Introduction to Mechanical Vibration

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Lecture - 31
Undamped Dynamic Vibration Absorber

So welcome to the lecture on vibration absorber. So today we will discuss the theory of undamped vibration absorber. So what is a vibration absorber? As from the name we can understand that it absorbs the vibration, so it means there is some system that is in vibration so it could be in vibration due to some external load and vibration absorber is another system that if we attach to the main system the vibration of the main system should reduce.

So if we have a main system and we can model it as a single degree of freedom system and - and the vibration absorber is also a dynamic system and it can be modelled as a single degree of freedom system, then the total system will be two degree of freedom system, so the vibration absorber they are used in several applications where there is vibrations and the system is running at a constant speed so like reciprocating engines.

Engine that are designed for running at constant speed with minimum fuel consumption, so these systems they can work in a very narrow band of frequency range, because they are tuned for certain frequency range and therefore we will also called damped the tune vibration absorber when they are tuned for certain frequency, so here secondly jumper is the transmission lines. So we can see here the transmission lines there are the cables and they are passing and due to the wind they get excitations and they can vibrate, so to reduce the vibrations.

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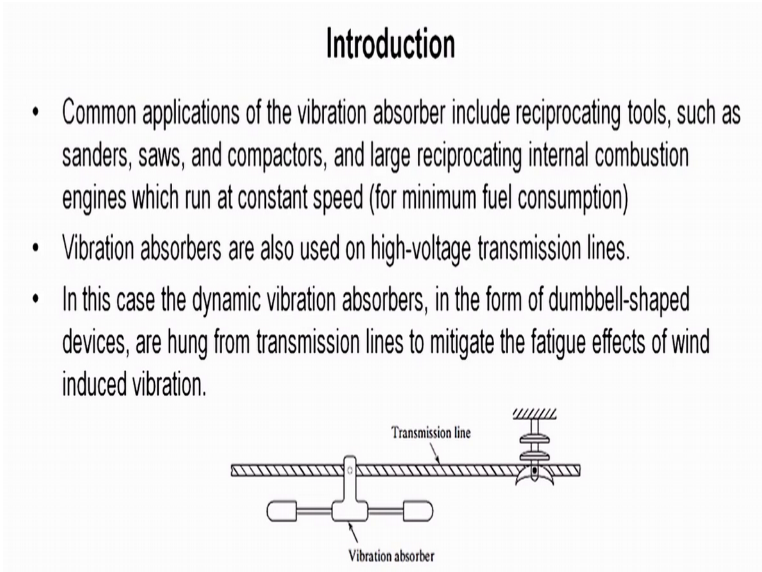

We can see that here we attach a vibration absorber, that is a dumbbell shaped device and this can mitigate the vibration due to the wind excitations. So any machine or any system that is subjected to some harmonic force, okay, and it is because we have already seen that when a system is subjected to any harmonic force it will start vibrate and the steady state vibrations will be governed by the harmonic force that is acting on the system.

So if we want to reduce the vibrations we have to design a vibration absorber and we have to attach this to that systems and this vibration absorber will be have capability to reduce the vibration it will shift the resonance frequency, so we will consider the two degree of freedom system, so this is our two degree of freedom system, so here we can see here is a machine and that is resting on some springs and on the rigid base.

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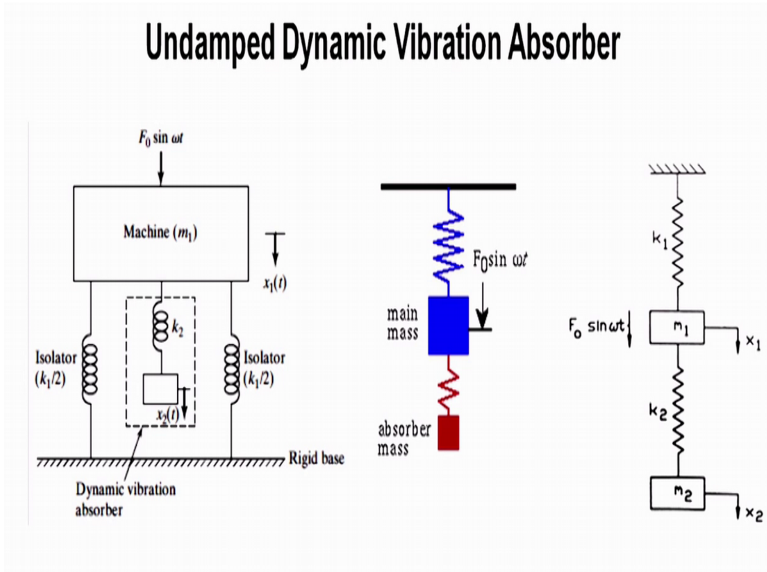

