

Operations Management
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Lecture – 15
Quantitative Methods-II

[FL] friends. Welcome to session 15 in our course on operations management. So, currently we are discussing the basic aspects of forecasting and if you remember we have covered 4 sessions. We have completed discussion in 4 sessions on forecasting.

We are trying to find out that how much we should produce from the operations point of view. We are trying to see that what can be the demand or what is the expected demand in the next year or in the next month. If you remember we have seen in the previous session that there are quantitative methods of making the forecasts. We have seen the simple average method, simple moving average method, weighted moving average method and we have found out that each one of them can be used in certain specific application areas.

Now, how to identify where which method can be used? That is really important and that decision is based on the scatter or the distribution or the variation of the data with respect to time. So, we have seen that on y axis usually we take demand and on x axis we take the time. We try to understand that how the demand has varied over the last 5 years. If we are taking the yearly demand into picture; if we are taking the monthly data into picture, we will say over the previous 5 months or over the previous 7 months or over the previous 12 months; how the demand has varied or how the actual demand has varied with respect to time?

And based on that, we decide on our method. Just to have a brief review in 4-5 sentences is what we have covered in the previous session. If the data is scattered over a central line; there is little variation above and below the central line. We can use a simple average method. Because, we see the data is scattered at around the central line. We can make use of simple average method, but in case there is an increasing trend or there is a decreasing trend observed in the data; our simple average method will not be suitable. All of us must keep in mind that simple average method will give us average value, which will be far away from the actual demand.

How we can see that? We can see that if there is an increasing trend, the average value may give us under forecast. If there is a decreasing trend, the average value may give us over forecast. So, we have to see that simple average may not be relevant wherever there is an increasing or decreasing trend. So, how to solve this problem? In that case we will give more weightage to the last 3 months data or the last 3 years data whatever is our time domain. If we are doing the calculation on yearly basis. We will consider last 3 years data.

If we are doing the calculation on monthly basis, we will cover the last 3 months data only and those 3 months we will do simple average and we will find out that what is going to be the forecast, but still there can be a problem. There can be an increasing or rapidly increasing trend in those 3 months or those 3 years also. Now, how to overcome this problem? We can overcome this problem by giving more weight to the last years data. For example, we are forecasting for 2018 and we have the actual demand data for 2017, we will give maximum weightage to the 2017 data point. Then slightly less weightage to 2016 data point. Then even lesser weightage to 2015 data point.

So, we are only considering the last 3 years data or in our previous session we have taken monthly data for the last 3 months data only and we are ignoring the rest of the data points. That are available with us why because, there is an increasing trend for the demand. We ignore the previous data points. We only consider the last 3 months data points only and within those 3 months also we are assigning maximum weightage to the last month data. For example, currently it is month of July. We want to forecast for August, we will give maximum weightage to July, slightly less weightage to June and even less weightage to May.

From May very less weightage, June slightly higher weightage, July maximum weightage and then we forecast for the month of August. So, all this we have covered in the previous session. We have done some calculations also. We have considered very simple problems also. With assigning weights to the previous months or previous years data and then doing the forecast for the next month or the year based on the problem that we have seen. Just one joke is coming to my mind that emphasizes the importance of averages and how sometimes averages can be misleading.

There was a person who was standing across a river and he wanted to cross that river. He was may be good at mathematics. He asked a passer by a person who was again passing through we can say the passage or the path. He asked him that what can be the average depth of this river. He said it is 3.5 feet. This person said the average height is 3.5. My height is 6 feet. Maybe it is higher than the average value. I must be easily able to cross this river. So, he entered into the river and never came out. The average is can sometimes be misleading and we should not blindly follow the averages.

So, we should follow the averages, but with the little pinch of salt maybe we should see the data that how the data is varying. So, if we can fit a central line or the data is scattered around the central line, we can follow the simple average method. But, if the data is showing a specific trend, we must not follow the simple average. We must focus on the moving average or the weighted moving average method. Now there is another advancement in the field of weighted moving average, which is called the exponential smoothing.

