Modeling & Simulation of Discrete Event Systems

Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture - 40
Problem Solving on Monte Carlo Simulation

Welcome to the lecture on Problem Solving on Monte Carlo Simulation. So, in the last lecture we discussed about how to solve the problem using Monte Carlo, and in this lecture also we will practice on some more questions so that we have even better understanding how to simulate the different kinds of problems which can counter, and we can use the generation of random numbers, and then we can predict the measures of performances.

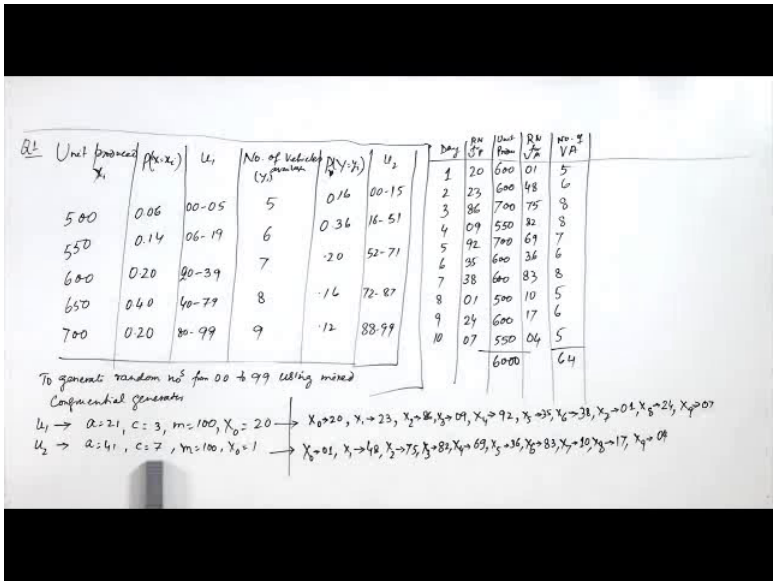
So, as we discussed that in the Monte Carlo simulation, basically you need to have the probability distribution or basically the probability values for certain outcomes for any particular parameter or variable. Now in that case if you are given the probability. So, you will find the cumulative probabilities. And then you will map the random numbers generated which are basically uniformly distributed random numbers between two numbers and then accordingly you are taking the numbers and solving the problems.

Now, as we discussed that many a times, you can have the generation of random numbers automatically by suitable programs and many a times you may use the different kinds of generation of random numbers using certain algorithm like congruential generators. So, how you can use some method of congruential generator and how can you use that random numbers which are generated by these congruential generators and you can use them for the solution of any problem.

So, let us deal with a problem where we will use the congruential generator that is mixed congruential generator first to find the random numbers. And then that random number is basically used for solving the problem. Many a time you will have the random numbers given that you can use this sets anyway those numbers also are distributed you know random numbers uniformly and that also is generated with certain algorithm certain method. So, if the problem is given like you predict certain measures of performance and use certain type of generator to find the random numbers, based on that you can predict you can take the values and then you can find the measures of performances.

So, let us deal with one such problem, which is related to the number of vehicles available and unit produced. So, the problem is like this.

(Refer Slide Time: 03:28)



That you have unit produced that is x and then you have also number of vehicles available. So, in a plant the units which are produced in the manufacturing facility that is given and based on that at a particular station what is the number of vehicle available that is having certain correlation. Now unit produced also in the plant has certain probability that can follow certain probability distribution, but then here the probability is given like if it is given that you have 500, 550, 600, 650 and 700.

So, suppose the probability of finding these I mean having these many units produced is given as suppose 6 percent. So, 0.06 then it is given that this is 14 percent 0.14, 20 percent; so 20, 40 percent so 40 and again 0.20. So, it is given that the probability that x is x i. So, this is basically x i you can say and probability that x equal to x I is given as this.

Similarly, the number of vehicles which are available are basically given that also ranges to 5, 6, 7, 8 and 9. So, this is 5 and then you have 6 7 8 and nine. So, basically if you have the unit produced 500 and you have number of vehicles which is available is 5 similarly on 550 it is 6 7 8 and 9, now for that also there is some probability if suppose this is yi and this is the random variable y. So, probability of y equal to y i. So, that value

is again given and this is given as 0.16. So, 16 percent then you have point 3 6 36 percent 0.20 then you have 0.16 and 0.12.

So, this is given that this is the probability, now in this case you are told that you should generate. So, the random numbers you should assign that what will be the random number and once the random number comes for that particular random number how can you say that this time you have the number of vehicles available is 5 or 6 or 7 like that. So, now, the thing is that they are telling that you generate the random variates or random numbers using the mixed congruential generator; so to generate random numbers between 0 and 100. So, excluding 100 basically from 0 to 99 basically that will be more correct from 0 0 to 99 using mixed congruential generator. So, mixed congruential generator you have to find. So, you have to find suppose u 1 here and you have to find u 2 here.

