Radiative Heat Transfer
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Module - 5
Lecture - 25
Zone Method

Hello friends, in the previous lecture, we discussed the method of spherical harmonics and discrete ordinate method. These 2 methods are approximate methods and are powerful methods to solve radiative transfer equation in many engineering applications, such as combustion, fire, radiative properties in reentry vehicles and other applications. The limitations we observed in these 2 methods are:

They are subjected to discretization errors as well as they perform poorly under optically thin conditions. In this lecture, we will study the zone method which is an accurate method. The method is similar to net radiation method that we discussed in case of flat surfaces. So, we write energy balance between flat surfaces. Here, the method has been extended, the net radiation method has been extended to include participating media.

So, we will have radiative transfer not just between surfaces, but also between surface and gas. Many things that we discussed, such as view factor while discussing the net radiation method between flat surfaces remains the same. The method is accurate in the sense you can discretize the domain as finely as possible. And you can solve for radiative heat flux and divergence of heat flux from energy conservation point of view. And the results are accurate for all wavelength regions irrespective of optically thick or thin regions of the spectrum.

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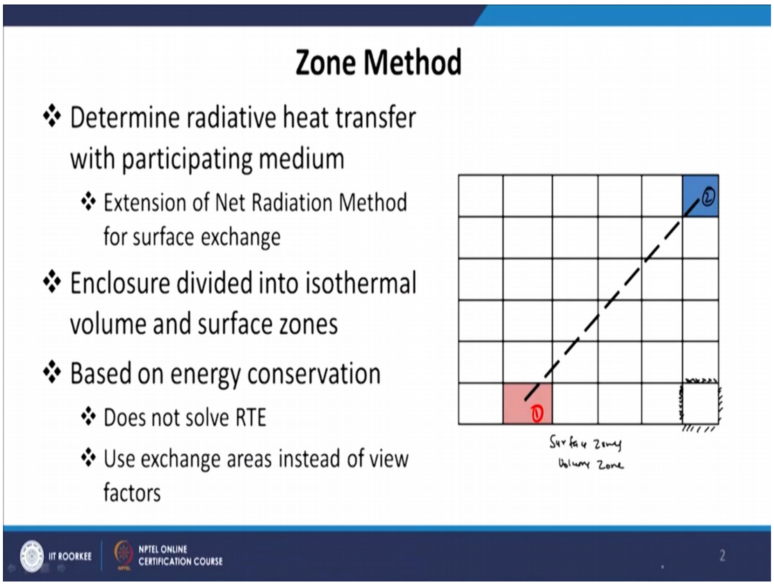

So, let us say we have a medium which is discretized, which is what we typically do. In engineering problems, we divide the domain into small cells, it is called meshing. So, we have divided this volume or area of interest where we want to solve the radiative transfer. We have divided this into number of cells. And we are interest in how much energy is emitted in 1 cell and how much energy is basically absorbed or emitted in cell 2. Okay.

So, we are interested in the overall energy balance between phases of this domain as well as volumes. So, this method is analogous to net radiation method that we discussed, where we are basically calculating the view factors between surfaces. And based on the view factor, we were able to calculate energy transfer between surfaces. Here we have to take into account how the participating media within the domain between the surfaces basically affects the view factor or how the radiation is absorbed between surfaces and volumes.

Now, we will call this, we will call surfaces of these small cells as surface zones and volumes as volume zones. So, each volume in this mesh is called a volume zone. While the surfaces of the volumes are called surface zones. Okay. Now, there are some real walls. These are real surfaces. And there may be some intermediate cell boundaries which are not real surfaces. Okay. So, we have to deal with this imaginary surfaces and real surfaces.

So, the method is based on energy conservation. We write energy balance equation between surface zones and volume zones; how much energy is emitted; how much energy is absorbed and what is the net radiative heat flux. The method can be applied to gray as well as non-gray absorption coefficient. Here, we will demonstrate this method for a gray absorption coefficient. Later on, we will extend our concepts on non-gray properties of the gas.

So, there is no RTE involved. So, we do not approximate on intensity, direction of intensity in any so manner. Okay. So, we will use what we called exchange areas as opposed to view factors that we introduced in the method of net radiation. We will use exchange areas. There is slightly different properties of these exchange areas.

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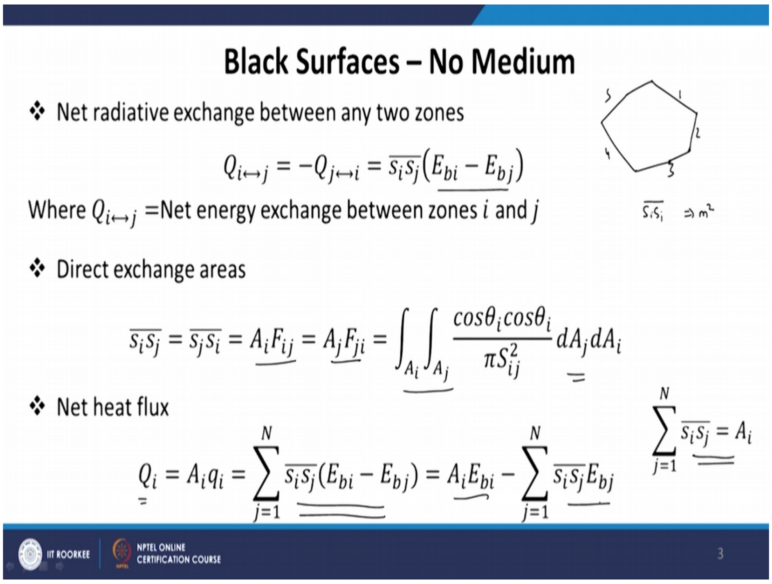

