Robotics and Control: Theory and Practice
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Lecture – 17
Biped Robot Basics and Flat Foot Biped Model

In this lecture, we shall see some basics on biped robots. We shall also see some results on the kinematics and trajectory planning for stable work of a Biped Robot with Flat Foot. So, a robot which can move from one place to another using two legs is called a biped robot as shown in this picture.

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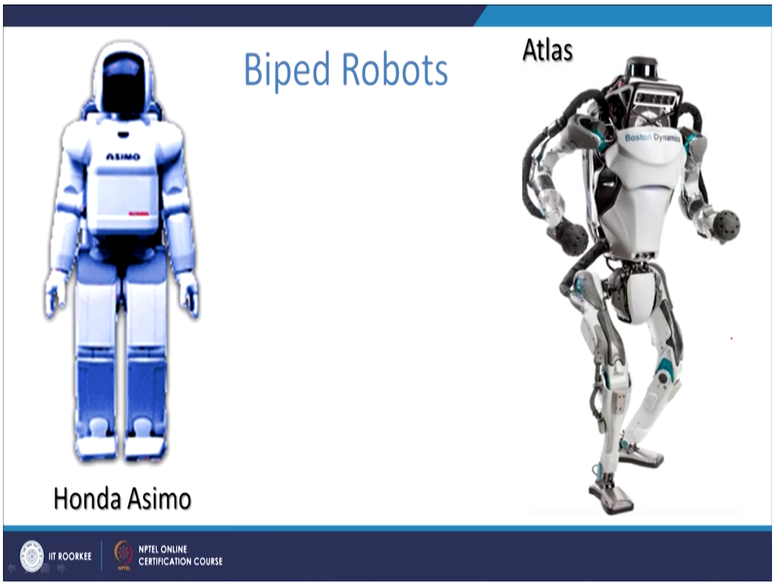

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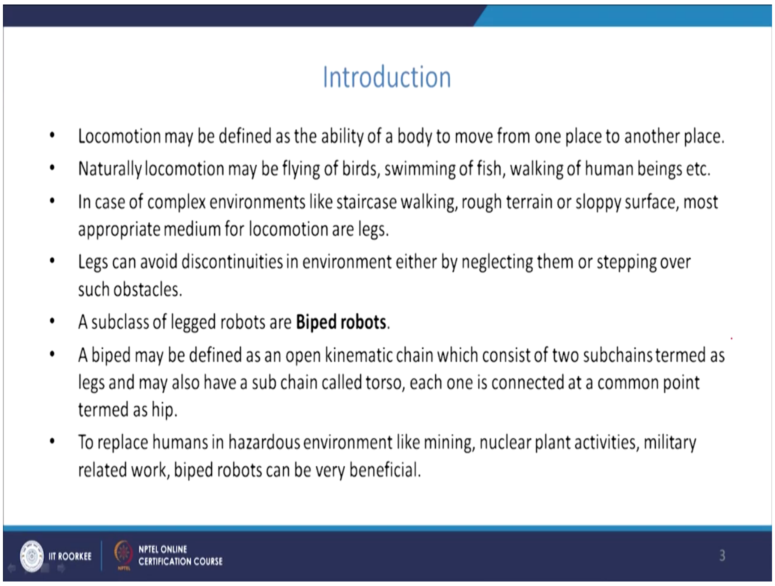

The ability of a body to move from one place to another is called locomotion. So, locomotion can be performed by flying, swimming, scrawling, rolling or walking. For moving on the ground; wheeled robot, wheeled mobile robots or legged robots are very useful.

Wheeled robots are suitable for a terrain which is very smooth; whereas, the legged robots are useful when the moving, the ground on which the robot has to move is a rough one or a staircase or a surface with discontinuities like ditches or wholes etcetera. There are several types of legged robots; four legged or six legged robots. So, they move like animals or insects, but two legged robots or the biped robot can walk like a human being.

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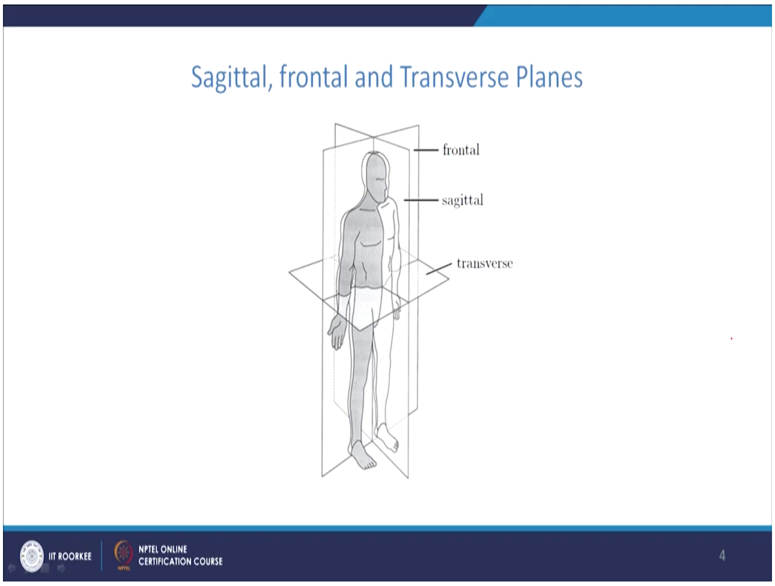

So, now we shall see some terminologies used in the biped robot literature; the sagittal plane, frontal plane and transverse plane. So, they are depicted in this picture. The sagittal plane is a plane which divides the body into two parts like the left side and the right side of the body. The frontal plane divides the body into two parts like the front side and the back side of the body. And the transverse plane is perpendicular to both these planes. So, it divides the body into upper portion and the lower portion.

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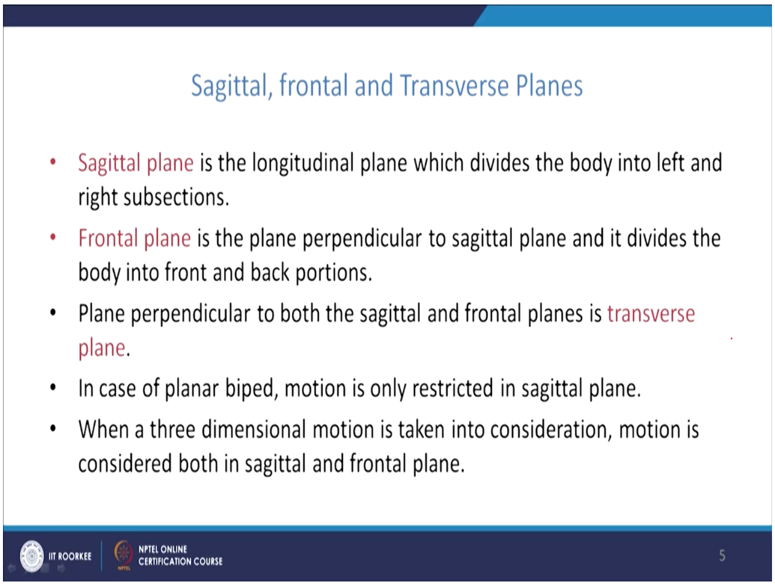

So, when we consider a biped robot walking on a plane, planar walking; so the walking is considered only in the sagittal plane and a three dimensional motion of a biped can be considered in the sagittal plane as well as the frontal plane. The motion of the biped if it is in both sagittal and the frontal plane; then it will give you a three dimensional motion, it looks like a natural human body motion.

