

## Micro Robotics

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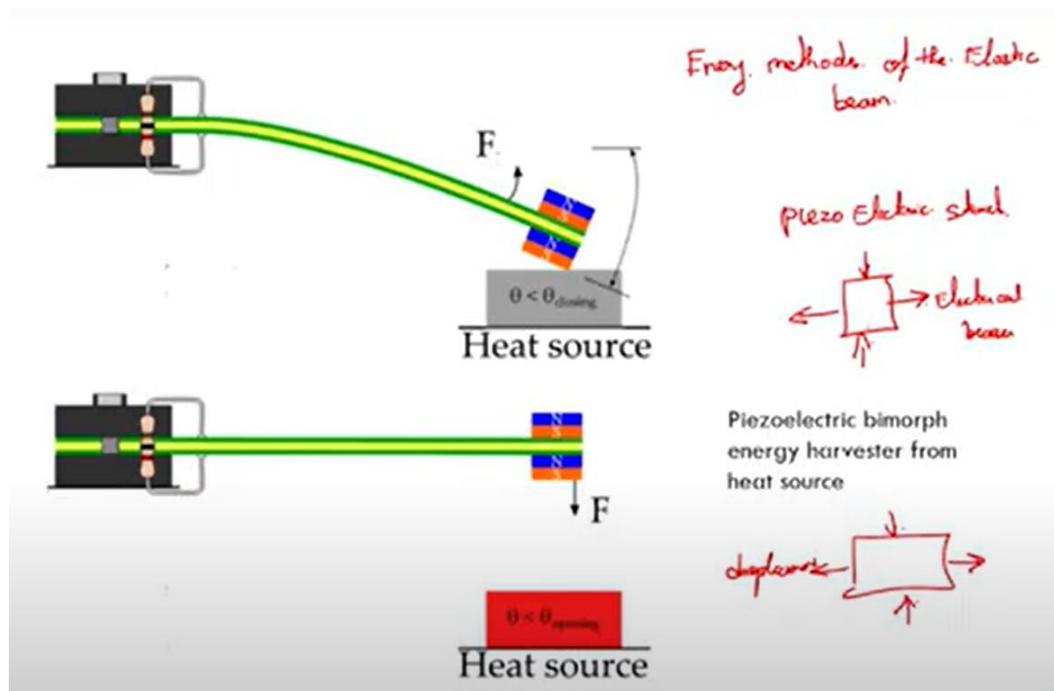
Indian Institute of Technology Indore

Week-2

Lecture-7

### Micro-Mechanics System Design ( Design of Microrobots) - Module 02

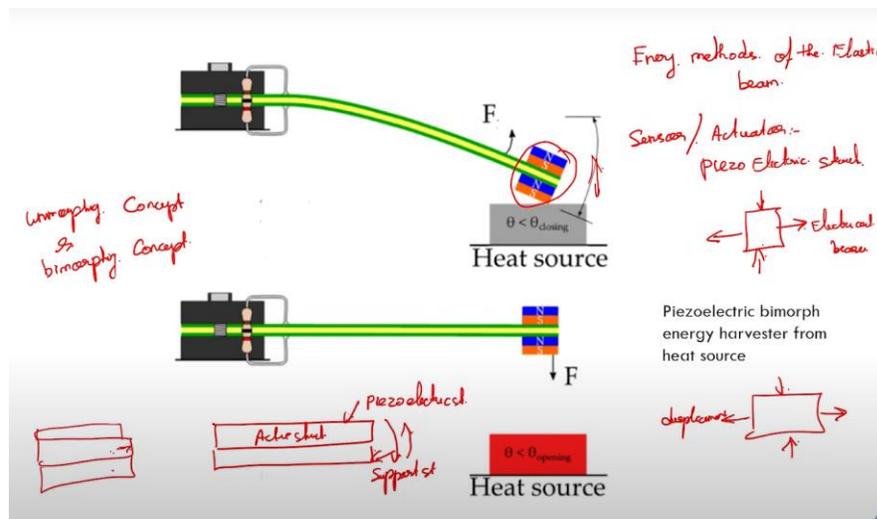
We have discussed this micromechanics system design in detail. So, here we will discuss the energy methods of the elastic beam. Before going into detail about the energy methods of this elastic beam, we need to understand energy harvesters. To understand the energy harvesters, let us take a simple piezoelectric structure. In these structures, when we try to apply a load, it generates an electrical bias. Similarly, when I try to apply an electrical bias in one direction, I can get a displacement in the other direction.