Now, the thing is that if the u 1 is now you can see that if the u 1value is from 0 0 to 0 5 in that case you have this value as 500. So, units which are produced are 500 because looking at this cumulative distribution, you can see that if you have the random number between 0 0 to basically 0 5. So, 0 1 2 3 4 5 there are 6 random numbers which if you are getting, you are basically assigning the units produced as 500. So, so since this cumulative probability will be 20 here. So, it will be from 0 6 to 19. So, there are 14 entities in between similarly is from. So, it will be 20. So, then again 20 to this is 40,s o 39. So, that number if comes you will have the basically number unit produced as 600; similarly then 40 to 79 and 80 to 99. So, if the u 1 is in this range you are going to take the unit produced as 500.

Now, similarly this tells that you have to generate the random from 0 0 to 99. So, if you are taking the mixed congruential generator, in the mixed congruential generator we have 3 parameters 1 is a 3 constant basically a c and m, a is the basically the constant and. So, that is multiplied basically a into x i minus 1 or x 0 for finding the next number x 1, and then you have the addition of c that is a constant and then you have modules of m. So, m is (Refer Time: 10:24) m will be basically 100 because you have to get the reminder. So, you have to get up to 99. Now similarly in the case of u 2 you will have number if you take random number which is will be valid for finding these values will be 0 0 to 1 5 then 16 to. So, this is 52. So, 16 plus 36 is 52. So, it will be 51 similarly you have 52. So, this

is 72 cumulative value is 72 here. So, it will be 71 then 72 to so 88. So, 87 will be there and then 88 to 99.

So, this way you are generating these you are assigning the random numbers, if they come on the way and then how can you assign these numbers for getting a particular unit produced or for getting the number of vehicles which are available at a particular iteration. So, now, how to generate these random numbers for that, for u 1 they have given that you should have a as 21, c as 3 and m as 100 and x naught or x 0 as 20 and similarly for u 2 they have given that the condition is given like this. So, for u 2 condition is a as 41, c as 7, m as again 100 because we have to generate between below 100. So, from 0 0 to 99 and then the seed value is taken as one.

Now, suppose this is how it is given. So, in that case you can generate the numbers. So, if you have. So, first of all if you go for this you will have x naught as basically you will has x naught as given as 20 then x 1. So, what will be x 1? X 1 will be a into x naught. So, this is 21 into 24 20 plus 3, 423 mod 100. So, it will be 423 mod 100. So, that is why it is 23 then x 2, now x 2 will be now taking 23, so 23 into 21. So, that is 483 plus 3, 486. So, it is 486 mod 100. So, this is coming as 86, similarly x 3 will be 86 multiplied by 21.

So, that is 1806 plus 3 1809 and that being taken mod by 100. So, it will be 0 9 similarly x 4 will be 0 9 multiplied by 21, 189 plus 3 192. So, it will be 192 mod 100 it will be 92, then you have x 5 as 92 multiplied by 21. So, that is 1 9 3 2 plus 3.1 9 3 5, 1 9 3 5 mod of 100 that is 35, x 6 will be 35 again multiplied by 21. So, 7 35 plus 3 38, x 7 38 multiplied by again 21, it will be 798 plus 3. So, that is 801, so 801 mod 100 that is 1. So, that becomes 1, then you have x 8. So, this way x 8 comes.

Then if you generate further 1 into 21 that is 21 plus 3 24 so that will be 24 further it will be 24 into 21 that is 5 0 4 and plus 3 5 0 7. So, it will be 7. So, this way you will have the numbers coming you have 10 numbers generated form x naught to x 9. So, this is how you can generate the numbers for even u 2. Now for u 2 if you look at you have x naught given as 1, x 1 becomes 1 into 41 and plus 7 mod of 100, so 48 mod 100. So, that is 48, x 2 48 multiplied by 41. So, that will be 1 9 6 8 and plus 7. So, that will be 1 9 7 5. So, that will be 1 9 7 5 mod 100. So, that will be 75, again 75 mod of 41. So, that will be 3 0 7 5 and 3 0 7 5 plus 7. So, that will be 3 0 8 2 and that further mod of 100. So, it will be 82, x

4 82 multiplied by 41. So, that will be basically 3 3 6 2 plus 7 3 3 6 9. So, it will be 69, x 5 will be 69 multiplied by 41.

