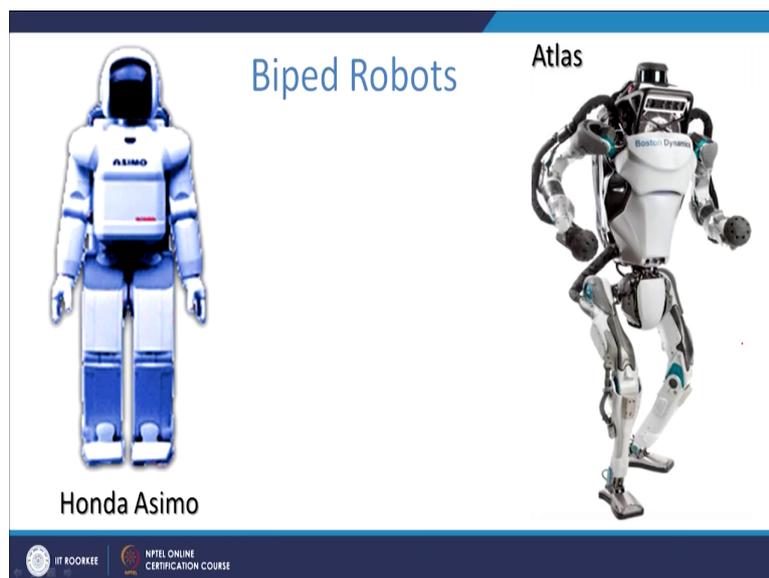


Robotics and Control: Theory and Practice
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Lecture – 17
Biped Robot Basics and Flat Foot Biped Model

In this lecture, we shall see some basics on biped robots. We shall also see some results on the kinematics and trajectory planning for stable work of a Biped Robot with Flat Foot. So, a robot which can move from one place to another using two legs is called a biped robot as shown in this picture.

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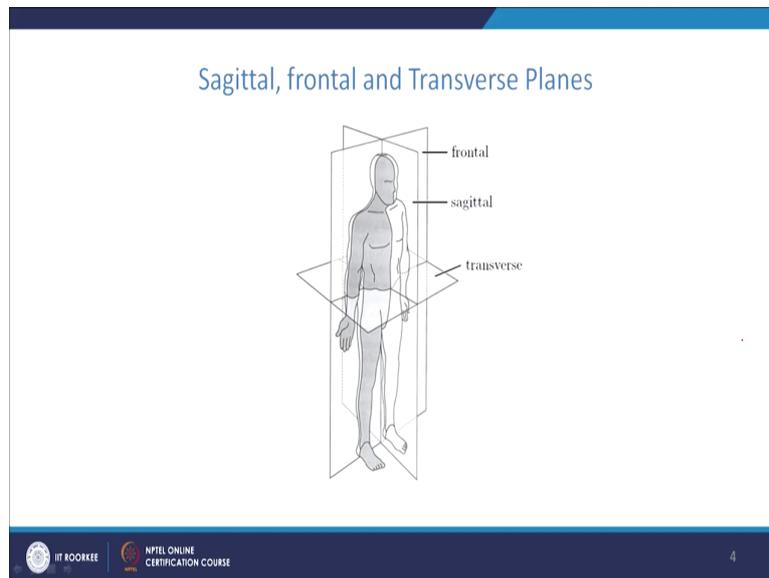
Introduction

- Locomotion may be defined as the ability of a body to move from one place to another place.
- Naturally locomotion may be flying of birds, swimming of fish, walking of human beings etc.
- In case of complex environments like staircase walking, rough terrain or sloppy surface, most appropriate medium for locomotion are legs.
- Legs can avoid discontinuities in environment either by neglecting them or stepping over such obstacles.
- A subclass of legged robots are **Biped robots**.
- A biped may be defined as an open kinematic chain which consist of two subchains termed as legs and may also have a sub chain called torso, each one is connected at a common point termed as hip.
- To replace humans in hazardous environment like mining, nuclear plant activities, military related work, biped robots can be very beneficial.

The ability of a body to move from one place to another is called locomotion. So, locomotion can be performed by flying, swimming, scrawling, rolling or walking. For moving on the ground; wheeled robot, wheeled mobile robots or legged robots are very useful.

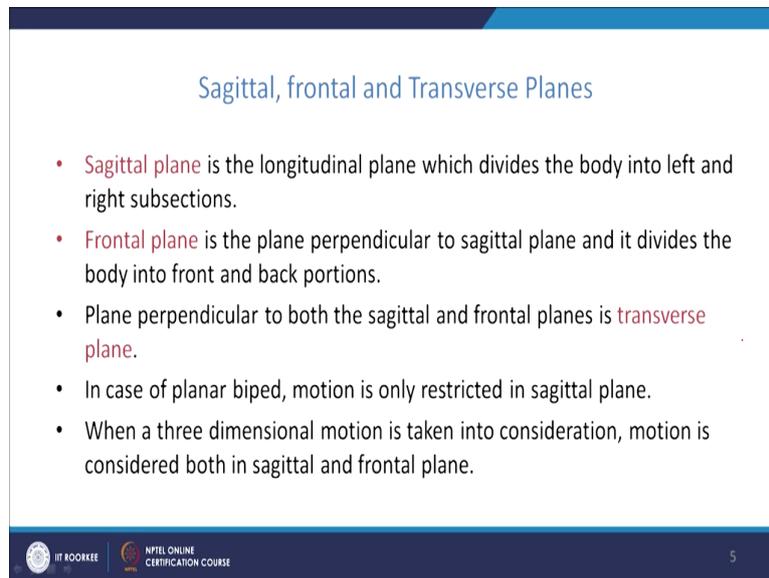
Wheeled robots are suitable for a terrain which is very smooth; whereas, the legged robots are useful when the moving, the ground on which the robot has to move is a rough one or a staircase or a surface with discontinuities like ditches or wholes etcetera. There are several types of legged robots; four legged or six legged robots. So, they move like animals or insects, but two legged robots or the biped robot can walk like a human being.

