Financial Mathematics
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Lecture – 49
Option Valuation

Welcome to the lecture on, Option Valuation. So this lecture, we are going to talk about the methods which are used for the valuation of the options. Before that we must know the; what are the determinants of the option value and what are the factors those factors on which these options value are basically depending upon. And basically, there are five major factors which determine the market value of an option and they are price of the underlying stock, so that is market price, then MP, so that is known as MP.

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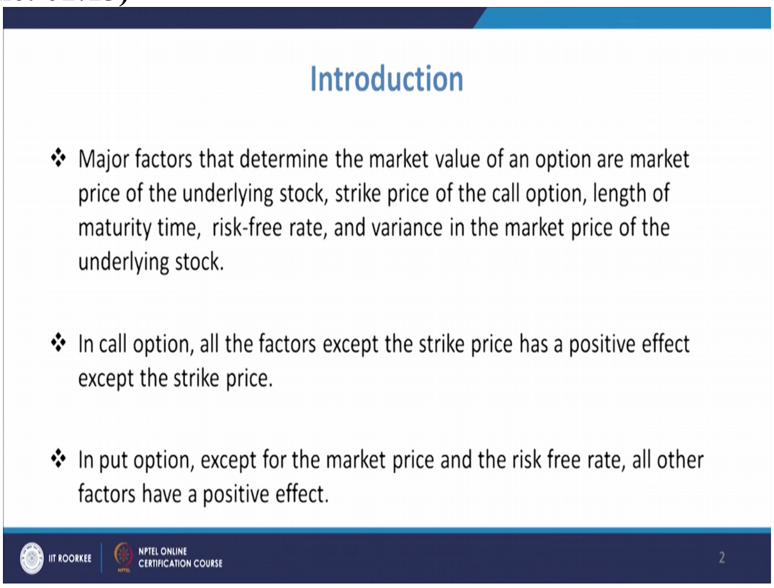

Then you have a strike price of the call option that is SP, length of the maturity time that we normally denote by t, risk-free rate, that we normally denote by RF, and also variance in the market price of the underlying stock, so that is known as you know Sigma MP, that is square Sigma square the variance for the market price.

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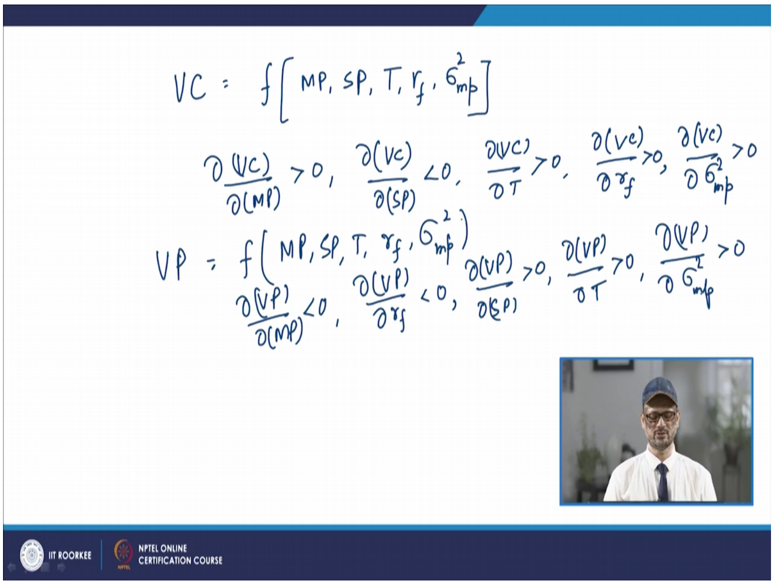

So basically what happens, that the, you know, the call value that is a value of a call option, so that is we denote it as VC, so that will be value of the call option. Now this will be depending, this will be a factor of the, we discussed that is equal to market price. Then it will be the factor of strike price, then it will be talking about depending upon the maturity time, similarly the risk-free return, so we normally denoted as RF, and then the variance of the market price.

So it depends upon these four, five factors. Now if you look at, you mean, if it is analyzed, then it is found that the so some of the factors have the positive value. So you know, if there is positive, you know increase in the, these factors, then that will be there will be positively increase in the, in this call value also, value of the call option.