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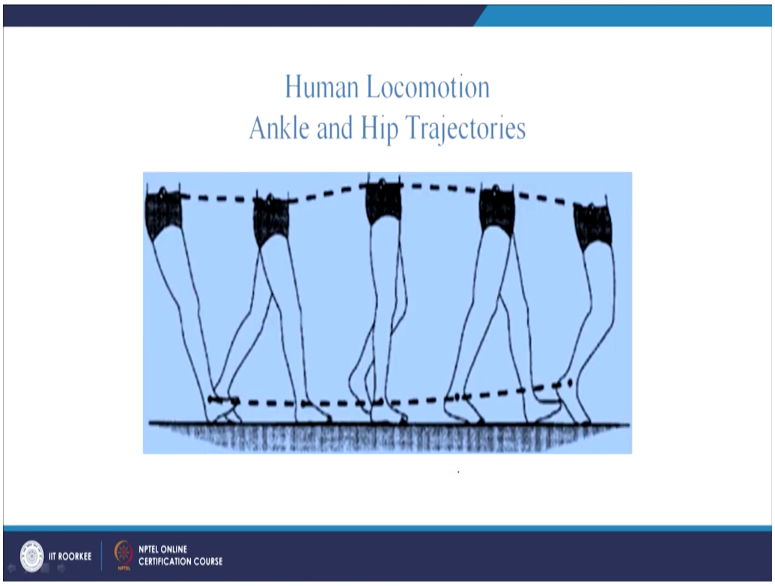

So, here the picture depicts the human locomotion. The ankle and hip trajectories are shown here in the dots. So, in this lecture we shall see some procedure for generating the kinematics and the trajectory planning of the ankle and hip trajectories of certain model of robot manipulators.

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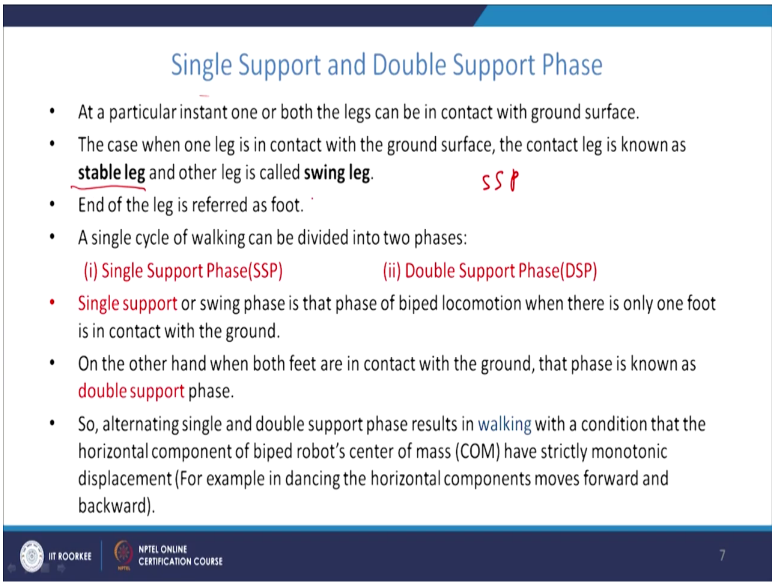

So, here the other terminology is the single support phase and the double support phase.

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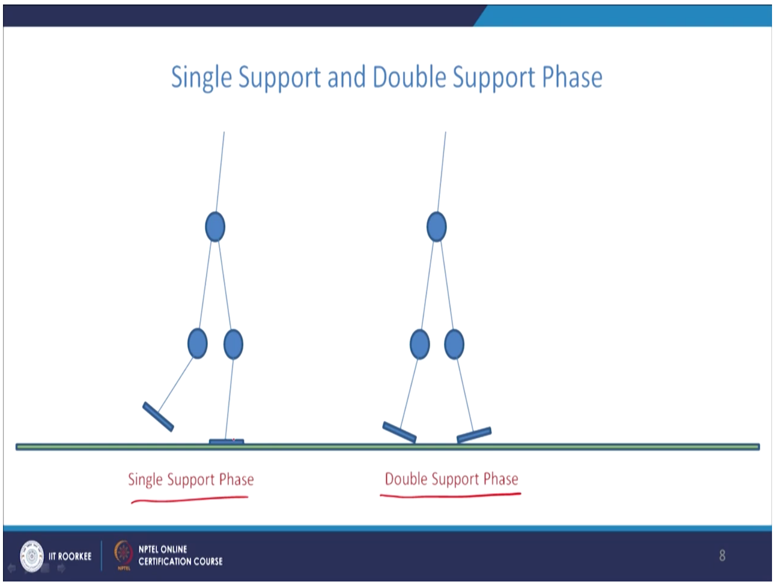

So, while walking there are two stages. Both the legs are on the ground in the contact of the walking surface; or one leg is on the surface and another leg is moving from the moving forward in the air.

So, the double support phase is one where both the legs are in contact with the ground and the single support phase is one leg is on the ground, another leg is moving in the air. So, the stable foot is the stable leg is the one which is stable on the ground during the single support phase. Single support phase, we consider the stable leg and the swing leg.