And this machine is subjected to some unbalanced forces maybe F0 sin omega t, that is acting on the machine and due to this these forces the machine will have a steady state vibrations, and so if we want to reduce this vibrations we have to attach another system we can see here, so the main system is here we the main system is represented by the mass m1 and stiffness k1, because here there are two springs k1 by 2 and k1 by 2 stiffness.

So the total stiffness because they are in parallel will be k1, so the main system is represented by a mass m1 and spring constant k1, while the absorber system is represented by mass m2 and stiffness k2 and this system is attached to the main mass, so this - this system can be represented in this way, so we have spring k1 than mass main mass that is m1, and then spring k2 that is the stiffness of the absorber and the mass of the absorber.

So we have this system, so we can write the equations and we can find that we have to derive that, how the response of the main system is being affected or it is being reduced due to the introduction of the secondary system that is the vibration absorber.

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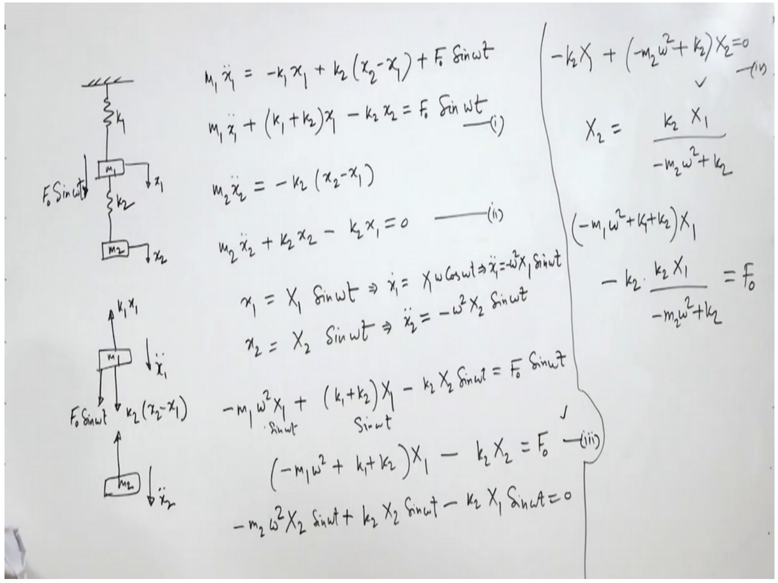

So this is our k1 m1, k2 m2, so k1 m1 is the main system and k2 m2 is the absorber system and this attached to the main system, now we this main system is subjected to some harmonic force that is F0 sin omega t and this is the response x1 of the main system and this is the response of the absorber that is x2. So now we can write the equation so we have to make the free body diagram of the masses and show the forces.

Then we have to apply the Newton's second law to write the equations of motion. So here we have m1 so this mass when it is pulling this direction so we assume this direction positive, so it is going to this phase so the spring force is working in opposite direction that is k1 times x1. Now we assume here x2 > x1, so the net force acting on the mass is in there this direction that is due to the spring k2 that is k2 x2 - x1.

And there is the harmonic force acting on the system so that is F0 sin omega t. now second march m2, so m2 will have the same force k2 x2 - x1 in opposite direction of the that of the mass m1 and no any other force and the direction of x less than we assume this as a positive. Now we can write the equations of motion, so we have for the first mass m1 x1 double dot = summation of the forces so - k1 x1 and + k2 x2 - x1 and + F0 sin omega t.

So here we can write m1 x1 double dot + k1 + k2 x1 - k2 x2 = F0 sin omega t. Now for the second mass so let us say this is equation number one, then the second mass we write m2 x2 double dot = - k2 x2 - x1. So we can rewrite like m2 x2 double dot + k2 x2 - k2 x1 = 0, this is equation two. Now as we have already discussed in the two degree of freedom systems that the complete system will vibrate with one frequency at one time and that will be the its natural frequency.

