpha(1-\alpha)A_{t-2} + \alpha(1-\alpha)^2 A_{t-3} + \alpha(1-\alpha)^3 F_{t-3}$$

For $\alpha = 0.2$:

$$= 0.2 + 0.2(1-0.2) + 0.2(1-0.2)^2 + \dots$$

$$= 0.2 + 0.16 + 0.128 + \dots$$

The final result shown is $0.2 + 0.16 = 0.36$.

So starting with the exponential smoothing method, in this case, it considers basically the two parameters, one is the, the most recent forecast and the most recent actual demand. So, forecast say, most recent forecast is like say F_{t-1} and most recent actual demand like say A_{t-1} . So one, we are using the forecast, like say, for February forecast if we have the data about the forecast of January, forecast of January and the actual demand of the January, then using these we can do the forecasting for February.

So, the most recent forecast and most recent actual demand data is used. Apart from these two parameters, it also considers the smoothing coefficient α which is represented by alpha. Alpha

value usually ranges from 0 to 1, most common value is used like say 0.1 to 0.3, however, we can use a higher values or the lower values, also depending upon the situation.

So, if we see, that the average of the demand is almost stable, average demand is stable, then we use the low alpha value and if the demand is going to change, is demand is, average demand is changing, then the higher value of the alpha is preferred.

So let us say, here, the new forecast means the last period's forecast into alpha into the last period's actual demand minus the last period's forecast. So, that is how we get a very simple equation for the next period, that is the last period's forecast F_{t-1} is used as it is plus the difference of the forecast and actual demand of the last period. So, A_{t-1} and F_{t-1} .

So the difference of the two A_{t-1} , the most recent actual demand and F_{t-1} , most recent forecast. Difference of these two will give us the next period's forecast. This can be written alternatively also if we just simplify this F_{t-1} plus alpha into A_{t-1} minus alpha F_{t-1} . So here, since we have the two F_t parameters, like, so we can rearrange this equation like, alpha into A_{t-1} alpha, that is the smoothing coefficient and the actual demand of the most recent period plus 1 minus alpha F_{t-1} .

So this is how it can be arranged, rewritten in arranged manner, so basically the newer way of writing becomes it like F_t that is the forecast for the next period will be alpha into A_{t-1} , that is the most recent actual demand plus 1 minus alpha F_{t-1} . So, this is the case when we are considering the actual demand and the forecast of the just one most recent period.

We are not considering in this case when this particular equation is being used, we in this case, we are considering just one period data and maximum weightage is being given to this data. If we see, if we want to use the data of the previous periods then the weightage given to the demand of the further, previous periods will be exponentially reducing.

Like here, F_t into alpha A_{t-1} plus 1 minus alpha F_{t-1} , if this is one equation, if we want to consider the demand of the two previous periods, then equation will be little bit modified and how it will be written, alpha A_{t-1} plus alpha into 1 minus alpha A_{t-2} plus alpha into 1 minus alpha, so this is the case when the data of the two previous periods is being used and then here 1 minus alpha F_{t-2} .

This is the case when the actual demand of the previous two periods A_{t-1} and A_{t-2} and the forecast of the A_{t-1} , F_{t-2} period is being used. If we want to, this is the case when the, the two period's demand and the forecast data is used for forecasting.

If you want to consider the three periods that demand and forecast data, then in that case it will be further modified like, $\alpha A_{t-1} + (1-\alpha)A_{t-2}$, so this is the second previous period actual demand plus $(1-\alpha)$ times A_{t-3} , A_{t-3} , then plus $(1-\alpha)^2 F_{t-3}$.

So this is the forecast of the third previous period, demand of the third actual, actual demand of third period, actual demand of second period and actual demand of the first period. So if we see the weight, weightage being given to the actual demand of the previous periods and the, to the forecast data is reducing exponentially.

This is about the, this is about the most recent period, this is about the second most recent period, this is about the third most recent period and this is the consideration of the forecast data of the third period. So if we see, let us assume the value of the 0.2 then how much weightage we are actually given?

Here α is equal to 0.2 and the second, if you are using the two period's data then the consideration is of $(1-\alpha)$ into $(1-\alpha)$. And for third period, it is α into $(1-\alpha)^2$. So if we just put the value of α then what we are getting? 0.2 into $(1-0.2)$, 0.2 into $(1-0.2)^2$. So 0.2, here 1 and then what we will have, 0.2 into 0.8 that will be 0.16 and here, 0.2 into 0.8 square.

So here, this will be coming equal to a 0.128. So if we see the kind of weight being given to third period data, is just 0.128, for second period data, 0.16, and for first, most recent period data is 0.2. So the weightage being assigned, weightage being given in this method to the previous period data of actual demand and the forecast is reducing exponentially. And that is why this method is called exponentially smoothing method.

