Radiative Heat Transfer
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Lecture - 16
Radiative Transfer in Participating Media

Hello friends. In this course on a radiative heat transfer. So far, we have discussed radiative heat exchange between surfaces and bounded by vacuum. So what do we mean by participating media that we are going to discuss in this lecture is that when radiation travels from one surface to another the medium which may be gas particles may interact with the radiation.

Now interaction may be of multiple forms the radiation may be absorbed by the media, radiation maybe scattered or the medium itself may emit radiation and augments the intensity of radiation. So the analysis of this type of problem where we have participating media is going to be much more challenging and interesting as well. Although the problem of radiative heat transfer between surfaces without any media was complicated enough.

But together with this participating media the analysis is going to be much more difficult and the analytical solutions would be very difficult to obtain. So most of the models that we will discuss in this course will be approximate models and we will also introduce some numerical methods to solve problems including the participating media. So the participating media the problem is very, very interesting.

We see a number of applications where we have gases and particles that absorb and emit radiation. For example, we have spacecraft flying it high altitudes or returning from deep space missions the velocity is very high. The velocity maybe in kilometer per second and the spacecraft are subjected to extremely hot environment. This leads to ionization and disassociation of gases at very high temperature.

Radiative heat transfer is the dominant mode of heat transfer in this case. Similarly, we have fires, the flames and combustion applications where we have different type of gases and particles radiating the white color basically comprises of wavelength of different wavelengths while we have typical yellow flame because of radiation coming from soot particles. So soot particles are basically small carbon particles which radiate energy like black-body.

And the radiation that appears from these types of flames which we have soot content will be yellowish in nature. We call it luminous flames. On the other hand, there are some flames which appear blue in nature.

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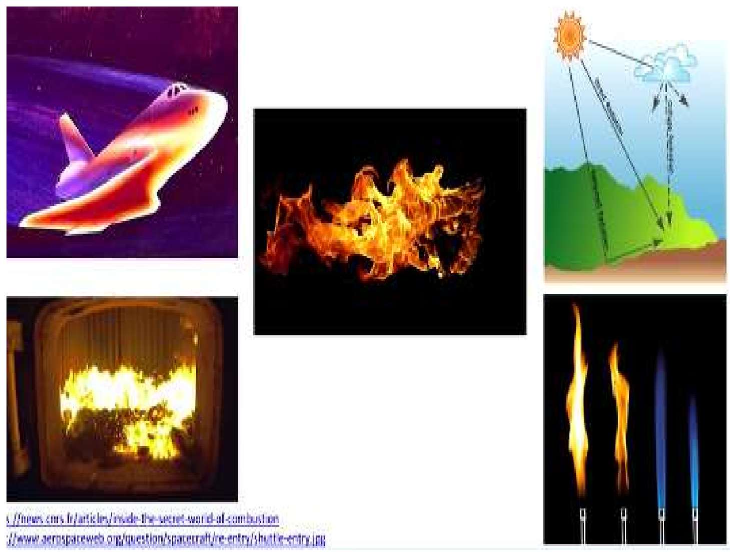

Now the color from this flames is a result of some radical of methane and Carbon monoxide and some other compounds like C_2 which have bluish and greenish color of the flame. We also find participating media in atmosphere where solar radiation travels through clouds, water vapor, suspended water vapor in the atmosphere and other gases. So this radiation solar radiation is observed by the atmosphere as well as it undergoes scattering. So all this problems require modeling of participating media in radiative heat transfer.

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Radiative Exchange in Participating Media

- Interaction of thermal radiation with an absorbing, emitting and scattering medium
- Examples :
  - Burning of fuel (gaseous, liquid or solid),
  - Rocket propulsion
  - Hypersonic shock layers
  - Ablation around reentry vehicles
  - Plasmas in fusion reactors
  - Absorption and scattering of solar energy
  - Fog: reflection by water droplet and back scattering

So there are as I said the medium the participating medium may absorb radiation. It may scatter radiation as well as it can lead to increase of intensity by emission. So large number of applications find radiative heat transfer in participating media such as combustion, rocket propulsion, hypersonic shock layers, ablation, fusion reactors and also in atmospheric sciences.