So some of the factors like the market price, if the market price is more, in that case, the value of the call option will be more. So basically dou of VC upon dou of MP so since market price being larger and larger self-image may be larger, the, the value of the call option will be more. So it will be more than 0. So slope will be basically 0. Then about the, you know, strike price you know that when there is strike price will be larger, then in the case of call option strike price being you know larger so the value of the call will be basically decreasing.

So you know what happens that dou of VC up or low of SP this will be basically less than 0. Then depending upon the maturity time, if it is higher then it has a positive effect. So dou of VC upon dou of t will be more than zero. Similarly dou of VC upon dou of RF will be more than 0 and also you know depending upon the variance, so if the variance is you know larger, so in that case also, your call value will be basically more.

So it has a positive effect on this so dou of VC upon dou of Sigma square MP this will be also be greater than 0. So these factors basically, you know, effect you know like this so what is it is written, that in the call option, all the factors except the strike price has a positive effect, you know. So you know that that is, what it is, then, you know, in the case of the you know other things like if you go for the put option now in the case of put option again put option will be also depending upon all these factors so in that case what you see is again we can write that the value of the put, you know, that will be again a function of, MP then SP then T RF and again Sigma MP square.

Now in this case when you are going to talk about the put option, in those cases, the MP if it MP will be higher in that case, the VP will be, you know, it has a negative effect. So dou of VP upon dou of MP. So this will be less than zero, and similarly when that is free rate of return, risk-free return, that is, you know, it has also a negative effect so dou of VP upon dou of RF, so that also has a negative effect.

Apart from that, all the other three factors, like strike price, then you have a maturity time, and also the variance of the market price, all have the positive effects. So dou of VP upon dou of SP will be more than zero, unlike to that of the value in the call option, and similarly dou of VP upon dou of T will be more than zero and dou of VP upon dou of Sigma Square and P, it will be also more than zero. So this way you know these factors are changing they have the different effects on the call or put valuation with respect to their own changes.

Now we will talk about the valuation of these calls and put by some different methods. So that is also clear from, you know, this expression that except for the market price and the risk-free rate, all the factors have a positive effect, because, you see that the rate of change of the you know value of the put will be changing and that will that has all has positive effect like strike price, maturity time, and the variance of the market price, and for the market so and with respect to market price and the risk-free rate of return it has a negative effect. So that is how it is effecting.

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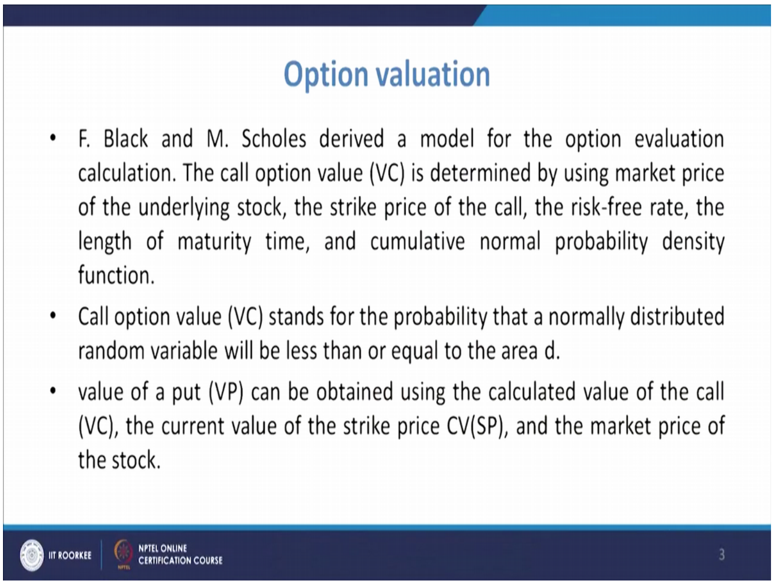

Now the revolution became I mean there was a very, you know, important finding by the Black and Scholes, so F. Black and M. Scholes, they have derived their model for the option valuation, and in that, the call option value is determined by using the market price of the underlying stock again MP, strike price of the call that is SP, the risk-free rate again that we know, the length of maturity time and the cumulative normal probability density function.

So in this case they have talked about another function, that is, cumulative normal probability density function, that is. So, so in that case you know we are talking about the call option value VC and it will be standing for the probability that a normally distributed random variable will be less than or equal to the area D that is you know D is that function.