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So, now we shall see some terminologies used in the biped robot literature; the sagittal plane, frontal plane and transverse plane. So, they are depicted in this picture. The sagittal plane is a plane which divides the body into two parts like the left side and the right side of the body. The frontal plane divides the body into two parts like the front side and the back side of the body. And the transverse plane is perpendicular to both these planes. So, it divides the body into upper portion and the lower portion.

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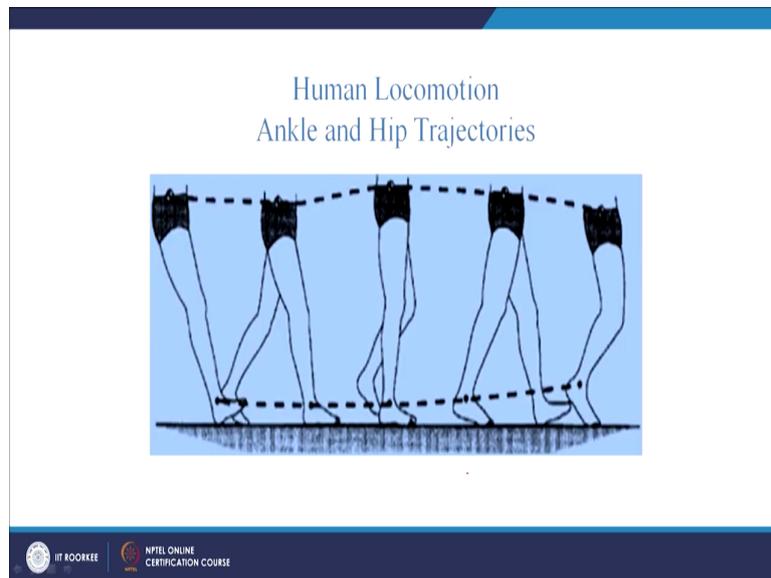
Sagittal, frontal and Transverse Planes

- **Sagittal plane** is the longitudinal plane which divides the body into left and right subsections.
- **Frontal plane** is the plane perpendicular to sagittal plane and it divides the body into front and back portions.
- Plane perpendicular to both the sagittal and frontal planes is **transverse plane**.
- In case of planar biped, motion is only restricted in sagittal plane.
- When a three dimensional motion is taken into consideration, motion is considered both in sagittal and frontal plane.

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So, when we consider a biped robot walking on a plane, planar walking; so the walking is considered only in the sagittal plane and a three dimensional motion of a biped can be considered in the sagittal plane as well as the frontal plane. The motion of the biped if it is in both sagittal and the frontal plane; then it will give you a three dimensional motion, it looks like a natural human body motion.

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So, here the picture depicts the human locomotion. The ankle and hip trajectories are shown here in the dots. So, in this lecture we shall see some procedure for generating the kinematics and the trajectory planning of the ankle and hip trajectories of certain model of robot manipulators.

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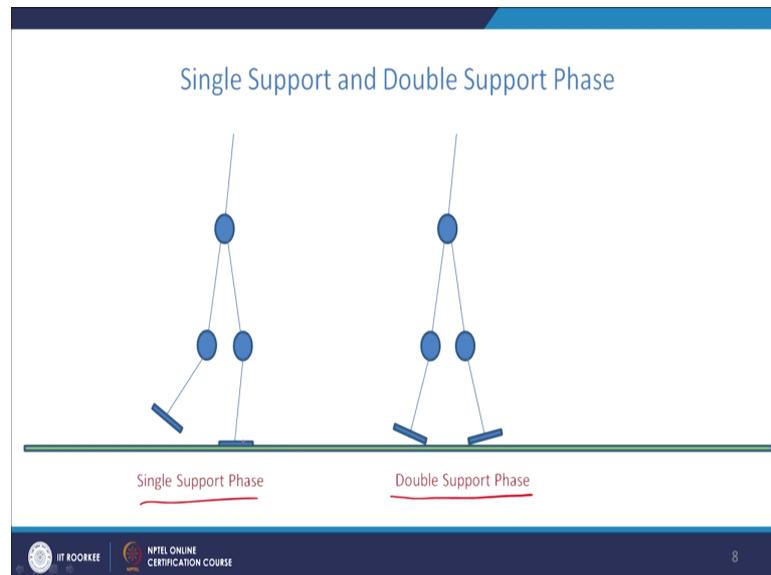
Single Support and Double Support Phase

- At a particular instant one or both the legs can be in contact with ground surface.
- The case when one leg is in contact with the ground surface, the contact leg is known as **stable leg** and other leg is called **swing leg**. SSP
- End of the leg is referred as foot.
- A single cycle of walking can be divided into two phases:
(i) Single Support Phase(SSP) (ii) Double Support Phase(DSP)
- **Single support** or swing phase is that phase of biped locomotion when there is only one foot is in contact with the ground.
- On the other hand when both feet are in contact with the ground, that phase is known as **double support** phase.
- So, alternating single and double support phase results in **walking** with a condition that the horizontal component of biped robot's center of mass (COM) have strictly monotonic displacement (For example in dancing the horizontal components moves forward and backward).

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So, here the other terminology is the single support phase and the double support phase.

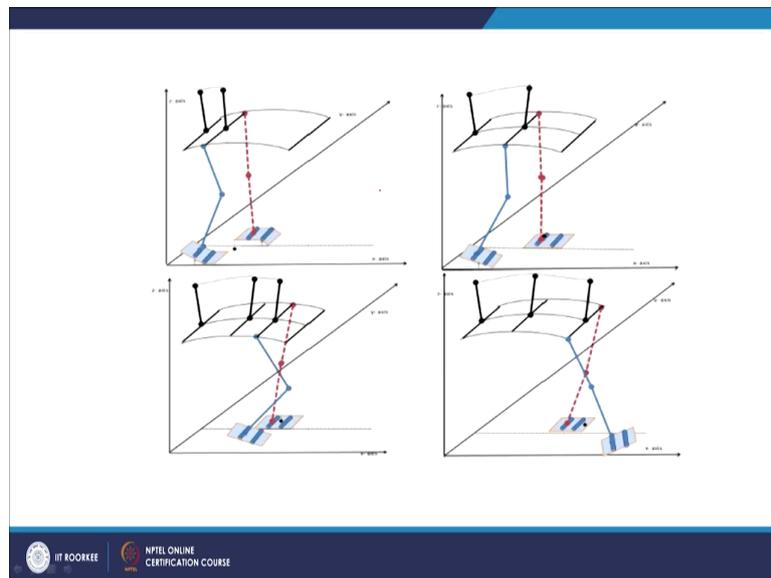
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So, while walking there are two stages. Both the legs are on the ground in the contact of the walking surface; or one leg is on the surface and another leg is moving from the moving forward in the air.