So, let us understand how it works. First, the method is demonstrated to a problem where we do not have participating media. Just for the purpose of understanding the basics of this method, we assume that there is no medium and we write energy balance between any 2 surfaces i and j. So, we have an enclosure. This could be divided into number of volume zones or it may not be divided into number of volume zones.

So, we have an enclosure, where we have different surface zones: 1, 2, 3 and 4 and so on. We are interested in finding the energy transfer between 2 surfaces i and j. So, Q i j is = – Q j i is = s i s j E bi – E bj. So, this is similar to what we did in a net radiation method. We write energy transfer between surfaces as difference between their emissive power, that is E bi – E bj.

And we, they are multiplied by a view factor and area. Here we are combined the view factor and area together and we call it exchange area. So, s i s j is defined as exchange area or direct exchange area between surface i and j. So, direct exchange area means, energy transferred from surface i and directly transmitted or absorbed by surface j without encountering any reflection. When we have participating media, this s i s j will be subjected to absorption but not scattering.

We will see how this will be affected when we have a participating media. But for the time being here, s i s j is direct energy transfer from surface i to surface j and the units of s i s j as opposed to view factors is meter square. So, s i s j is having units as meter square as opposed to view factor which was dimensionless. So, we defined s i s j. It is just related to the view factor A i F ij and A j F ji. And it obeys the reciprocity rule.

And it will be double integral over A i A j cos theta i cos theta j pi S ij square d A j d A i. So, this is something we have already done. Okay. This is same expression for the view factor that we developed earlier. Now, just multiplying by the area we call it direct exchange area. So, for black surfaces, this is the direct exchange area. When we have participating media, the expression for the direct exchange area will change.

We will see how it will change. But for no medium, this is same as we did in the previous lectures. Similar to the summation rule, the summation rule applies to exchange areas also. So, summation for all the surfaces j is = 1 to N s i s j is simply = area A i. And net heat flux on surface i contains a surface exchange with all other surfaces. So, we write summation j is = 1 to N s i s j E bi – E bj.

So, that is, radiative exchange between surface i and j. And we have integrated over all the surfaces j. This can be written, this, we can take E b item separately also. So, we write summation s i s j E bi as simply A i E bi. And then, – j is = 1 to N s i s j E bj. So, both the expressions are equivalent. And we can use this expression to find out radiative transfer from surface i.

So, at this point there is no difference as such between the net radiation method and the zone model. The only thing is, that we are representing the view factor in terms of a exchange areas in this method.

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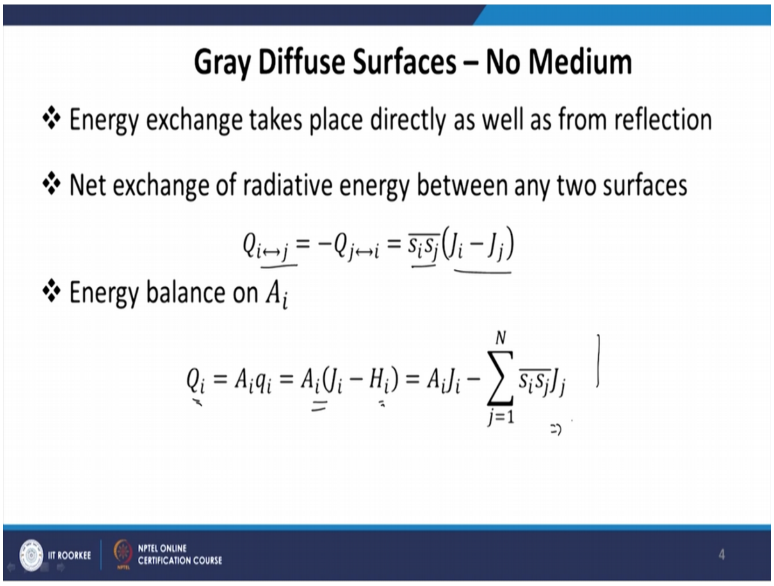

For gray surfaces, again we have same thing. Here we have radiosities in terms of in replace of emissive power. In black cases we have emissive power. Here in gray cases, we have radiosity. We can write down net exchange of radiative energy between any 2 surfaces as the difference between their radiosities J i – J j and the exchange area s i s j. And similarly, the flux on surface Q i is simply = A i q i and energy emitted.