In which the weights keep on decreasing exponentially. Maybe you suppose we have the 10 years or 10 months data available with us, we will assign maximum weightage to the latest data and then the weights will keep on reducing and as we are going far off from today that weights will keep on reducing. For example, if we want to forecast for the month of 2008 or for the year of 2018. We will give supposed 0.9 weightage to the data of 2017 and then for 2016 lesser weight, 2015 even lesser weight, 2014 even lesser weight.

So, the weights will keep on reducing as we are going further and further from today's date. That is what is the basic concept of exponential smoothing. We have understood that we give weights to the data points in the previous session and how these weights are assigned. That is adding an element of subjectivity. Nobody has questions or maybe you may question in the discussion board, that is are how we can decide that 0.5 should be given to the as a weightage to the previous months data. Why it is 0.5, 0.3 and 0.2 only? Why it is not 0.7, 0.2 and 0.1?

So, these kinds of questions can come and these decisions regarding how much weight must be given as you can say; element of subjectivity into the objective methods of forecasting or into the quantitative methods of forecasting. There are mathematical tools,

there are statistical tools that can help us in the calculation or in the finding out of these weights also. But, in exponential smoothing it is assumed that the weights will vary exponentially.

So, we will see that how exponentially the weights will vary with the help of certain slides and when are the weights varying exponentially? Where these methods can be used? Or exponential smoothing method where it can be used? It can be used where we want to give the maximum weight to the previous months or previous years data and we want that the furthest readings must be given minimum weight. So that they have the minimum influence on the forecast that we are going to make for the future; we will try to understand the exponential smoothing method today.

Then it is a bridge into an equation, which can be used for making a forecast. But the basic concept is that the weights will vary exponentially over a period of time. The most recent will be given the maximum weight and then the weight will exponentially reduce as we go further and further from the forecast period. So, let us try to understand this with the help of a presentation and start our discussion. In our simple average method, we have seen all the previous years data are considered and equal weights are assigned.

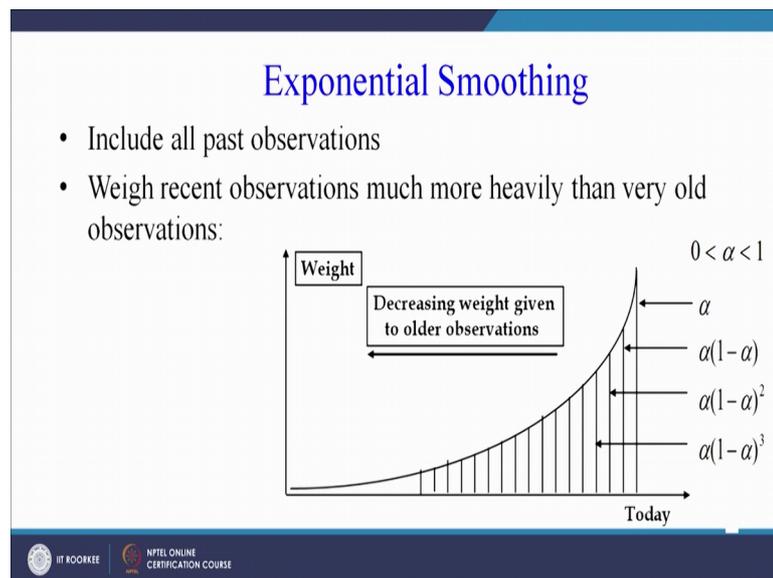
In case of moving average, we neglect some of the readings or some of the data points. We consider only the latest data points, but we give equal weightage to all the previous data points or all the data points being considered for making a forecast. In weighted moving average we considered the last 3 months or 3 years data points only, but we assign specific weights to the 3 data points.

So, all the 3 data points are not given the equal weights. Please keep in mind that 3 I am taking as an example. You can use a 4 period moving average also; you can use a 5 period moving average also. Suppose we have 20 years data available with you, you want to focus on last 5 years only.

So, you can make use of a 5 period moving average method. n is 5 in this case. In my example that I have taken n can be 3. That is the purely your decision that what is the period you are going to select, but the important point to note is, in simple average we give equal weightage to all the previous years data or previous months data. In moving average we consider the previous 3 or 4 or 5 years data only.

