Radiative Heat Transfer
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Lecture – 13
Radiative Heat Exchange between Partially-Specular Gray Surfaces

Hello friends in this course so far we have learned how to find radiative heat exchange between the diffuse surfaces. In this lecture we will learn radiative heat exchange between specular and partially specular surfaces. So diffuse reflection basically we have what we introduce the in this previous lectures quantity called Radiosity.

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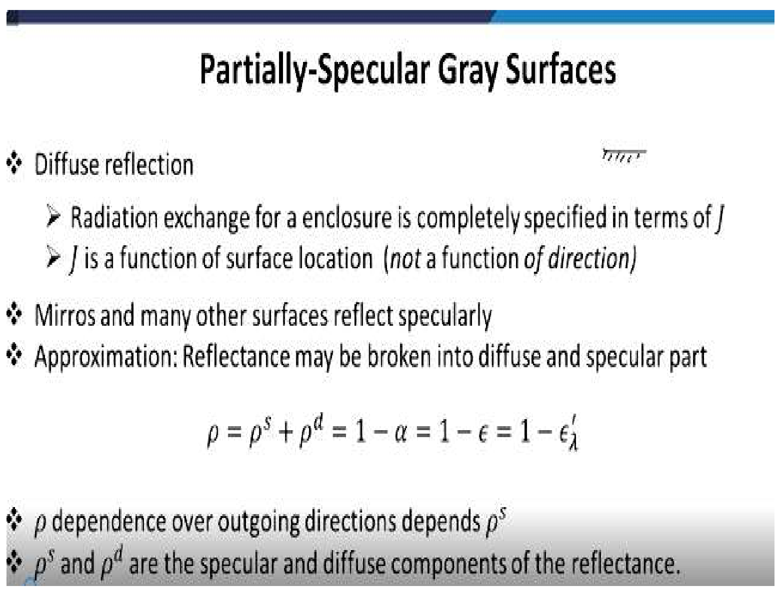

So the good thing about the diffuse surface is that we do not have to distinguish between emission and reflection. So let us say we have a surface and this is a diffuse emitter that means it emits radiation uniformly in all direction. And the same surface receives some radiation it may be coming from a single direction or it may be diffuse also. But the reflected radiation is diffuse okay so we have diffuse emitter.

And diffused reflection and the combined can be diffuse also okay. So this is also combined so we do not have to distinguish what is emitted and what is it reflected we call it Radiosity okay. So that was a powerful approach of dealing with the diffuse surfaces. But when a surface is partially specular the method for analysis becomes quite different okay what do we mean by partially specular surfaces is we have a surface.

And the radiation incidents on this circle leads to some radiation reflected like a diffuser, a diffuser reflection and some goes in a particular direction okay. So there are two components of this radiation or reflected radiation one is diffused and one is reflected. For example, mirrors, now mirrors are finding applications in many problems especially in solar energy. So these mirrors and many other surfaces they behave like specular surfaces or partially specular surfaces.

So as such the method of analysis is complicated unless and until we do some simplification. So the simplification here is that the diffuse the surface maybe having a component for specular reflection but it emits radiation and it absorbs radiation diffusely. So epsilon (ϵ) and alpha (α) do not depend on angle so $\epsilon = \alpha = \epsilon` = \alpha`$ So emission and absorptance emittance and absorptance do not depend on direction they are diffused.

And by this logic the total reflectance rho is also going to be diffused. Okay, now we divide the total reflectance into two components the diffuse component and the specular component okay. Now one may ask so how the directional effect of specular reflection is coming into plate. So the directional effect of specular reflection is coming into plate just by its dependence on ρ_s which is not continuous is in a given direction. This point will be clear in the next slide.

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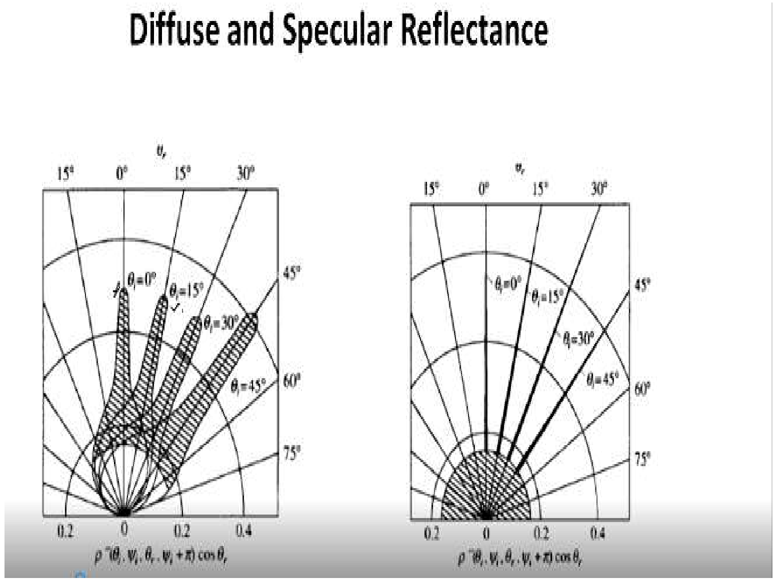

So let us say we have a surface as is shown on the left hand side this surface is a partially specular surface the reflectance is strongly dependent on direction as is seen it is strongly reflects at an angle of theta (θ)=0 degree and at an angle of theta (θ)=15 degree and so on. But it also reflects at other angles between 0 and 15 degree. But the reflectance decreases gradually so this is the real behaviour of a surface partially specular surface.