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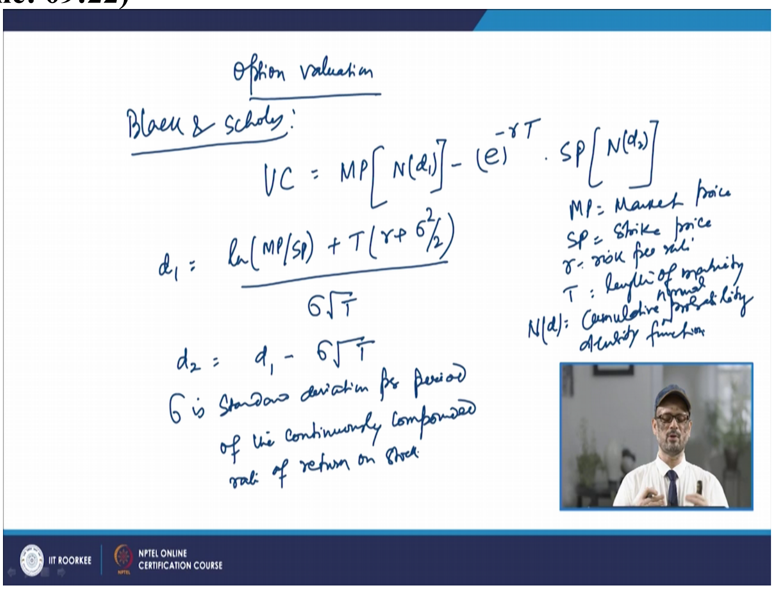

So according to that if you talk about these option valuation by Black and Scholes, so that will be by Black and Scholes, so they have basically given the relationship like VC so-called value that will be equal to MP and to multiplied by, it will be n d1 and that will be minus e raised to the power minus R into T and then into SP that is strike price multiplied by n d2. So the here we know that MP is the market price and here in this case we know that SP is the strike price and R is the risk-free rate, T is the length of maturity time.

And now in this case the nd basically this is the new thing which has been given by Scholes. So that will be talking about the cumulative probability density function. So this is normal probability density function, so cumulative normal probability density function. So now that is what it is defined, that this, we see in this case, will be standing for the probability that a normal distributed random variable will be less than or equal to the area d. So d is calculated, basically d1 and d2 and that will be seen from a table and that value is used for finding the, the VC.

Now what we do for that we first find d1 so we have n d1 and n d2 so d1 is found as Ln of MP by SP and plus T * R + Sigma square by 2, so divided by Sigma root T. So this is the equation for d1 and similarly we find d2 and d2 is calculated as, once you calculate d1, you can find d2, d1 - Sigma root T. So we know that the Sigma is the standard deviation. So this Sigma is standard deviation per period of the continuously compounded rate of return on stock.

So what you do, that once you know, these you need to know the; you will be, knowing the market price, the strike price, the maturity time, the risk-free rate, and the you know this variance. So once you know that, you know, you can find the d1 value and once you know the d1, and then you will also find the d2, and once you know that, then in that case you can find the valuation calculation, in such cases. This is for the call options you can also go for the; you know the put option. So if you go for the put option, now in that case, if you find the VP, so VP will be VC plus CV into SP.

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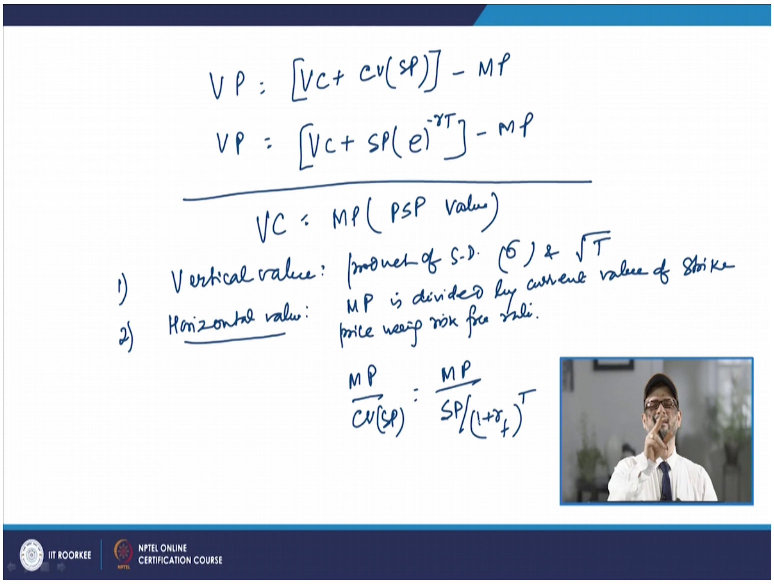