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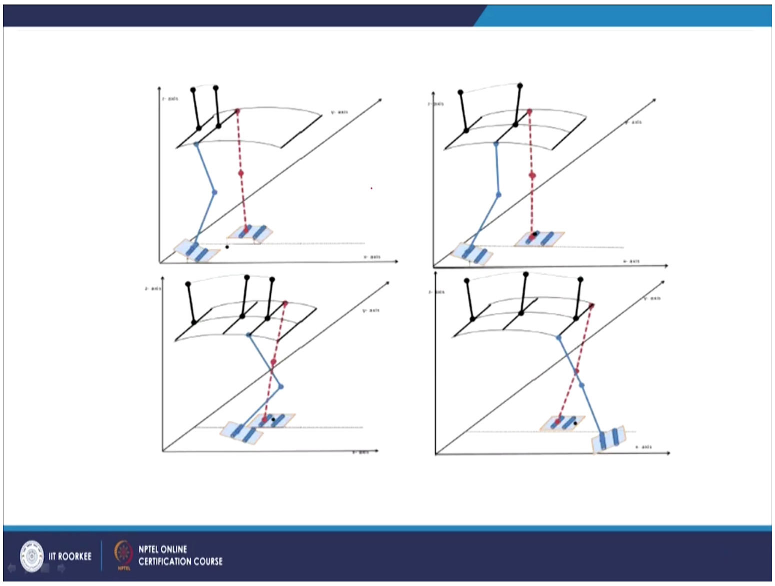

So, this picture depicts the various stages of motion. So, while walking the three dimensional motion where the motion is in the sagittal plane as well as in the frontal plane. So, here the foot of the robot is moving in the sagittal plane; the upper body moves in the frontal plane in this direction. So, the first position is shown here; after certain instant of time the position of the upper body is shown here and again the third position is shown here.

So, it shows that the upper body is moving swinging from one side to the other during the walking. While the swing leg is taking its position from one position to the other position like this; so the end position is given here, the upper body motion at various stages are shown in this picture.

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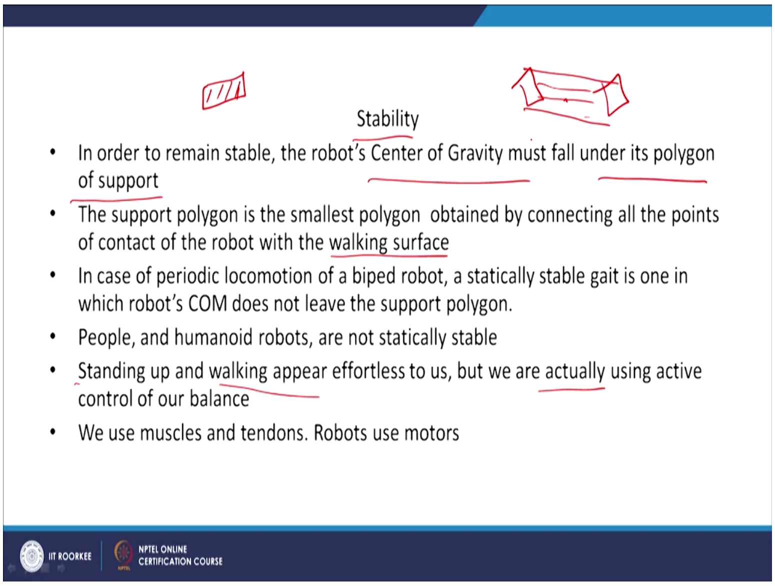

In order to remain stable, the robots center of gravity should lie on the support region; the polygon of support. So, the polygon of support is nothing, but the smallest polygon that can be constructed by connecting the points of contact of the robot with the surface.

So, for example, in the double support phase both the foot are touching the ground. So, the polygon which can be constructed by connecting all the support point is shown by this picture; whereas in the single support phase only one foot is on the ground, another foot is on the air it is moving. So, the support polygon is only below the foot which is on the contact of the ground. So, the support polygon is very important for considering the stability of motion of the biped robot. So, the center of gravity should lie within the support region during walking whether it is in double support or the single support phase.

So, normally the standing without motion or walking; so this for a human being it appears to be very obvious or very simple task. But even for standing at a position without falling we need to control ourselves, so that the center of mass lies on the within the polygon of support. So, for doing that we should constantly control ourselves, so that the person do not fall down. So, the same thing should be done for a biped robot manipulator. So, the control should be devised for whether for standing or walking in a particular manner.

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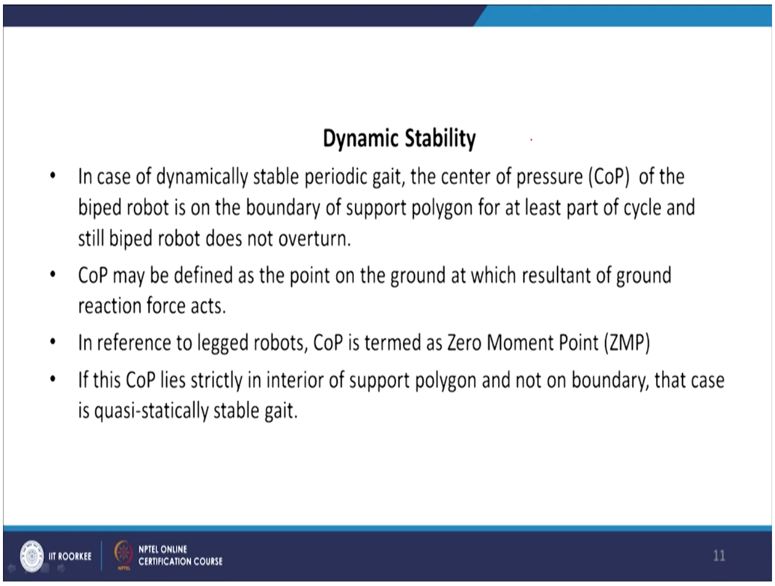

So, the dynamic stability is the one in which the center of pressure lies in the support polygon; within the support polygon or in the boundary of the support polygon. So, when the center of pressure; the center of pressure is defined as the point on the ground at which the resultant of the ground reaction forces acts.

So, the ground reaction forces acts wherever there is a contact point with the leg and the ground; the ground reaction force act in the opposite direction. So, the resultant of all such forces acts at a particular point; that point is called the point of contact, the center of pressure. The point at which the resultant force acts is called the center of pressure; so the it is also termed as the zero moment point. So, when the zero moment point lies within the polygon of support, the stability of the biped robot is ensured.

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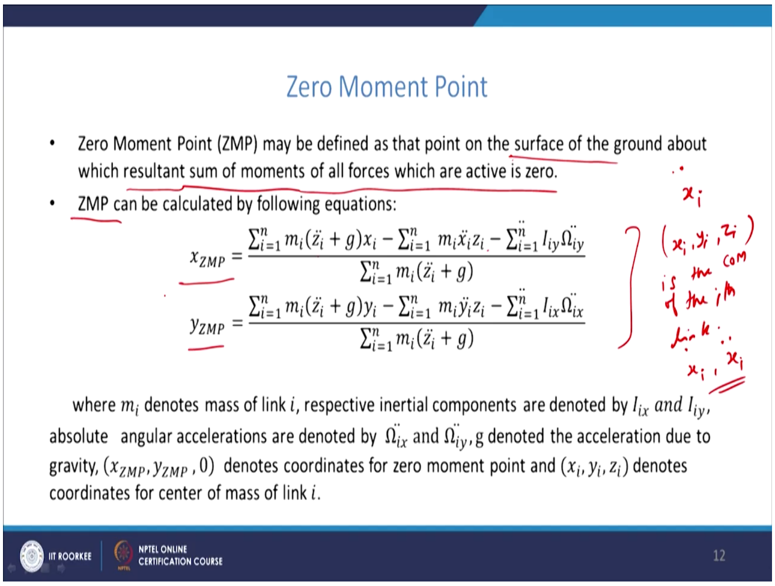

So, the contact the center of pressure or the ZMP, the zero moment point is computed or calculated using this expression. The ZMP is considered to be a point on the surface of walking. So, here the x coordinate of these ZMP is calculated using this expression; in which the double derivative the x double dot, y double dot, z double dot denotes the acceleration of the center of mass of the i th link.