Now as such it is very difficult to solve a problem if the properties the reflectance properties are behaving in such a way. So we have to approximated it and the way we approximated is we write the total reflectance rho (ρ) as a sum of diffused and specular component. So the term so this is the diffused part that is uniform in all directions and then there is a specular component which is a finite value in a given direction.

Okay so there is no continuous variation of this in versus theta only we have a specular reflection in theta (θ)=0 degree, 15 degree, 30 degree, 45 degree, and so on. But between 0 and 15 degree there is no reflection no specular reflection there is diffuse reflection between 0 and 15 degree but there is no specular reflection. So this simplification we have to do if you want to solve this problem analytically.

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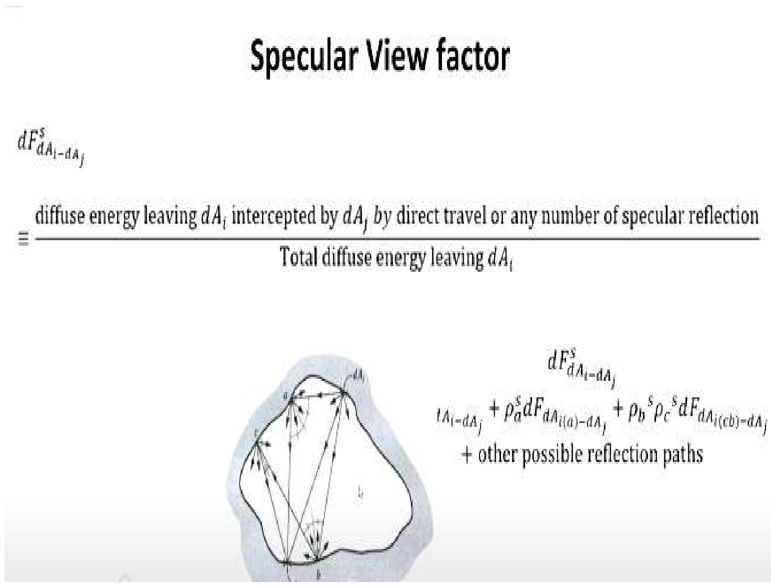

Okay just like we have defined diffuse view factor we can also define a specular view factor. So specular view factor is defined as the total amount of diffused energy leaving an element in dAi and received by another element dAj directly as well as after any number of specular reflection. So please note we are only interested in specular reflection while defining the specular view factor.

The diffused reflection is not accounted here in the diffuser view in specular view factor and in the denominator we have total diffuse energy leaving dAi. So as its clear in this image. let us say we have a surface dAi, we have a surface the dAi okay and energy is leaving diffusely in entire solid angle 2π some of these energy directly travels towards dAj. Some energy travels towards a and specularly reflected towards dAj.

Some energy first travels towards b undergoes a specular reflection at b received by surface c. Undergoes reflection specular reflection from c and then arrives at j, dAj. So there are direct reflection multiple reflection that are accounted in the definition of specular view factor. So we can write down a small element specular view factor (dF^s) dF superscript s for specular the dAi to dAj as = the direct view factor $dF_{dAi-dAj}$,this is the direct and diffuse view factor.

So this is diffused and direct okay between dAi and dAj. we also have contribution from a reflection so we write the specular reflectance ρ_a^S ,for the radiation coming from a after reflection from a, and then view factor from dF small view factor dAi(a)-dAj. I will explain that the term

$dF_{dAi(a)\text{-}dAj}$ and then double reflection from b and c and similarly the view factor okay all these view factors are diffused view factors and we have not used any superscripts S.

While the view factor on the left hand side is a specular view factor and we have used the superscript S. We may have other possible reflection paths also but in this particular picture we have only three paths direct, single reflection from a and double reflection from b and c respectively. So the quantity $dF_{dAi(a)\text{-}dAj}$ is defined as diffuse view factor between image of dAi now which image reflected from surface a and dAj.

So we are talking about the view factor between the two surfaces where one surface is real surface dAj while the other surface is basically an image of dAi. So dAi a is basically an image okay it will be clear in the next image next figure.

