

**Modeling & Simulation of Discrete Event Systems**  
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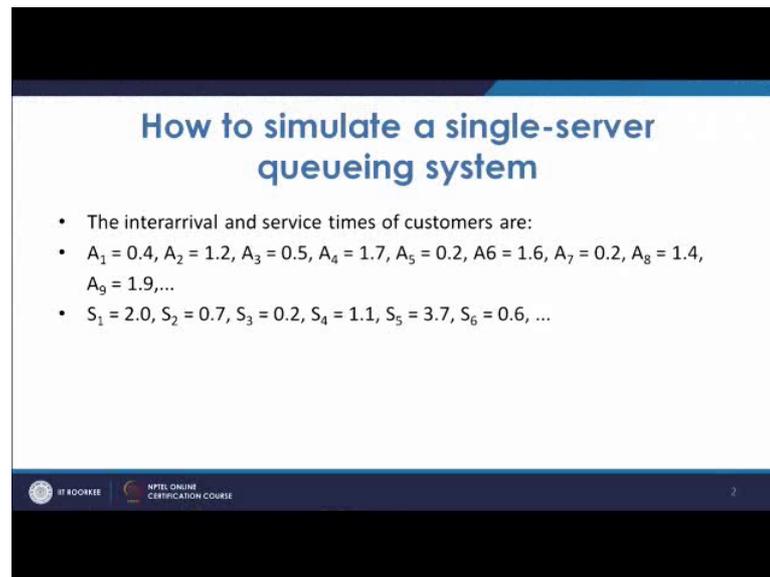
**Lecture – 15**  
**Computer Representation of Simulation of Single Server Queuing System**

Welcome to the lecture on computer representation of simulation of single server queuing system. So, in the last lecture we discussed about the single channel queuing system, where we found the Qt Bt and delays all the calculations where done by finding the graphs on the axis. And we try to find that how these performance measures are computed. Now we will have an overview or we will have to see that how in the computer when we do the simulation, how the things are updated as we move from time to time.

So, as we know that the computers basically read whatever is fed to them. So, being a you know transient case you have the events occurring at different times. So, there will be you know initialized and initially at 10 equal to 0. And then the computer has to read the subroutines or routines or programs. So, it will invoke the main program to take the data from the respective subroutines. Based on that it will update the counters update the variables list, and then slowly with respect to the earlier cycles or previous you know values it will try to update the statistical counters.

So, we will have in this lecture, we will have the snapshots of how the computer proceeds by you know by every iteration at every time it will move and it will tell you that how the different you know values are updated. So, the same problem which we discussed the last class that we will see in this case.

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**How to simulate a single-server queueing system**

- The interarrival and service times of customers are:
- $A_1 = 0.4, A_2 = 1.2, A_3 = 0.5, A_4 = 1.7, A_5 = 0.2, A_6 = 1.6, A_7 = 0.2, A_8 = 1.4, A_9 = 1.9, \dots$
- $S_1 = 2.0, S_2 = 0.7, S_3 = 0.2, S_4 = 1.1, S_5 = 3.7, S_6 = 0.6, \dots$

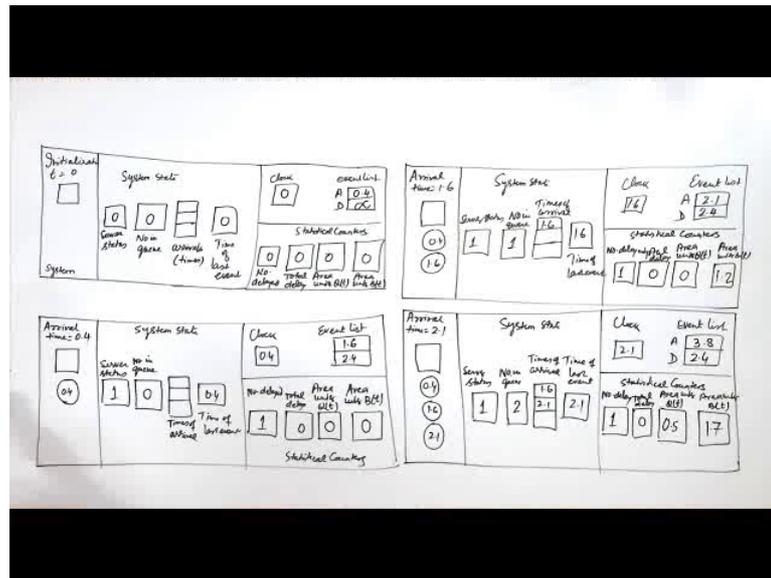
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So, how the computer is going to do the analysis. Now in the computer when the computer is there it will have basically the you know one is that it will talk about the system. What system is telling you will have system states which will talk about how many persons are there in the system, how many arrivals are there.