For example, x i double dot; it represent the acceleration of the center of mass in the x direction. So, when we substitute those values in this expression, we get the x coordinate and y coordinate of the zero moment point of the robot. So, it is nothing, but the resultant some of the moments of all active forces is zero. The point at which the resultant of the sum of all moments of the active forces becomes zero is called the zero moment point. So, when the zero moment point lies within the support region, the stability of the robot is ensured.

So, the aim of finding a control is to keep the zero moment point within the support region. So, whenever we design a trajectory; the trajectory should ensure that the zero moment point lies within the support region.

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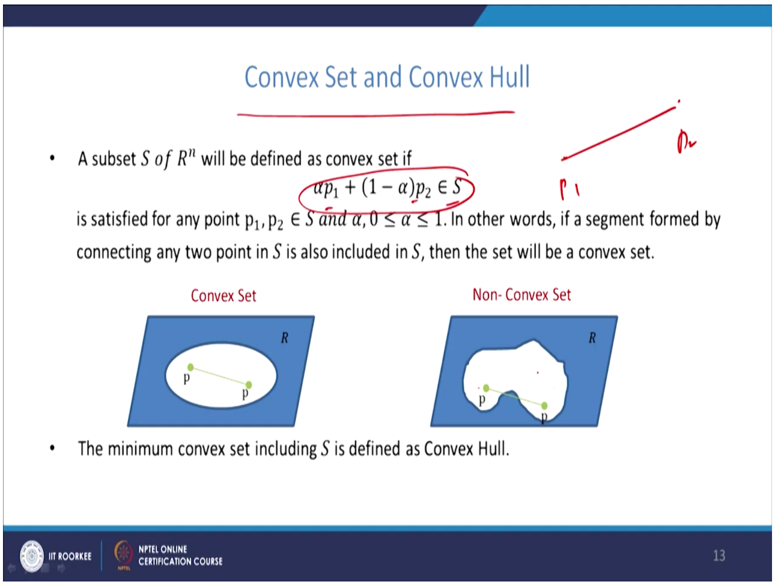

So, let us consider the definition of the convex set. Convex set is any set in which if p 1 and p 2 are two points; then the line joining that point should also lies within that particular set. So, if p 1 and p 2 are in S; then alpha times p 1 plus 1 minus alpha times p 2 also should belong to S. So, this represent the line segment between the two point p 1 and p 2.

So, if the entire line lies that is the convex set. So, for example, this one is not a convex set; because the p 1 and p 2, the line segment goes outside the given set. So, the support polygon when we construct it is always a convex set.

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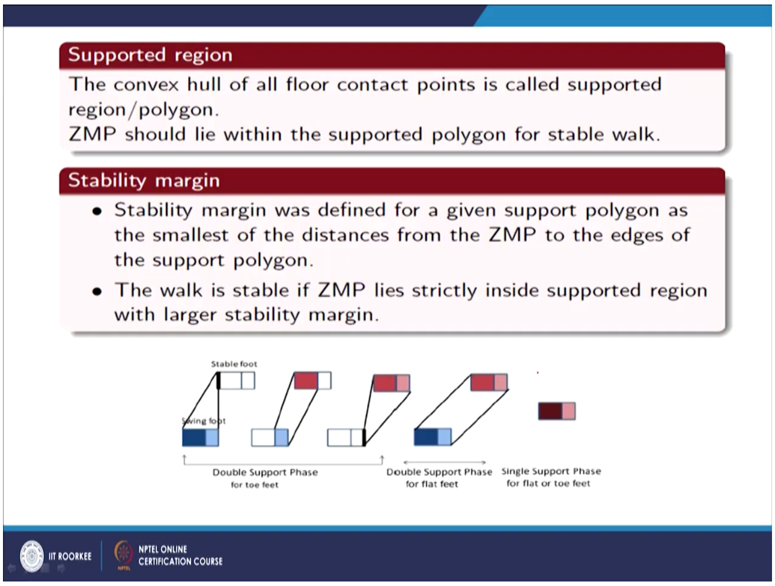

And it is required for the stability of, for analyzing the stability of a biped robot. So, for these stable walk, the ZMP should lie within the support polygon.

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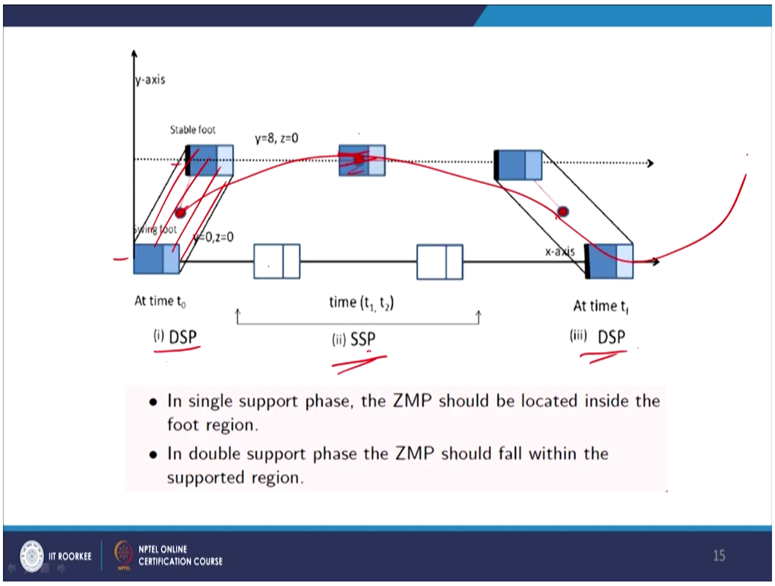

So, this picture depicts that during single support and double support phase, where the ZMP should lie. So, here in the double support phase; both the feet to the first and second feet are in contact with the ground. So, the support polygon is given by this region and the if the ZMP lies within this region; then the stability is ensured.

So, while the single support phase where one leg is on the ground, another leg is moving on the air. So, then the ZMP is expected to lie within the foot here. Similarly, after coming to the double support phase again, the ZMP should move in this direction.