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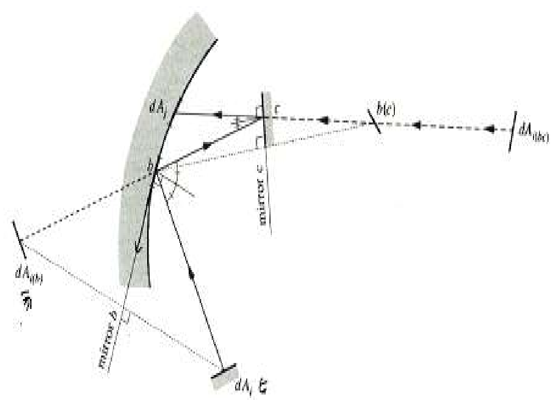

❖ $dA_i(bc)$ is the image of dA_i as mirrored by surface b and c

So here this is the surface element dAi it is reflected from b so this is the mirror surface its image in mirror b is formed here. So when we talk about the view factor between an image and surface we have to take into account this area okay. After the second reflection in mirror c the image appears here. So this is the image of the area dAi after twice reflection from mirror b and mirror c. So the definition of view factor dF between dAi(bc)-dAj means, we are talking about the area dAi after reflection from b and c and this is the image.

So this view factor we are interested in between this and between this area okay.

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Specular View Factor
(Reciprocity)

$$dA_i dF^s_{dA_i-dA_j} = dA_j dF_{dA_j-dA_i} + \rho_a{}^s dA_j dF_{dA_j-dA_{i(a)}} + \rho_b{}^s \rho_c{}^s dA_j dF_{dA_j-dA_{i(bc)}}$$

$$= dA_j dF_{dA_j-dA_i} + \rho_a{}^s dA_j dF_{dA_{j(a)}-dA_i} + \rho_b{}^s \rho_c{}^s dA_j dF_{dA_{j(bc)}-dA_i}$$

$$= dA_j dF^s_{dA_j-dA_i}$$

$$dA_i F^s_{di-j} = dA_j dF^s_{j-di} \quad (J_j = \text{const})$$

$$A_i F^s_{i-j} = A_j F^s_{j-i} \quad (J_i, J_j, = \text{const})$$

❖Summation relation

$$\sum_{j=1}^{N} (1 - \rho_j^s) F^s_{i-j} = 1 \qquad i = 1,2,\ldots N.$$

So our specular view factor can be written as multiplied by dAi and applying the reciprocity theorem dAj $dF_{dAj-dAi}$, this is direct, this is between Aj and the first image between Ai and mirror a and double reflection again we have applied the reciprocity theorem. Now one thing you should keep in mind is as from just purely the geometric point of view we have we can define dAj-dAi(a) that means the view factor between an element Aj and an image of Ai.

This is again the same area as of that Ai is basically equal to dAj the image of Aj-dAi that means we can take an image of any of the surfaces and the view factor will not change okay. Either we take image of Ai in mirror a or we take image of Aj in mirror A it is the same okay so that relation we have invoked here. So we have converted dAj to dAi(a) an into dAj(a) to dAi the same thing here okay.

So this relates to the dAj the entire these three terms basically are equal to dAj.dF^S the specular reflector from Aj to dAi and looking at these two terms what basically it appears is that reciprocity theorem. So reciprocity theorem is also valid on specular view factors, we have proved it earlier that it is valid for diffuse view factors and in this slide we have also proved that reciprocity theorem also holds on specular view factors.

Okay now the summation relation for the diffuse surfaces was summation $\sum F_{ij}$ =1 that was a summation rule for diffuse view factor. For specular view factor it can be proved that the relation (1-ρ_j^S)F_{ij}^S summed over surface is 1 to N is going to be =1. So the summation rule is different for specular surfaces and the diffuse surfaces.

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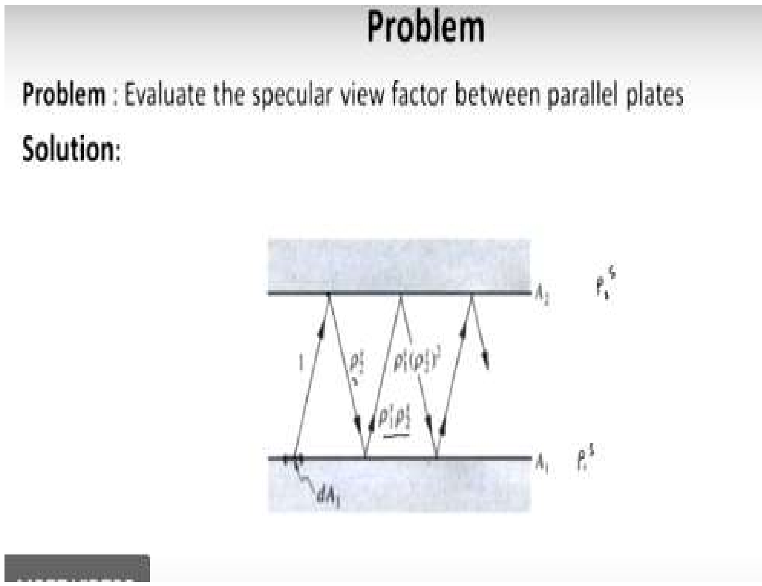

so let us do one problem and find out the specular view factor for this geometry we have two parallel plates in finite parallel plates the first surface has a reflectance $\rho_1^S$the specular reflectance and the second surface has reflectance ρ_2^S okay so any radiation leaving surface A_1 the small element dA_1 will directly travels towards A_2 it will undergoes reflection with the reflectance ρ_2^S It may undergo reflection again with magnitude ρ_1^S, ρ_2^S and so on. so there may be multiple reflection between these two plates. So we have to find out the combined specular view factor between these two plates.