So, this is a current value of the strike price and so that will be and minus MP. So that is your VP. So VP can further be, you know, for finding the current value, you will have to you know for continuous compounding you have to multiply with the factor. So it will be VC plus you know you will find the SP into e raised to the power minus RT. That will be giving you because it is continuous compounding, so you are multiplying with e raised to the power minus RT, and that is again and from that you will find the; you will subtract the market price.

So this way now you are able to calculate the value of the put, you know, in that case, also, this methods were simplified you know further because it looks somewhat you know complex. So since they are looking there, but they can be simplified and the simplified way is to calculate this VC as the MP into PSP value. So this PSP value is nothing but, the percentage of the share price value. Now in this case basically you have to calculate these d1 and d2 and then from the table you have to find the PSP value.

So there once you get the PSP value you can multiply with the MP and you can get it. Now how can you get these values? So far that, first of all, you will be referring to a table and from that table you have a vertical column and you have a horizontal column, horizontal row. So you have vertical you know value and we have horizontal value and that, once you get these two values, then you can get you know the values like the vertical value.

So how to refer so for the vertical value you know for that you are going to have the product of standard deviation that is Sigma and root T. So that is a square root of the maturity time. So that will be calculated and then second is the horizontal value and horizontal value will be calculated by dividing the market price to the current value of so that will be of the strike price. So this will be market price divided is divided by current value of a strike price using risk-free rate so the risk-free rate is taken and using that basically you have to find the you know the current price.

So you will be dividing it with it is something like, that factor and in that case you are you know getting and for that the actual maturity time, you know, it will be calculated. So this will be MP divided by CV into SP. CV of SP, so current value of the strike price. Now this will be MP upon SP by 1 plus RF. So this is for the for the maturity time of T.

So this is done for the whole maturity time span and for that time span when you have to calculate the current value of that you know strike price for that maturity time, taken as the basis, in that case, you have to divide it with 1 plus R raised to the power T. So that is how you calculate the present value from any future value you have to divide it with 1 plus interest rate raised to the power the periods. So here you have the maturity time.

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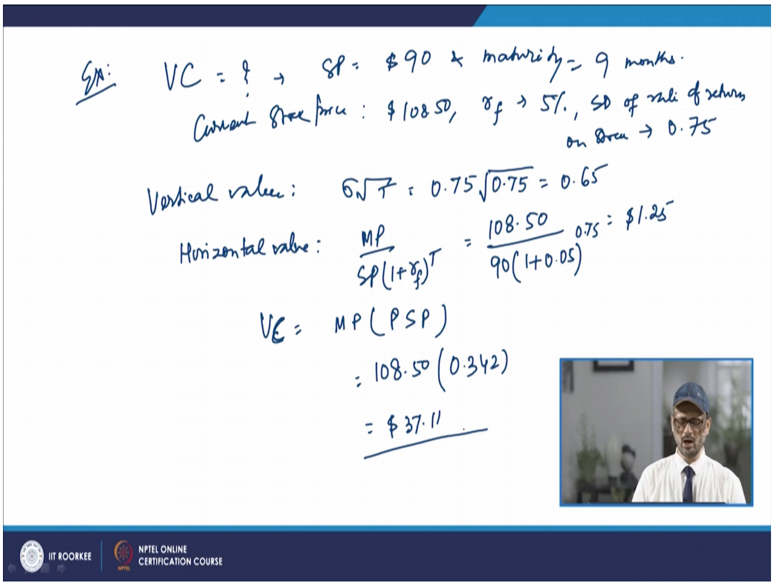

So this way you calculate these horizontal and vertical values, and from the horizontal and vertical values, you can calculate these PSP value, from the, you know, table itself. So that is how, you try to calculate these values from the table. You can see that suppose you know if you have an example, say, that you have to calculate the value of the call option and you have strike price is given as 90 and maturity is nine months.