That is A i J i by emission and reflection and energy absorbed. And this will be equal A i J i – summation j is = 1 to N s i s j J j. Now, 1 thing you should observe here from this expression is, the emittance has been absorbed in exchange area itself. That is, s i s j now contains the effect of emittance also. So, that has been absorbed in this expression.

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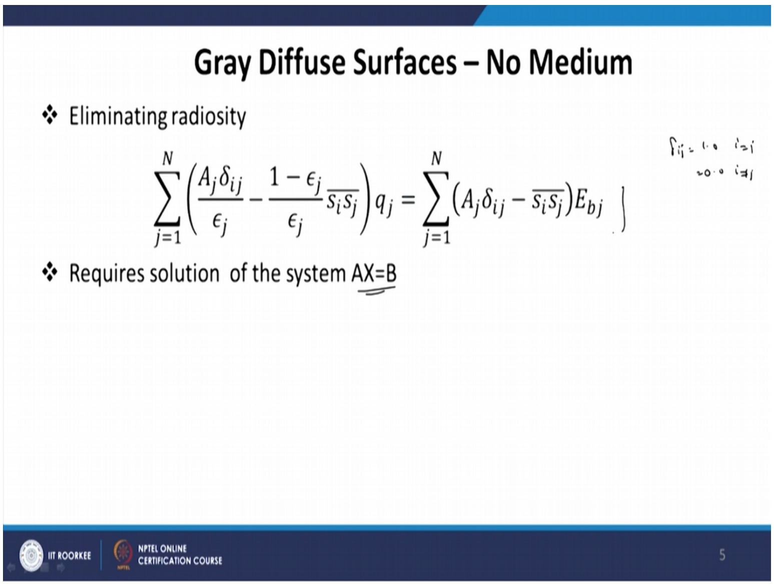

Now, if we eliminate the radiosity from this expression, we can write down the heat flux or the governing equation for gray enclosure as summation j is = 1 to N A j delta ij, where delta i j is simply = 1 for i is = j and is = 0. For i nought = j, it is Kronecker delta. And we can write down the expression as A j delta ij upon epsilon j – 1 – epsilon j upon epsilon j s i s j. And on the right-hand side, we have the emissive power for the surfaces for which temperature is known.

So, in this way, we have written our governing equation for radiative transfer between gray surfaces. And this gives you a system of equations which can be solved using any method for the solution of the linear system AX is = B. So, all these things, we have discussed earlier. The only thing is, we have rebottled our expression in terms of exchange areas rather than the view factor.

Now, we introduce another concept which is used in zone model. And that is the concept of total exchange areas. Now, this is different from what we done earlier in the case of a view factor and net radiation method. Here, what we do is, we define the energy transfer between any 2 surfaces as;

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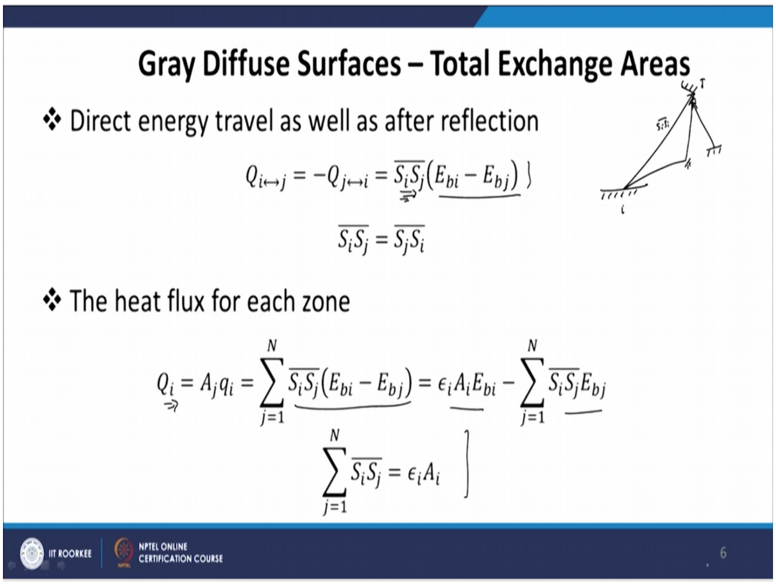

Capital S i S j, which is called total exchange area, E bi – E bj. Now, what is S i S j? S i S j is defined as the energy transfer from i to j directly, as well as after reflection from other surfaces. So, now S i S j contains all the reflections also. So, let us say we have surface i. This is surface j. And there are some other surfaces also. So, i to j, S i S j capital S i S j includes direct contribution. That is s i s j, small s i s j + reflected part also from other surfaces.

So, you can see that, although S i S j is related to small s i s j; that is, total exchanged area is related to direct exchange area, but there are additional components due to reflection. So, the good thing about this method is that we can directly calculate the radiative energy transfer by just taking the difference between emissive power and multiplying by the direct total exchange area. We do not have to solve for this equation again and again.