These kinds of piezoelectric structures can be efficiently used as sensors as well as actuators. These are called stacks. Further, in order to have a freestanding support for these piezoelectric structures, there are two concepts known as the unimorphing concept and the

bi-morphing concept. In the unimorphing concept, let us consider a piezoelectric structure and a supporting structure.

When this piezoelectric structure gets deflected, one-way actuation is observed in the form of a forward stroke by a piezoelectric layer and a return stroke by the supporting structure. If we want to perform this actuation at a higher frequency, like a few kilohertz, then these support structures should be capable enough that they do not create any kind of deflection or deformation to the structure. Generally, it works closely with the main structure and is known as an active structure. Hence, we can obtain the required displacement and frequency based on necessity. Now the morphing is classified into two categories: one is unimorphing, where we have a single support structure; the other is bimorphing.



In the case of a bimorphing, we have a two-support structure in place that adheres to an active structure. The bonding between them is efficient enough that the appropriate deflection can be achieved. Let's take a simple case study where a piezoelectric bimorph is used as an energy harvester from a heat source. Here, these piezoelectric structures are actuated through a thermal system. The figure shows a heating element and a resistor, where the main function of the resistor is to provide actuation to the system.

Furthermore, a magnetic field and a heat source are shown. With reference to the heat source, there is a deflection in the magnetic field that is efficiently supported by the beam shown in the figure. This beam further tries to deflect back and forth in such a way that some energy is generated, and this energy can potentially be used. From a micromechanics point of view, there is a beam, a structure, and a heat source that are available here. The

heat source results in the overall actuation of the structure; hence, a deflection is observed, and the deflection has the capability to harvest energy.

Now, in this particular case, let us discuss the micro-mechanism of this particular process. Let's consider an energy method for an elastic beam. Statistically, in an indeterminate beam, we analyze the structure. This has a force that is exerted in this direction. It is the fixed guide beam with the transverse load guided at the end, and a statically indeterminate beam is present.

Such systems are basically solved using an energy method. The strain energy stored in the beam of length L is given by SE. Now, thus for a bar A and a bar B, Castigliano's theorem is defined. So by considering Castigliano's first theorem, if the strain energy of a linearly elastic body is expressed in terms of its displacement, then the force at a point is equal to the partial derivative of the strain energy with respect to the displacement in the direction of the force at the point where the force is applied. So, mathematically in as follows.

### Micro-mechanics

### Energy Methods for Elastic Beams

#### Statically indeterminate beam

- The unknown reactions and forces can't be determined from the static equilibrium equations. Thus, the beam is an indeterminate beam.
- Such systems are solved using energy methods.
- Strain energy stored in the beam of length L is given by,

$$SE = \int_0^L \frac{\sigma \epsilon}{2} A dx = \int_0^L \frac{M^2}{2EI} dx$$

Where,

$$M = \frac{EI}{2} \left( \frac{d^2 \Delta}{dx^2} \right)$$

Thus, for a bar and a beam Castigliano's theorems are defined.

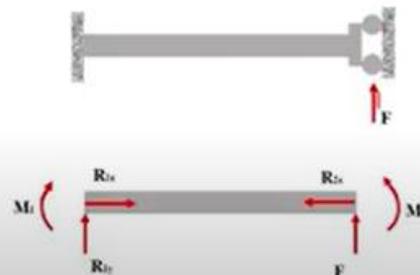


Fig 11: Fixed guided beam with a transverse load at guided end. A statically indeterminate beam

Similarly, when we consider Castigliano's second theorem, if the strain energy of a linearly elastic body is expressed in terms of force, then the force at a point is equal to the partial derivative of the strain energy with respect to the force at the point where the displacement is considered in the direction of the displacement. Mathematically, we can represent it as u equals the sum of f by the f. Both equations are general such that force can be replaced by moment and displacement can be replaced by rotation. In micromechanics, let's solve for the intermediate beam in the figure. The following equations are  $x = f$ ,  $x = r_1 x - r_2 x = 0$ ,  $f y = r_1 y + f = 0$ ,  $\epsilon m_2 = -m_1 + m_2 + f l = 0$ , and  $u x = l = \frac{d u}{d x} r_2 x$ , which

is the axial displacement.