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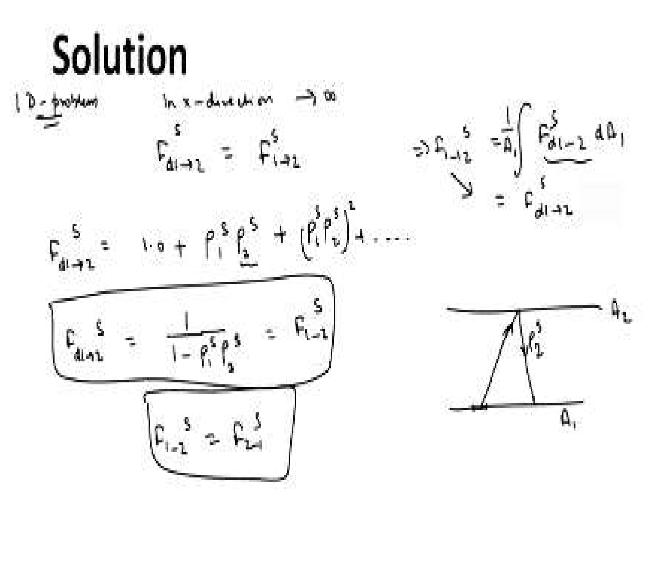

So let us do the solution for this problem we have a one dimensional problem here okay so the dimension of the plate in x direction okay, x tends to infinity it is a one dimensional problem okay. So we from this we can basically write that F^S_{d1-2} is basically $=F^S_{1-2}$. now why? because F^S_{1-2} is basically integrated over F_{d1-2} over dA_1 and because of the one dimensionality of the problem this does not depend on the location.

So we can take it out so F_{d1-2} any location okay so F_{d1-2} is specular same as F^S_{1-2} for the for this particular configuration. So now let us a write down the factor we have already defined this relation. So view factor between F_{d1-2} is made up of a number of components the first component is direct (F_{d1-2}) this is going to be 1 okay. As you can see from this geometry all the radiation leaving this surface will actually reach the top plate.

This is A1 this is A2, there is no other way it can escape so all the direct radiation will reach surface A2 so this is going to be 1. Then the second part is the reflected component so we have the reflected part this is going to be ρ^S_2okay and this is going to be ρ^S_1 times ρ^S_2. Just look at the relation once again so we have ρ^S_a times the view factor between the image dAi and dAj okay.

So that same thing we have done. so we have taken the reflection from the surface 2 okay and this will be + similarly $(\rho^S_1\rho^S_2)^2$ and so on. So this is how we are basically defining this view factor and we can this is a geometric series we can write down this as $\frac{1}{1-\rho_1^S\rho_2^S}$ okay. So this is going to be our view factor okay so F_{d1-2}^S is simply = this value ($\frac{1}{1-\rho_1^S\rho_2^S}$) and this is = F_{1-2}^S okay.

And also because of symmetry $F_{1-2}^S = F_{2-1}^S$ okay. So all these view factors are basically the same okay. So again just look at this one energy directly leaving has a magnitude =1 energy leaving after two reflections is $\rho_1^S\rho_2^S$ times the original intensity and similarly this is going to be $(\rho_1^S\rho_2^S)^2$and so on okay so there will be multiple reflections so after multiple reflections this is going to be the view factor okay.

Now one thing that you should observe here is quite a special case let us say ρ_1^S= ρ_1^S= let us say 0.5 okay.

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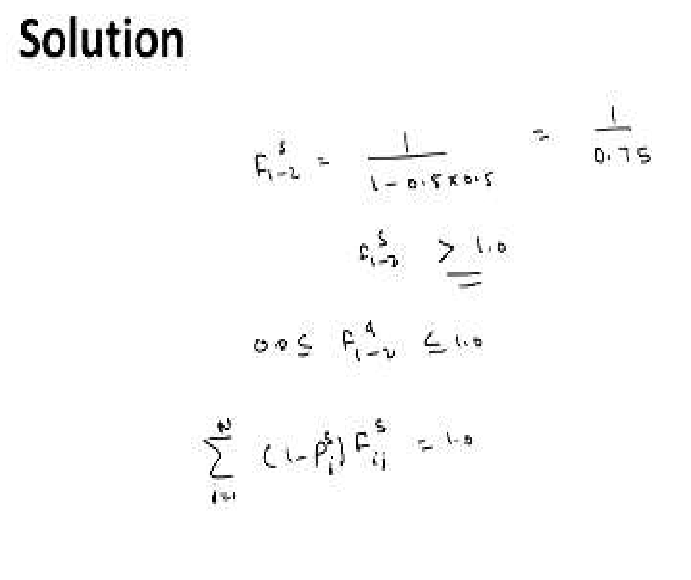