So and also the current stock price so that is given as dollar 108.50 and risk-free rate is 5%, also the standard deviation of rate of return on a stock, so that is also given as 0.75. So if suppose, such are the data, which are given to you, and you have to find the valuation of the you know call in that case, what you do is, you know first of all we have as we know that you have to you know value is horizontal as well as vertical values.

Now vertical value is you will see that will be Sigma root T, so Sigma is 0.65, so Sigma is 0.75 and then you will take the square root of the imaginary time, that is in years, so it will be again 0.75. So this will be coming out to be zero point you know six five. Now once you so this is the vertical value which is calculated, now you have to calculate the horizontal value now horizontal value will be you know MP by M so current value of SP.

So horizontal value will be MP by SP into 1 plus RF raised to the power T. So you know that the market price so that is given as 108.50 and strike price is shown as 90 and this will be 1 plus 0.05 and T is given as 0.75 years. So now if you take that, it will be, you know 1.25. Now you will be given a table and from that table basically with the vertical score of 0.65 and the horizontal score of 1.25, you have to find the percentage share price that is PSP.

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Call option values and percentage of share price

$\sigma\sqrt{T}$	MP/CV(SP) .40	.45	.50	.55	.60	.65	.70	.75	.80
.05	.0	.0	.0	.0	.0	.0	.0	.0	.0
.10	.0	.0	.0	.0	.0	.0	.0	.0	.0
.15	.0	.0	.0	.0	.0	.0	.1	.2	.5
.20	.0	.0	.0	.0	.0	.1	.4	.8	1.5
.25	.0	.0	.0	.1	.2	.5	1.0	1.8	2.8
.30	.0	.1	.1	.3	.7	1.2	2.0	3.1	4.4
.35	.1	.2	.4	.8	1.4	2.3	3.3	4.6	6.2
.40	.2	.5	.9	1.6	2.4	3.5	4.8	6.3	8.0
.45	.5	1.0	1.7	2.6	3.7	5.0	6.5	8.1	9.9
.50	1.0	1.7	2.6	3.7	5.1	6.6	8.2	10.0	11.8
.55	1.7	2.6	3.8	5.1	6.6	8.3	10.0	11.9	13.8
.60	2.5	3.7	5.1	6.6	8.3	10.1	11.9	13.8	15.8
.65	3.6	4.9	6.5	8.2	10.0	11.9	13.8	15.8	17.8
.70	4.7	6.3	8.1	9.9	11.9	13.8	15.8	17.8	19.8
.75	6.1	7.9	9.8	11.7	13.7	15.8	17.8	19.8	21.8
.80	7.5	9.5	11.5	13.6	15.7	17.7	19.8	21.8	23.7
.85	9.1	11.2	13.3	15.5	17.6	19.7	21.8	23.8	25.7
.90	10.7	13.0	15.2	17.4	19.6	21.7	23.8	25.8	27.7
.95	12.5	14.8	17.1	19.4	21.6	23.7	25.7	27.7	29.6
1.00	14.3	16.7	19.1	21.4	23.6	25.7	27.7	29.7	31.6
1.05	16.1	18.6	21.0	23.3	25.6	27.7	29.7	31.6	33.5
1.10	18.0	20.6	23.0	25.3	27.5	29.6	31.6	33.5	35.4
1.15	20.0	22.5	25.0	27.3	29.5	31.6	33.6	35.4	37.2
1.20	21.9	24.5	27.0	29.3	31.5	33.6	35.5	37.3	39.1
1.25	23.9	26.5	29.0	31.3	33.5	35.5	37.4	39.2	40.9
1.30	25.9	28.5	31.0	33.3	35.4	37.4	39.3	41.0	42.7
1.35	27.9	30.5	33.0	35.2	37.3	39.3	41.1	42.8	44.4
1.40	29.9	32.5	34.9	37.1	39.2	41.1	42.9	44.6	46.2
1.45	31.9	34.5	36.9	39.1	41.1	43.0	44.7	46.4	47.9
1.50	33.8	36.4	38.8	40.9	42.9	44.8	46.5	48.1	49.6
1.55	35.8	38.4	40.7	42.8	44.8	46.6	48.2	49.8	51.2
1.60	37.8	40.3	42.6	44.6	46.5	48.3	49.9	51.4	52.8
1.65	39.7	42.2	44.4	46.4	48.3	50.0	51.6	53.1	54.4
1.70	41.6	44.0	46.2	48.2	50.0	51.7	53.2	54.7	56.0
1.75	43.5	45.9	48.0	50.0	51.7	53.4	54.8	56.2	57.5
2.00	52.5	54.6	56.5	58.2	59.7	61.1	62.4	63.6	64.6
2.25	60.7	62.5	64.1	65.6	66.8	68.0	69.1	70.0	70.9
2.50	67.9	69.4	70.8	72.0	73.1	74.0	74.9	75.7	76.4
2.75	74.2	75.4	76.6	77.5	78.4	79.2	79.9	80.5	81.1
3.00	79.5	80.5	81.4	82.2	82.9	83.5	84.1	84.6	85.1
3.50	87.6	88.3	88.8	89.3	89.7	90.1	90.5	90.8	91.1
4.00	92.9	93.3	93.6	93.9	94.2	94.4	94.6	94.8	94.9
4.50	96.2	96.4	96.6	96.7	96.9	97.0	97.1	97.2	97.3
5.00	98.1	98.2	98.3	98.3	98.4	98.5	98.5	98.6	98.6