In simple smoothing method, we consider just one period demand and forecast data that is corresponding to this one. Like $F_t = \alpha A_{t-1} + (1-\alpha)F_{t-1}$, that is most period, most recent

period actual demand and the F_{t-1} is the most recent period forecast and alpha is the coefficient.

So here, that is how we can get the details of how to calculate and how to consider the actual demand and the forecast data for calculating the forecast of the next period using the past data. The interesting thing is that in this method actually it requires very less data, so the record keeping of the actual demand and the forecast data of the previous periods is not much needed. Maybe most recent data of one period be also be okay to a to get the forecast of the next period.

(Refer Slide Time: 13:51)

Exponential Smoothing

New forecast = $\frac{\text{Last period's forecast} + \alpha (\text{Last period's actual demand} - \text{Last period's forecast})}{1}$

$$F_t = F_{t-1} + \alpha (A_{t-1} - F_{t-1})$$

$$F_t = \alpha A_{t-1} + (1 - \alpha) F_{t-1}$$

Where F_t = new forecast
 F_{t-1} = previous forecast
 α = smoothing (or weighting) constant ($0 \leq \alpha \leq 1$)

Forecast current (F_{t-1}) = 100 units
 Actual current demand (A_{t-1}) = 120 units
 Smoothing constant (α) = 0.20

High α when average is likely to change
 Low α when underlying average is stable

Handwritten calculation:
 $100 + 0.2(120 - 100)$
 $= 100 + 0.2 \times 20$
 $= 104$

So here, we will go, this is what is there in F_t , next period's data and F_{t-1} is the most recent period, previous forecast and smoothing coefficient alpha. So just one example, this is the rewritten form, alpha into A_{t-1} plus $(1 - \alpha) F_{t-1}$. So, if we want to, just for an example like say if this is the data then the forecast for the next period considering this situation like alpha assuming 0.2 plus A_t .

So alpha into actual demand is 120, so 0.2 into 120 plus 1 minus 0.8, 0.2 into F_{t-1} , F_{t-1} that is the forecast, forecast is 100. So this is one way to calculate, another simple way was the right, like if you put the data in this equation also, so F_t is 100, F_{t-1} is a 100 and alpha is 0.2, and the actual demand is 120 and the forecast for the most recent period F_{t-1}

is 100, so that is how it becomes very simple into 20, so it becomes 4, 104 will be the next period's data.

That is how we can calculate, so when we have a high alpha value is used when the average of the demand period average of average demand is going to change and the low value is used when the underlying, the average is almost stable.

(Refer Slide Time: 15:42)

Example $\alpha = 0.3$

Period	Demand	Forecast	$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$
Jan	100	120 ✓	
Feb	110	114	$120 + 0.3(100-120) = 120 - 6 = 114$
Mar	120	113	$114 + 0.3(110-114) = 114 - 1.2 = 112.8 \sim 113$
April	118	115	$113 + 0.3(120-113) = 113 + 2.1 = 115.1 \sim 115$
May	121	116	$115 + 0.3(118-115) = 115 + 0.9 = 115.9 \sim 116$
June			

When we, so if we see, this is another example when we are considering the many data for the previous periods also, like say, for the January data, for the January, the actual demand is 100 and the forecast is 120. So, the 120, 0.3 if assuming that alpha is 0.3 so alpha, so 100 minus 120, so this will be giving us 120.6, so here 120, so this is like in this particular case, forecast is more and the demand is less.

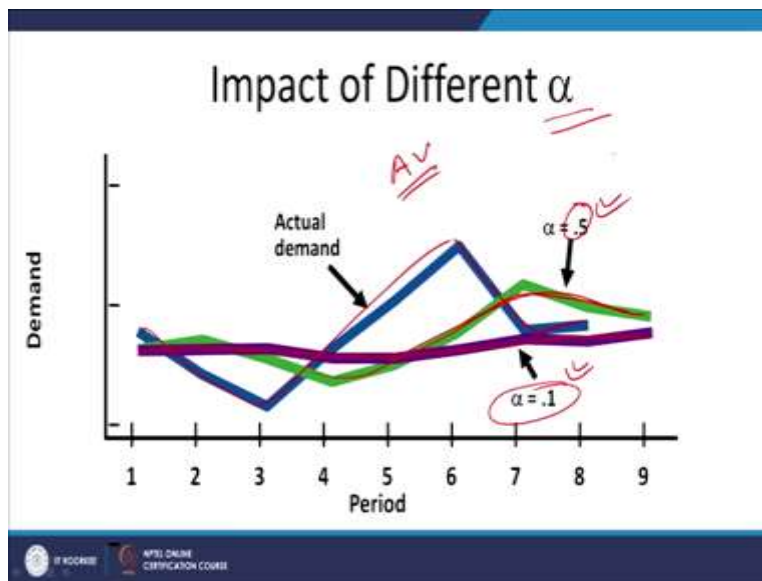
So here, the next period's demand will be 120 minus 6, so the next period's, the demand will be 114, so this demand for the next period is 114, this becomes the forecast for the February. So this is what has been calculated based on the January data, we got 114, so the forecast for February was 114 and if the actual data is 110, so the forecast 114, 0.3 and now actual data is 110, so 110 and 114 difference of this will be giving us 113.