**Castigliano's second theorem:**

If the strain energy of a linearly elastic body is expressed in terms of forces, then the force at a point is equal to the partial derivative of the strain energy with respect to the force at the point where displacement is considered in the direction of displacement. Mathematically,

$$u = \frac{\partial SE(F)}{\partial F}$$

Both equations are general, such that force can be replaced by moment and displacement can be replaced by rotation.

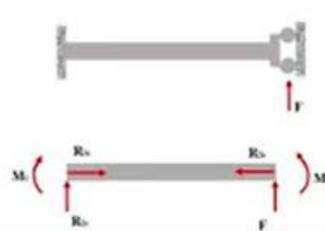
Now, though  $\frac{\partial u}{\partial x}$  equals  $\frac{\partial^2 u}{\partial x^2}$ , which is the slope, and by implementing Castigliano's theorem,  $\frac{\partial u}{\partial F}$  equals though  $\frac{\partial^2 u}{\partial F^2}$ . Thus,  $\frac{\partial u}{\partial F}$  equals  $\frac{m^2 L^2}{2EI} + \frac{2FL^3}{6EI}$ . So,  $m^2$  equals  $\frac{FL^3}{2EI}$ . So, the  $\frac{\partial u}{\partial F}$  can be represented as  $\frac{FL^3}{4EI} + \frac{FL^3}{3EI}$ , which is nothing but  $\frac{5FL^3}{12EI}$ .

**Micro-mechanics**  
 Thus, to solve the indeterminate beam in figure, we have following equations

$$\sum F_x = R_1 - R_2 = 0$$

$$\sum F_y = R_1 + F = 0$$

$$\sum M_z = -M_1 + M_2 + FL = 0$$

$$U_x: L = \frac{\partial SE}{\partial R_2} = 0 \text{ (Axial displacement)}$$


This equation is considered to solve the indeterminate beam as shown in the figure. This is a kind of generalized equation that can be used to form or establish a certain amount of parameters for any kind of structure that has a similar domain. Now, let us take an example of a two-axis micromirror that is supported by a serpentine beam. When we try to talk about a beam, it is a kind of cantilever beam that exhibits. But suppose I have a structure something like this: it is an inner structure and the outer structure.

It's a kind of concentric circle. In these concentric circles, we have four different supports. So the main function of this support will be helpful for us to deflect this structure, especially considering this as structure A and this as structure B. So we deflect structure A with different degrees of freedom. Now, based on the types of degrees of freedom, we can make the beams to our required dimensions.

So, if we move only in one plane, we can replace this beam with a spring. The spring will be capable enough that the structure can move only in one plane. If we are interested in deflecting the beam in a particular direction, we either have to consider a serpentine structure or a simple structure. Here, we can have a kind of cantilever arrangement that will be capable of deflecting the mirror based on our requirements. The main advantage of opting for the serpentine structure is that it has the capability to precisely manipulate this base structure.

So not only with reference to degrees of freedom, it also has a good advantage in manipulating the beam based on the applications. This is one advantage of this serpentine structure. Let us take a simple case study where we are using a kind of two-axis micro mirror. This micro mirror needs to be manipulated with a serpentine spring structure. The

**Micro-mechanics**  
 Example: Two axis micromirror

For serpentine spring structure, q is in plane width, p is length of vertical beams and t is out of plane thickness.  
 Strain energy of spring,

$$SE = \int_0^L \frac{M^2}{2EI} dx = \frac{M^2 L}{2EI}$$

by Castigliano's second theorem  
 the rotation is

$$\theta_{x:z} = \frac{\partial SE}{\partial M} = \frac{ML}{EI}$$

Fig 12: Two axis micromirror supported by serpentine beams

serpentine structure is quantified as Q for plane width, P for the length of the vertical beam, and T for the out-of-plane thickness.

Now, the strain energy of the system can be represented as SE equals L m squared by 2Ei dx, which is nothing but m squared L by 2Ei. So, by Castiglione's second theorem, the rotation is theta x equals l, which is dou SE by dou m, nothing but ml by ei. So, the total angular displacement for torque T equals m is the sum of all 6 beams. These 6 beams are 4 long beams plus 2. So, the angular stiffness constant is calculated as theta equals 2 Tp by Ei plus 4 T2p by Ei, which is 10 Tp by Ei.