Now for this the view factor F_{1-2}^S will be =(1/1-0.5*0.5)=(1/0.75). So this F_{1-2}^S can be >1, okay. So as compared to diffuse view factor, the diffuse view factor will always be <1 and >0 the specular view factor F_{1-2}^S can be >1 and that is why the relation for summation for specular view factor is given by summation j=1 to N (1-ρ_j^S) F_{ij}^S =1 and the basic logic why this is coming >1 is because radiation leaving a particular surface is undergoing multiple reflection.

And all those contributions are being added up before they finally reaches the other surface. So that is why they are specular view factor can be >1.

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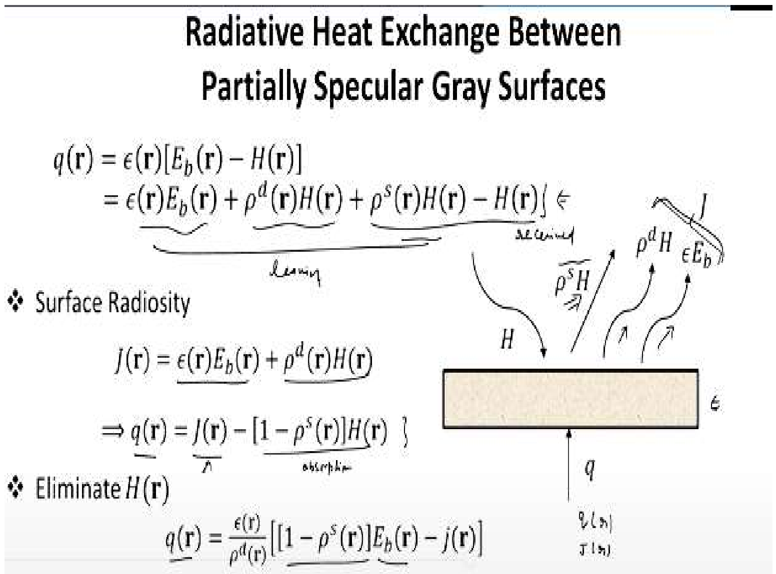

Now based on this information and this knowledge of view factors for specularly reflecting surfaces we will solve for heat transfer between these surfaces. The theory is a pretty much similar to what we have done for diffuse surfaces. We have a surface this surface is partially specular it emits radiation diffusely that is why epsilon times the black body emits power. It emits it reflects radiation diffusely.

So there is a reflected component or diffuse reflected component together with the we call this Radiosity just like redefine Radiosity for a diffuse surface. The other term that additional term that is only for this specular partially specular surfaces the partially the specular refracted component ρ^Stimes H where H is irradiation. So this is the additional component so we write an energy balance for this equation.

Heat flux = energy emitted- energy absorbed, energy emitted, diffused energy reflected sorry this is diffused reflection this is specular reflection. So this is the total amount of energy leaving and this is the energy amount received okay. We can define surface Radiosity as we have done it for the diffuse surfaces just by including the emission and diffuse reflection. So in the definition of Radiosity we do not include specular reflection.

Substituting this value of J into this equation we get heat flux = diffuse Radiosity that is energy leaving diffusely -the 1- the specular reflection times Hr okay. So this is the absorption term okay so we can eliminate Hr the irradiance and we can solve for the heat flux just like we did it earlier. So substituting the value of or eliminating the Radiosity between these equations we can write down the heat flux q as epsilon(ϵ)/the diffuse reflectivity reflectance $\rho^d(r)$.

(1-$\rho^s(r)$) times (Eb-Jr) okay so here we have unknowns in the form of flux and Radiosity okay so we have to eliminate a Radiosity also.

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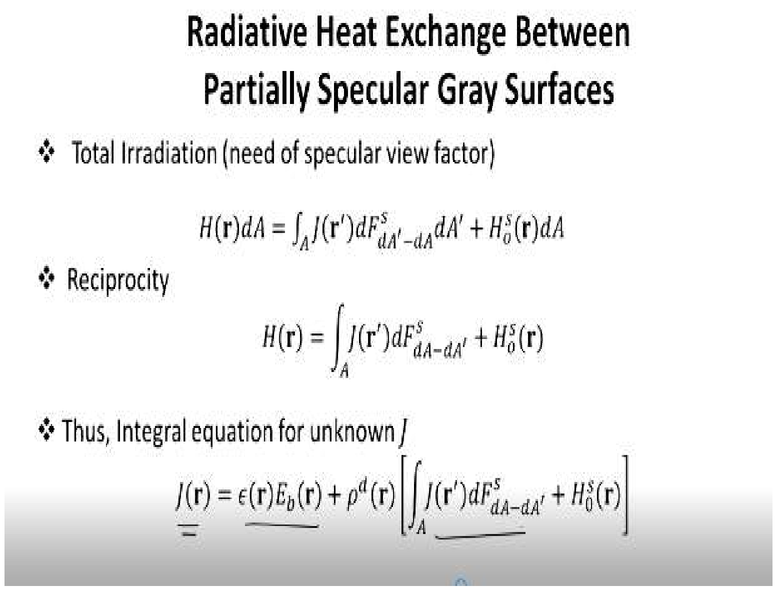