So, it is expected that the ZMP while walking, it should follow a path as shown here in this particular manner; while the leg is moving from double support to single support and again to the double support phase. So, this should be repeated for further walking. So, this picture shows only one step of the walking and for more steps the legs, the ZMP should be having a trajectory as shown in this particular picture.

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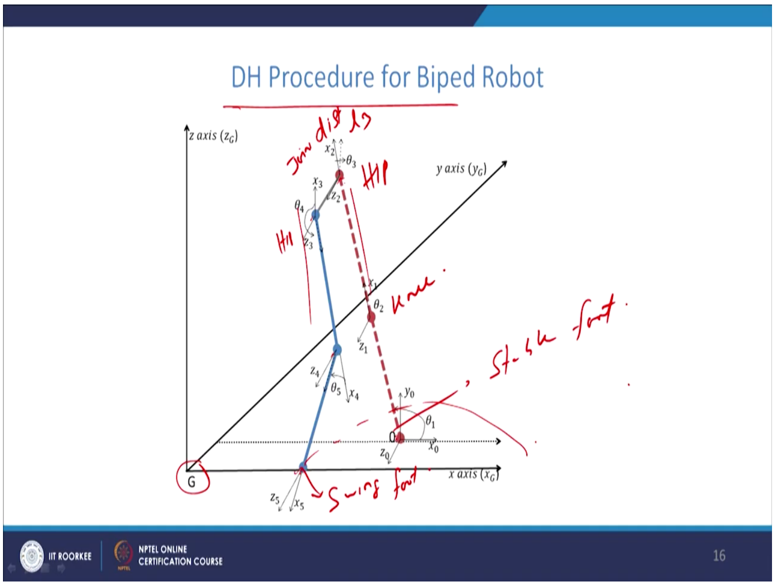

So, now we will see before going to the kinematics and the trajectory planning of the flat foot robot. We will see how to write the kinematic equation using the DH procedure. So, the DH algorithm we have already seen how to fix the coordinate frames at each joint. So, in this picture a robot with 5 degrees of freedom is shown here. So, the angle x 0, y 0, z 0 is shown as the base.

This is the stable foot and this is the swing foot. So, z 0 is the axis of rotation at the ankle of these stable foot; z 1 is the axis of rotation at the knee joint of the stable foot; and the hip joint z 3 is the axis of z 2 is the axis of rotation; and z 3 is the axis of rotation at the second hip. And z 4 is the axis of rotation in the knee of the swing leg.

So, these are the five rotation axis of rotations shown here; all of them are parallel. During the entire walking, the axis z 0, z 1, z 2, z 3, z 4 all of them maintain a parallel direction during the walking.

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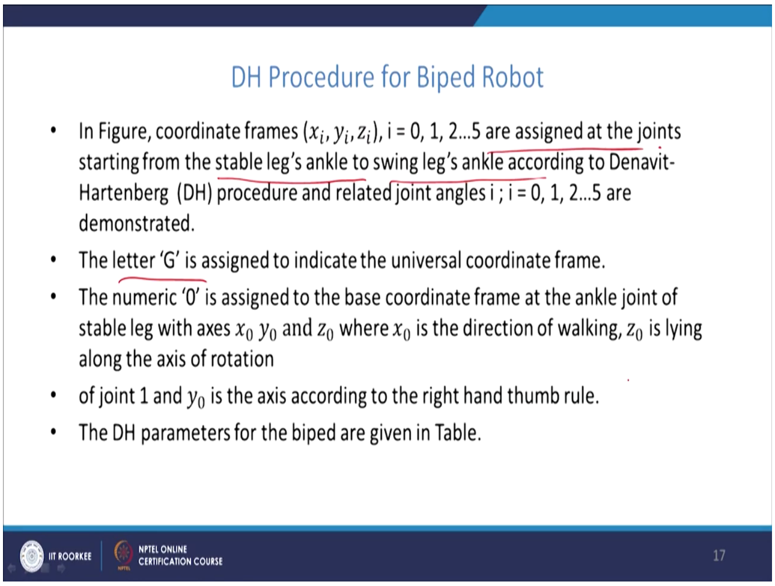

So, it is now easy to fix the x direction and y axis at each joint. And then, find the coordinate homogeneous transformations relating one joint to the another joint. So, here it shows the x i, y i, z i; i equal to 0, 1, 2, 3, 5 are assigned at the joints starting from the stable legs ankle to the swing legs ankle. So, the stable leg ankle is considered as the base and the swing legs ankle is considered as the end effector of the robot manipulator. So, the steps of the DH procedure can be considered.

Here G denotes the universal coordinate frame. So, here the fixed frame is considered as G and in this x, y, z, G frame, the biped robot is walking, and the coordinates of each joint is measured with respect to the G frame.

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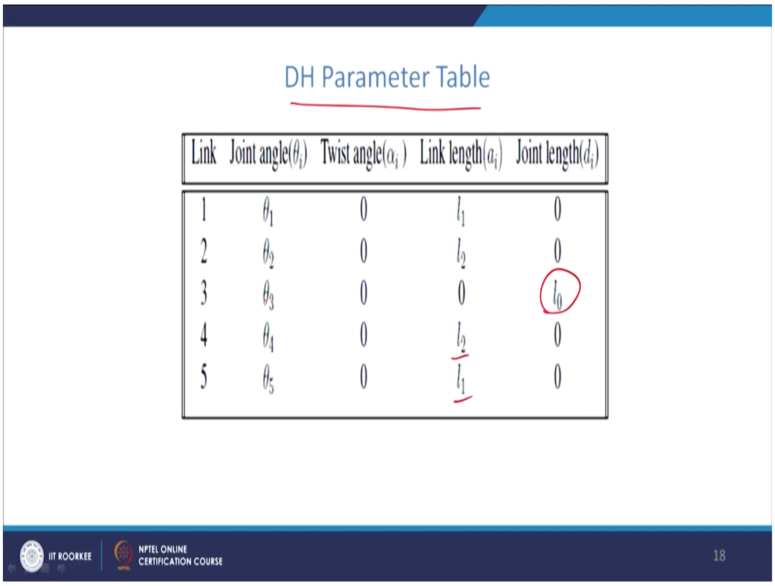

So, now, we can assign the coordinate frames and the parameters of the joints as given in this particular table. So, the joint angles are theta 1 to theta 5; because all the joints are revolute joints. And by following the DH procedure we can easily see the twist angles are all 0; because all the z axis are parallel to each other, so the all the twist angles are 0.