Source: R. A. Brealey and C. M. Stewart (1991). *Principles of Corporate Finance*. McGraw-Hill, New York.

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Now that can be referred from such table, this table is by the, you know, from the book McGraw Hill. So here this table is given as Sigma root T, is you know in the vertical side and your MP by CV is given as this. So you have a sigma root two coming as 0.65 but the MP by CV is coming as 1.25.

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	MP/CV(SP)									
$\sigma\sqrt{T}$	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.75	2.00	2.50
.05	16.7	20.0	23.1	25.9	28.6	31.0	33.3	42.9	50.0	60.0
.10	16.8	20.0	23.1	25.9	28.6	31.0	33.3	42.9	50.0	60.0
.15	17.4	20.4	23.3	26.0	28.6	31.1	33.3	42.9	50.0	60.0
.20	18.5	21.2	23.9	26.4	28.9	31.2	33.5	42.9	50.0	60.0
.25	19.8	22.3	24.7	27.1	29.4	31.7	33.8	42.9	50.0	60.0
.30	21.2	23.5	25.8	28.1	30.2	32.3	34.3	43.1	50.1	60.0
.35	22.7	24.9	27.1	29.2	31.2	33.2	35.1	43.5	50.2	60.0
.40	24.3	26.4	28.4	30.4	32.3	34.2	36.0	44.0	50.5	60.1
.45	25.9	27.9	29.8	31.7	33.5	35.3	37.0	44.6	50.8	60.2
.50	27.6	29.5	31.3	33.1	34.8	36.4	38.1	45.3	51.3	60.4
.55	29.2	31.0	32.8	34.5	36.1	37.7	39.2	46.1	51.9	60.7
.60	30.9	32.6	34.3	35.9	37.5	39.0	40.4	47.0	52.5	61.0
.65	32.6	34.2	35.8	37.4	38.9	40.3	41.7	48.0	53.3	61.4
.70	34.2	35.8	37.3	38.8	40.3	41.6	43.0	49.0	54.0	61.9
.75	35.9	37.4	38.9	40.3	41.7	43.0	44.3	50.0	54.9	62.4
.80	37.5	39.0	40.4	41.8	43.1	44.4	45.6	51.1	55.8	63.0
.85	39.2	40.6	41.9	43.3	44.5	45.8	46.9	52.2	56.7	63.6
.90	40.8	42.1	43.5	44.7	46.0	47.1	48.3	53.3	57.6	64.3
.95	42.4	43.7	45.0	46.2	47.4	48.5	49.6	54.5	58.6	65.0
1.00	44.0	45.2	46.5	47.6	48.8	49.9	50.9	55.6	59.5	65.7
1.05	45.5	46.8	48.0	49.1	50.2	51.2	52.2	56.7	60.5	66.5
1.10	47.1	48.3	49.4	50.5	51.6	52.6	53.5	57.9	61.5	67.2
1.15	48.6	49.8	50.9	51.9	52.9	53.9	54.9	59.0	62.5	68.0
1.20	50.1	51.3	52.3	53.3	54.3	55.2	56.1	60.2	63.5	68.8
1.25	51.6	52.7	53.7	54.7	55.7	56.6	57.4	61.3	64.5	69.6
1.30	53.1	54.1	55.1	56.1	57.0	57.9	58.7	62.4	65.5	70.4
1.35	54.6	55.6	56.5	57.4	58.3	59.1	59.9	63.5	66.5	71.1
1.40	56.0	56.9	57.9	58.7	59.6	60.4	61.2	64.8	67.5	71.9
1.45	57.4	58.3	59.2	60.0	60.9	61.6	62.4	65.7	68.4	72.7
1.50	58.8	59.7	60.5	61.3	62.1	62.9	63.6	66.8	69.4	73.5
1.55	60.1	61.0	61.8	62.6	63.3	64.1	64.7	67.8	70.3	74.3
1.60	61.4	62.3	63.1	63.8	64.5	65.2	65.9	68.8	71.3	75.1
1.65	62.7	63.5	64.3	65.0	65.7	66.4	67.0	69.9	72.2	75.9
1.70	64.0	64.8	65.5	66.2	66.9	67.5	68.2	70.9	73.1	76.6
1.75	65.3	66.0	66.7	67.4	68.0	68.7	69.2	71.9	74.0	77.4
2.00	71.1	71.7	72.3	72.9	73.4	73.9	74.4	76.5	78.3	81.0
2.25	76.3	76.8	77.2	77.7	78.1	78.5	78.9	80.6	82.1	84.3
2.50	80.7	81.1	81.5	81.9	82.2	82.6	82.9	84.3	85.4	87.2
2.75	84.6	84.9	85.2	85.5	85.8	86.0	86.3	87.4	88.3	89.7
3.00	87.8	88.1	88.3	88.5	88.8	89.0	89.2	90.0	90.7	91.8
3.50	92.7	92.8	93.0	93.1	93.3	93.4	93.5	94.0	94.4	95.1
4.00	95.8	95.9	96.0	96.1	96.2	96.2	96.3	96.6	96.8	97.2
4.50	97.8	97.8	97.9	97.9	97.9	98.0	98.0	98.2	98.3	98.5
5.00	98.9	98.9	98.9	98.9	99.0	99.0	99.0	99.1	99.1	99.2