So, this way there will be a system. There will have we will have the you know clock which will talk about the timings. It will tell that what is the present time what is the time at which the last event has occurred or present event is occurring. So, what is the present simulation clock time, it will talk about the event list it will tell that which of the event is most eminent you know in the computer, you fill you give the you know use a subroutines you have the programs by which you feed these values that when there will be first arrival when they will be second arrival or so.

Now, it will talk a I mean on logical basis, it has to decide that which one will be occurring next. And then based on that every time once it moves from one time 2 other the statistical you know counters counter is there. So, basically this parameters or performance measures the updated one by one. So, we will see that how that is updated. So, what we see in the computer representation you have basically.

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In normal computers representation you have one is your system. So, this is your system then you have the system state here.

Now, in this side you have the computer representation. In the computer representation you have system state. So, you have the system state, which will have few you know parameters, that one is you know server status. Second is persons in queue. So, it is number in queue. Third will be arrivals. So, that is times of the arrivals. And then you have time of last event. So, this is your system state. Then you will have another portion in which you have the calculation of a statistical counters. So, you will have a statistical counters, and in the statistical counters you have the delay.

So, that is number delayed. Then you have total delay. Then you have area under  $Q_t$  and area under  $B_t$ . So, we have found this  $Q_t$  and  $B_t$  curve in the previous lecture. So, from there how the computer represents and you directly get in this you are having one clock. And another is event list. So, in the even list you will have 2 events basically one is arrival and another is departure. So, this is how, now this is your system which will. So, that talks about what the situation is. It will talk about the event which is occurring, whether there is any arrival or there is departure. So, if there is arrival it will be arrival at certain time. If there will be departure it will be departure at certain time. And initially you are initializing the system at time  $t$  equal to 0.

So, first of all certainly we start with initializing the system at time equal to 0. So, the main program invokes the initialization routine. So, initialize initialization time to  $t$  equal to 0, you have a server you do not have any customer in the system. So, it means that the server is at  $t$  equal to 0 which is there is no customer your server is idle it will be 0. So, that is why there is no person in the queue this is no arrival nothing is. So, we have time 0, this is clock time is 0.

Now, at time 0 it has to say from the event list, the computer has to read this. And then it has to decide what is the next most eminent event. Now if you look at the data as we have seen your first arrival is at 0.4. So, it means the first is only arrival there cannot the first departure there can be only first arrival. So, we see is the next arrival which is scheduled is 0.4. Now at this time, because in the next iteration once the computer has to go to next iteration, it has to see that whether it has to take the arrival or departure. So, anyway it cannot afford to take departure. So, initially it is poisoned the depth the departure is poisoned. So, in the first iteration the time of departure is taken as infinity very high value.

So, basically that poisson said this is known as poisoning of the departure. There is no person in the system. So, there is no delayed. So, it will be 0, total delay is 0 there is no other value previous. So, each it will all be 0. Now the computer will look to the next time the next event, it will look for the next event time from the event list, it will see to the event list and it will see the minimum of the 2. And the minimum of the 2, whatever it is it is either arrival or departure. So, that will be taken as the next event. So, we will go to next case now in this case again we have this is the system.

Now, the in this case we are coming to arrival time equal to 0.4. So, you have the server which is represented by the box. And one person has arrived there is one arrival in the system. So, that will be represented by a circle here. He is coming at 0.4 time. So, it means that in the system in front of the server there is one person who is standing in front of the server. He is coming and he has stood in front of the server to get the service.

Now, we are coming to system state in the system state, you have server status. So, as there is a customer in front of the server. So, server state becomes busy. So, it will become one, then number in queue. Now number in queue means as we have already discussed number in queue means the person as long as he is not on the service counter

he is in the queue. Once he goes he is already in the system, but not he said to be in the queue. So, it remains still number in queue is 0. Because he is already at the at the service counter.

The arrivals times of arrivals is represented by the array of. So, you have some arrive define because you will be having number of customers we will be coming you know and they will be grouping themselves. So, you have times of arrival. So, the person who is already there he is already here there is no person who has arrived had been another arrival it will come the time here time of last event will be again 0.4. Now we will come to clock. So, clock time is set as 0.4 we are coming to the event list.