So, the double support phase is one where both the legs are in contact with the ground and the single support phase is one leg is on the ground, another leg is moving in the air. So, the stable foot is the stable leg is the one which is stable on the ground during the single support phase. Single support phase, we consider the stable leg and the swing leg.

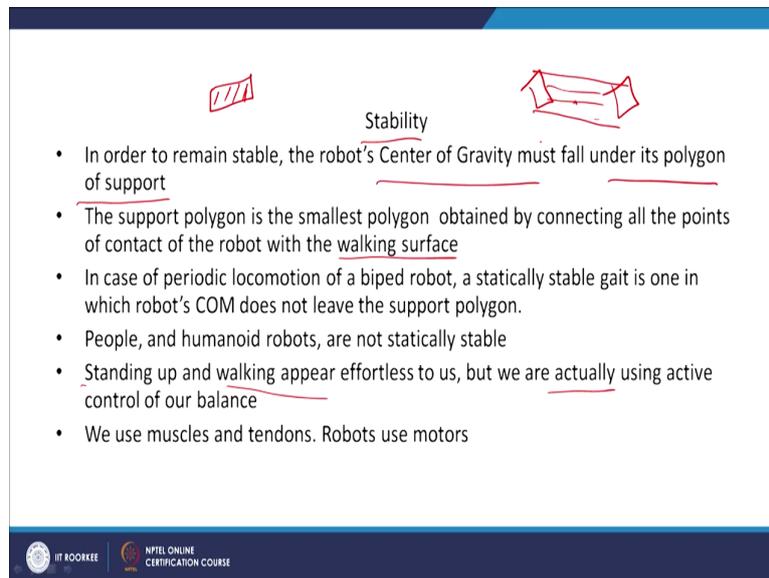
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So, this picture depicts the various stages of motion. So, while walking the three dimensional motion where the motion is in the sagittal plane as well as in the frontal plane. So, here the foot of the robot is moving in the sagittal plane; the upper body moves in the frontal plane in this direction. So, the first position is shown here; after certain instant of time the position of the upper body is shown here and again the third position is shown here.

So, it shows that the upper body is moving swinging from one side to the other during the walking. While the swing leg is taking its position from one position to the other position like this; so the end position is given here, the upper body motion at various stages are shown in this picture.

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The slide is titled "Stability" and features two hand-drawn diagrams in red ink. The left diagram shows a simple quadrilateral representing a support polygon. The right diagram shows a more complex, elongated shape representing a support polygon for a biped robot in a double support phase. Below the diagrams is a bulleted list of points regarding stability and support polygons.

Stability

- In order to remain stable, the robot's Center of Gravity must fall under its polygon of support
- The support polygon is the smallest polygon obtained by connecting all the points of contact of the robot with the walking surface
- In case of periodic locomotion of a biped robot, a statically stable gait is one in which robot's COM does not leave the support polygon.
- People, and humanoid robots, are not statically stable
- Standing up and walking appear effortless to us, but we are actually using active control of our balance
- We use muscles and tendons. Robots use motors

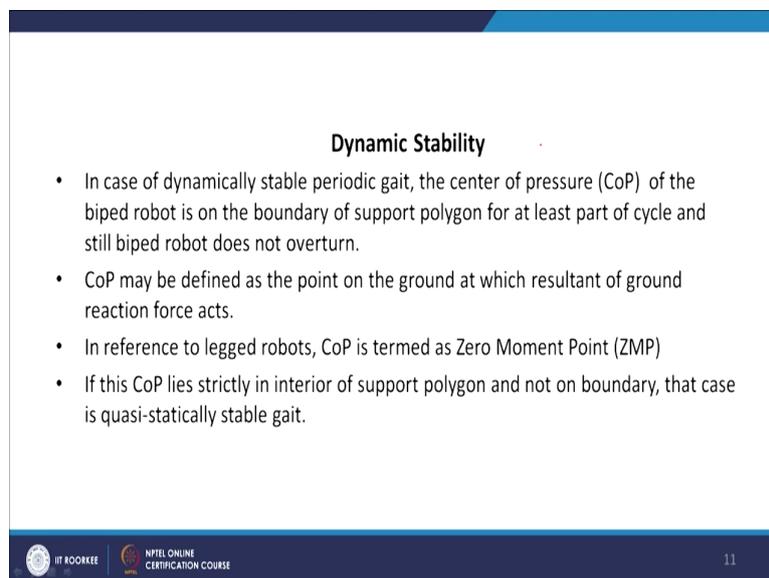
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In order to remain stable, the robots center of gravity should lie on the support region; the polygon of support. So, the polygon of support is nothing, but the smallest polygon that can be constructed by connecting the points of contact of the robot with the surface.

So, for example, in the double support phase both the foot are touching the ground. So, the polygon which can be constructed by connecting all the support point is shown by this picture; whereas in the single support phase only one foot is on the ground, another foot is on the air it is moving. So, the support polygon is only below the foot which is on the contact of the ground. So, the support polygon is very important for considering the stability of motion of the biped robot. So, the center of gravity should lie within the support region during walking whether it is in double support or the single support phase.

So, normally the standing without motion or walking; so this for a human being it appears to be very obvious or very simple task. But even for standing at a position without falling we need to control ourselves, so that the center of mass lies on the within the polygon of support. So, for doing that we should constantly control ourselves, so that the person do not fall down. So, the same thing should be done for a biped robot manipulator. So, the control should be devised for whether for standing or walking in a particular manner.