Again, I will like you to understand the advantage of this method is; suppose s i s j that is direct exchange area is not changing. In many problems, what you have to do is, you have to solve the problem iteratively. For example in combustion, you are solving a problem iteratively. And there, the temperature of the chamber, temperature of the domain is changing. So, the exchange area is not going to change.

So, we do not want that we should solve the linear system of equation AX is = B again and again. If we just have to find temperature and we know that the exchange areas are not changing, we should not solve for this equation again and again. So, what we do is, we have calculated total exchange area once and for all. And then just calculate the emissive power for the new iteration and multiply by the total exchange area.

And we can directly calculate the radiative transfer between surfaces i and j. So, we do not have to solve for the, this matrix AX is = B, we do not have to solve. So, for such problems, total exchange area is very good. We can calculate this total exchange area matrix once and then do simple calculation. And this will give you more efficient way of solving the problem. So, in terms of total exchange areas, we can define radiative heat flux or radiative transfer from surface i as summation j is = 1 to N S i S j, that is totally exchange area multiplied by the difference in emissive power.

And this can also be written similar to the total exchange area as epsilon i A i E bi – summation j is = 1 to N S i S j E bj. And total exchange area now follows S i S j is = epsilon i A i. So, this is very similar to the concept of direct exchange areas. The only thing is, the reflection part, the reflected edition is taken into account in the exchange area itself. And we do not have to solve for the matrix again and again.

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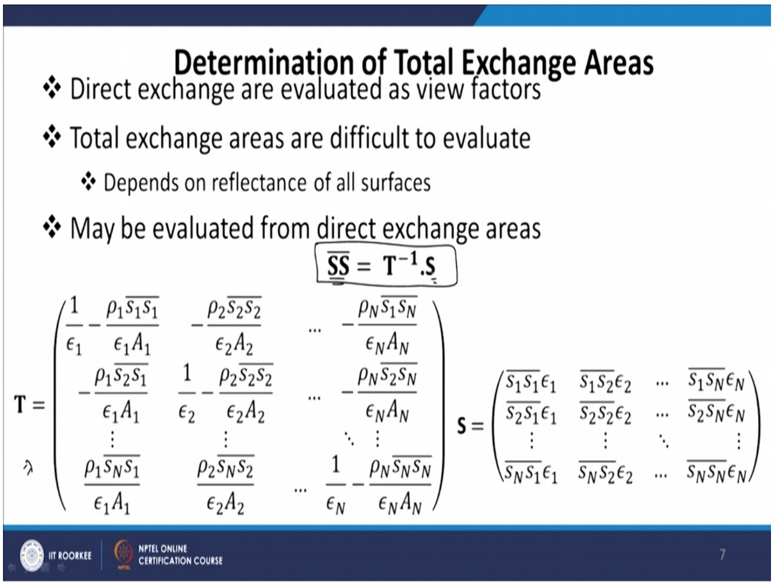

Now, how to evaluate this total exchange areas. So, direct exchange areas are evaluated as view factors. So, there is no difference. Because direct exchange areas are directly related to the view factors. However, the calculation of total exchange area is not straight forward, because it depends on reflections from other surfaces. So, total exchange areas cannot be calculated in an any easy manner. Okay.

But, we can make use of energy balance and we can derive equations to find out total exchange areas in terms of direct exchange areas. So, let us say the derivation of this is given in the textbook, but I will just summarize the result. So, let us say SS represents the matrix or vector of total exchange areas and S represents the matrix of direct exchange areas. And we have a matrix T.

Then, using this relation SS is = T inverse S, we can convert direct exchange areas into total exchange areas. This we will demonstrate with the help of an example. So, the S contains direct exchange areas and T also contains direct exchange areas. When the inverse of T is multiplied by S, we can convert this into total exchange areas. And we will demonstrate it with the help of an example.

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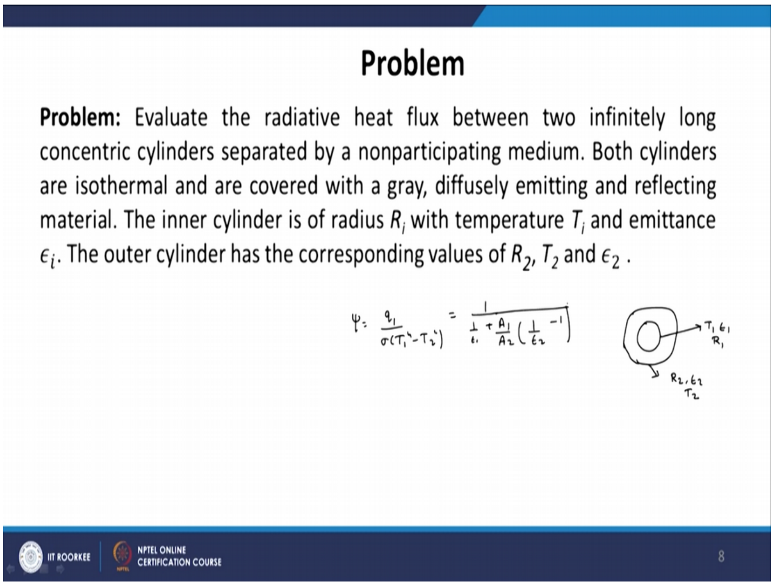

So, let us say we have to evaluate radiative heat flux between 2 infinitely long concentric cylinders separated by a nonparticipating media. So, we have concentric cylinders. This problem we have already solved using net radiation method. I will give you the result. And both the cylinders are isothermal at temperature T 1 and T 2. Okay. So, the inner cylinder has temperature T 1 emittance epsilon 1, radius R 1.