Okay we need to find out irradiation before we solve for Radiosity the expression the mathematics involved is pretty similar to what we have already done. The Radiosity is basically made of emission and diffuse reflection. So ρ^d is the reflectance and this is total amount of irradiation H okay in the bracket we have total amount of irradiation and this undergoes diffuse reflection.

So Radiosity is made up of these two components so we can calculate the Radiosity using the relations that we have already developed for Gray diffuse surfaces. So there is not difference here this entire thing is same as done for diffuse Gray surfaces. Okay the only thing that has changed is basically the specular reflectance that appears basically in this relation. In the relation for heat flux the expression of J does not change okay.

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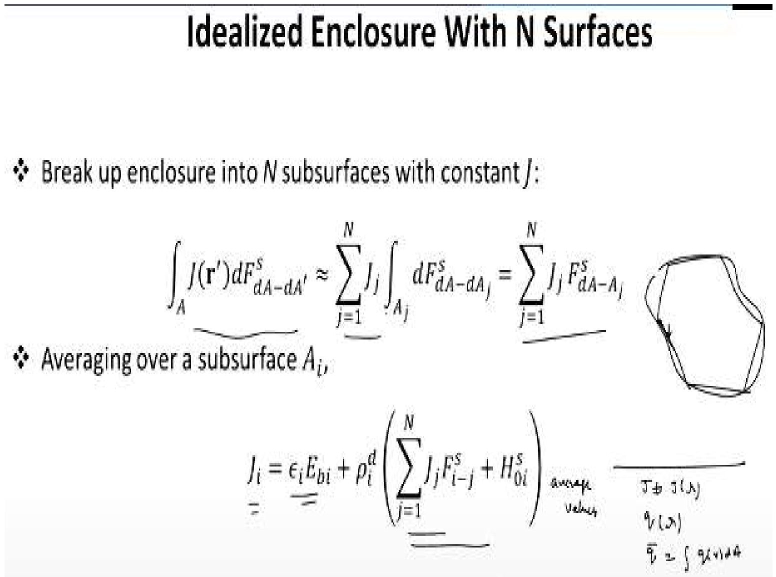

So again as we did in the previous cases of diffuse black and Gray and closures we divide the enclosure into number of finite length or finite area flat surfaces. Okay so the integral can be now converted into a summation so j=1 to N. So N surfaces, we have divided the enclosure to N surfaces. Okay so integral and then converted into simply summation Radiosity times the specular view factor okay.

And when as from previous argument the Radiosity maybe uniform over the surface but heat flux is not because of view factor changes over the surface. So Radiosity may be same it may not be a function of r over the surface but heat flux is a function of r. Because the view factor is the function of r so we have to take an average okay so we have to define an q average by taking an average over each flat surface.

And when we do that we define Radiosity surface Ji as averaged emission from the surface average irradiation on the surface multiplied by diffuse reflectors. So this is average values okay this is local values of Radiosity but on a given surface.

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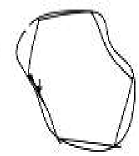
Idealized Enclosure With N Surfaces

- Break up enclosure into N subsurfaces with constant J:

$$\int_A J(\mathbf{r}')dF^s_{dA-dA'} \approx \sum_{j=1}^{N} J_j \int_{A_j} dF^s_{dA-dA_j} = \sum_{j=1}^{N} J_j F^s_{dA-A_j}$$



- Averaging over a subsurface A_i,

$$J_i = \epsilon_i E_{bi} + \rho_i^d \left(\sum_{j=1}^{N} J_j F^s_{i-j} + H^s_{0i} \right)$$

Now with just a few manipulation the final equation for relative transfer so this is the final equation for a radiative transfer between Gray is specular partially specular surfaces okay. So this is the exhibition for partially specular surfaces this energy balance even applied to solve a problem. Now this problem we have solved in the case of Gray surfaces also when we solved this problem.

The vertical plate that was used as a reflector to increase the efficiency of this collector plate was not a mirror it was basically a diffusely reflecting surface. So in this example that we have solved earlier also we have a collector plates that receives solar radiation and we have already noted earlier that the absorptance or the amount of energy received by this collector is very poor if there is no reflector plate put adjacent to it.