And this is the two degree of freedom system so it will have two natural frequencies, so the two frequencies and two natural frequencies and two mode shapes corresponding to each frequency. So here we assume that the system is vibrating with frequency omega, the same frequency as the first frequency because the steady state response of the system will be governed by the external excitation force that is F0 sin omega t.

So we assume that x1 = x1 sin omega t and x2 = x2 sin omega t, now we can put these values here in this equation one and two, so x1 is x1 sin omega t, so it means x1 dot is x1 omega cos omega t. And so x1 double dot = omega square - x1 sin omega t, so - omega square x1 sin omega t, similarly x2 double dot = - omega square x2 sin omega t, now we put this x1 double dot value here in equation one and x2 double dot value here and x1 value here and here and x2 value here and here.

So what do we get so here if we put so - m1 omega square x1 + k1 + k2 x1 of course here will be sin omega t and here also sin omega t - k2 x2 sin omega t that is equal to F0 sin omega t, so we can cancel out sin omega t or we just take this spot, so - m1 omega square + k1 + k2 x1 - k2 x2 = F0 this is equation number three, similarly we can have equation two.

So - m2 omega square x2 sin omega t + k2 x2 sin omega t - k2 x1 sin omega t = 0 again we can write here - k2 x1 + this is k2 so we have - m2 omega square + k2 x2 = 0, so this is equation four. so we have now these two equations, equation three and four, so from here we can find x2 = - k2 x1 so it will be +, so if we take this other side so k2 x1 by - m2 omega square + k2. Now we put the value of x2 in equation number 3 here.

So we have - m1 omega square + k1 + k2 x1 - k2 into x2 so here x2 is k2 x1 by - m2 omega square + k2 and that is equal to F0, okay. So we can solve this equation for x1.

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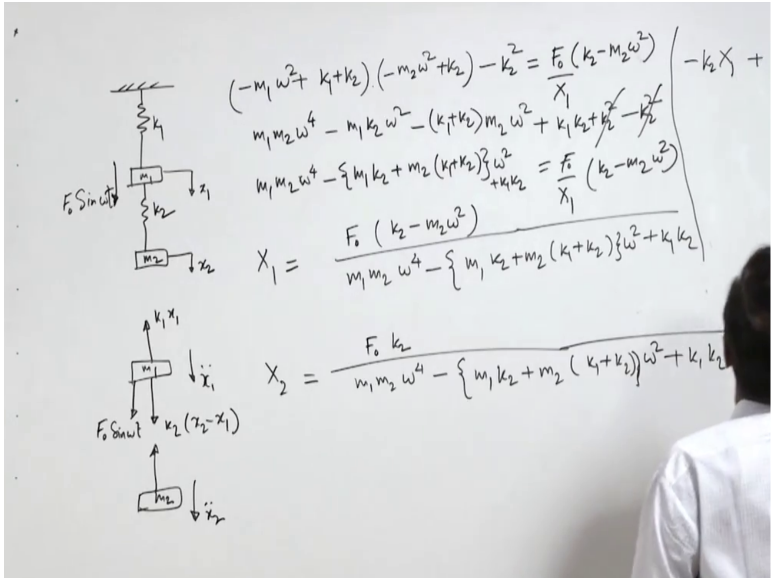

So - so if you solve for x1 we will have - m1 omega square + k1 + k2 into - m2 omega square + k2, because this term will multiply this term - k2 square = F0 into so this time will multiply here and here so it is k2 - m2 omega square and here we can write x1, because x1 is common here and here so it can go in the denominator. So we can solve this so that is m1 m2 omega power 4 and then it is - m1 k2 omega square.

And this is + k1 so it is here - k1 +k2 m2 omega square + k1 k2 + k2 square - k2 square and that is equal to F0 by x1 k2 - m2 omega square, so now this k2 square will cancel out and we can rewrite this term so m1 m2 omega power 4 - m1 k2 + m2 k1 + k2 omega square + k1 k2 so that is equal to this term. So we can write x1, so x1 is equal to F0 k2 - m2 omega square upon m1 m2 omega power 4 - m1 k2 + m2 k1 + k2 omega square + k1 k2, okay.