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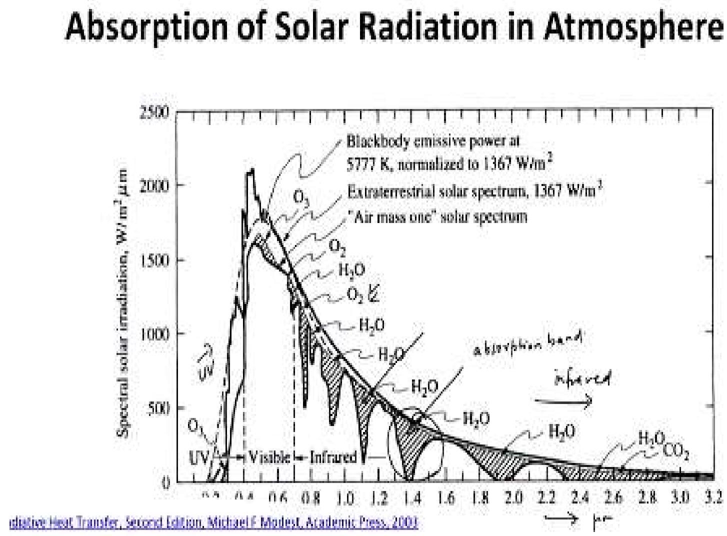

So we start by looking at the solar spectrum because solar spectrum is most widely studied topic in radiative heat transfer in participating media. So if we look at the solar spectrum outside the earth atmosphere that is extraterrestrial solar spectrum as received from sun, but not encountering the atmosphere. The solar spectrum behaves like a black body okay at 5770 Kelvin okay and the total heat flux that we receive and on top of the atmosphere is 1367 watt per meter square.

Now this radiation is extraterrestrial, it is not subjected to absorption and scattering by molecules and particles in the atmosphere. When the same radiation penetrates our atmosphere it is subjected to absorption and scattering by a number of gases and the most important gas is that absorb solar radiation are basically water vapor and carbon dioxide which are in major amount in our atmosphere.

So we see that the radiation is absorbed that means the intensity of this solar spectrum does not follow any more the black body spectrum. We find that the intensity has decreased at a number of wavelengths. The wavelength on the x axis is in microns. So the significant absorption we find is in ultraviolet which is due to ozone we all are familiar with ozone layers that absorbs harmful ultraviolet radiation.

Then in infrared we have mostly this is infrared region where water vapor is important gas that absorbs solar radiation and then we have absorption by oxygen molecules and carbon dioxide as well. The one thing that you should observe here is that the absorption is not uniform throughout the spectrum, somewhere the absorption is high somewhere the absorption may be low.

These type of absorptions patterns are called bands okay. So each absorption pattern is basically called a absorption band okay and we will discuss when we discuss the molecular radiative properties of molecular and atomic gas is how these bands basically are formed. But one should keep in mind that the absorption by gases is not uniform it is centered around certain bands and these bands for the solar spectrum are located in visible and infrared design. While ozone absorbs radiation in the ultraviolet part of the system.

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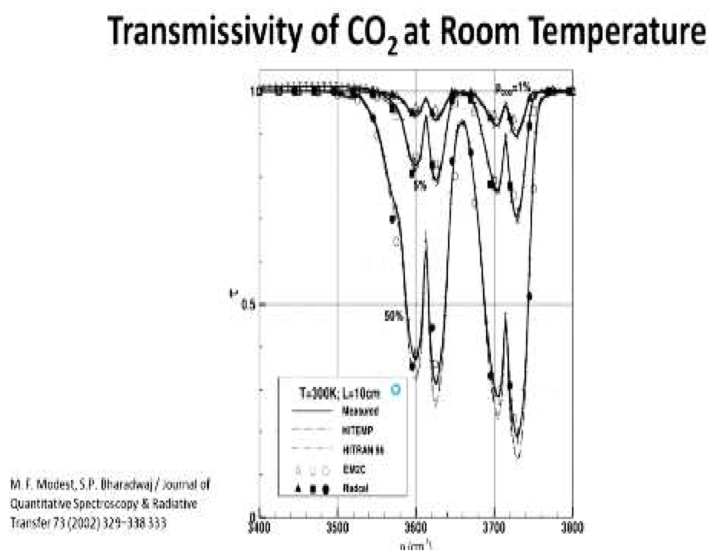

Lot of interest has been in the transmissivity of carbon dioxide which is a major pollutant also in large number of combustion applications. So CO2 has been of great interest not only in atmospheric sciences, but also in combustion because carbon dioxide is a major absorbing gas of radiation in atmosphere where the temperature is low but as also in combustion where the temperature is high.

So lot of measurements experimental studies have been done on CO2 transmissivity at low and high temperatures both low resolution and high resolution. In this plot taken from Modest and Bharadwaj the x axis is in wave number this is another way of representing the color or wavelength or the frequency a wave number and on the Y axis is the transmissivity and again we see that there are certain bands of CO2 which has low transmissivity value and certain bands which have high transmissivity value.