We neglect all the other data points, but we give equal weightage to all these 3 or 5 data points that we have considered. In weighted moving average, we further fine tune our forecast by giving specific weights to the previous years demand. Maybe the most recent may be given the maximum weight and we have also seen this with the help of an example in our previous session. In exponential smoothing we will include all the past observations.

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But, the weight assigned will be more for the recent observations. So, as is given on the slide, the weight recent observations much more heavily than very old observations.

So, on your screen you can see on y axis we have the weight. It is not the demand. Sometimes, the students or the learners confuse it with demand. This is the weight and this is today we are here. The data which is most recent, we are giving the maximum weight and the weight is exponentially reducing and this is maybe the oldest observation or the oldest data point is here and we are assigning the minimum weight to the oldest data point and we are assigning the maximum weight to the most recent data point.

So, the decreasing weight given to older observations. Suppose, we give alpha value to our, we can say the most recent data point the next years or data point will be given lesser weight and that distribution will be exponential distribution. It will reduce exponentially.

So, the value of alpha which we call as the smoothing coefficient, the name of the method is exponential smoothing where the recent observations much more heavily than the very old observations. I have already highlighted. Now, suppose we give the smoothing coefficient value alpha as 0.7. So, the latest observation is given the way weightage of 0.7. Then it will keep on changing. Next year it will be alpha into 1 minus alpha, because, it is varying exponentially. Then for next year it will be alpha into 1 minus alpha to the power 2. It will keep on decreasing. The weight is keep on decreasing.

So, if in these values suppose you take alpha is 0.7. The value of 0.7 into 1 minus 0.7 will certainly be less than 0.7. Similarly, 0.7 into 1 minus 0.7 to the power whole square will be even lesser. The values or the weights are reducing exponentially. Now suppose we as we have considered in our previous session we have taken last 3 months data point only. If we give weightage alpha as 0.7.

The month if you remember in our previous session we were forecasting for the month of December. If we give alpha equal to 0.7 for the month of November, if we are forecasting for December, exponentially the weight will come down for the month of October and further it will come down for the month of September. We will assign minimum weight to the month of September or to the value or the demand data for the month of September, then slightly higher weight in for the demand data for the month of October and the maximum weight for the demand data for the month of November.

So, September, October, November weight is increasing. November, October, September weight is decreasing. How it is decreasing? It is decreasing exponentially and this method will give us you can say easier method, way to decide on the weights that if we decide on one value that this is going to be my alpha value. The other values can be calculated based on alpha into 1 minus alpha. Then alpha into 1 minus alpha to the power 2. Very easily we can calculates the weights that can be assigned to the older observations.

So, as per the forecast we have seen the most recent.

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The slide is titled "Exponential Smoothing" in blue text. It contains three equations for forecasting demand. The first equation is $F_t = \alpha D_t + \alpha(1-\alpha)D_{t-1} + \alpha(1-\alpha)^2 D_{t-2} + \dots$. The second equation is $F_t = \alpha D_t + (1-\alpha)[\alpha D_{t-1} + \alpha(1-\alpha)D_{t-2} + \dots]$. The third equation is $F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$. At the bottom of the slide, there are logos for IIT ROORKEE and NPTEL ONLINE CERTIFICATION COURSE.

We are giving the value alpha. F_t if we want to forecast for the period t plus 1. So, we will see alpha into demand for the previous year then alpha into 1 minus alpha demand for the older observation; alpha into 1 minus alpha to the power 2 for the even older observation. So, as the observations are getting older, the weights are getting lesser and lesser. As we have seen in exponential smoothing method when we are making a forecast F_t represents the forecast.