Similarly, now the 113 becomes the forecast for the March and if the actual demand is 120, so the forecast for the March is 113 plus point 3 is the smoothing coefficient, so actual demand 120, 120 minus 113. So 113 plus (point) 2.1, this will be giving us the, the forecast of 115.

So forecast for 115, this is the forecast for the next period that is April, so if you have got April forecast 115 and actual demand is 118, this can be used to calculate the demand of the May, the forecast of the May. So forecast of May can be calculated using this one, the forecast of the most recent period April 115.3, so 118 minus 115, this will be giving us 0.3 into 3, 0.9 plus 115.

So this will be giving us the forecast for May, forecast for May becomes 116. So this is how we can keep on calculating the forecast for the next period. So forecast for the next period, similarly can be, so this is how we can continue to do, to determine the forecast of the next period using the exponential smoothing method.

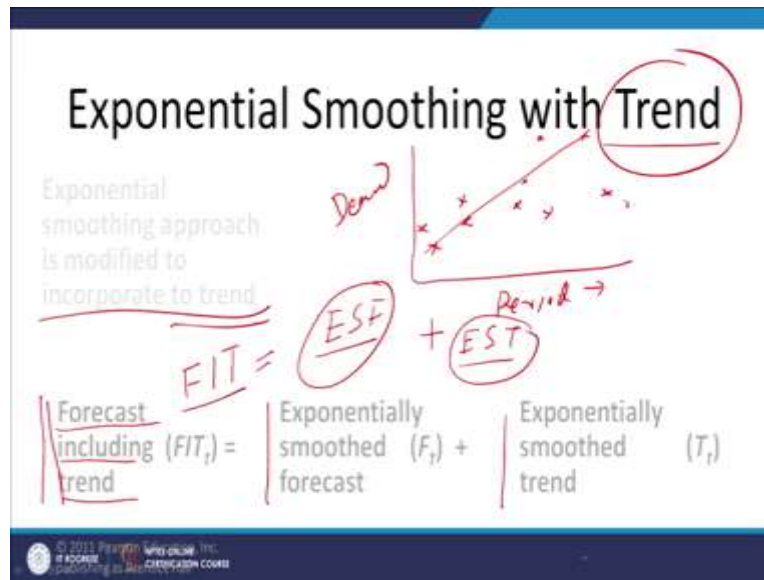
(Refer Slide Time: 18:58)



Now what this diagram shows, what happens when the different alpha values are used? Say, this is the actual demand variation as a function of time and if we see, the forecast becomes, shows the trend similar to that of the demand when the higher value of the alpha is used and while the forecast remains almost flat, when the lower alpha value is used and it, it incorporated less variation in the actual demand.

So the, the variations which are taking place in actual demand are less incorporated when the lower alpha value is used and it reflects in somewhat better way in forecast when the higher value of the alpha is used. So when the average is shifting during the forecast, actual demand average is shifting significantly so it is, it is more appropriate to use a higher alpha value so that it reflects the fluctuating demand variation in the forecast also.

(Refer Slide Time: 20:13)



Then we have the second aspect that is about application of the exponential smoothing with the trend. Trend we know, in case of the trend, there is a, like say, as a function of the period, maybe quarter, month or years. The demand may show a some kind of the trend, means the demand is a kind of a change is consistent either increasing or in decreasing way.

So to incorporate this trend, trend means the particular pattern of a change in the demand as a function of a, a period we need to consider the trend. So, because if it is like simply static, then using the moving average or simple moving average, weighted moving average or exponentially smoothing, it can be done in somewhat better way.

But if there is a trend component, then this trend component must be incorporated where data is consistently increasing as a function of period or time or it is decreasing. Now, we will see how to incorporate the trend in the forecasting. So for that, exponential smoothing approach is

modified to incorporate the trend component, so here, what it considers, the forecast including trend, forecast including trend.