The total angular displacement for torque  $T$  is the sum of all six beams  
(four long + 2 short beams)

The angular stiffness constant is calculated as

$$\theta = \frac{2Tp}{EI} + 4 \frac{T(2p)}{EI} = \frac{10Tp}{EI}$$

$$\text{Then } k = \frac{T}{\theta} = \frac{EI}{10p} //$$

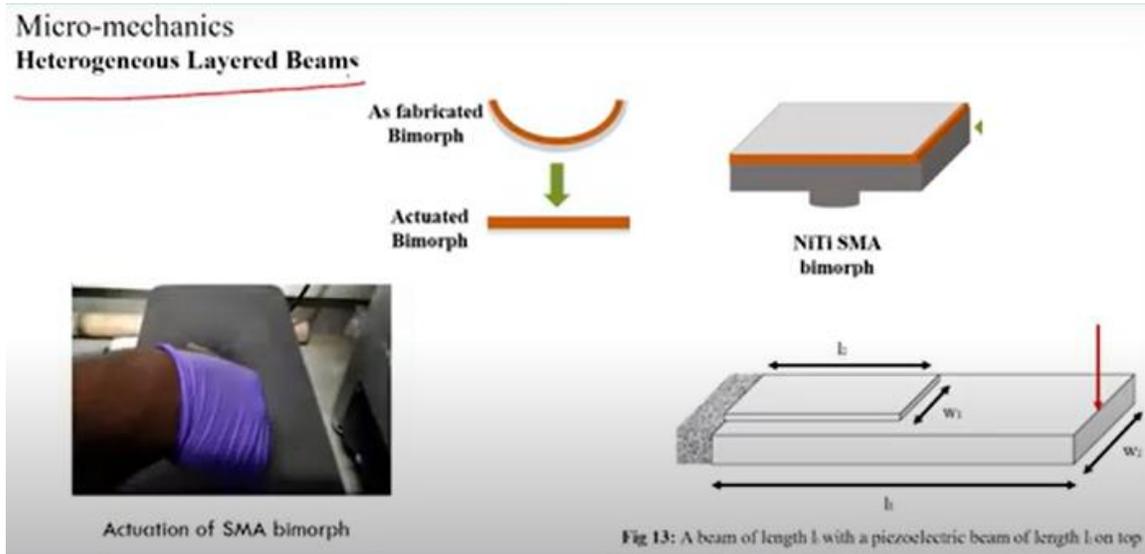
So, this  $k$  equals  $T$  by  $\theta$ , which is  $EI$  by  $10p$ . This is a kind of generalized equation that is derived for a two-axis micromirror based on the requirements, and the extension of this can be suitably applied for a certain amount of micromanipulation or a certain amount of micro-deviation related applications. Let us see the next case, where it is called a heterogeneous layered beam. In concern of this heterogeneous layered beam, we discussed the unimorph and bimorph concepts. The thought process is that there is an active layer; on the active layer, we will be placing an active system, basically an active actuator.

So, this active actuator activates and results in a certain amount of displacement. Now, the shape memory alloy also participates, as well as the active layer, in displacement. That is, it might be either a bimorph layer or a unimorph layer. Both will participate to create a certain amount of functionality. Here, one particular case study shows the key aspects with reference to this microactuator: developing a microactuator using a shape memory alloy, as shape memory alloys have a good amount of energy density.

However, there is a major drawback regarding the frequency at which it is generated. In the case of piezoelectric structures, the frequency is on the order of a few kilohertz. However, in the case of SME-based actuators and shape memory alloy-based actuators, the frequency is restricted to less than 15 hertz. In reports, it is shown that people have attempted up to 25 hertz, so 25 hertz is the maximum frequency. However, in order to enhance the frequency as well as to use it for certain applications, we need to integrate this Kapton polyimide with the shape memory alloy structures.