And the link lengths are given by l 1, l 2 and the fourth one is of the same length. So, it is l 2 and l 1; because the both the legs have the same measurement. Here the length is l 1, here also l 1 and l 2 is this; the link lengths are given, and the joint length is given by this expression l

3. That can be seen from here the, this particular thing is the joint distance l 3 according to the DH algorithm.

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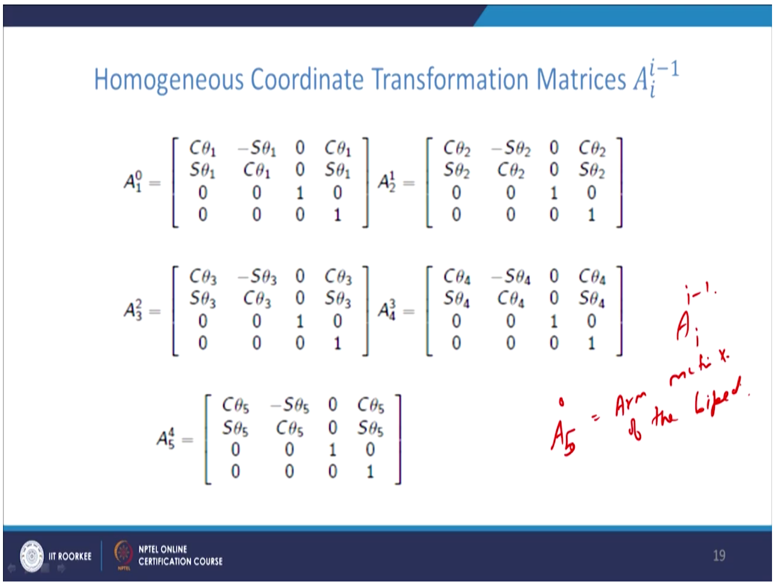

So, after constructing this parametric table, we can construct the homogeneous transformation relating the A i i minus 1. That is the I th framed with respect to the i minus 1 th frame as shown in this slide. So, this gives a relation. Finally, when we multiply all of this, we can get to the relation between the zeroth frame with respect to the A 5 with respect to zeroth frame. So, this is the arm matrix of the biped.

Considering the biped as a robot manipulator with the five degrees of freedom; we can get the arm matrix with respect to the base, the end of vector with respect to the base. So, during the walking of the robot manipulator, we can consider the kinematics equation using the arm matrix. So, now, we will see a particular example of a biped robot with flat foot.

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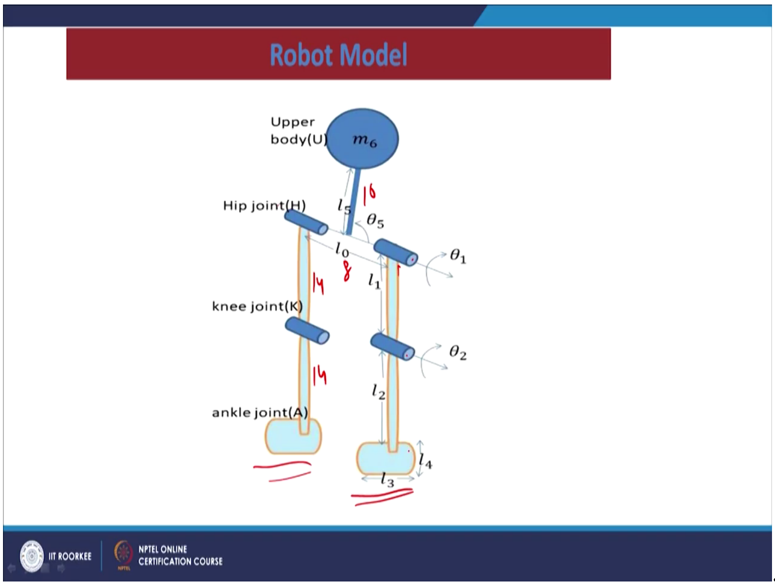

So, here we consider the biped with two legs and with four degrees of freedom. So, here the angle of rotation is at the knee joint and the hip joint for one leg. Similarly, at knee joint and hip joint for the second leg. The foot is considered to be a flat one. So, it is slightly different from the previous model where a rotation is considered at the ankle joint also. So, if you remove the rotation at ankle joint. So, the model which we are considering now is with the flat foot.

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Parameters

Link	Length	Value	Mass	Value
HK	l_1	14 inches	m_1	4kg
KA	l_2	14 inches	m_2	4kg
HU	l_5	10 inches	m_6	50kg
HH	l_0	8 inches	m_5	4kg

Table 3.1: Parameters

So, now, we consider the biped model with the flat foot as given in this picture. And the parameters of the biped robot is given here; the length of the various joints, various links are given here. HK, this length HK is given to be 14 and KA is given to be 14; the length of this one. And HU is considered to be 10 inches, and this is 8. So, here it is 8 and then here it is 10 units of length.

Similarly, the mass of each link is also given as m 1, m 2 and m 6 and m 5. And it is considered that the two legs are identical; whether the length and masses are identical to each other. So, using this particular model, we will construct a trajectory for stable walking of the biped robot.

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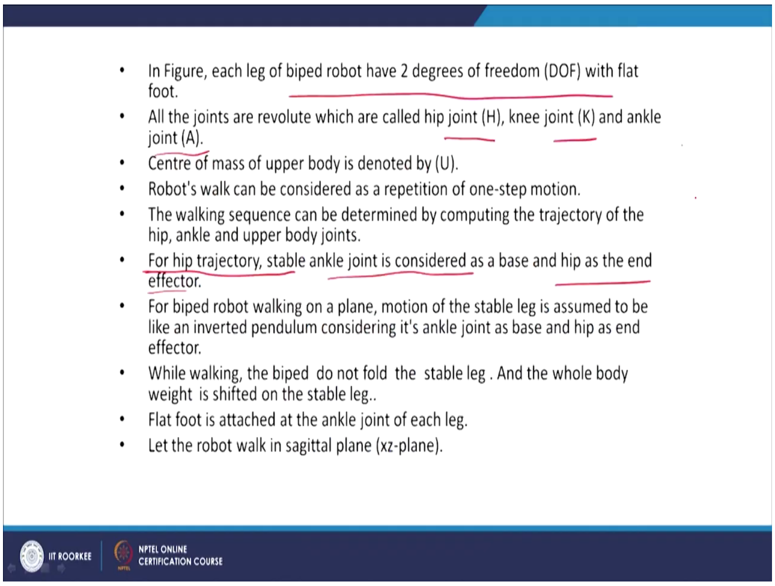

So, we consider that each leg has two degrees of freedom. It is considered a two arm manipulator; each, each leg is considered as a two arm manipulator separately and all the joints are revolute. So, the hip joint is considered as H, the knee joint is K and the ankle joint is A as shown in the picture.

So, during the walking; the hip traject, for generating the hip trajectory. We consider the stable ankle joint is the base and the hip joint as the end effector of the robot manipulator.