In the event list, the next arrival because this arrival we have only taken. So, next arrival is scheduled, at the time inter arrival time is given as 1.2. So, next arrival is scheduled at 1.6 and the departure because the first customer has come. So, for him the departure will be 0.4 plus 2. So, 2.4. So, his departure will be at 2.4. Now number delayed. So, this is as one person has come into the system the number delayed is taken as one. When this reaches to 6 the simulation stops that is the stopping condition for the simulation total delay he has not experienced any delay. So, is this value is 0, area under  $Q_t$ .

Now, area under  $Q_t$  will be the area under  $Q_t$  value from the previous time plus number in queue times the time lag between this and this event. So, this is how area under queue is updated every time. Whatever you had the earlier value plus number in queue times the time which has elapsed between this event and the previous event. So, anyway it is 0 plus again it is 0 here. So, it will be again 0. Now it will be area under  $B_t$ , busy time function. Now area under busy time function if you look at it will be the previous value plus whether the server is busy or idle.

So, server is if you look at that in this case the server is idle. So, it is having 0. So, 0 plus 0 times this time lags. So, it will be 0. So, so far area under  $B_t$  is coming out to be 0. Now we will go to the third case. So, what we see is from here the computer again seize is from the event list of the arrival and the departure arrival time is less. So, it will take next event as the arrival. So, in the system it will tell again arrival time is 1.6. So, the system representation will be like this you have a server. In that you had already a customer with at 0.4 and there will be another customer at 1.6.

The system state updation will be done. In the system state you have server status, it will be 1. Because server is now since in the in front of the server you have customers. So, server is anyway busy then you have number in queue number. In queue if you look at it was 0 here because there is only one person in front of the server who is getting the service. Now you have 2 customers in front of the server. So, this person is said to be in queue not in the service. So, that is why you are number in queue is coming as 11.

Then you have times of arrival. Now what we see is that this persons arrival time is written here. Because once this person moves this person will come and join the server. So, the person who is in queue it will talk about the time of there you know their arrival in the queue who have been in the system. Then you have the time. So, this is your time of last event again as it is 1.6. Now we will come to clock. So, clock is clock time is 1.6 and event list.

Now, event list you have 2 arrival and departure. What we see is in the earlier case you have already taken further the arrival. So, the next arrival will be if you look at the next arrival it will be 2.1. And departure as usual it will be here placed. Now we will update the a statistical counters. So, in the statistical counters. You have number delayed. Number delayed is still one there is no second customer who has come into the queue second is total delay total delay is still is 0. Then you have area under  $Q_t$  and area under  $B_t$ .

So, area under  $Q_t$  is here 0, and here the number of person in queue is 0. So,  $0 + 0$  into  $1.6 - 0.4$ . So, since it is 0 and as well as this is 0. So, this will again come as 0. Now the server was busy. So, busy function value was 1. So, area under  $B_t$  will increase area under  $B_t$  in the earlier case was 0, but now in this case it was 1. So, it will be  $0 + 1$  into this time  $1.6 - 0.4$ . So, it will be  $1.6 - 0.4$ ,  $1.2$  multiplied by 1. So, it will be 1.2.

So, under the  $B_t$  function because there is server is now busy. So, this value will increase. Now let us see the next. So, the next event will be again arrival. As you see among the event list you have further an arrival. So, there will be further arrival at time 2.1. So, the system looks like this, you have the server in front of that you had one customer first customer at 0.4 second customer came at 1.6 and third customer has come

at 2.1. Now we will update the system state. The system state server status will certainly be busy, because you have many customers in front of the server.

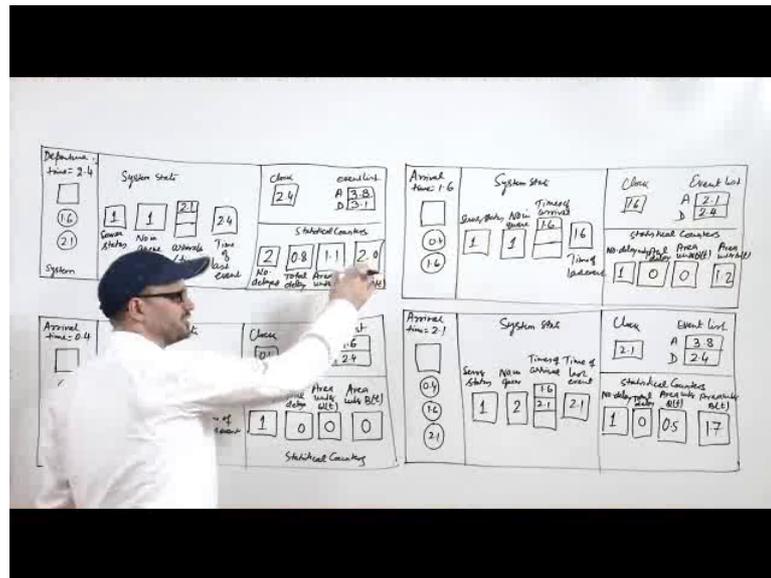
Number in queue as we see the number in queue is 2 persons. So, it will be 2. Then you have times of arrival. So, what we see is you had 1.6 and then another has come at 2.1. This is just to store the additional arrivals which is occurring and then the time of the last event is 2.1. Now we will move to the clock is at 2.1 time and from the event list we will see arrival and departure.