Now the plots obtained or plotted in this figure have been taken. The data for this plot have been taken from number of standard resources like HITEMP and HITRAN 96. These 2 are data basis which have large number of spectral data that means information to calculate the transmissivity of the gas at the function of wavelength the data has been corrected by a number of researchers over a period of many years at different resolutions, different temperatures.

The data contains both experimental as well as theoretical data so we will see how this data can be used to calculate the properties of gases not just CO2. The data base has large number of gases and we can calculate the transmissivity of these gases using these data base.

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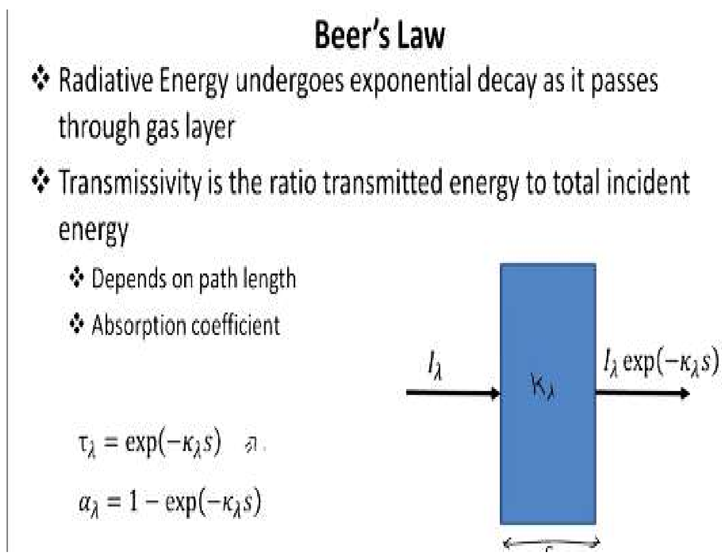

So we start our analysis of participating media with the well-known Beer's Law. So Beer Law says that when a radiation of wave of photons enters a medium. It undergoes exponential decay and exponential decay depends on 2 quantities one is the length of the path travelled and the property of the gas kappa lambda that is called absorption coefficient. So amount of energy absorbed within the medium depends on the path travelled.

And the absorption coefficient of the medium. So the transmissivity is defined as exponential of $-k_\lambda S$ where $k_\lambda$is the absorption coefficient and S is the path travelled so that is the Beer's Law and similar to the properties of surface is the absorptivity that means amount of radiation absorbed by this gas layer is defined as 1- transmissivity which is 1-(expoenential–$k_\lambda S$).

So Beer's Law has found many applications in chemical engineering where the concentration of solution is found by passing some radiation through a solution and based on how much radiation is absorbed by the solution one can calculate the concentration of the solution.

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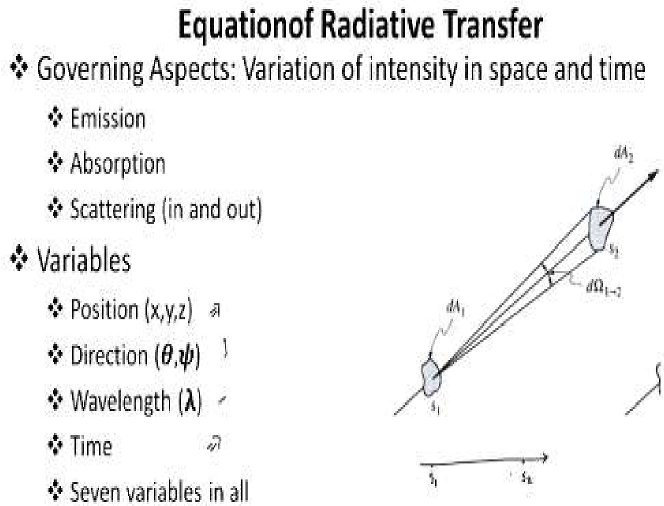

So for a detailed analysis of radiative heat transfer in participating media we will develop the equation of radiative transfer. So equation of radiative transfer basically tells how the intensity of a single wave travelling within a solid angle small solid angle $d\Omega$ undergoes emission, absorption and scattering. These 3 modes basically alter the intensity of the radiation as it passes through space.

The variables in this problem we have total 7 variables in this problem the intensity will vary in space as a function of x, y and z. The intensity also depends on two angle, polar and Azimuthal as we have already discussed and it is a function of wavelength and also time. So intensity at two location let us say S_1 and S_2 the distance between S1 and S2 maybe of the order of meters for typically combustion applications.

It may of the order of kilometer if you are talking about atmospheric sciences the time requires for radiation to reach from S_1 to S_2, but there are certain applications as we will see where time maybe a parameter and important parameter especially lizards of femtosecond and nanosecond where transient radiation modeling maybe of importance.