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Dynamic Stability

- In case of dynamically stable periodic gait, the center of pressure (CoP) of the biped robot is on the boundary of support polygon for at least part of cycle and still biped robot does not overturn.
- CoP may be defined as the point on the ground at which resultant of ground reaction force acts.
- In reference to legged robots, CoP is termed as Zero Moment Point (ZMP)
- If this CoP lies strictly in interior of support polygon and not on boundary, that case is quasi-statically stable gait.

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So, the dynamic stability is the one in which the center of pressure lies in the support polygon; within the support polygon or in the boundary of the support polygon. So, when the center of pressure; the center of pressure is defined as the point on the ground at which the resultant of the ground reaction forces acts.

So, the ground reaction forces acts wherever there is a contact point with the leg and the ground; the ground reaction force act in the opposite direction. So, the resultant of all such forces acts at a particular point; that point is called the point of contact, the center of pressure. The point at which the resultant force acts is called the center of pressure; so the it is also termed as the zero moment point. So, when the zero moment point lies within the polygon of support, the stability of the biped robot is ensured.

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Zero Moment Point

- Zero Moment Point (ZMP) may be defined as that point on the surface of the ground about which resultant sum of moments of all forces which are active is zero.
- ZMP can be calculated by following equations:

$$x_{ZMP} = \frac{\sum_{i=1}^n m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^n m_i \ddot{x}_i z_i - \sum_{i=1}^n l_{iy} \ddot{\Omega}_{iy}}{\sum_{i=1}^n m_i(\ddot{z}_i + g)}$$

$$y_{ZMP} = \frac{\sum_{i=1}^n m_i(\ddot{z}_i + g)y_i - \sum_{i=1}^n m_i \ddot{y}_i z_i - \sum_{i=1}^n l_{ix} \ddot{\Omega}_{ix}}{\sum_{i=1}^n m_i(\ddot{z}_i + g)}$$

x_i
 (x_i, y_i, z_i)
 is the com of the ith link
 x_i, x_i

where m_i denotes mass of link i , respective inertial components are denoted by l_{ix} and l_{iy} , absolute angular accelerations are denoted by $\ddot{\Omega}_{ix}$ and $\ddot{\Omega}_{iy}$, g denoted the acceleration due to gravity, $(x_{ZMP}, y_{ZMP}, 0)$ denotes coordinates for zero moment point and (x_i, y_i, z_i) denotes coordinates for center of mass of link i .



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So, the contact the center of pressure or the ZMP, the zero moment point is computed or calculated using this expression. The ZMP is considered to be a point on the surface of walking. So, here the x coordinate of these ZMP is calculated using this expression; in which the double derivative the x double dot, y double dot, z double dot denotes the acceleration of the center of mass of the i th link.

For example, \ddot{x} ; it represents the acceleration of the center of mass in the x direction. So, when we substitute those values in this expression, we get the x coordinate and y coordinate of the zero moment point of the robot. So, it is nothing, but the resultant of the sum of all active forces is zero. The point at which the resultant of the sum of all moments of the active forces becomes zero is called the zero moment point. So, when the zero moment point lies within the support region, the stability of the robot is ensured.

So, the aim of finding a control is to keep the zero moment point within the support region. So, whenever we design a trajectory; the trajectory should ensure that the zero moment point lies within the support region.

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Convex Set and Convex Hull

- A subset S of R^n will be defined as convex set if $\alpha p_1 + (1 - \alpha)p_2 \in S$ is satisfied for any point $p_1, p_2 \in S$ and $\alpha, 0 \leq \alpha \leq 1$. In other words, if a segment formed by connecting any two points in S is also included in S , then the set will be a convex set.

Convex Set

Non-Convex Set

- The minimum convex set including S is defined as Convex Hull.

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So, let us consider the definition of the convex set. Convex set is any set in which if p_1 and p_2 are two points; then the line joining that point should also lie within that particular set. So,

if p_1 and p_2 are in S ; then $\alpha p_1 + (1 - \alpha)p_2$ also should belong to S . So, this represents the line segment between the two points p_1 and p_2 .

So, if the entire line lies that is the convex set. So, for example, this one is not a convex set; because the p_1 and p_2 , the line segment goes outside the given set. So, the support polygon when we construct it is always a convex set.

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Supported region
The convex hull of all floor contact points is called supported region/polygon.
ZMP should lie within the supported polygon for stable walk.

Stability margin

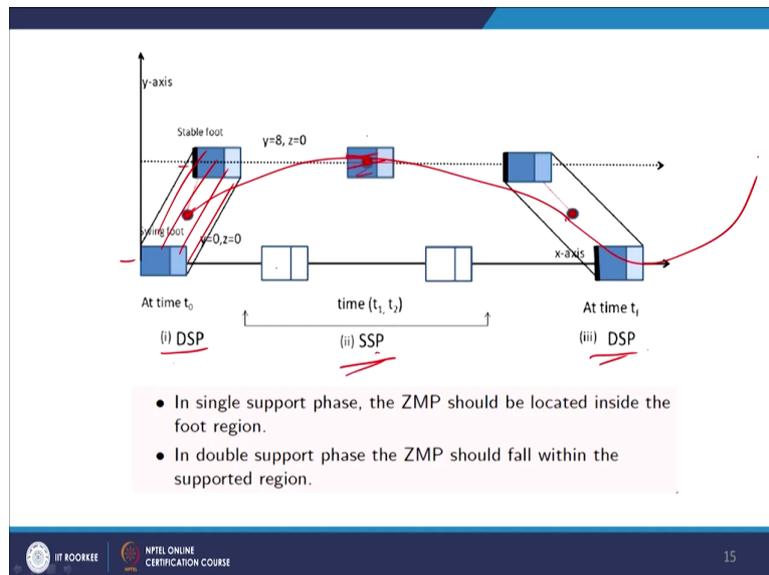
- Stability margin was defined for a given support polygon as the smallest of the distances from the ZMP to the edges of the support polygon.
- The walk is stable if ZMP lies strictly inside supported region with larger stability margin.

The diagram shows a sequence of foot positions during a gait cycle. It labels 'Stable foot' and 'Swing foot'. It identifies three phases: 'Double Support Phase for toe feet', 'Double Support Phase for flat feet', and 'Single Support Phase for flat or toe feet'. The feet are represented by colored rectangles (blue and red) on a white surface.