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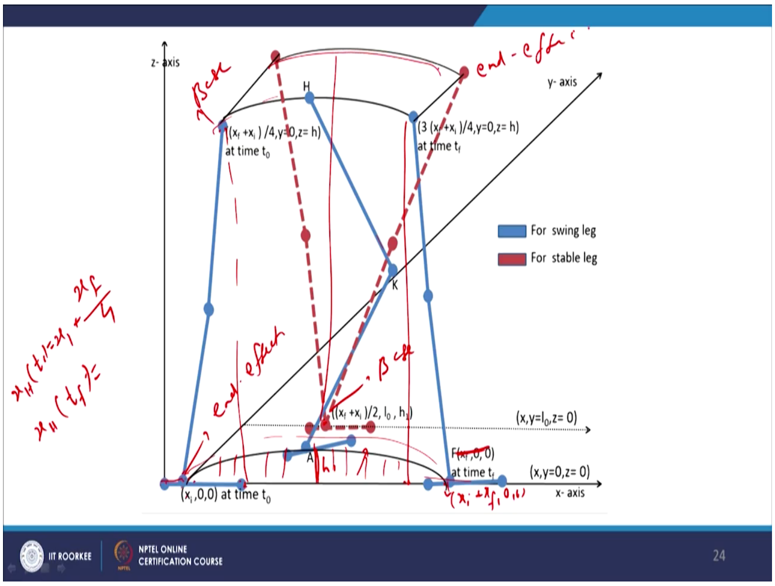

In this picture we can see that the stable leg which is shown in the red color; the ankle is considered as the base and the hip joint is considered as the end effector. Similarly, for the swing leg which is in the blue color; the hip is considered as the base and the ankle is the end effector. And they are considered as a two arm manipulator and their kinematics and inverse kinematics can be calculated very easily as we have already seen.

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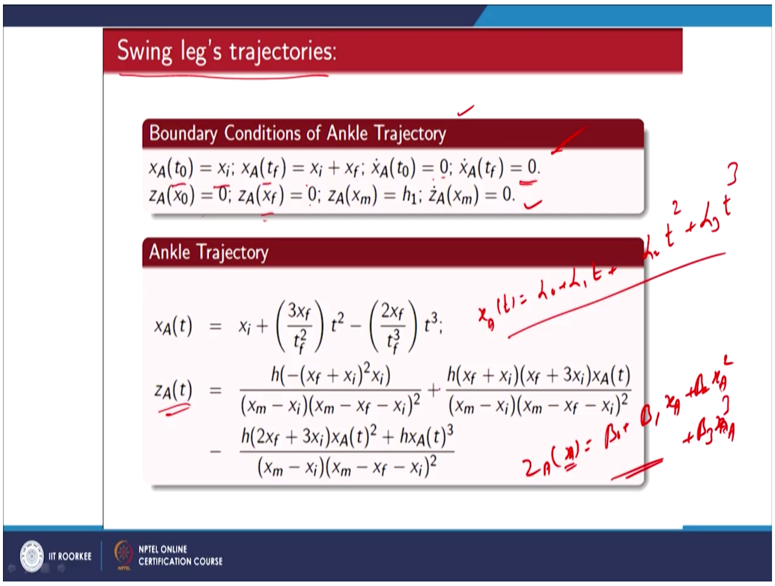

So, now let us consider the trajectory of the swing leg. Swing leg has two coordinates that is X at X coordinate and the Z coordinate. The boundary conditions for the ankle trajectory for the swing leg we consider it to be like this. At time initial time t 0, the x coordinate of the ankle is at X i and at the final time t f, the X coordinate of the ankle is at X i plus x f.

So, as shown in this picture this is the point X i is the X coordinate and X i plus X f is the X coordinate at this position; this is X i plus X f 0 0, I think this is the value. And the velocity at the initial time is 0 and the velocity at the final time is 0 in the X direction. And the Z coordinate of the ankle is it depends on the X coordinate. So, it is a curve which z which has a relation between z and X here in the Z X plane.

So, X Z A at initial position of X 0 is 0 and Z A ankle Z coordinate of the ankle at final position is 0 again. Z A at the middle position is given by h 1; that is at the middle position the height is h 1. And the velocity at the middle position of the Z component is 0; because the Z component is slowly increasing and the maximum value is h 1, then it should start decreasing Z value. So, the derivative is equal to 0 at that point the middle point X m.

So, using this boundary condition we can define a polynomial of X A t. We can write it as alpha 0 [noise, alpha 1 t, alpha 2 t square, alpha 3 t cube. Now, by putting all this four boundary condition, we will get a trajectory like this. Similarly, Z A of x can be considered as beta 0 beta1 X beta 2 X square beta 3 X cube.

Now, by substituting this four boundary condition in terms of X, we will get Z A in terms of X; whereas, X is in terms of t; here X A. So, by substituting X A in terms of t in this expression, we will get Z A in terms of t as given in this picture in this particular expression.

So, this gives for all time in instance the position of X A and Z A. So, we can get the entire trajectory as a function of time. So, we get the x and z coordinates of the swing leg trajectory as given in this expression.

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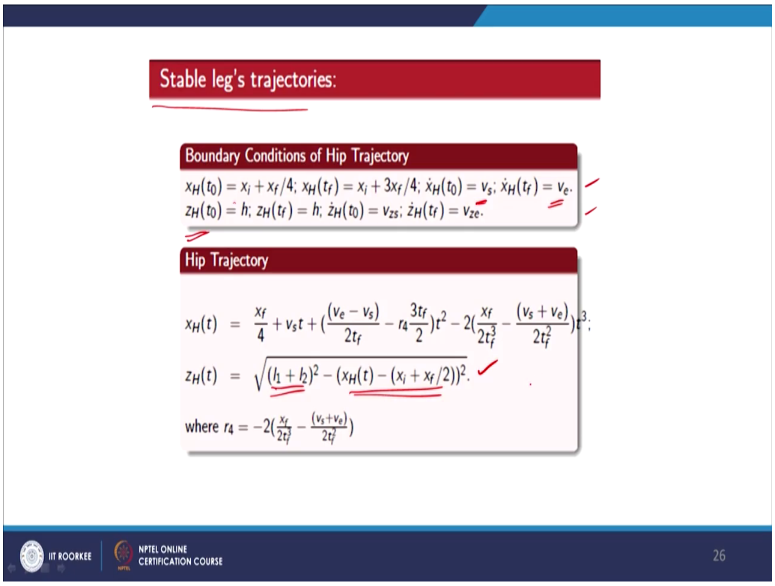

Similarly, we can get this stable legs trajectory as given in this two expression. The boundary, initial and the boundary condition is given by this one. At time, initial time t 0, the hip X coordinate is given by this expression; because we can see from this picture the hip is at this point, the x component is at one fourth of the total length. That is the X f is the foot length, total foot length by 4 is this length and then x i is the initial distance from here; so X i plus X f by 4 that is the hip initial position.