So, last case was the arrival. So, next arrival is at what time, the next arrival as it is seen will be at as we see at 3.8 the next arrival which is coming at 3.8 and departure is 2.4. Now we will update the statistical counters. So, what we see is we got this 3.8 from this figure you have the fourth customer who is coming at 2.1 plus 1.7, 3.8 and anyway departure time from the event list is unchanged. So, you have 2.4.

Now, we will update the statistical counters in that you have number delayed. So, there is no change in the number delay is still it is one, there is no departure. So, far then you have the total delay that is 0. Then comes area under  $Q_t$ . Now area under  $Q_t$  will be now changing. The earlier value was 0 plus since now there is one customer in the queue. And there is time lag of 0.5, as we see the time is 2.1 and this time is 1.6 So, there is there is lag of time of 0.5 minutes. So, 0.5 multiplied by 1. So, it will be 0.5.

Similarly, area under  $B_t$  area under  $B_t$ , will be previous value plus it was busy. So, one times this manage this time that is 0.5. So, 1.2 plus 0.5 that is 1.7. So, this way the value is updated as the time progresses. Let us see in the next event what happens. So, we will see here.

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So, after this state after time 2.1. What we see is there is of the 2 event in the event list the lesser value is 2.4. So, next event will be. So, it was earlier 1 2 3 4. And the fifth reading was now here you have a departure.

So, departure time and this time is 2.4. So, after this you will have this snap shot coming. At this time if you look at you have the server. And the per the customer which was just in front of the sever it has departed. So, now, the customer who has come who is arrival time was 1.6. And he is followed by a customer who is there in the system whose arrival time was 2.1. In this case whether we see that server is busy. Then there is number is queue is 1. Earlier it was 2 and as there is a departure the number of customer in the queue will be 1.

From this also array also one value will decrease. So, this 1.6 arrival time customer is anyway in the service he is getting service should now he will you will have 2.1. Here time of last event is 2.4 clock is 2.4. Now the departure was the event. So, anyway earlier arrival time will not be changed. Now the customer who has come at 1.6. Now he will have the service and he will depart.

So, what will be the next departure. So, that is to be seen. Now in this case you have arrival is 3.8 and since it is getting free at 2.4 and it takes 0.7 time to serve. So, the next departure will be at 0.7 time. So, so 2.4 is a departure time. Next if it is departure the next departure will be for 0.7 time units and it will be 2.4 plus 0.7. So, it will be 3.1 next

eminent departure will be 3.1. Now we will update the statistical counters. So, number delayed now number delayed since there is a departure the second person will go and join the queue. So, number delayed will be 2.

When this will reach to 6 the simulation will stop then total delay now what will be the total delay for this customer. So, the person who came at 1.6 and departed at 2.4. So, here has to you know wait for point 8 times. So, total delay is point 8 then area under  $Q_t$  area under  $Q_t$  the last  $Q_t$  value is 0.5. And number in queue is 1. And number in queue here is 2. So, 0.5 plus 2 into time lag that is 0.3. So, 0.5 plus 2 into 0.3. So, 0.5 plus 0.6. That is 1.1 and then area under  $B_t$  will be again 1.7 plus time lag that is 0.3. So, 2.

So, this way you have to continue this till you end. So, this is left upon you to exercise and you can follow the book of you know famous book for studying this modeling and simulation and book by law and Kelton. So, you can refer this book to study and you can see that how the problem is solved. And you can go up to the end of time equal to 8.6. So, this way you can solve such problems. And finally, you will see that you will get area under  $Q_t$  and area under  $B_t$  as the values which we computed in the earlier or previous you know examples. So, you will see that at that time you will have those values coming at 9.9 and 7.7 or so. In this cases and that is how the computer is doing the simulation. So, you can practice more and more problems by taking the inter arrival times and so, and service times and you can get the results.

Thank you very much.