Outer cylinder is R 2, epsilon 2 and T 2. Okay. So, we have to find out the radiative heat flux between these 2 cylinders. And the result, let me just tell you the result, because we have already solved this problem. So, you can recall that the non-dimensional heat flux for this case on the inner cylinder can be written as = 1 upon epsilon 1 A 1 by A 2 epsilon 2 – 1. So, you may recall this relation we have found this result before also.

So, 1 upon 1 upon epsilon 1 + A 1 by A 2 multiplied by 1 by epsilon 2 – 1. So, this was the result we calculated using the net radiation method and view factors. Now, the same problem, we will solve using the total exchange area.

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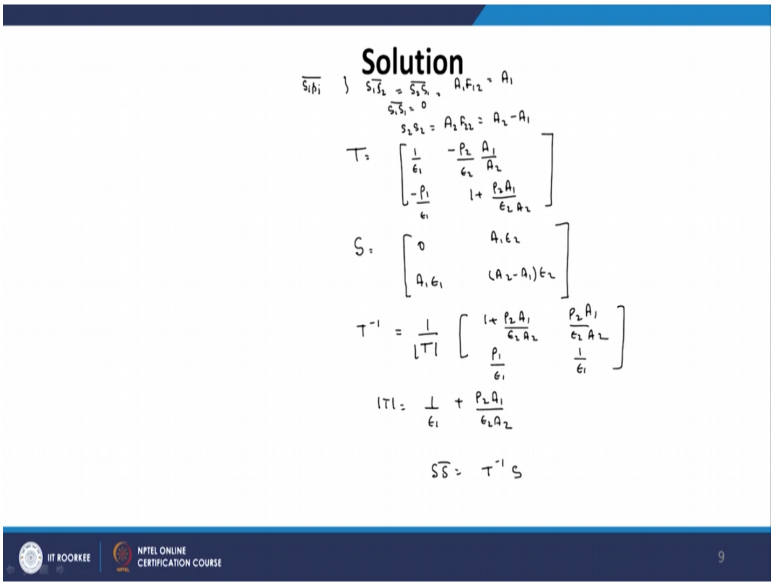

So, let us observe certain things and then we will solve. So, for this particular configuration, let us write down the exchange areas s i s j. Okay. So, we have S 1 S 2. That is, to a direct exchange area between 1 and 2 as = S 2 S 1. Okay. This will be simply = A 1 F 1 2. And F 1 2 is simply = 1. So, this will be = A 1. So, S 1 S 2 is = S 2 S 1 is = A 1. Okay. Now, S 1 1 because of convex geometry is = 0.

And using the summation rule, we have S 2 S 2 as; outer cylinder is concave, so S 2 S 2 is simply = A 2 F 2 2 is = A 2 – A 1. Using the summation rule, we get A 2 – A 1. So, we have calculated the direct exchange areas. So, now let us see how we can evaluate the total exchange areas using the matrix notation. So, the matrix T is = 1 by epsilon 1 – rho 2 epsilon 2 A 1 by A 2 and – rho 1 by epsilon 1 1 + rho 2 A 1 epsilon 2 A 2.

This is the T matrix. And similarly, we have matrix S as 0 A 1 epsilon 2 A 1 epsilon 1 A 2 – A 1 epsilon 2. Now, we have to find out total exchange areas. So, total exchange areas can be calculated. First, we have to find out the inverse. T inverse will be = 1 upon the determinant of T, 1 by T. And this will be = 1 + rho 2 A 1 epsilon 2 A 2 rho 2 A 1 epsilon 2 A 2 rho 1 by epsilon 1 and 1 by epsilon 1.

So, this is the T inverse. And determinant of this T matrix is simply = 1 by epsilon 1 + rho 2 A 1 epsilon 2 A 2. Okay. So, we have calculated the T inverse. Now, we will use this relation SS is equal T inverse S. So, SS is = T inverse S. So, this relation we will use to find out the total exchange area.