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And it is required for the stability of, for analyzing the stability of a biped robot. So, for these stable walk, the ZMP should lie within the support polygon.

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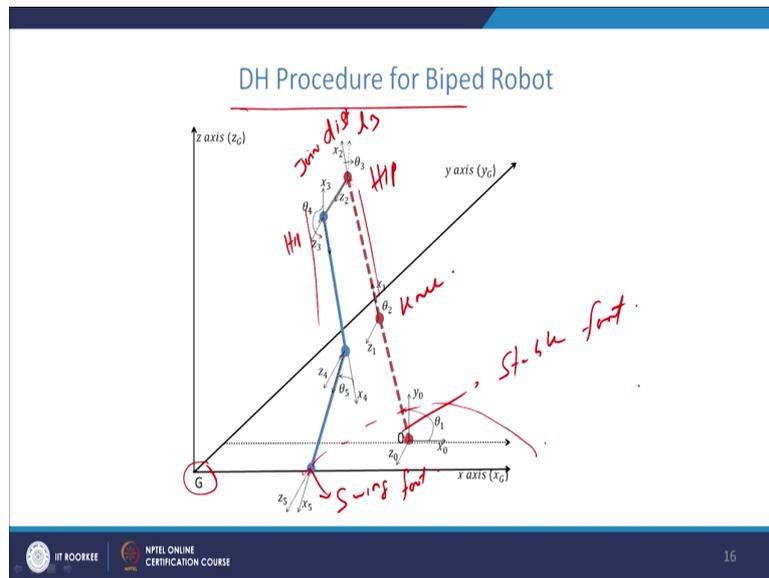
So, this picture depicts that during single support and double support phase, where the ZMP should lie. So, here in the double support phase; both the feet to the first and second feet are in contact with the ground. So, the support polygon is given by this region and the if the ZMP lies within this region; then the stability is ensured.

So, while the single support phase where one leg is on the ground, another leg is moving on the air. So, then the ZMP is expected to lie within the foot here. Similarly, after coming to the double support phase again, the ZMP should move in this direction.

So, it is expected that the ZMP while walking, it should follow a path as shown here in this particular manner; while the leg is moving from double support to single support and again to the double support phase. So, this should be repeated for further walking. So, this picture

shows only one step of the walking and for more steps the legs, the ZMP should be having a trajectory as shown in this particular picture.

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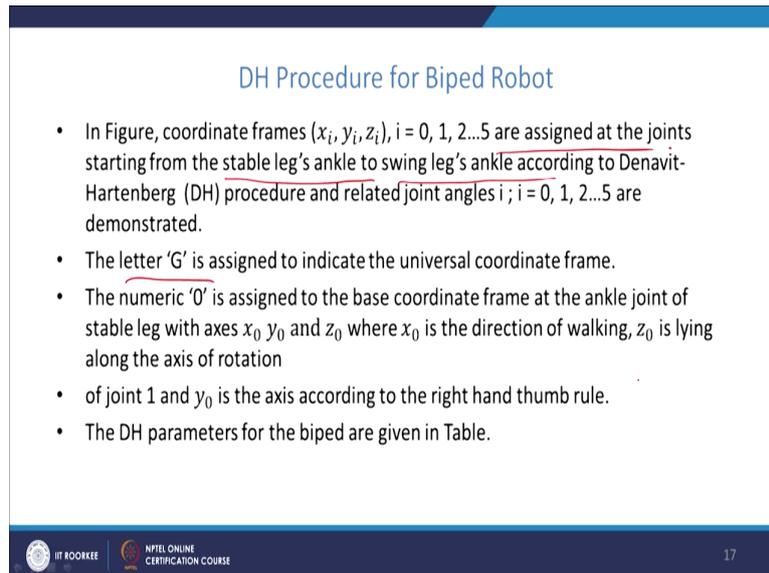


So, now we will see before going to the kinematics and the trajectory planning of the flat foot robot. We will see how to write the kinematic equation using the DH procedure. So, the DH algorithm we have already seen how to fix the coordinate frames at each joint. So, in this picture a robot with 5 degrees of freedom is shown here. So, the angle x_0 , y_0 , z_0 is shown as the base.

This is the stable foot and this is the swing foot. So, z_0 is the axis of rotation at the ankle of these stable foot; z_1 is the axis of rotation at the knee joint of the stable foot; and the hip joint z_3 is the axis of z_2 is the axis of rotation; and z_3 is the axis of rotation at the second hip. And z_4 is the axis of rotation in the knee of the swing leg.

So, these are the five rotation axis of rotations shown here; all of them are parallel. During the entire walking, the axis z_0, z_1, z_2, z_3, z_4 all of them maintain a parallel direction during the walking.

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The slide is titled "DH Procedure for Biped Robot" and contains a bulleted list of five points. The first point discusses coordinate frames (x_i, y_i, z_i) for $i = 0, 1, 2, \dots, 5$ assigned at joints, starting from the stable leg's ankle to the swing leg's ankle, following the Denavit-Hartenberg (DH) procedure. The second point states that the letter 'G' indicates the universal coordinate frame. The third point explains that the numeric '0' is assigned to the base coordinate frame at the ankle joint of the stable leg, with axes x_0, y_0 and z_0 , where x_0 is the direction of walking and z_0 is along the axis of rotation. The fourth point notes that for joint 1, y_0 is the axis according to the right-hand thumb rule. The fifth point states that the DH parameters for the biped are given in a table.

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So, it is now easy to fix the x direction and y axis at each joint. And then, find the coordinate homogeneous transformations relating one joint to the another joint. So, here it shows the x_i, y_i, z_i ; i equal to 0, 1, 2, 3, 5 are assigned at the joints starting from the stable legs ankle to the swing legs ankle. So, the stable leg ankle is considered as the base and the swing legs ankle is considered as the end effector of the robot manipulator. So, the steps of the DH procedure can be considered.

Here G denotes the universal coordinate frame. So, here the fixed frame is considered as G and in this x, y, z, G frame, the biped robot is walking, and the coordinates of each joint is measured with respect to the G frame.