Similarly, hip final position is given by this one. So, with X final position is given by this particular point; where X f by 4 should be subtracted from here, so as given in this expression that is X i plus 3 by 4th of X f. Similarly, the velocity at the initial point is V s and the velocity of the hip at the final point is V e; that is starting and end velocities.

And in the similar manner, the z coordinate of the hip trajectory is given by these boundary conditions. We can observe that the hip will move in the circular path with the center at this position and radius to be like this. So, it is simply a circular path. So, the equation of a circle is given here, the radius is l 1 plus l 2; the total length of the leg is the radius and the center is at this particular position. So, the X and Z are related by the circle with radius l 1 plus l 2 and center at this position, it gives the trajectory of the hip; the swing trajectory.

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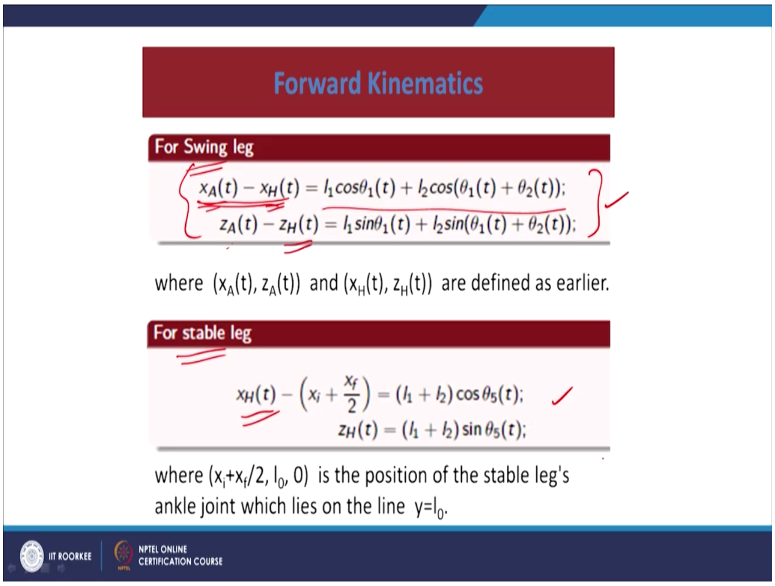

So, now we can write the swing legs equation in terms of the angle. Whatever is given in this particular slide, in the previous slide are only the position of the end effector. Now for achieving the end effector position, the angles theta 1 and theta 2 should be adjusted. So, we write the forward kinematic equation of the two arm manipulator equation. So, here for the swing leg the hip is the base and X the ankle is the end effector. So, as the hip is moving constantly, we will get the coordinate X A t minus X H t given by the formula l 1 cos theta 1 plus l 2 cos theta 1 plus theta 2.

And similarly, the Z component of the end effector that is the ankle Z component and with respect to the base that is Z H of t is given by this formula, so this can be easily verified. Because as time changes the base also keeps on moving; the X H and Z H is moving. So, for finding the X A t and Z A t in the moving frame, we have to use this particular expression. Similarly, the hip joint X H of t of the stable leg, it is moving in the form of a circle as given here.

Up to this we have seen that how to generate the trajectory of the swing leg and this stable leg in terms of the angles. Now, if you solve the inverse kinematics of these two arm manipulators; suitably we can get how the angles should be calculated for achieving a particular trajectory as given by all these boundary conditions. But we have to verify whether these trajectories are stable or unstable. So, for doing that we should adjust the upper body motion suitably, so that the ZMP lies in the support polygon.

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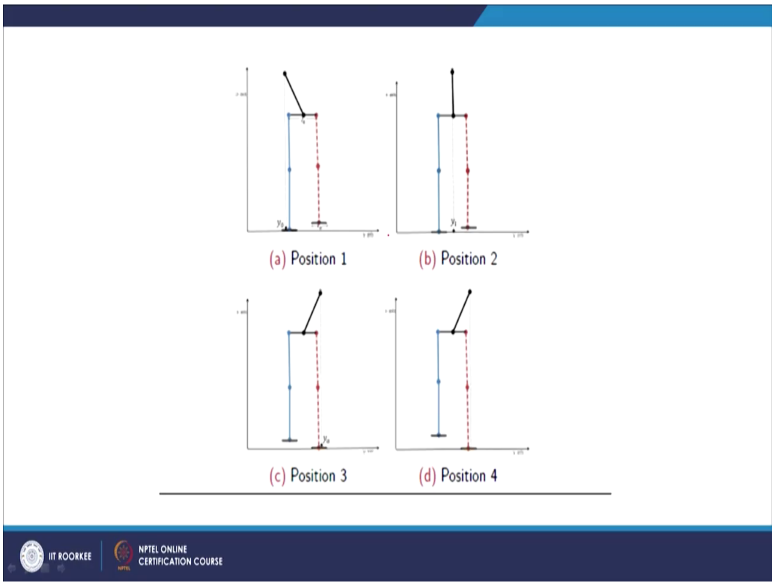

So, this portion we will see in the next lecture when we consider other types of robot manipulators.

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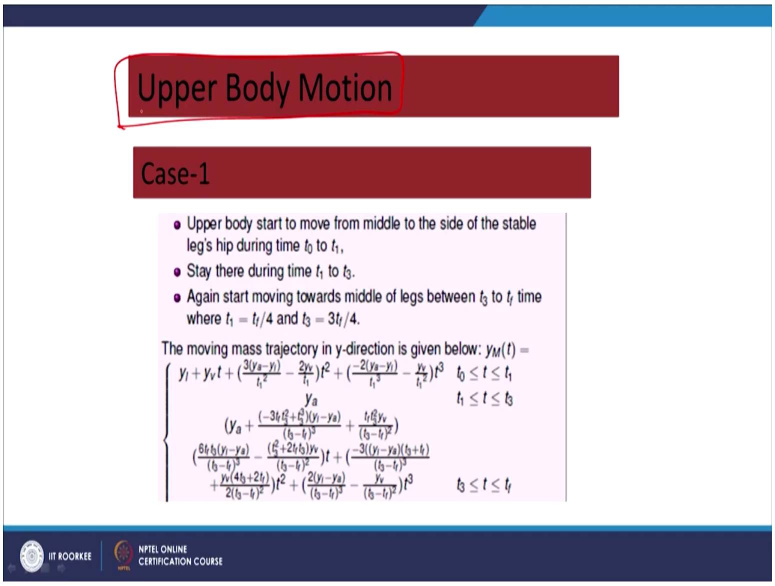

So, the upper body motion is very important to make the robot stable, so that the ZMP lies within the support region. So, this particular point we will see in the next lecture.

Thank you.