So if we finally we have got the x1- we have got x1, now we put the value of x1 here and we will get x2, so x2 = we put the value of x1 so we put the value of x1 here. So here in denominator is k2 - m2 omega square and in the nominator - numerator here is k2 - m2 omega square so they will cancel out and we will get F0 k2 upon m1 m2 omega power 4 - m1 k2 + m2 k1 + k2 omega square + k1 k2, so this is what we have got, okay.

So we have got x1 and x2. Now we can rewrite this equations as because here we have omega 1 is the natural frequency of the first system.

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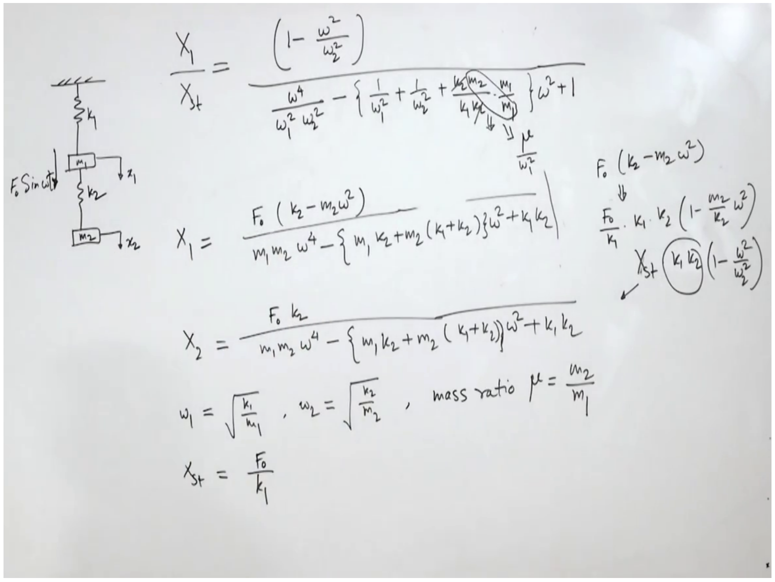

So that is root k1 by m1 and omega 2 is the natural frequency of the second system that is root k2 by m2 and we define mass ratio utter mass ratio that is mu and it is m2 by m1, so mu is defined as the ratio of the two masses that is the mass of the absorber and divided by the mass of the main system that is m2 by m1. Now we define one more term Xst, so Xst is defined as the zero frequency deflection of the main system due to the force F0 that is the zero frequency deflection.

Now we can rewrite these x1 and x2 in terms of these parameters, so we can write here so X1 so we can take k2 outside here, so this equation we can convert like F0 k2 - m2 omega square we can do like F0 by k1 into k1 into k2 1 - m2 by k2 omega square, so here F0 by k1 is Xst into k1 k2 into 1 -, so here k2 by m2 under root is omega 2 square. So we can write omega square by omega 2 square and this is k1 k2.

Because this will come in the numerator we can take here and we can divide here, so we can write and we take this Xst term to other side so here we write x1 by Xst = 1 - omega square by omega 2 square and k1 k2 will divide this denominator terms so this is omega power 4 and if we divide this will be by k1 - by k1 k2. So they will be k1 by m1 is omega 1 square and k2 by m2 is omega 2 square.

So we will have omega 1 square and omega 2 square - here we will again k1 by k2, so k2 will cancel out we will have m1 by k1, so we can write 1 by omega 1 square here again we will have m2 so here m2 k1 by k1 k2, so m2 by k2 so it is 1 by omega 2 square and this is k2 m2 by k1 k2, so it is k2 m2 by k1 k2 so we can cancel out here this k2 and we divide it and multiply with m1 by m2.

So what will happen these terms we can write so this is m2 and m2 k2 here by k1 k2, so we will have here m2 by m1 that is mu - m2 by m1 is mu this term m2 by m1 and k1 by m1 is omega 1 square and this term is multiplied by omega square and then this is k1 by k2 by k1 by k2 so it is 1, so here we can have - we can have this equation again modified so we will have.