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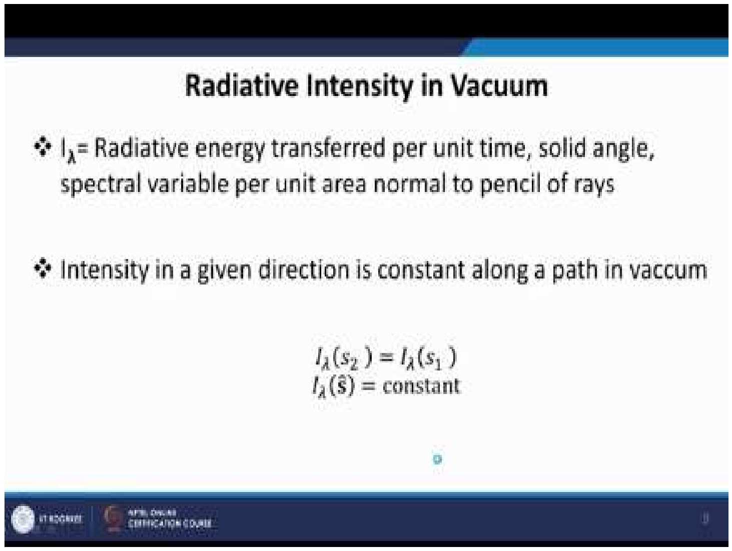

Before we start writing the equation of radiative transfer for participating media. We recapitulate the definition of intensity so radiative intensity is basically defined as energy transferred per unit time per unit solid angle per unit area normal to pencil of rays and spectral intensity also has per unit wavelength. So this is a definition we have already discussed and we will see how this intensity changes over space and time.

Intensity in vacuum because in vacuum there is no participating media so intensity will remain constant. The intensity will not suffer absorption, radiation will not suffer emission from any gas or particles and there will not be any scattering. So under the absence of participating media the intensity will remain constant while in participating media the intensity will continuously change in space and time because of these 3 factors absorption, emission and scattering. So we will take this factors one by one.

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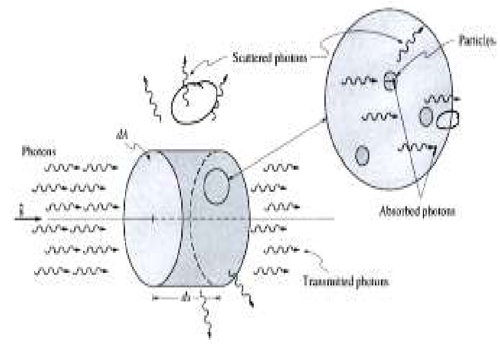
Attenuation by Absorption and Scattering

- Participating medium:
- Intensity attenuated by absorption and scattering



Source: Radiative Heat Transfer, Second Edition, Michael F. Modest, Academic Press, 2003

The first one we will take is absorption in scattering. Now absorption and scattering may lead to attenuation, attenuation or decay of intensity. So intensity will decrease because of absorption and scattering okay. In this schematic you see that large number of photons are travelling through this medium which has thickness ds and cross section area dA and the direction is *S*. Now when this photon encounter participating media some of the photons are absorbed within that medium and some photons are basically scattered.

So these photons are scattered away so they definitely will result in decrease in intensity and some photons are basically absorbed within the medium and definitely this will increase the energy of the medium. So the absorption of radiation within a medium will increase the internal energy of the medium and that intensity will decrease.

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Attenuation by Absorption

- The absolute amount of absorption is directly proportional to magnitude of incident energy as well as the distance travel through the medium

$$dI_\lambda)_{abs} = -k_\lambda I_\lambda ds$$

$dI_\lambda|_{abs} \propto -I_\lambda s$

$= -k_\lambda I_\lambda ds$

Where k_λ = linear absorption coefficient (measure of attenuation)

$-ve$ sign as the intensity decreases

- **Mass and pressure based absorption coefficient for gases**

$$dI_\lambda)_{abs} = -k_{\rho\lambda} I_\lambda \rho ds$$

$k_\lambda = k_{\rho\lambda}\rho$

$$dI_\lambda)_{abs} = -k_{p\lambda} I_\lambda p ds$$

So we defined change in intensity dI_λ due to absorption. So $dI_\lambda$is changed in intensity due to absorption. So we will write separately change in intensity dI_λ for absorption and scattering. So this is proportional to the incoming intensity and the path travelled ds. So the change in intensity of radiation when it travels through a small path ds depends on the incoming intensity and we can replace this proportionality sign with a constant this becomes $k_\lambda I_\lambda ds$.