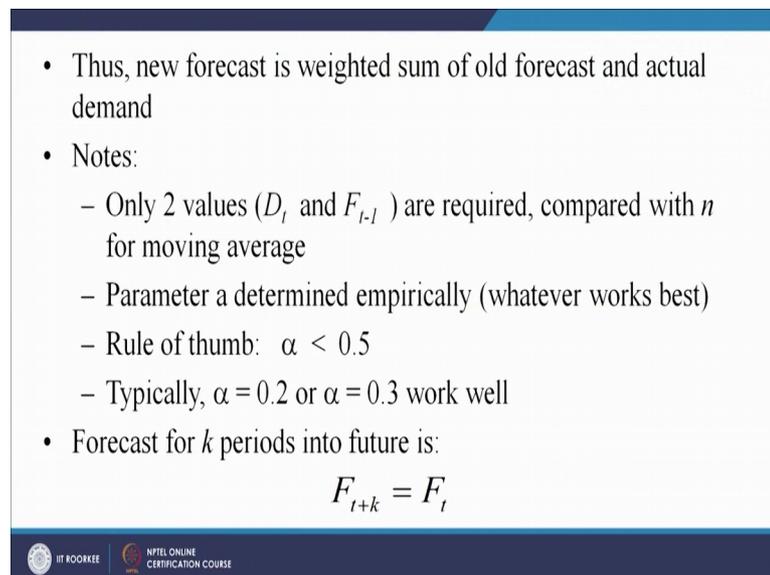
So, if that we are forecasting for t that is t may be the forecast for the month of December as we have seen in the previous session when we are forecasting for the month of December. We should have here the value t minus 1 or we can write here F_{t+1} . So, $t+1$ becomes equal to December and your t become then November. We should have we can make use of the previous months data for making a forecast and this is basically then the forecast for the month of November.

So, this is a forecast for the month of November. This is the demand for the month of November and this forecast we are doing $t+1$ for the month of December and this we can see that the previous forecast and the previous demand if is known to us, we can make the forecast for the next month. So, just to simplify we should not bogged down by these equations. To simplify the things we can just take an examples. That suppose we want to make a forecast for the month of December, what is required? We require the demand for the month of November and the forecast for the month of November.

So, if we know the forecast for November and demand for November, very easily we can make a forecast for the month of December. Now for calculating the forecast for the month of November we require the demand for the month of October and the forecast for the month of October. So, this equation then simplifies into F_t . Please focus on this equation only this is the correct equation. If t is December, we can say we require the demand for t minus 1. That is November and we require a forecast for t minus 1. That is November.

So, as I have given the example if we want to forecast for the year 2018. We require the demand for 2017 and the forecast for 2017. If we are doing based on monthly data points, if we want to forecast for January, we require the demand for December and the forecast for December. That much if you can remember, you can very easily use the forecasting technique. That is exponential smoothing. Alpha value definitely has to be assumed or there are method through which we can have you can say good estimate of the value of alpha. We will try to understand this with the help of an example in our subsequent slides. Thus the new forecast is weighted, sum of old forecast and the actual demand.

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- Thus, new forecast is weighted sum of old forecast and actual demand
- Notes:
 - Only 2 values (D_t and F_{t-1}) are required, compared with n for moving average
 - Parameter α determined empirically (whatever works best)
 - Rule of thumb: $\alpha < 0.5$
 - Typically, $\alpha = 0.2$ or $\alpha = 0.3$ work well
- Forecast for k periods into future is:
$$F_{t+k} = F_t$$

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So, actual demand basically we will only know when that period has passed.

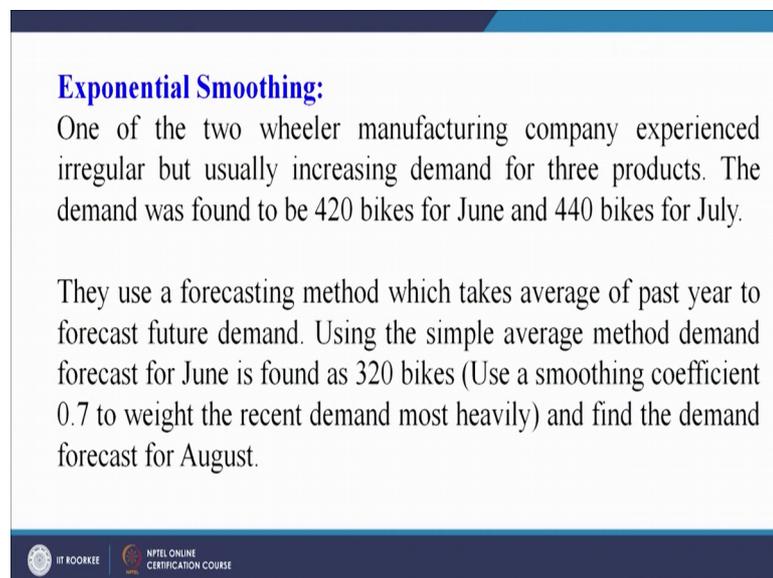
So, if we are forecasting for December, we will require the forecast for November as well as the actual demand for November. Only 2 values d_t we should say d_{t-1}