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DH Parameter Table

Link	Joint angle(θ_i)	Twist angle(α_i)	Link length(a_i)	Joint length(d_i)
1	θ_1	0	l_1	0
2	θ_2	0	l_2	0
3	θ_3	0	0	l_0
4	θ_4	0	l_2	0
5	θ_5	0	l_1	0


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So, now, we can assign the coordinate frames and the parameters of the joints as given in this particular table. So, the joint angles are theta 1 to theta 5; because all the joints are revolute joints. And by following the DH procedure we can easily see the twist angles are all 0; because all the z axis are parallel to each other, so the all the twist angles are 0.

And the link lengths are given by l_1 , l_2 and the fourth one is of the same length. So, it is l_2 and l_1 ; because the both the legs have the same measurement. Here the length is l_1 , here also l_1 and l_2 is this; the link lengths are given, and the joint length is given by this expression l_1

3. That can be seen from here the, this particular thing is the joint distance l 3 according to the DH algorithm.

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Homogeneous Coordinate Transformation Matrices A_i^{i-1}

$$A_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4^3 = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 & C\theta_4 \\ S\theta_4 & C\theta_4 & 0 & S\theta_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = \begin{bmatrix} C\theta_5 & -S\theta_5 & 0 & C\theta_5 \\ S\theta_5 & C\theta_5 & 0 & S\theta_5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

*A₅⁰ = Arm of the biped.
A_iⁱ⁻¹ = matrix.*

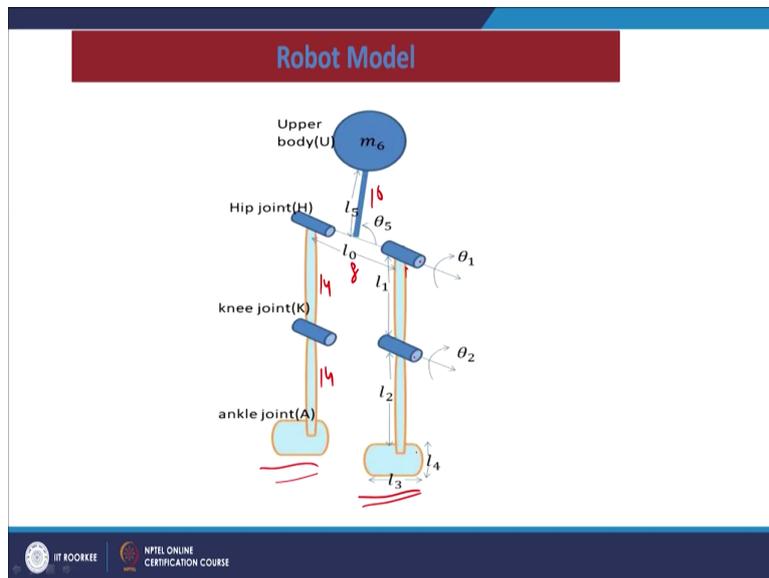


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So, after constructing this parametric table, we can construct the homogeneous transformation relating the A_i^{i-1} . That is the i th frame with respect to the $(i-1)$ th frame as shown in this slide. So, this gives a relation. Finally, when we multiply all of this, we can get to the relation between the zeroth frame with respect to the A_5 with respect to zeroth frame. So, this is the arm matrix of the biped.

Considering the biped as a robot manipulator with the five degrees of freedom; we can get the arm matrix with respect to the base, the end of vector with respect to the base. So, during the walking of the robot manipulator, we can consider the kinematics equation using the arm matrix. So, now, we will see a particular example of a biped robot with flat foot.

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So, here we consider the biped with two legs and with four degrees of freedom. So, here the angle of rotation is at the knee joint and the hip joint for one leg. Similarly, at knee joint and hip joint for the second leg. The foot is considered to be a flat one. So, it is slightly different from the previous model where a rotation is considered at the ankle joint also. So, if you remove the rotation at ankle joint. So, the model which we are considering now is with the flat foot.

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Parameters				
Link	Length	Value	Mass	Value
HK	l_1	14 inches	m_1	4kg
KA	l_2	14 inches	m_2	4kg
HU	l_5	10 inches	m_6	50kg
HH	l_0	8 inches	m_5	4kg

Table 3.1: Parameters



So, now, we consider the biped model with the flat foot as given in this picture. And the parameters of the biped robot is given here; the length of the various joints, various links are given here. HK, this length HK is given to be 14 and KA is given to be 14; the length of this one. And HU is considered to be 10 inches, and this is 8. So, here it is 8 and then here it is 10 units of length.

Similarly, the mass of each link is also given as m_1 , m_2 and m_6 and m_5 . And it is considered that the two legs are identical; whether the length and masses are identical to each other. So, using this particular model, we will construct a trajectory for stable walking of the biped robot.

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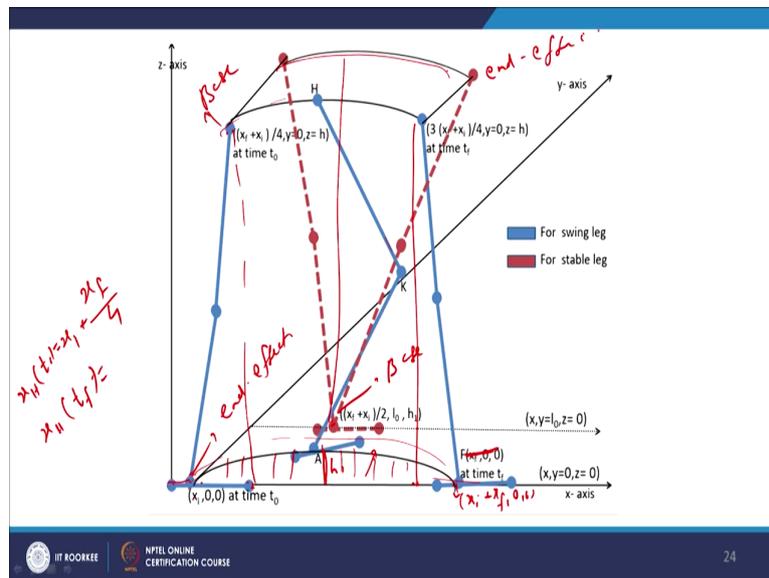
- In Figure, each leg of biped robot have 2 degrees of freedom (DOF) with flat foot.
- All the joints are revolute which are called hip joint (H), knee joint (K) and ankle joint (A).
- Centre of mass of upper body is denoted by (U).
- Robot's walk can be considered as a repetition of one-step motion.
- The walking sequence can be determined by computing the trajectory of the hip, ankle and upper body joints.
- For hip trajectory, stable ankle joint is considered as a base and hip as the end effector.
- For biped robot walking on a plane, motion of the stable leg is assumed to be like an inverted pendulum considering it's ankle joint as base and hip as end effector.
- While walking, the biped do not fold the stable leg . And the whole body weight is shifted on the stable leg..
- Flat foot is attached at the ankle joint of each leg.
- Let the robot walk in sagittal plane (xz-plane).