So change in intensity of radiation as it passes through a small participating media of thickness ds=$-k_\lambda I_\lambda ds$. k_λ is defined as linear absorption coefficient which is a measure of attenuation. Now in many combustion application as well as in atmospheric sciences linear absorption coefficient is not preferred. So in these areas mass based absorption coefficient is preferred.

So mass based absorption coefficient is nothing but kappa lambda=kappa rho lambda times rho. So the units basically we have changed we have added another parameter to define the absorption coefficient similarly there is a pressure based absorption coefficient we can define as kappa p lambda times p okay this is pressure based absorption coefficient. So when you are dealing with different type of absorption coefficient you keep in mind the units of the absorption coefficient.

Because the units of linear absorption coefficient is different than pressure based absorption coefficient and when we are using pressure based absorption coefficient you have to multiple by the pressure of the medium.

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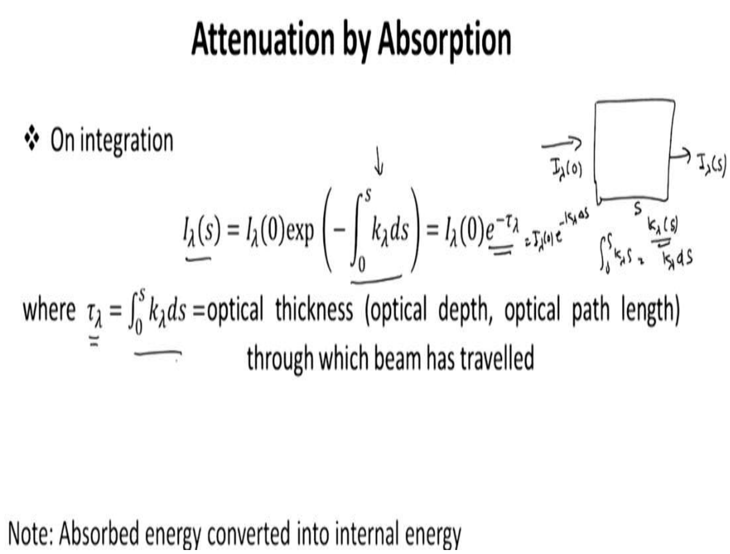

We can integrate this equation we can integrate this equation and we find that $I_\lambda(s) =$

$I_\lambda(0)\exp(-\int_0^s k_\lambda ds) = I_\lambda(0)e^{-\tau_\lambda}$ which is seen as a Beer's Law. So intensity this is the intensity $I_\lambda(0)$ and this is the intensity $I_\lambda(s)$ of path s. So it follows an exponential decay has also been observed by the Beer's Law. Now we have used integration here because the property kappa lambda maybe a function of x or it may be uniform. So kappa lambda absorption coefficient may continuously change over the path or it may be uniform.

If it is uniform, we can take it out of the integral and integral $\int_0^s k_\lambda ds$ will be simply=$k_\lambda ds$ okay and then it will be simply=$I_\lambda(0)e^{-k_\lambda \Delta s}$ okay. So we have used integration here because we are making a general derivation. So we assume that k_λ may also vary with the coordinate okay. So this quantity tau lambda.

Now please bear in mind this τ_λ and this transmissivity we define the Beer's Law is not the same okay. The tau lambda here was transmissivity okay while the τ_λ that is defined here 0 to S, $k_\lambda ds$ is not transmissivity it is optical thickness okay optical depth or optical path length. This is basically a non-dimensional number which we find by multiplying the absorption coefficient with the path length okay.

So this thing many times you may get confused for the same use of symbol, but τ_λ here is optical depth or optical thickness.

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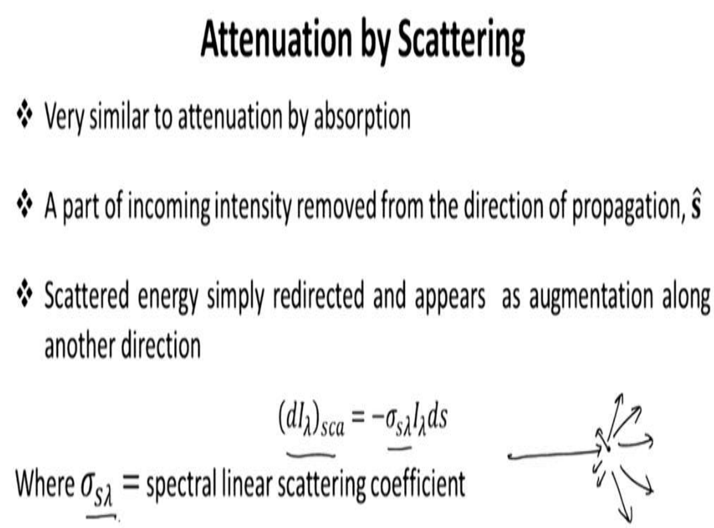