here and F_{t-1} are required compared with n for moving average. Here we require only 2 values as I have already highlighted. Whereas in previous methods we require n values. n means maybe last 3 years, last 5 years or last 10 years. n can be 3, 5, 10. We require lot of data, but in exponential smoothing 2 values.

The previous years data demand data as well as a previous years forecast data, if we are using a yearly calculation. If we are using a monthly calculation, we require previous months demand and the previous month forecast and both these 2 data points can help us to make a forecast using the exponential smoothing method. So, parameter alpha is determined empirically. I have told you that empirically alpha can be determined. Rule of thumb says that alpha value can be equal to or less than 0.5 and many times alpha 0.2 or 0.3 is assumed for doing the exponential smoothing calculations.

So, forecast for k periods into future F_{t+k} is actually the forecast F_t . Now, let us try to take this example and I think it will clarify all the doubts that may have occurred or that may have come to your mind over the period of today's discussion.

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Exponential Smoothing:

One of the two wheeler manufacturing company experienced irregular but usually increasing demand for three products. The demand was found to be 420 bikes for June and 440 bikes for July.

They use a forecasting method which takes average of past year to forecast future demand. Using the simple average method demand forecast for June is found as 320 bikes (Use a smoothing coefficient 0.7 to weight the recent demand most heavily) and find the demand forecast for August.

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One of the 2 wheeler manufacturing company experience irregular, but usually increasing demand for 3 products. The demand was found to be 420 bikes for June and 440 bikes for July.

So, you have the demand data available for the month of June and July, 420 and 440 respectively. They use a forecasting method which takes average of the past year to forecast future demand. They are using simple average method. Also using the simple average method demand forecast for the June is found to be 320 bikes. Now, you have 3 data points that are available with you. The demand forecast for June is 320 bikes. The actual demand for June is 420 bikes and the actual demand for July is 440 bikes. Use a smoothing coefficient of 0.7 to weigh the recent demand most heavily and find the demand forecast for August.

Now, how we can do the calculation here? Very easily we can say that as per our equation for the month of August if we want to forecast, what are the 2 important values required as per equation? We require the forecast for July and we require the actual demand for July. Now actual demand for July as you know is given 440 bikes; we require the forecast for July and for forecast of July.

What are the 2 things required for forecast of July? We required the actual demand for June and the forecast for June. Forecast for the month of June has been calculated using the simple average method and the value given is 320. The actual demand for June is 420 bikes which is given in the problem. Therefore, for calculating the forecast for the month of August, we need to first calculate the forecast for the month of July. Which is not given here, but the data points required to calculate the forecast for July is given. We have the demand for June and the forecast for June.

So, very easily we can first calculate the forecast for July and then using that forecast and the actual demand that is already known for the month of July as 440 bikes. We can easily calculate the forecast for August. So, you can see using the exponential smoothing method and the exponential smoothing coefficient as 0.7. We can calculate F_t for the August month.

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$$\hat{F}_t = \alpha D_{t-1} + (1 - \alpha) F_{t-1}$$

where α = Smoothing Coefficient
 D_{t-1} = Actual Demand for Recent Period
 F_{t-1} = Demand Forecast for Recent Period
 F_t = Forecast of Next Period Demand

for July:

$$\begin{aligned} &= 0.7(420) + (1 - 0.7)320 \\ &= 294 + 96 \\ &= 390 \text{ units} \end{aligned}$$

for August:

$$\begin{aligned} &= 0.7(440) + (1 - 0.7)390 \\ &= 308 + 117 \\ &= 425 \text{ units} \end{aligned}$$


The equation is exponential smoothing coefficient alpha into the demand of previous month plus 1 minus alpha into the forecast of the previous month. So, D_{t-1} is the actual demand for the recent period. F_{t-1} is a demand forecast for the recent period and F_t is the forecast of the next period demand. For July, first we have to do the calculation smoothing coefficient is 0.7. 420 is the actual demand for the month of June and 1 minus alpha into 320 which is the forecast for the month of June based on the simple average method. This comes out as 390 as the forecast for the month of July.