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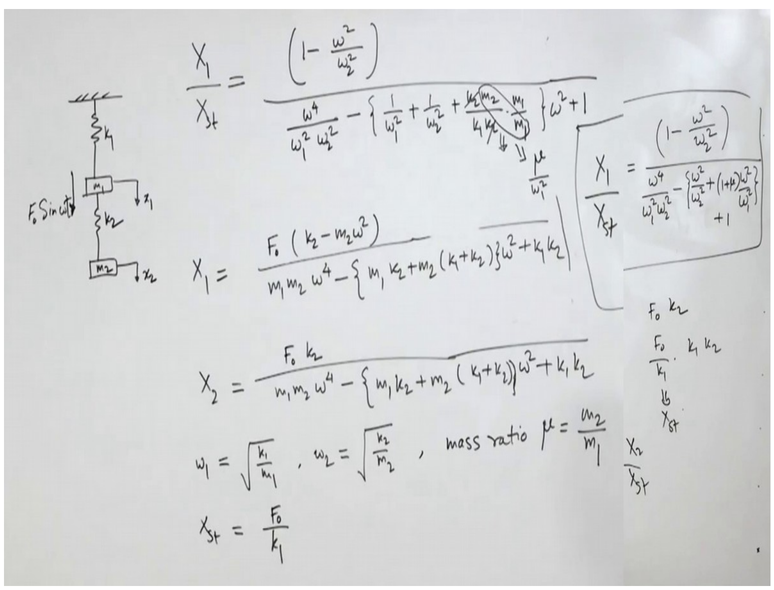

So x1 by Xst = 1 - omega square by omega 2 square upon omega 4 by omega 1 square omega 2 square - omega square by omega 2 square + 1 + mu omega square by omega 1 square + 1, so this is x1 by Xst, similarly we can write x2 in this form so if we write x2 so F0 k2 we can write like F0 by k1 into k1 k2, so F0 by k1 is Xst and k1 k2 we can divide the denominator terms so we will have the same denominator term, but because Xst will go, we write x2 by Xst so in the numerator there will be 1.

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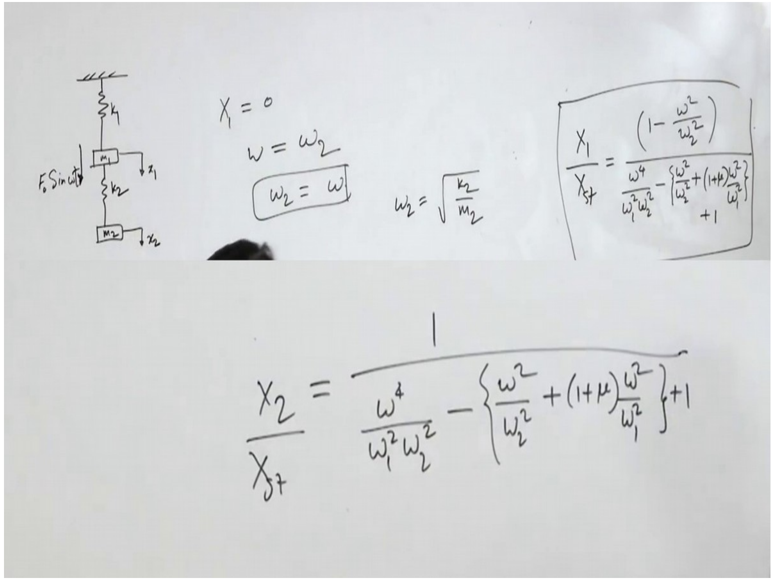

So we can we can write x2 here so we can write x2 = 1 so x2 by Xst = 1 upon omega 4 by omega 1 square omega 2 square - omega square by omega 2 square + 1 + mu omega square by omega 1 square, so we can see that this is the response amplitude of the first mass the main system. So x1 is 0 when omega = omega 2 when omega = omega 2, so it means that the natural frequency of the absorber is equal to the excitation frequency we will get the zero response of the main system.

So therefore if we select because omega 2 is root k2 by m2, so if we can select k2 and m2 such that the under root of k2 by m2 is equal to the excitation frequency, then the main system will have zero vibration so it zero amplitude vibration so it will be at still it will stop so vibration will be mitigated of the main system. So now I would like to stop here and see you in the next lecture, thank you.