Now attenuation by scattering is very similar to attenuation by absorption. We may have radiation coming from certain direction, it may encounter some molecule or some particle and energy maybe scattered in different directions. The directional dependence may be different some particles tend to scatter more radiation in the forward direction and less radiation in the backward direction. Some particles or molecules tend to scatter more radiation in the backward direction and less radiation in the forward direction okay.

So the point is that when ray of radiative and photons when radiative path encounter any particles or gas and undergoes a scattering its intensity will change. And this intensity will change according to the same relation $(dI_\lambda)_{sca} = -\sigma_{s\lambda} I_\lambda ds$ where we have defined this coefficient the proportionality constant adds spectral linear scattering coefficient just like we defined k_λ as spectral linear absorption coefficient this is defined as spectral linear scattering coefficient.

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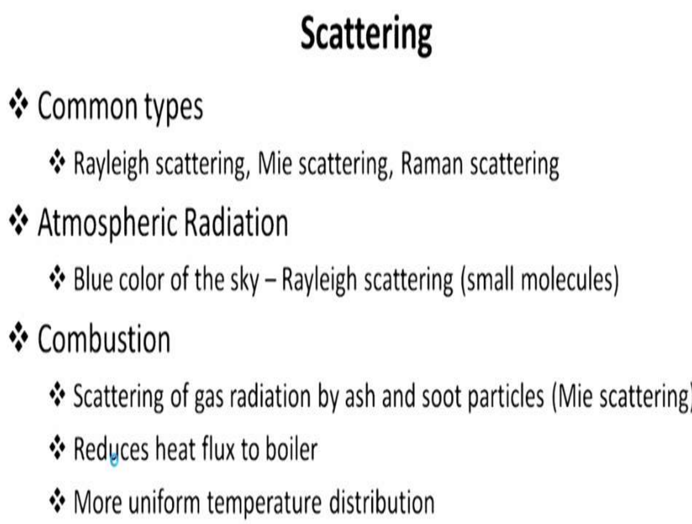

A quick note although we will be dealing with the scattering in detail in later chapters. There are many type of scattering that being encounter in normal applications. The 3 common types are Rayleigh scattering, Mie scattering and Raman scattering. There may be other type of scatterings. Now scattering can be elastic or non-elastic. Elastic scattering means that the photon does not lose any energy after scattering. Inelastic scattering means photons lose energy after scattering okay.

So Rayleigh scattering and Mie scattering are elastic type of scattering where the energy of the photon is not changed and when the energy of the photon is not changed during scattering the wavelength of the photon is not changed okay so they preserve wavelength. So atmospheric radiation in atmospheric radiation we have molecules of water in the atmosphere and these molecules scatter solar radiation.

This type of phenomena is called Rayleigh scattering and it gives blue color to the sky. The molecules are of very small size so small particles scattering or small molecule scattering is Rayleigh scattering that we have application in atmospheric radiation. In combustion we have scattering by soot and ash particles especially in coal-fired boilers. There we have scattering by large particles relatively large particles and we call it Mie scattering.

And it finds application in boilers where scattering by particles blocks the radiating heat transfer and it does not allow boiler surfaces to get heated from combustion gases okay so it reduces the heat flux and reduces the efficiency of the boiler, but scattering as I said redistributes the energy in all the directions. So with scattering you will find generally more uniform temperature distribution.

The gradients in temperature will be much severe when scattering is not there while the gradients in temperature will be much less when scattering is there because scattering basically acts to redistribute the energy in different directions.

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Total Attenuation

- The total attenuation of intensity in a pencil of rays by both absorption and scattering is known as *extinction.*
- The extinction coefficient

$$\beta_\lambda = k_\lambda + \sigma_{s\lambda}$$

- The optical distance based on extinction

$$\tau_\lambda = \int_0^s \beta_\lambda ds$$

So we combined absorption in scattering into a single coefficient this coefficient is called extinction coefficient which is sum of liner absorption coefficient and linear scattering coefficient. β_λ is the extinction coefficient and this also can define optical depth, optical path length based on this extinction coefficient as $= \int_0^s \beta_\lambda ds$.