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So, we consider that each leg has two degrees of freedom. It is considered a two arm manipulator; each, each leg is considered as a two arm manipulator separately and all the joints are revolute. So, the hip joint is considered as H, the knee joint is K and the ankle joint is A as shown in the picture.

So, during the walking; the hip traject, for generating the hip trajectory. We consider the stable ankle joint is the base and the hip joint as the end effector of the robot manipulator.

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In this picture we can see that the stable leg which is shown in the red color; the ankle is considered as the base and the hip joint is considered as the end effector. Similarly, for the swing leg which is in the blue color; the hip is considered as the base and the ankle is the end effector. And they are considered as a two arm manipulator and their kinematics and inverse kinematics can be calculated very easily as we have already seen.

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Swing leg's trajectories:

Boundary Conditions of Ankle Trajectory

$$x_A(t_0) = x_i; x_A(t_f) = x_i + x_f; \dot{x}_A(t_0) = 0; \dot{x}_A(t_f) = 0.$$

$$z_A(x_0) = 0; z_A(x_f) = 0; z_A(x_m) = h_1; \dot{z}_A(x_m) = 0.$$

Ankle Trajectory

$$x_A(t) = x_i + \left(\frac{3x_f}{t_f^2}\right) t^2 - \left(\frac{2x_f}{t_f^3}\right) t^3; \quad x_A(t) = h_1 \cdot h_1 \cdot t^2 + h_1 \cdot t$$

$$z_A(t) = \frac{h(-x_f + x_i)^2 x_i}{(x_m - x_i)(x_m - x_f - x_i)^2} + \frac{h(x_f + x_i)(x_f + 3x_i)x_A(t)}{(x_m - x_i)(x_m - x_f - x_i)^2} - \frac{h(2x_f + 3x_i)x_A(t)^2 + hx_A(t)^3}{(x_m - x_i)(x_m - x_f - x_i)^2}$$

$z_A(x) = \text{for } \theta, x_f, x_i \text{ and } h_1$

So, now let us consider the trajectory of the swing leg. Swing leg has two coordinates that is X at X coordinate and the Z coordinate. The boundary conditions for the ankle trajectory for the swing leg we consider it to be like this. At time initial time t 0, the x coordinate of the ankle is at X i and at the final time t f, the X coordinate of the ankle is at X i plus x f.

So, as shown in this picture this is the point X i is the X coordinate and X i plus X f is the X coordinate at this position; this is X i plus X f 0 0, I think this is the value. And the velocity at the initial time is 0 and the velocity at the final time is 0 in the X direction. And the Z coordinate of the ankle is it depends on the X coordinate. So, it is a curve which z which has a relation between z and X here in the Z X plane.

So, X Z A at initial position of X 0 is 0 and Z A ankle Z coordinate of the ankle at final position is 0 again. Z A at the middle position is given by h 1; that is at the middle position

the height is h_1 . And the velocity at the middle position of the Z component is 0; because the Z component is slowly increasing and the maximum value is h_1 , then it should start decreasing Z value. So, the derivative is equal to 0 at that point the middle point X_m .

So, using this boundary condition we can define a polynomial of $X_A t$. We can write it as $\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$. Now, by putting all this four boundary condition, we will get a trajectory like this. Similarly, Z_A of x can be considered as $\beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$.

Now, by substituting this four boundary condition in terms of X , we will get Z_A in terms of X ; whereas, X is in terms of t ; here X_A . So, by substituting X_A in terms of t in this expression, we will get Z_A in terms of t as given in this picture in this particular expression.

So, this gives for all time in instance the position of X_A and Z_A . So, we can get the entire trajectory as a function of time. So, we get the x and z coordinates of the swing leg trajectory as given in this expression.

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Stable leg's trajectories:

Boundary Conditions of Hip Trajectory

$$x_H(t_0) = x_i + x_f/4; \quad x_H(t_f) = x_i + 3x_f/4; \quad \dot{x}_H(t_0) = v_s; \quad \dot{x}_H(t_f) = v_e.$$

$$z_H(t_0) = h; \quad z_H(t_f) = h; \quad \dot{z}_H(t_0) = v_{zs}; \quad \dot{z}_H(t_f) = v_{ze}.$$

Hip Trajectory

$$x_H(t) = \frac{x_f}{4} + v_s t + \left(\frac{v_e - v_s}{2t_f} - r_4 \frac{3t_f}{2} \right) t^2 - 2 \left(\frac{x_f}{2t_f^2} - \frac{v_s + v_e}{2t_f^2} \right) t^3.$$

$$z_H(t) = \sqrt{(h + h)^2 - (x_H(t) - (x_i + x_f/2))^2}.$$

where $r_4 = -2 \left(\frac{x_f}{2t_f^2} - \frac{v_s + v_e}{2t_f^2} \right)$

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Similarly, we can get this stable legs trajectory as given in this two expression. The boundary, initial and the boundary condition is given by this one. At time, initial time t_0 , the hip X coordinate is given by this expression; because we can see from this picture the hip is at this point, the x component is at one fourth of the total length. That is the X_f is the foot length, total foot length by 4 is this length and then x_i is the initial distance from here; so X_i plus X_f by 4 that is the hip initial position.