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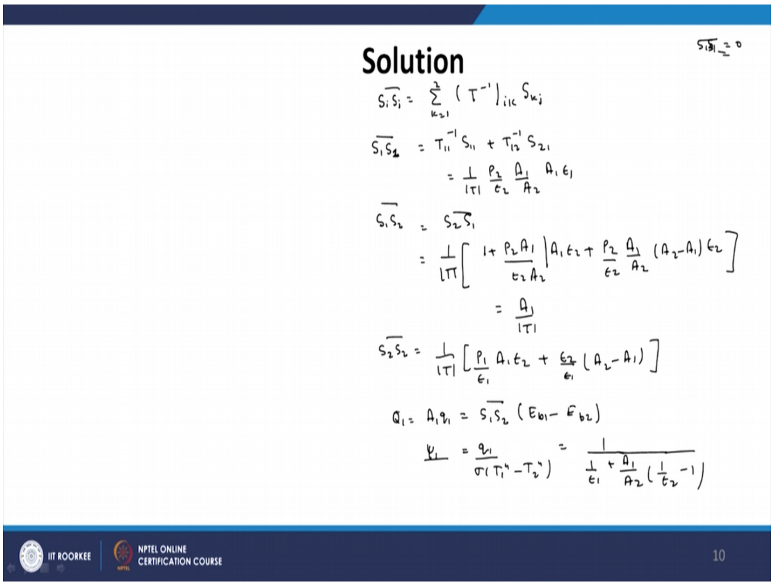

So, S i S j is simply = summation K is = 1 to 2. T inverse iK S Kj. We have to multiply this 2 matrices T inverse and S matrix. So, I have written the expression for the total exchange area S i S j in terms of product. So, there will be 2 terms, K is = 1 to 2 T inverse iK and S Kj. So, from this rule, S 1 S 2; that is, totally exchange area S 1 S 2 is = T 1 1 inverse S 1 1, where T 1 1 inverse is the element number 1.

First row, first column of the T inverse matrix and S 1 1 is the first element of the S matrix. + T 12 S 21. And this will be simply = 1 by T magnitude rho 2 by epsilon 2 A 1 by A 2 times A 1 epsilon 1. Okay. And similarly; sorry, this will be S 1 S 1. This is S 1 S1 and this will be S 1 S 2 which will be = S 2 to S 1. Because of symmetry, this will be = 1 by T 1 + rho 2 A 1 epsilon 2 A 2 A 1 epsilon 2 + rho 2 epsilon 2 A 1 upon A 2 A 2 – A 1 epsilon 2.

And on simplification, this will be simply = A 1 upon T. And the last thing is S 2 S 2. This will be = 1 by T rho 1 by epsilon 1 A 1 epsilon 2 + epsilon 2 by epsilon 1 A 2 by A 2 – A 1. So, we have calculated all the total exchange areas using this system of matrix. 1 thing you should note is that, as opposed to direct exchange area S 1 1 S 1 1 which was = 0, the capital S 1 S 1 is not 0. Because now it includes reflection also.

So, the inner cylinder, cylinder 1 will not receive any radiation from itself directly. But it will receive radiation after reflection from the outer cylinder. So, that is why, the capital S 1 S 1 is not 0. That is, total exchange area from 1 to 1 is not 0. So, the heat flux Q 1 is = A 1 q 1 is = S 1 S 2 E b1 – E b2. Okay. And this will be = non-dimensional q 1 upon sigma T 1 power 4 – T 2 power 4 is = 1 by epsilon 1 A 1 by A 2 1 upon epsilon 2 – 1. Okay.

So, this is the expression for the radiative heat flux for the first surface q 1. That is the net radiation transfer between surface 1 is = A 1 q 1 is = S 1 S 2 E b1 – E b2. And substituting the expression for S 1 S 2 we get the result which we discussed earlier also. And we get the same result. Although the calculation time here is much more involved and we have to go through complicated algebra, but the purpose was to demonstrate.

The concept of total exchange areas may not appear useful to you at this moment. But when we have to solve radiative transfer problem again and again; for example, in combustion applications, we would like to avoid any repetition of calculations that typically we encounter while using the direct exchange areas. So, at that moment the use of total exchange areas will be much useful. Now, how to extend; so, the basic idea of zone method was to apply the net radiation method or extend the net radiation method to absorbing emitting medium.

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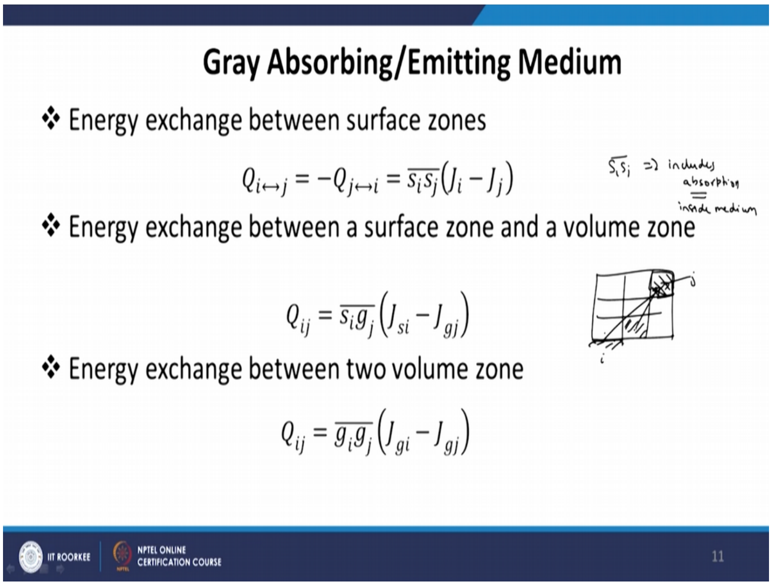

So, how do we extend it? This method. So, we write energy exchange between surface zones as before. So, Q i j is = – Q j i is = s i s j J i – J j. This is as before. So, the net radiation, net radiation exchange between surface i and j is written similar to what we did before. The only thing is, now s i s j, that is the direct exchange area includes absorption and emission. Okay. So, direct exchange area is basically includes absorption within the medium.