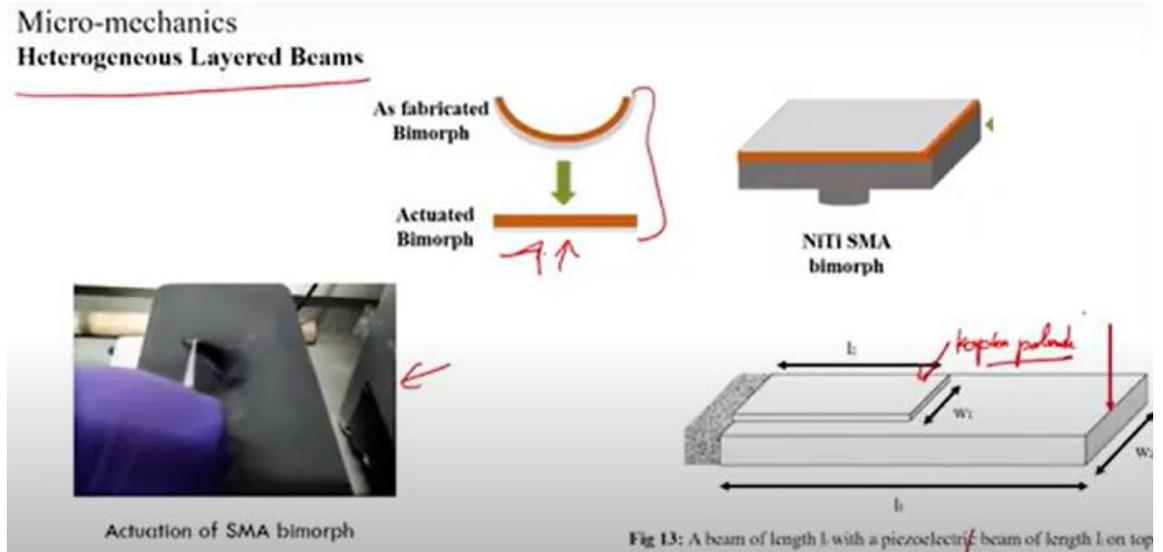
One-way actuation is done by the shape memory alloy, and the return stroke is achieved by Kapton polymer. These are kinds of heterogeneous layers where two different materials are brought together. So, with reference to these two layers, this shows an efficient actuation. So we can have control over the frequency as well as over the actuation

characteristics related to the displacement characteristics. We will be discussing this in one module related to microfabrication, where we fabricated a Stuart platform using an SMA bimorph for optoelectronic applications.



Now, in this case, we can program the displacement based on our requirement. So the programming is done by applying an appropriate strain to the system. By applying a strain to the system, we can program the structure to deform as needed. Now, this is an SMA-based actuation on a hot plate. This is a flexible SMA thin-film-based structure.

We are trying to place this SMA thin-film-based structure on a hot plate. It is unfolding because of the actuation of the heat. This unfolding is happening because of the actuation in the shape memory alloy. When it is out of the hot plate, it folds back again. This folding back is happening because of the participation of the Kapton polyimide in this particular structure.



This is as mentioned in the overall video. This is an actuated biomorph. It has a curled structure as it was fabricated. When we try to actuate it, it means it becomes straightened by the application of heat. After we try to take it back, it comes back to shape.

There are different phenomena that exhibit over here. One phenomenon is that there is no need to completely coat a Kapton polyamide on the SMA structure. So, here we have a limited layer; hence, we have control over the overall curvature, the overall dynamic, as well as the frequency. In this case, let us consider that this is a beam length of  $L$ . In this case, we have used a piezoelectric material.

However, we can also consider an SMA beam based on our requirements, and this is kept on a polyamide structure that is integrated with the shape memory alloy structure so that the deflection in these structures can be appropriately addressed. Now, the overall configurations and the overall usage of these heterogeneous layer beams will be discussed in the next module. Let us consider a heterogeneous layer as discussed previously regarding the SMA bimorph-based structures. In these SMA bimorph-based structures, we are trying to fabricate a bimorph, study the incorporation of this Kapton polymer with the shape memory alloy, and investigate the actuation behavior and its characteristics. This terminology is called a heterogeneous layered beam or heterogeneous layered structures.