Similarly, once we have the forecast for July. Actual demand for July is already known as 440. So, we can calculate using the same equation, we can calculate the forecast for the month of August 0.7. That is a smoothing coefficient into the demand for the month of July. That is 420 already known to us and forecast for the month of July we have already calculated as 390. 425 is the forecast for the month of August. So, using a simple equation F_t is equal to alpha into the demand of $t-1$ plus 1 minus alpha into the forecast of the period $t-1$. We can calculate the forecast for the period t .

Now, many times we required to calculate the forecast error also or the accuracy of our forecast. So, for that we usually make use of a term called MAD which is Mean Absolute Deviation.

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$$\text{MAD} = \frac{\text{sum of the absolute value of forecast error for all periods}}{\text{number of periods}}$$
$$= \frac{\sum_{i=1}^n |\text{forecast error}_i|}{n}$$
$$= \frac{\sum_{i=1}^n |(\text{forecasted demand} - \text{actual demand})_i|}{n}$$

where n is the number of periods.

Now, what is Mean Absolute Deviation? This is sum of the absolute value of forecast error for all the periods divided by the number of periods. It is the summation of the forecast errors for all the periods divided by the total number of periods. We can see the forecast error can be calculated using the forecasted demand. As we have learnt in the last 2-3 sessions, how we can make a forecast.

We have seen simple average, moving average, weighted moving average, exponential smoothing method. Using any of the method we can forecast the demand. Then there will be actual demand has experienced in the market. The difference between the forecast and the actual demand will give us the forecast error.

Suppose we have the forecast error available with us for the last 5 years. So, we will add up the forecast error for the last 5 years and then we will divide it by 5 and we will get a mean absolute deviation value. Now, how where this value can be useful? Or how we can make use of this value? We can use this value for comparing 2 or 3 different methods of forecasting.

Suppose for any given data, we make use of simple average method, weighted moving average method and the exponential smoothing method. So, we will get 3 forecast for every year and then we can calculate the forecast error using each method and then sum up of all the errors and find out the mean absolute deviation. So, the method which gives us more accurate forecast or maybe the less value of the mean absolute deviation, we can

select that method for making a forecast for that particular segment. With this we come to the end of our discussion on forecasting. In forecasting we have covered 5 different sessions. Week 3 was focused on forecasting only and we have seen that how we can make use of simple technique for developing a forecast.

We have seen in the first session what is the need requirement and importance of forecasting, how the forecasted demand can be used for making decisions (Refer Time 30:08) by the managers of an organisation. Then we have seen the forecasting system, that how an accurate forecast can be made or what are the various elements of the forecasting system. Then we have seen the qualitative methods of forecasting. That is estimated surveys or the Delphi method.

Finally, we have seen the quantitative methods of forecasting. In which we have covered the simple average, the moving average, the weighted moving average and the exponential smoothing method. Also, there is a time series forecasting model, in which we take care of the 4 important elements. That is the trend component, the seasonal component, the cyclic component and the random component. So, this time series model is also very relevant and used widely for making the forecast. The only complication in using this model is the calculation of the seasonal index and the cyclic index and random you can say index.

We can calculate easily the trend component using the regression method, but there are mathematical tools for calculating the seasonal and the cyclic components. Also basically the forecast in time series model is made up of the 4 components, the trend component, the seasonal component, the cyclic component and the random component. The main issue is related to the calculation of the seasonal indices and the cyclic we can say index.

Basically, we can use the time series models also for forecasting the demand, but as the time is limited we have to cover the topic of forecasting in 2 and half hours only. Because, our overall objective is to learn the basic aspects of operations management and sales forecasting is one aspect of operations management.

We have tried to give due weightage to this topic in our discussion. And for further you can say doubts and clarifications. You can always write on the discussion board and we would be more than happy to give you the replies to the best of our abilities.

Thank you very much.