So, there will be in absorption in side medium. Okay. We will discuss this in the next lecture, how this is calculated. In addition to surface-to-surface exchange, we write surface-to-gas zone exchange. So, Q ij is energy transfer between surface zone i and gas zone j, which is written as the product of s i g j. Where s i g j is direct exchange area between surface and gas. Surface is i and gas is j.

So, direct exchange area, suppose we have a domain. This is surface i and this is gas volume zone j. So, we are basically trying to understand how much energy from this surface is absorbed in this cell. So, s i g j represents the direct exchange of radiative energy between surface i and gas zone, volume zone j. J si is the radiosity of the surface and j underscore J subscript g represents the radiosity of the volume zone g.

Now, energy exchange between 2 volume zones can be written in similar fashion. So, let us say we have 1 volume zone this. So, between these 2 volume zones, how much energy is basically exchanged. We can write down this as Q ij is = g i g j, where g i g j is basically volume to volume direct exchange area. And J gi – J gj is the radiosity of the volumes i and j respectively. We can write down the energy balance for a surface zone. So, surface receives radiation from other surfaces.

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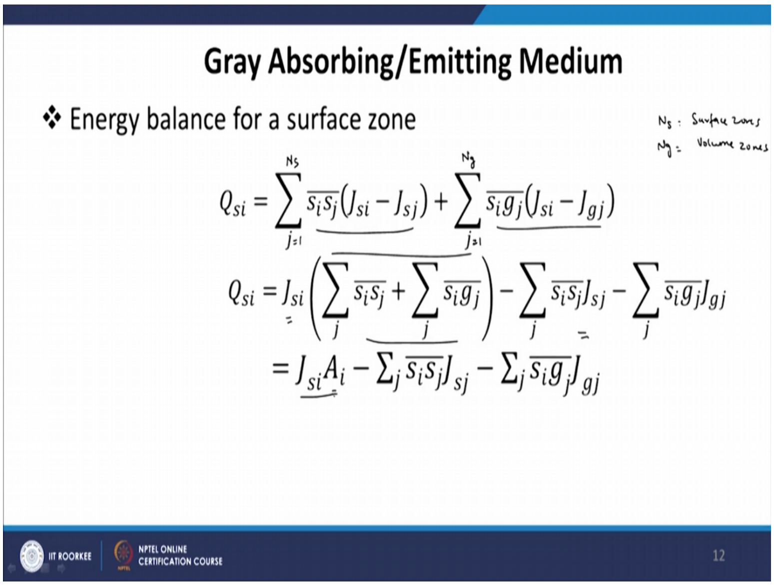

So, direct exchange area s i s j between surfaces and the difference in radiosity. So, that is surface-to-surface radiative exchange. Then surfaces also receive radiation from the volumes. So, we write down s i, summation over all the volume zones s i g j, J si J gj. Here, I will just like to point out that we have to, sum j is = 1 to N s, where N s is the number of surface zones. And here, j is = 1 to N g, where N g is number of volume zones.

N s is surface zones and N g is volume zones. Okay. Now we write this, we simplify this expression. And we can write down Q si is = J si. By combining the first terms in the 2 summations, s i s j + s i g j. And then, we have termed summed over j s i s j J sj – sum j s i g j and J gj. And there is a equality that we will discuss in the next lecture. The summation over s i s j, that is the summation of surface-to-surface direct exchange area + surface-to-gas direct exchange area is simply = A i.

And this comes from, directly from energy conservation, that all the energy from surface i is either going to surface j, all the surfaces j or it is going to the gases. So, the energy is conserved. So, that is why the summation in this, the 2 terms in the bracket basically add up to A i. And we can write down this as J si A i. And the rest 2 terms remains the same. So, this is the final expression for energy transfer from surface zone i.

Same type of energy balance, we can do for a volume zone. So, for a volume zone, Q gi, that is energy transfer from volume zone i can be taken as energy transfer from surface-to-gas and between gas-to-gas.