So you have to go to next table for 1.25 and 4.65, this is coming out to be 32.6. So you know this is 34.2, so 34.2 is the value. So this 34.2 will be so you have to go for the VC calculation it will be MP into percentage share price. So percentage share price. So MP is 108.40 and percentage share price will be 0.342. 34.2 was the value which was for the value of you know Sigma root two as T as 0.65 and the horizontal value as you know 1.25.

So for that, basically you can get this value as PSP as 0.342, so you multiply that with, and you are going to get dollar 37.11. So this is your; you know calculation of the evaluation of the call, you know, option, with the with this method. You can even calculate this for a put option, and suppose you know if you have to find the; you know put option for this example then in that case, you have the call value.

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Value of the put = ?

$$VP = [VC + CV(SP)] - MP$$

$$CV(SP) = \frac{SP}{(1+r_f)^T} = \frac{90}{(1+.05)^{0.75}} = 86.77$$

$$VP = (37.11 + 86.77) - 108.50$$

$$= 15.38\ \$$$

So that you know, you know, if you have to calculate you know the value of the put, so you know, that we know the value of you put becomes VP VC + CV into SP and then it will be subtracted with the MP. So that is how, you calculate the value of the put. Now in this case, what happens is, so this will be current value of the strike price. So it will be strike price divided by 1 + RF raised to the power T.

So, this will be becoming so strike price is about 90 so 90 will be divided by 1 + 0.05 raised to the power 0.75. So it will be coming to 86.77. So you can find the value of the put and that will be the value of the call that has come to 37.11 + 86.77 and then you will be multiply by differencing, taking the subtract to this market price, that is 108.50 so this is coming out to be 15.38 dollars.