Let us consider this as a generalized system. In this generalized system, instead of a shape memory alloy, we are considering a piezoelectric beam, which has a length  $L$ , and on top of it, we have a layer of length  $L_2$ , with width 1 and width 2 being different. Let us consider  $E_1$  and  $E_2$  to be the Young's modulus,  $T_1$  and  $T_2$  to be the thickness of the beam, and  $y_n$  to be the height of the neutral axis from the bottom of the bigger beam. The stresses are  $\sigma_1 = E_1 (y - y_n)$  for  $y \leq t_1$  and  $\sigma_2 = E_2 (y - y_n)$  for  $t_1 < y \leq t_1 + t_2$ . So, in pure bending, no axial force acts on the cross-section.  $\int_0^{t_1} \sigma_1 w_1 dy + \int_{t_1}^{t_1+t_2} \sigma_2 w_2 dy = 0$ , which is 0. The  $y$  power  $n$  is  $e_1 a_1 y_1 + e_2 a_2 y_2$  divided by  $e_1 a_1 + e_2 a_2$ .  $y_1$  equals  $T_1$  divided by 2 and  $y_2$  equals  $(T_1 + T_2)$  divided by 2. To consider the bimorph effect, consider a cantilever beam made of two different materials with different thermal expansion coefficients. So, on heating, both layers try to expand differently but are adhered to each

other due to the thermal strain.

**Micro-mechanics**  
**Heterogeneous Layered Beams**  
 Let  $E_1$  and  $E_2$  be the Young's modulus;  $t_1$  and  $t_2$  be the thicknesses of the beams.  $y_n$  is the height of neutral axis from the bottom of the bigger beam. Stresses are,

$$\sigma_1 = \frac{E_1}{P} (y - y_n) \text{ for } y \leq t_1$$

and

$$\sigma_2 = \frac{E_2}{P} (y - y_n) \text{ for } t_1 < y < t_1 + t_2$$

in pure bending, No axial force acts on the cross section.

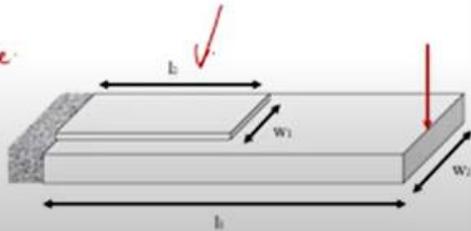


Fig 13: A beam of length  $l$  with a piezoelectric beam of length  $l_1$  on top

So, the thermal strain is relieved by bending, and the beam shown in the figure is heated to a temperature of  $\Delta T$ , and as there is no external force or moment, the equilibrium equations are  $P_1 = P_2 = P$ .  $P = t_1 + t_2$  divided by 2 equals  $m_1 + m_2$ . Let  $r$

$$y_n = \frac{E_1 A_1 y_1 + E_2 A_2 y_2}{E_1 A_1 + E_2 A_2}$$

where  $y_1$

be the radius of curvature of the bent beam, and  $m_1 = -E_1 L_1 / r$ , and  $m_2 = -E_2 L_2 / r$ . Now, by representing the free body diagram, this is the free-body diagram of the bent bimorph beam. The elongation of the layer has three components: a thermal expansion component, an axial deformation component due to a force  $P$ , and an elongation component due to bending.

## Micro-mechanics

### Bimorph Effect

Consider a cantilever beam made of two different materials with different coefficient of thermal expansion.

- On heating, both layers try to expand differently but are adhered to each other leading to thermal strain.
- Thermal strain is relieved by bending.
- The beam given in figure 17 is heated by temperature  $\Delta T$ .
- As there is no external force or moment, the equilibrium equations are,

$$P_1 = P_2 = P$$

$$P \frac{(t_1 + t_2)}{2} = M_1 + M_2$$

Let  $r$  be the radius of curvature of bend beam,

$$M_1 = -\frac{E_1 t_1^3}{\gamma} \quad \& \quad M_2 = -\frac{E_2 t_2^3}{\gamma}$$



Fig 17: A thermal bimorph

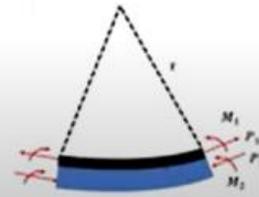


Fig 14: Free body diagram of a bend bimorph beam

Thus, the radius of curvature of the bending can be determined by for one  $\Delta T$  plus  $P$  by  $E_1 T_1 b$  plus  $T_1$  by  $T_2$  by  $r$  is equal to  $\alpha_2 \Delta T$  minus  $P$   $E_2 T_2 b$  minus  $T_2$  by  $2$  by  $r$ . Where  $\alpha_1$  and  $\alpha_2$  are the coefficients of thermal expansion of the bottom and top of the layer, since these two are kinds of heterogeneous structures and are participating in the overall actuation. In fact, we have seen a video with reference to SMA bimorph-based actuation. We have seen that when the SMA tries to deflect, the actuation happens on the SMA. Similarly, when we bring it into an open environment, it returns to its original shape because of the participation of the polymer.