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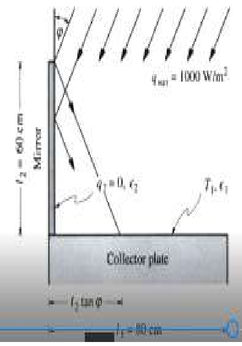
Problem

Problem: A very long gray-diffuse solar collector plate is to collect energy at T_1 = 350 K. A highly polished mirror is placed to improve the collector efficiency as shown. Assume ϵ_1=0.8 and the mirror is specular with $\epsilon_2 = 1 - \rho_2^s$=0.1. Determine the energy collected if solar irradiation is 1000 W/m^2 at incidence angle of 30^0.



In the example that we did earlier the reflected the reflector plate was made up of Gray surface and it increase the efficiency of the collector significantly. Now in this particular example the same configuration but the reflector plate the vertical refractor plate instead of Gray is basically specular purely specular and has a reflectance a specular reflectance of 0.9 or the emittance of 0.1.

And we will see that when we put mirror a specular surface instead of a Gray surface the efficiency of the collector is going to significantly improve. So let us solve this problem so we will first write down the energy equation for the surfaces and then we will so we will use this equation. So this equation we will use and we will write this for the two surfaces.

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Solution

Surface-1 →

$$[1-(1-\rho_1^s)F_{11}^s]E_{b1} - (1-\rho_2^s)F_{12}^s E_{b2} = \left[\frac{1}{\epsilon_1} - \frac{\rho_1^d}{\epsilon_1}F_{11}^s\right]q_1 - \frac{\rho_2^d}{\epsilon_2}F_{12}^s q_2 + H_{o1}^s$$

surface-2

$$-[(1-\rho_1^s)F_{21}^s]E_{b1} + [1-(1-\rho_2^s)F_{22}^s]E_{b2} = -\frac{\rho_1^d}{\epsilon_1}F_{21}^s q_1 + \left[\frac{1}{\epsilon_2} - \frac{\rho_2^d}{\epsilon_2}F_{22}^s\right]q_2 + H_{o2}^s$$

$F_{11}^s = 0$, $F_{22}^s = 0$

F_{12}^s = direct + ~~specular reflection~~ (0) = F_{12}^d

$F_{21}^s = F_{21}^d$

$F_{12} = \frac{1}{4}$; $F_{21} = \frac{1}{3}$

So surface 1 so we write this as [(1-ρ_1^S) F_{11}^S]E_{b1}- [(1-ρ_2^S) F_{12}^S]E_{b2}.this is the left hand side =$\left[\frac{1}{\epsilon_1} - \frac{\rho_1^d}{\epsilon_1} F_{11}^S\right] q_1 - \frac{\rho_2^d}{\epsilon_2} F_{12}^S q_2 + H_{01}^S$ (the collector plate is Gray q_1).so this is the energy balance equation for the surface 1 and same thing if you do for surface 2 so: -(1-ρ_1^S) F_{12}^S E_{b1}+[(1-ρ_2^S) F_{22}^S]E_{b2}= $\frac{-\rho_1^d}{\epsilon_1} F_{12}^S q_1 + \left[\frac{1}{\epsilon_2} - \frac{\rho_2^d}{\epsilon_2} F_{22}^S\right] q_2 + H_{02}^S$

So we have just written the energy equation energy balance equation for the two surfaces okay now first thing that we will do is look at the view factors. Now this particular configuration is a very specially chosen because although we have a specularly reflecting surface. But the specular view factors all turn out to be 0. So the first thing that we will look at is F_{11}^S that is any radiation leaving surface an any diffuse radiation leaving surface 1 that is the collector plate.

After specular reflection or directly is it coming back to 1. So let us look at the configuration the relation leaving diffusely from the surface it travels towards mirror but after mirror it will just escape it will not come back okay and directly there is no way it can come back okay so this will be = 0. Now F_{22} any radiation leaving surface 2 diffusely and after a specular reflection. But surface 1 is not specular it is diffused so this is also 0.

Because surface 1 is not specular okay now F_{12}^Sso what is this F_{12} any radiation leaving the collector plate diffusely and arriving at 2 okay after directly or reflection. So it will have direct component + reflection specular reflection component as we have already done it. But since there is only one specular surface this is going to be 0.

So F_{12} is basically = direct component so F_{12}^S=F_1^dokay. and similarly F_{21}^S=F_{21}^d okay. So diffuse view factors we can calculate from the table we already know that okay the value of the view factor basically for this configuration is equal to. So let me give you the value F_{12} =1/4 and F_{21} =1/3. Okay so we have calculated the view factors in this equation and let us know solve this problem further.