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Augmentation by Emission and Scattering

- A light beam gains energy by emission and scattering from other directions into the direction of travel $\hat{s}$

Augmentation by Emission

- The emitted intensity must be proportional to the length of path and local energy contained in the medium

$$(dI_\lambda)_{em} = k_\lambda I_{b\lambda} \widetilde{ds}$$

- complete equation of transfer for absorbing-emitting medium

$$\frac{dI_\lambda}{ds} = k_\eta (I_{b\lambda} - I_\lambda)$$

Radiative transfer Equation for absorbing and emitting medium

So these were two physical phenomena that leads to attenuation of radiation. Now there are two phenomena that leads to augmentation or increase in intensity. The first one is emission and the second one is scattering. So we discussed in the first lecture itself that all substances emit radiation solid surfaces, liquids as well as gases. So when radiation ray passes through a medium the medium itself will emit some radiation and this radiation will basically adds to the ray and total intensity may increase.

So we write the increase in intensity dI_λ due to emission which depends on path length again and it also depends on the energy content of the medium which is generally given by the black-body function or plank function $I_{b\lambda}$ and the absorption coefficient k_λ. So the total amount of energy emitted and due to which the intensity of the radiation or single ray changes is = $k_\lambda I_{b\lambda}$ds.

Now we can assume that scattering is not there if we assume that the scattering is not there we can combine the absorption and emission part and just write down the simplified energy equation or radiative transfer equation. So this is radiative transfer equation for absorbing an emitting medium. In this there is no scattering. So scattering part is not there, but we want to derive a general expression for absorbing, emitting and scattering medium.

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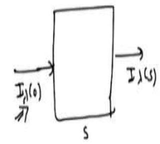
Augmentation by Emission

- The equation of transfer for an isothermal gas layer of thickness s

$$I_\lambda(s) = \underbrace{I_\lambda(0)e^{-\tau_\lambda}}_{\text{absorption}} + \underbrace{I_{b\lambda}(1 - e^{-\tau_\lambda})}_{\text{emission}}$$



- If only emission considered $I_\lambda(0) = 0$

- Emissivity: amount of energy emitted over a certain s path in a given direction

$$\epsilon_\lambda = I_\lambda(s)/I_{b\lambda} = 1 - e^{-\tau_\lambda} \quad \text{emissivity of gas layer}$$

We can integrate this equation as we did for the simply absorbing medium so we have intensity coming in from the left hand side that is called the intensity $I_\lambda(0)$ and intensity going out and this path length is s. So it will increase due to emission this is the term emission where $I_{b\lambda}$ is the plank function and $1 - I_{b\lambda}(1 - e^{-\tau_\lambda})$is the emissivity. So this is the emission part or augmentation part and this is the attenuation part or absorption part.

So intensity will change due to absorption it will decrease due to absorption and it will increase due to emission okay. If only emission is considered and absorption is not there we basically assume that there is no intensity coming in if we assume that then the emissivity can be defined as $I_\lambda(s)/I_{b\lambda} = 1 - e^{-\tau_\lambda}$. So this is defined as emissivity of gas layer. So this is amount of energy emitted over certain path in a given direction. So this is the emissivity spectral emissivity.

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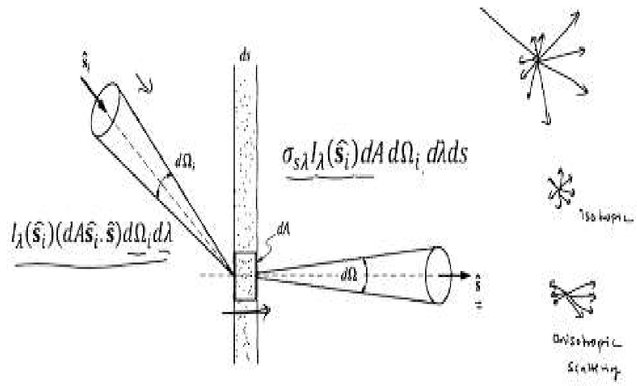
Augmentation by Scattering

❖ Augmentation due to scattering has contribution from all the direction



Now augmentation by scattering. Now scattering is a complicated topic so here we have basically let us say we may have a single particle and radiation comes and strike these particles and as a result it may scatter in different directions okay. The magnitude of scatter may be different in different directions okay. If it is uniform, uniformly scattered in all directions it is called isotropic scattering or it may be non-uniform more forward scattering less backward scattering okay.

There may not be sides scattering at all okay this is called an isotropic scattering. So for our interest let us say we have a radiation ray coming from certain directions $\hat{s}_i$ within a solid angle $d\Omega_i$. So total amount of energy within this ray is as per the definition of intensity $I_\lambda(\hat{s}_i)$ projected area times solid angle times the small wavelength. So this is the total amount of energy coming from a certain direction.