Similarly, hip final position is given by this one. So, with X_f final position is given by this particular point; where X_f by 4 should be subtracted from here, so as given in this expression that is X_i plus 3 by 4th of X_f . Similarly, the velocity at the initial point is V_s and the velocity of the hip at the final point is V_e ; that is starting and end velocities.

And in the similar manner, the z coordinate of the hip trajectory is given by these boundary conditions. We can observe that the hip will move in the circular path with the center at this position and radius to be like this. So, it is simply a circular path. So, the equation of a circle is given here, the radius is $l_1 + l_2$; the total length of the leg is the radius and the center is at this particular position. So, the X and Z are related by the circle with radius $l_1 + l_2$ and center at this position, it gives the trajectory of the hip; the swing trajectory.

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Forward Kinematics

For Swing leg

$$\begin{aligned} x_A(t) - x_H(t) &= l_1 \cos \theta_1(t) + l_2 \cos(\theta_1(t) + \theta_2(t)); \\ z_A(t) - z_H(t) &= l_1 \sin \theta_1(t) + l_2 \sin(\theta_1(t) + \theta_2(t)); \end{aligned}$$

where $(x_A(t), z_A(t))$ and $(x_H(t), z_H(t))$ are defined as earlier.

For stable leg

$$\begin{aligned} x_H(t) - \left(x_i + \frac{x_f}{2}\right) &= (l_1 + l_2) \cos \theta_5(t); \\ z_H(t) &= (l_1 + l_2) \sin \theta_5(t); \end{aligned}$$

where $(x_i + x_f/2, l_0, 0)$ is the position of the stable leg's ankle joint which lies on the line $y=l_0$.


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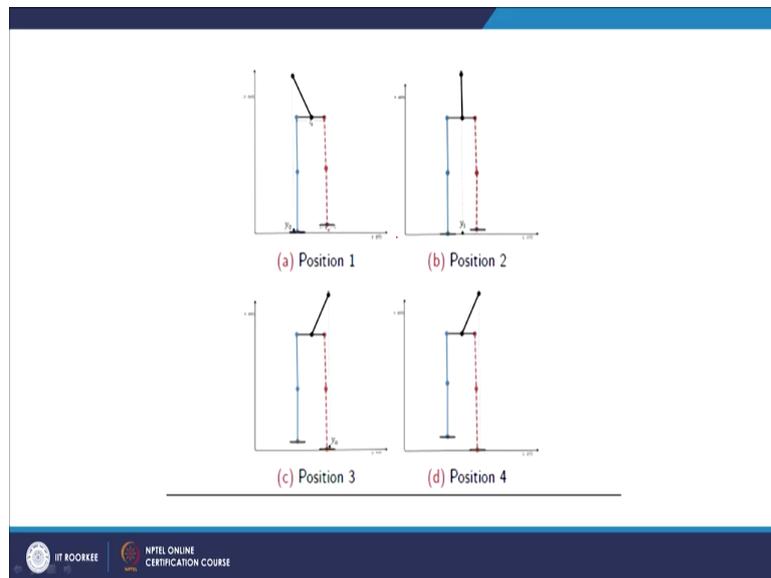
So, now we can write the swing legs equation in terms of the angle. Whatever is given in this particular slide, in the previous slide are only the position of the end effector. Now for achieving the end effector position, the angles theta 1 and theta 2 should be adjusted. So, we write the forward kinematic equation of the two arm manipulator equation. So, here for the swing leg the hip is the base and X the ankle is the end effector. So, as the hip is moving

constantly, we will get the coordinate $X_A t$ minus $X_H t$ given by the formula $l_1 \cos \theta_1$ plus $l_2 \cos \theta_1$ plus θ_2 .

And similarly, the Z component of the end effector that is the ankle Z component and with respect to the base that is Z_H of t is given by this formula, so this can be easily verified. Because as time changes the base also keeps on moving; the X_H and Z_H is moving. So, for finding the $X_A t$ and $Z_A t$ in the moving frame, we have to use this particular expression. Similarly, the hip joint X_H of t of the stable leg, it is moving in the form of a circle as given here.

Up to this we have seen that how to generate the trajectory of the swing leg and this stable leg in terms of the angles. Now, if you solve the inverse kinematics of these two arm manipulators; suitably we can get how the angles should be calculated for achieving a particular trajectory as given by all these boundary conditions. But we have to verify whether these trajectories are stable or unstable. So, for doing that we should adjust the upper body motion suitably, so that the ZMP lies in the support polygon.

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So, this portion we will see in the next lecture when we consider other types of robot manipulators.

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Upper Body Motion

Case-1

- Upper body start to move from middle to the side of the stable leg's hip during time t_0 to t_1 ,
- Stay there during time t_1 to t_2 .
- Again start moving towards middle of legs between t_2 to t_f time where $t_1 = t_f/4$ and $t_2 = 3t_f/4$.

The moving mass trajectory in y-direction is given below: $y_M(t) =$

$$\begin{cases}
 y_i + y_v t + \left(\frac{3y_a - y_i}{t_f^2} - \frac{2y_v}{t_f} \right) t^2 + \left(\frac{2(y_a - y_i)}{t_f^3} - \frac{y_v}{t_f} \right) t^3 & t_0 \leq t \leq t_1 \\
 y_a & t_1 \leq t \leq t_2 \\
 \left(y_a + \frac{(-3t_f t + t_f^2)(y_i - y_a)}{(t_f - t)^2} + \frac{t_f y_v}{(t_f - t)} \right) \\
 \left(\frac{8t_f(y_i - y_a)}{(t_f - t)^3} - \frac{(5 + 2t_f t)y_v}{(t_f - t)^2} \right) t + \left(\frac{-3(y_i - y_a)(t_f + t)}{(t_f - t)^3} \right. \\
 \left. + \frac{y_v(4t_f + 2t)}{2(t_f - t)^2} + \left(\frac{2(y_i - y_a)}{(t_f - t)^2} - \frac{y_v}{(t_f - t)} \right) t \right) t^2 & t_2 \leq t \leq t_f
 \end{cases}$$




So, the upper body motion is very important to make the robot stable, so that the ZMP lies within the support region. So, this particular point we will see in the next lecture.

Thank you.