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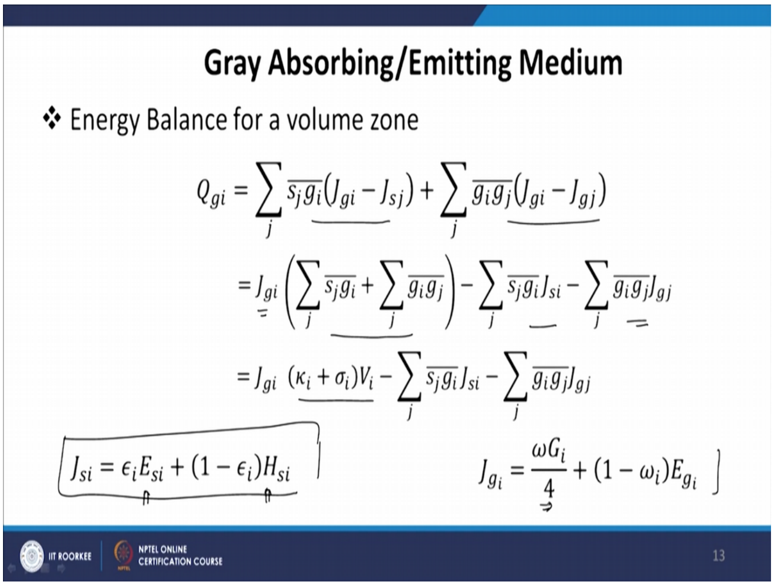

So, we have surface emitting and absorb radiation absorbed by the gas zone i. And we can have energy emitted from a different volume zone and absorbed by the volume zone i. Now, again we simplify this expression, taking the first 2 terms and we write down in terms of radiosity of the volume zone i. s i g i + summation j g i j j. And then, summation over the surfaces and summation over the volumes.

Now, we will again make use of energy conservation. All the energy from a volume zone is either going to a surface J or it is going to another volume g j. So, energy is conserved. And this total amount of energy is basically given by beta. That is, extinction coefficient times V i multiplied by the radiosity of the volume zone. So, this is the total amount of energy emitted by the volume zone by emission and as well as scattering.

So, J gi kappa I + sigma i Vi represents total amount of energy emitted by a volume zone by emission as well as scattering. So, these 2 equalities we can basically prove J si is = epsilon i E si where surface radiosity is basically emission part and reflection part. We use this expression before also, for surface radiosity. Similarly, we define gas radiosity J gi as the amount of scattered radiation.

So, that is amount of energy scattered by the volume. So, that will be part of the volume radiosity + emitted radiation. So, just like a surface leave, radiation leave a surface by either emission or reflection. Radiation leaves a volume by either emission or by scattering or reflection. So, we have omega G i by 4 where G i is the incident radiation and omega is the single scattering albedo. So, we can define a volume radiosity in this manner.

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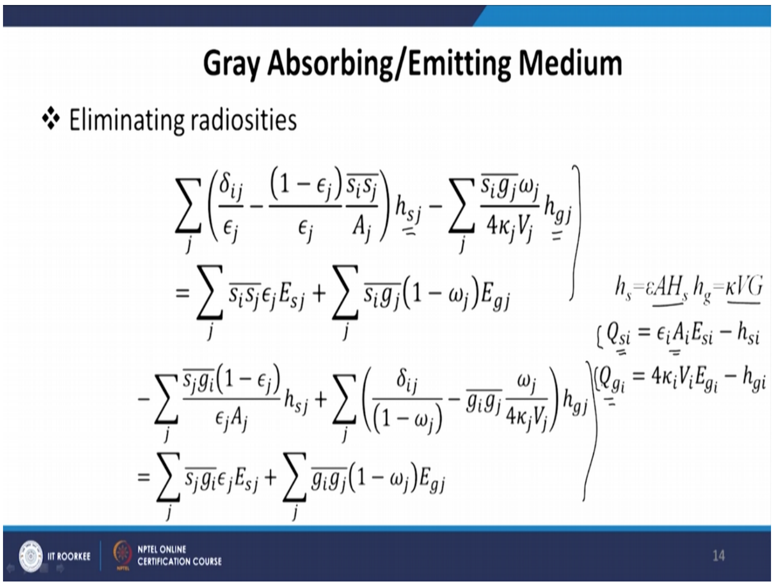

So, now simplifying eliminating the radiosity in this expression, we get 2 equations. So, these are the 2 equations we get. We will discuss these equations in more detail in the next lecture. Where now h sj and h sg h gj represents the amount of energy transfer from a surface and volume zone respectively. Okay. So, Q si is = emission from the surface epsilon i A i E si – h si. And Q gi is = 4 kappa i V i E gi – h gi. So, h si and h gi are 2 unknown variables which have been introduced here.

h s is epsilon AH s and h g is kappa times V times G. So, for 2, for these 2 variables we will solve these 2 equations. And once we have solved these 2 equations, we can calculate the radiative heat flux at the surface and radiative transfer from the volume using this expression. So, this gives a system of linear equations which we have to solve using some kind of matrix method.

And after solving this, we will get an expression, we will get the values of h s and h g. And substituting this values of h s and h g, we can calculate the radiative heat flux Q si and radiative transfer from the volume Q gi. We will discuss again this, in the next lecture this how these equations are derived and how to solve it. So, thank you for your time. We will discuss the direct surface-to-surface exchange areas, surface-to-gas exchange areas and gas-to-gas exchange areas.

And how the governing equation is derived for zone method for participating media. And how we can solve for this in a system of linear equation, how this can be implemented on a computer. We will discuss this in the next lecture. Thank you.