It considers the two things, the one is very simple like what we have done earlier, exponentially smoothed forecast, exponentially smoothed forecast and plus exponentially smooth trend. So the exponentially smooth trend is calculated separately and exponentially smooth forecast is calculated separately and then their algebraic sum is used to determine the forecast considering the trend.

So how to incorporate, how to determine the exponentially smooth forecast and how to determine the exponentially smooth trend?

(Refer Slide Time: 23:04)

The slide displays the following equations and handwritten notes:

$$F_t = \alpha(A_{t-1}) + (1 - \alpha)(F_{t-1} + T_{t-1})$$
$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

Handwritten notes include:

- FIT
- $\alpha = 0.2$
- $\beta = 0.3 / 0.4$

Basically, this uses the two equations, one is similar to what we have done earlier like the exponentially smooth forecast considers the actual demand of the most recent data, the forecast of the most recent period, actual demand of the most recent period, forecast of the most recent period and as a trend data of the most recent period.

Whether it, the how it is increasing or decreasing. Then what we do, the exponentially smooth trend is calculated, exponentially smooth trend is calculated using the forecast of the current period, forecast of the most recent period and the trend of the most recent period. So these are the, the data (value), these are the parameters which are used.

Apart from this, if we see, it also uses the two different means this approach, like the forecast including or considering the trend, considers the two coefficients, one is exponential smoothing coefficient for forecast and another is beta, that is exponential smoothing coefficient for the trend. Alpha and beta, say alpha, we may use like 0.2 or 0.3 and beta value, depending upon the trend, maybe use 0.3 or 0.4. So suitably, the value of the alpha and beta are selected.

(Refer Slide Time: 25:12)


Period	Actual Demand A_t	Smoothed Forecast F_t	Smooth Trend T	Forecast including trend
Jan	18	16	2	
Feb	18			

$\alpha = 0.2$
 $\beta = 0.4$

(i) $F_t = \alpha A_t + (1-\alpha)F_{t-1} + T_{t-1}$
 $= 0.2 \times 18 + (1-0.2)(16+2)$
 $= 3.6 + 0.8 \times 18$
 $= 3.6 + 14.4$
 $= 18$

(ii) Trend $= \beta (F_2 - F_1) + (1-\beta)T_1$
 $= 0.4(18-16) + (1-0.4)2$
 $= 0.8 + 1.2 = 2$

(iii) $FIT = F_2 + T_2 = 18 + 2 = 20$



Let us say here, if we have for a certain periods, like say, January, February, like this. So, the actual, what we have, the actual demand A_t and smoothed forecast F_t , a smooth trend T , then the forecast including trend. So if we have some data for the previous one, say 18 is the actual demand, and the smoothed forecast is 16, smooth trend is 2, then what we can determine considering this, considering this we can determine the forecast including the trend.

How it can be done, for this we need to use the equation first, smooth forecast, exponentially smooth forecast that is the first step. What it considers alpha A_t that is actual demand plus 1 minus alpha F_t minus the T_1 that is the trend that we have. And we will be putting these values in equation alpha say, alpha is 0.2, beta is 0.4. So 0.2 into 18 plus 1 minus 0.2 into F_t , F_t exponentially smooth forecast for January was 16 minus 2, trend is 2.

So, here we can calculate this, 0.2 means 3.6 plus, it will be 0.8 into 14. So here, it will be 3.6 plus, so here this is the 16 plus 8, this is plus. So here, now it will be 18 and it will become 14.4.

So here total data will be 18. So exponentially smoothed forecast now will be 18. That will be for the next period with the trend.

So exponentially smoothed forecast here it is 10, this is the first step, basically then the second step is about the trend. Exponentially smoothed trend is to be determined and the trend will be calculated from the beta into $F_2 - F_1 + 1 - \beta$ into T_1 . So here, what we have, 0.4 into $18 - 16 + 1 - 0.4$ into 2 that is the trend. So here what we get basically the value of $0.8 + 1.2$ so it becomes 2 .

So finally, the third step here is to calculate the FIT, means forecast including the trend becomes the F_2 , now the exponentially smooth forecast plus the T_2 . So here we have got the exponentially smooth forecast 18 and the exponentially smooth trend was 2 , so here $18 + 2$, it becomes 20 . So the 20 becomes the exponentially, 20 becomes the exponentially smoothed forecast including the trend becomes the 20 .

So these are the 3 steps, basically, we have to calculate the exponentially smoothed forecast, exponentially smoothed trend and then calculation of the forecast considering the exponentially smoothed forecast including the trend. So the forecast plus the trend, these two gives us the forecast including the trend.

Now I will summarize this presentation. In this presentation, basically, I have talked about the exponential smoothing method and the calculation of the forecast considering the trend. Thank you for your attention.