## Micro-mechanics

The elongation of the layers have three components- A thermal expansion component, axial deformation component due to force  $P$  and elongation component due to bending.

Thus, radius of curvature of bending can be determined by following equation;

$$\alpha_1 \Delta T + \frac{P}{E_1 t_1 b} + \frac{(t_1/2)}{\gamma} = \alpha_2 \Delta T - \frac{P}{E_2 t_2 b} - \frac{(t_2/2)}{\gamma}$$

Where,

$\alpha_1$  and  $\alpha_2$  are coefficients of thermal expansion of top and bottom layers respectively.

$E_1$  and  $E_2$  are Young's Modulus of top and bottom layers respectively.

$t_1$  and  $t_2$  are thickness of top and bottom layers respectively.

$b$  is width of beam.



So ideally, both the coefficients of thermal expansion need to be taken into consideration, and  $E_1$  and  $E_2$  are the Young's moduli of the top and bottom layers, and  $T_1$  and  $T_2$  are the

thicknesses of the top and bottom layers, respectively.  $B$  is the width of the beam. Now, in transverse loading, let's consider the effect of residual stress in this particular case. Since we had two different compositions or two different heterogeneous layers, there is an actuation exhibited; ideally, it may result in a kind of residual stress within the system. When we try to study the overall failure or degradation behavior of any such type of shape memory alloy structures One of the key aspects is the degradation behavior of the shape memory alloy bimorph.

**Micro-mechanics**  
Effect of Residual Stresses

Residual stresses in micromechanical systems may arise due to following reasons:

- Thin film deposition.
- Thermal expansion
- Oxidation, substitutional doping or ion implantation during microfabrication.
- Lattice mismatch in epitaxial growth.

Shear force acts at the right face of the beam in case (c).

The diagram shows three horizontal beams labeled (a), (b), and (c). Beam (a) is fixed at the left end and free at the right end. Beam (b) is fixed at both ends. Beam (c) is fixed at the left end and free at the right end, with a dashed box at the right end indicating a magnified view of the beam's cross-section and its adhesion to a substrate below it. A red arrow points from this magnified view to a larger, detailed view of the beam's cross-section at the bottom right, showing the beam's adhesion to a substrate and the resulting shear force.

**Fig 15:** Effect of residual stress along the length (a) a fixed-free beam simply expands or contracts (b) Both side fixed beam with no bottom constraint holds the stress within and (c) A fixed-free beam adhering to the substrate at bottom

So when we are trying to integrate these two structures, there is continuous movement happening, which will result in a certain amount of residual stress. This residual stress will slowly build up, resulting in hysteresis. Then there will be a reduction in the overall performance of this shape memory alloy structure. This is quite evident for all kinds of bimorphs that are being used functionally.

A certain amount of preconditions is taken. Further, to consider the residual stresses, we try to strain the film after it is deposited or pre-strain the film during heating conditions to

account for such residual stress. So, those are the things we will be discussing in the upcoming lecture.

### Micro-mechanics

The elongation of the layers have three components- A thermal expansion component, axial deformation component due to force P and elongation component due to bending.

Thus, radius of curvature of bending can be determined by following equation:

$$\alpha_1 \Delta T + \frac{P}{E_1 t_1 b} + \frac{(t_1/2)}{r} = \alpha_2 \Delta T - \frac{P}{E_2 t_2 b} - \frac{(t_2/2)}{r}$$



Where,

$\alpha_1$  and  $\alpha_2$  are coefficients of thermal expansion of top and bottom layers respectively.

$E_1$  and  $E_2$  are Youngs Modulus of top and bottom layers respectively.

$t_1$  and  $t_2$  are thickness of top and bottom layers respectively.

$b$  is width of beam. ✓