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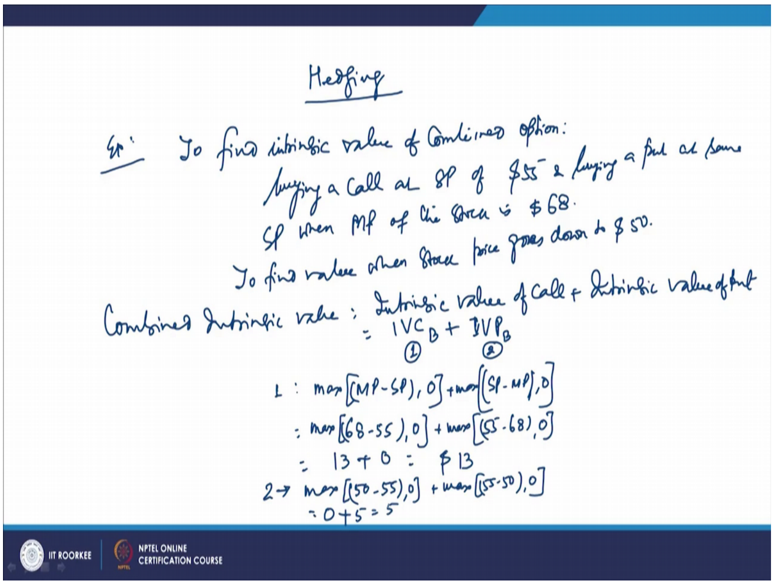

So this way you can calculate the value of the put, as well the value of the call using such you know, methods. Now many a times you have to combine so basically you know one of the best way to protect your assets, against the business, business risk is also the the method of hedging. So you know in that case what you do is you are combining these options and creating a mix of the possibilities and so you have different, you know, so so and then in combined way you are finding the intrinsic values.

So basically we are doing this combining of this intrinsic value of the options and in that basically, we have already studied that you have two ways, one is straddle, another is butterfly. So suppose how so what we do basically we are having the individual you know value is calculated, and then we are basically calculating the final value of the call, or an intrinsic value of the call or intrinsic value of the put.

So coming to the example of you know something we are taking for a straddle say suppose if you have an example suppose you have to find the intrinsic value of the combined option so to find intrinsic value of combined option, so you have you know, two options, I mean I have a combination of the option, which is buying a call so that at strike price of suppose dollar 55 and you know and buying a put at same strike price when market price of the stock is dollar 68.

So you have to find you know the value of so you have to find value when a stock price goes down, goes down to dollar 50. Now suppose so you have to you have the combined option you have to call an option and it is then at the same strike price, that is an example of straddle and how you calculate the you know value of you know this you know call in this case. So value of, you know, combined increasing value how you call, so basically if you try, to call find the combined intrinsic value, so it will be basically the intrinsic value of the call and intrinsic value of the put.

So both of these has to be, are to be added. So this will be intrinsic value of call plus intrinsic value of put. So you know in this case, you have intrinsic value of call, it will be denoted by IVC and then you have intrinsic value of put so IVP, that is. Now this is buying basically so that is why you have B as the subscript which is written here, now if you try to calculate this, so intrinsic value as we can recall it will be so if you this is 1 and this is 2.

Now 1 is basically, so it will be maximum of MP minus SP and zero so that will be and and plus maximum of SP minus MP and zero. So this will be your intrinsic value of the call for the buy and if you take that so it will be maximum of you know 68 MP, your market price is 68. So it will be 68 minus strike price is 55 and then it will be 0 and similarly maximum of the strike price is 55 and - 68 and 0. So it will be 13 and it will be 0. So, that will be dollar 30.

So that is your intrinsic value of the call and intrinsic value of the put if you look at, if so this will be calculated as the maximum value of, so in this case, you know, the stock price has gone down to 50. In that case now it will be 50 - 55 and 0 + maximum of 55 - 50 and 0. So this will be, so if you take maximum of -5 and 0 it will be 0, and if you take the next value it will be 5 + and 0 out of that maximum will be 5.

So you have 5. So basically this way, your, you know the value of the intrinsic value of the call and it is value of put function. So that can be calculated and they can that can be added and the value of the call can be, you know, value of the, value can be calculated in such combined cases. So this is basically, when the strike price is remaining the same, in that case this is the case, when you have the case of straddle and you can calculate the value.

Similar will be the kind of cases of calculation in which you have the case of you know butterfly, so in that case as we know that, you have you know the strike price is different and in that case you know the values are you know, so for the butterfly case, you can calculate even those values. So now with the different strike prices you will have a case of you know butterfly and in that case situation may be that you have to calculate the; you know individual you know values.

And then you have to add them and you can get the value of that you know option. So, this way you calculate the valuation you know you know in the in the case of the options. Thank you very much.