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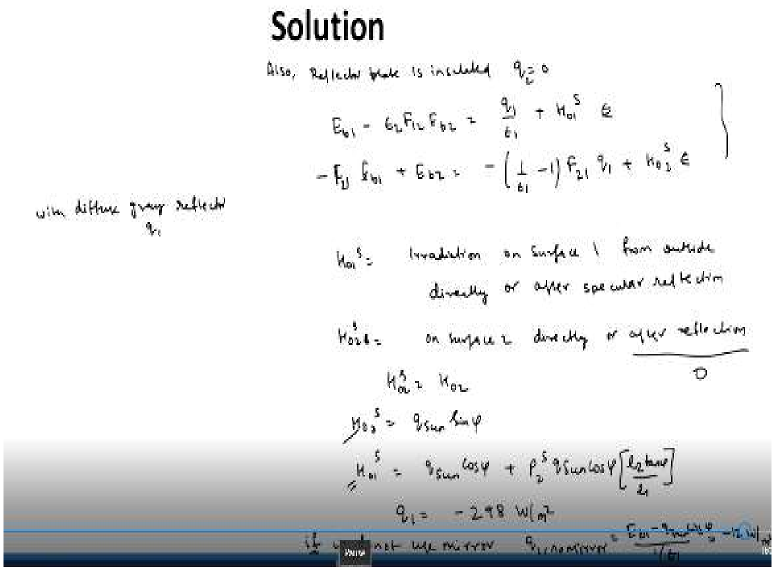

So with this simplification also the reflector plate is insulated so q =0. So with this simplification the energy equation reduces to $E_{b1} - \epsilon_2 F_{12} E_{b2} = q_1/\epsilon_1 + H_{01}^S$ and $-F_{21} E_{b1} + E_{b2} = -(1/\epsilon_1 - 1) F_{21} q_1 + H_{02}^S$ So there is almost the effect of specular reflection we have kind of all eliminated okay. most of the terms in this x in these two equations do not depend on anything specular except H_{01}^S and H_{02}^Sokay.

So what is H_{01}^S and H_{02}^S so H_{01}^S is irradiation on surface one from outside okay directly or after specular reflection and this is basically the what makes the difference okay because surface 1 the collector will receive irradiation directly as well as after the reflection from the mirror okay H_{02}^S is similarly irradiation on surface 2 directly or after reflection specular reflection okay.

Now surface 1 is not a reflection so this term will be 0 okay so H_{02}^S is basically the directly received radiation only. So H_{02}^S is basically H02 that is it actually there is no reflected part into this. So let us a write down this the value of H_{01}^S and H_{02}^S and then some problem will be pretty easy to understand. So H_{02}^S is simply = q_{sun} which is given us 1000 watt meter per square and sin psi (ψ)where psi (ψ)is given as in this angle.

Psi (ψ) this is the angle and this value is given as 30 degree okay. now one more thing that is needed in the valuation of H_{01}^S. H_{01}^S is includes the direct radiation from the sum plus anything that is reflected from the mirror. The mirror after reflection the mirror will not reflect the radiation on the entire collector plate only this part of the collector plate will receive radiation after reflection.

As you can see from this image radiation hitting here will hit here and radiation you are leaving here are hitting here on the mirror will reach here okay. So this part of the plate will not receive any radiation after reflection only the first or the length $l_2 \tan\phi$ well receive radiation the reflected radiation. So we have to take an average over this value okay so we define H_{01}^S as a direct component.

This is the direct irradiation of collector plate from the sun and then the reflected part $\rho_2^S q_{sun} \cos\psi$ okay this is the reflected part. But we have to take an average because the plate is receiving radiation only at fraction of its length. So we have to define an average so we define $l_2 \tan\phi / l_1$,wherel_2 is given as 60 centimetre and l_1 is given as 80 centimetre so we have to take an average of these values okay.

So now we have all the information so we put the value of H_{02}^S and H_{01}^S in these equation. E_{b1}, q_2 is 0. the temperature of the surface the collector plate we know, so everything is known and when we solve it we get a value of q_1 as = -298 W/m^2. -sign the note that the energy is basically in incoming energy into the collector if it is plus that the energy is leaving the collector so total -298 W/m^2.

Out of thousand is basically received by that collector okay if we do not use mirror if you do not use mirror then energy simply $q_{1(\text{no mirror})}$ will be = $(E_{b1} - q_{sun})\frac{\cos\psi}{(\frac{1}{\epsilon_1})} = -12$ W/m^2.this is nothing okay only very less amount of energy to receive if you do not use mirror okay and that much amount of energy may be actually lost it due to convection and other losses.

So we can understand the importance of mirror in this problem we solve this problem with the Gray reflector also in the in one of the previous classes and the result we got was -172 okay so i will write down this result here with diffuse Gray reflector we got q_1 as -172 W/m^2. Okay so compared to -172 we are receiving -298 so that basically explains how important the mirror is in this example.

If you use a diffuser reflector and if you use a specular reflector it makes hell lot of a lot of difference. So specular reflector is going to increase the collector efficiencies significantly the corrective radiation from - W/m^2has increased to -298 $W/m^2$and this is significant improvement. So I end this lecture here in the next lecture we will discuss combined mode of heat transfer.

Where we have thermal radiation together with other modes of heat transfer like conduction and convection and we will understand how the combined radiative heat transfer with conduction and convection affect the performance of a equipment thank you very much.