So, that will be 2 2 9 plus 7 2 8 3 6. So, it will be 36, x 6 will be 36 multiplied by 41 that will be 1 4 7 6 plus 7, 1 4 8 3. So, it will be 1 4 8 3 mod 100 83. Similarly x 7 will be 83 multiplied by 41. So, it will be 3 4 0 3 plus 7 3 4 1 0. So, it will be 1 0, x 8 will be 10 multiplied by 41, 4 10 plus 7 417. So, it will be 17 and x 9 will be 17 multiplied by 41, 697 plus 7 that is 7 0 4. So, you get 7 0 4 mod 100 that will be 0 4.

So, this is how you can generate the random numbers you know that in this case you will have a maximum period of 100 random numbers because you have the period of 100. So, once any number comes in that case that will further a cycle will be generated, but here we have you have not seen that that stage has come and we feel that this is. So, anyway this numbers require to be tested for uniformity and dependence, but then that is a separate issue that can be done.

Now, we can do the simulation. So, let us see we will take this numbers in order, now we have the table. So, you have the day number. So, you will have you will have the simulation now. So, simulation will be day number what is day will be 1 2 3, then you will have the random number for unit production. Similarly you will have unit produced according to that number, then you have further random number for vehicle availability V A similarly you will have number of vehicle available. So, now, let us see we have the random numbers starting from 20. So, first of all day 1 2 3 4 5 6 7 8 9 10 suppose you have to do the simulation for 10 days in that case you can see like this in the first case random number for unit produced will be 20, 23, 86. So, this 20, 23, 86 then you have 9 9, 92, 35 similarly 38 1 and 24 and then you have 7.

Now, again this number if you compare from here. So, twenty comes in this. So, 20. So, here you will have (Refer Time: 20:40) 600, 23 also comes here. So, 600, 86 comes here. So, 700, 9 comes in the second row that is 550, 92 will be here. So, that will be 700, 35 will be here that is 600, 38 will be again here 600, 0 1. 0 1 will be in this 500, 24. 24 will be in this range. So, 600 and 7. So, 7 will be in this in that second one. So, that will be 550. Similarly random number for the vehicle availability, now random number vehicle availability is as 148 75 then 82 69 36, 83 10 17 and 0 4. So, for 0 1 it is 5 days 5 number of vehicles for 48 it is here. So, it is 6 for 75, it is here 8, for 82 it is 8, for 69 it is 7 for 36

it is 6, for 83 it will be 8, for 10. So, for 10 it will be here 5, for 17 it will be again 6, for 4 it will be 5.

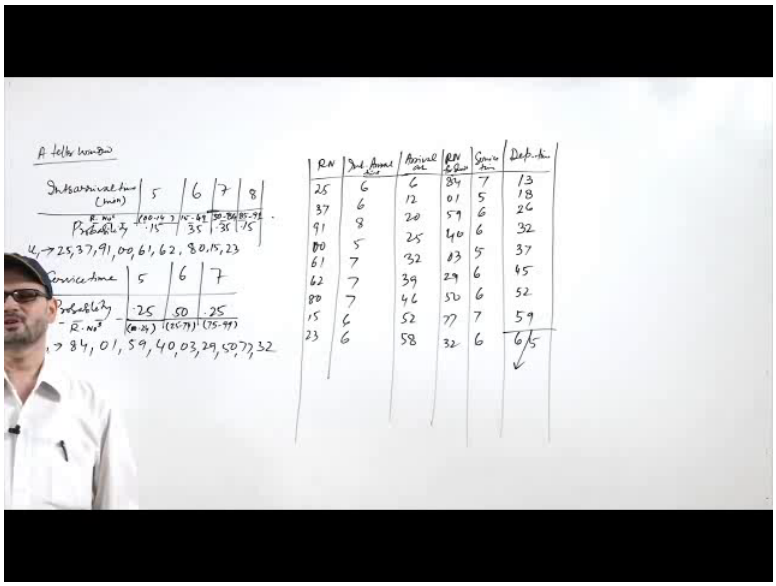
So, like that you have to generate. So, the purpose was to make you aware that how you have to generate the different. Now there may be another method you may be told to generate using the mid square method. So, in the mid square method you are told some number and then further you have to go and take the middle of the numbers. So, that can be done, but in that case it is not we are not sure that what will be the how many numbers you can get in continues that you have check it.

Now, in may such congruential generators, you know that how you can achieve the maximum period. So, that is basically there are certain rules that we had discussed. So, this way if you say if you see the total number of unit produced, if you add them it is coming around 6000 yes. So, it is 6000 similarly if you add them number of vehicles which are available; so 11 1927 7 34, 6 48 53 then 59 plus 5 64.