Now out of this the energy absorbed within this medium. So this intensity strikes here and scatters in all the directions and some amount of it travels in this direction. This direction is S and the amount of energy absorbed due to scattering is simply $\sigma_{s\lambda} I_\lambda(\hat{s}_i)\, dA\, d\Omega_i\, d\lambda\, ds$. So this is the amount of energies scattered and absorbed within that path of thickness ds.

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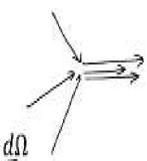
Scattering Phase Function

- Scattering occurs in all direction
- It is non isotropic (forward and backward scattering)
- Fraction scattered into the cone $d\Omega$ around the direction into $\hat{s}$

$$\Phi_\lambda(\hat{s}_i, \hat{s})d\Omega/4\pi$$



Φ_λ = scattering phase function

- Amount of energy from cone $d\Omega_i$ scattered into the cone $d\Omega$

$$\sigma_{s\lambda} I_\lambda(\hat{s}_i)dA\, d\Omega_i d\lambda ds \frac{\Phi_\lambda(\hat{s}_i.\ \hat{s})}{4\pi} d\Omega$$

Now as I said scattering occurs in all directions if it is uniformly scattered it is called isotropic scattering otherwise it is called Anisotropic scattering. We defined the behavior of scattering using a function called scattering function or scattering phase function phi lambda. Now phi lambda is a scattering phase function which depends on two directions which direction the radiation is coming here Si and which direction the radiation is going s.

If it is independent of s, then it is isotropic okay. So ϕ_λ is a scattering phase function. So out of all the energy scattered let us say this fraction is amount of energy scattered in a given direction s because we are writing radiative transfer equation for this direction s. So we want to know how much radiation coming from a certain direction $\hat{s}_i$ is actually scattered and adds to the intensity in direction s.

So this is the fraction of radiation that is basically coming from certain direction $\hat{s}_i$ and adds to the intensity in direction s. So with this the amount of energy from cone $d\Omega_i$ is scattered into the cone $d\Omega$ that is radiation coming from this direction and scattered into this direction will basically leads to increase in intensity and this can be written by this expression. Now we want to basically include all the directions okay.

So this is the direction of radiation coming in and going in this direction we want to include all the directions. So how much radiation coming from this direction is going into this direction how much radiation coming from this direction is going into this direction and so on. So we want to integrate this expression over the entire solid angle for Ω. So please note the solid angle in this case.

Because this is we are dealing with space this is not a surface where the solid angles will be limited to 2Ω. Here the solid angle will be limited to 4π the entire solid angle we have to take. So we integrate over the entire solid angle.

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Augmentation by Scattering

- Energy flux scattered into direction $\hat{\mathbf{s}}$ from all incoming directions $\hat{\mathbf{s}}_i$

$$(dI_\lambda)_{sca}(\hat{\mathbf{s}}_i)dAd\Omega d\lambda = \int_{4\pi} \sigma_{s\lambda} I_\lambda(\hat{\mathbf{s}}_i)\, dAd\Omega_i\, d\lambda ds \Phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})\frac{d\Omega}{4\pi}$$

$$(dI_\lambda)_{sca}(\hat{\mathbf{s}}) = ds\frac{\sigma_{s\lambda}}{4\pi}\int_{4\pi} I_\lambda(\hat{\mathbf{s}}_i)\Phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})d\Omega_i$$

$$\frac{1}{4\pi}\int_{4\pi}\Phi_\lambda(\hat{\mathbf{s}}_i, \hat{\mathbf{s}})d\Omega \equiv 1$$

- isotropic scattering ($\Phi_\lambda \equiv 1$)

And once we do that we get the intensity augmentation that is increase in intensity due to in scattering this is called in scattering because radiation coming from certain direction is in scattered or scattered in the direction $\hat{\mathbf{s}}_i$ okay. So this is the expression for the augmentation by scattering. And we have an integration appearing into this expression and this basically is the problematic part which makes the radiative transfer equation as integral equation integral differential equation.

And we will see how this basically appears in the equation because the energy is conserved. So the total integration of this scattering phase function should satisfy the energy conservational loss or amount of total energy scattered in any direction integrated over all solid angles should be=1 and for scattering isotropic scattering the phase function is simply=1.

So most of the time we assume that the scattering phase function is isotropic or ϕ_λ=1 otherwise the calculations may be difficult to carry out. So thank you we basically in this lecture started some basics of radiative heat transfer in the participating media. We will carry forward the derivation of radiative transfer equation and solve some problem and look into the cases of special interest for participating media in the next lecture. Thank you.