So, what we see is the average number of unit produced is basically 6000 here, and that if you take the average unit it will be 6000 by 10 that is 600. Similarly average number of vehicles which are available that will be 64 and divided by 10. So, that will be 6.4 this you can basically find and similarly if you have the other data given like what is the total capacity which it can accommodate that way, the production plant.

So, you can have different tables and from there you can have the shortages what is expected like that and you can find all these majors of performances in such problems. So, it is all it depends that you must understand how to generate these random numbers using those methods. You can generate using the uniform distribution method by using the generator like excel by using programs by using the softwares like Erina input analyzer or. So, so you can have the generation of numbers and you can have all these values mapped into and predict the measures of performances.

(Refer Slide Time: 25:35)



We can further deal with another question, another question which is like this that you have a teller window and in that the probability of inter arrival time is given. So, inter arrival time in minutes is given. So, and its probability is also given. So, it is from varying from 5 to 8. So, 5 6 7 and 8 and its probability is coming out to be 0.15, 0.35, 0.35 and 0.15. So, suppose it is this type of distribution function where you have 0.15 on the side and then it is coming like 0.35, 0.35 and 0.15 similarly you have the service time.

So, then service time is given and then that is given as and its probability is also given. So, service time varies from 5 to 7. So, 5 6 and 7 and in that case its probability is 0.25, 0.50 and 0.25 and you can generate the random numbers for this also, but here that numbers are given and the this is random number u 1 is given for this as 25 37 91 0 0 61, 62, 80, 15 and 23; similarly for this the random number is given as 84, 0 1, 59, 40, 0 3 29 50, 77 and 32. So, suppose this is given we know that the numbers which will be assigned for 5 we can do it from here itself.

So, as we know that we will have the random numbers for the range will be we can write here just or in the bracket we can write, here the spaces less. So, we can write in the brackets itself random numbers which will be assign falling in this, similarly you will have random numbers. So, as you see that this is 15.15. So, 0 0 to 1 4 similarly you will have 1 5 to 34 and 35 to 69, no there is some mistake here it will be 1 4, 1 5 to it will be

35 numbers. So, it is 50. So, 49 then it will be 50 to 85. So, 50 to 84, 50 to 84 and then 85 to 99.

Similarly in this case you will have 0 0 to, 24 25 to 74, 75 to 99. So, this way the random number you can assign and then you can see that how. So, so the random numbers will be coming and accordingly you can assign. So, the random number for the inter arrival time and that is seen as 25, 37 37, 91, 0 0, 61, 62, 80, 15 and 23. So, these are the numbers which are given suppose and then you have for this you can have the inter arrival time.

So, you can have for this, what is the inter arrival time arrival time you can write from here. So, inter arrival time will be for 25 suppose it is coming in this it will be 6. So, this way 6; 6 8 5 then you will have 7 7 7 6 6. So, this way you can just see any number like 15. So, for 15 it is here. So, it is 6, then for 62 it is 7 like that.

Similarly it will be arrival time. So, arrival will be at what time arrival at. So, you can simply write 12, 20 it will be you have to add them. So, this will be 20, 25, 32, 39, 46, 52 and 58 next we will go beyond 60. So, we have to go for one hour simulation that is why we are stopping here, and then in that case you will have the random number for departure service. So, service as you know we have 84, 0 1, 59 then you have 40, 0 3, 29 and then we have 50 77 and 32.

So, for that again you can see the service time service time will be further written and service time will be for 84, it will be 7. So, similarly you will have other numbers you can write. So, this will be 5 6 6 5 then further you will have 6 6 7 6. So, this way you will have those service times, now you can find the time of departure. So, departure time you can find now this person comes at 6 and his service time is 7. So, he will depart at 13. So, this 13 this person comes at 12, but this person departs at 13. So, he will depart at 18 because he has 5 minutes of service time.

Then he is coming at20. So, anyway the server will be free for that time and this 20 will come and go at 26. Now it 26 it will be he will have to wait for one minute. So, 26 plus 6 32, now this 32 is same. So, it will be 37, 37 here again 39. So, 2 minutes is the waiting I mean server is waiting. So, 39 plus 6 45 similarly 46 plus 6 52, then you will have 59 and then you will have 35. So, 65 anyway it is going. So, it is going below beyond the 60 minutes. So, we are stopping here.

So, this way now you can find from here whatever you have to calculate that how much is the waiting time of the server, what is the utilization of the server percentage all these can be found out. So, what we see that you can use the different methods of generating random numbers, this method could have been generated by any congruential generators or any other type of random number random variate generators using any typical distribution or so, and you can solve such methods solve problems and gain more and more confidence.

So, this is the end of this last lecture of this course, I hope that you enjoyed this course and I wish you a very good luck for those who are appearing for the exams of this course.

Thank you very much.