Microrobotics

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Week- 1

Lecture No- 02

Scaling Laws for Designing Micro Robots

While implementing micro robots, there is a need to understand the overall environment to design the micro robot. As discussed earlier on a macro scale, when we design a robot, it has several other features and one of the features is a macro level actuator. So, we have sensors that can be incorporated appropriately, and the necessary feedback can be collected and appropriately the design changes and the considerations can be taken into the integration of these robots and then we can deploy these robots for different applications. However, with reference to a micro-robotic perspective, there are several constraints that will affect the design of the robots. To understand, let us take an example of a simple Automated Guided Vehicle (AGV). When we develop an AGV at a macro level, the propulsion system of the AGV, i.e., wheels, is incorporated, and different controllers are stacked for appropriate communication. For instance, we have an antenna in place for communication. Further, the important aspect is locomotion. Here, an external power source or a battery arrangement can be deployed for charging and then used for the movement of the system. However, when we try to reduce it to a micro level, there are different challenges that persist. Some of the key challenges include the influence of surface tension, inertia, friction, the influence of air flow, and the influence of viscosity, to name a few. One of the key challenges involved in fabricating micro robots is that we may need to scale down from a macro level to a micro level, considering the losses associated with scaling while designing this micro robotic system. So, the main motto of this module is to give necessary exposure to the participants about the different scaling laws and the parameters involved in the scaling laws.

So, we have selected a few specific domains, and we will be discussing the scaling laws that are implemented for this domain. So, when we talk about scaling laws, the scaling laws describe the attributes of a system that changes as its size increases or decreases. But a question arises, i.e., why do objects, creatures, and physical phenomena act differently at various scales? The differences occur because of external physical forces that the body is

exposed to, which include gravity, surface tension, inertia, friction, etc., that do not scale the same way with size and change with reference to different parameters. Now when we consider the scaling, the traditional engineering principles do not apply at small scales. Basically, it may require a new approach. For instance, when we study the acting forces on a body, the various types of forces behave differently; at times, surface forces dominate and gravity becomes negligible. Even the motion in micro robots requires alternate strategies.

For instance, certain microbots would use helical propulsion, while a few may use an oscillating motion strategy depending on the application. Another key challenge is the power requirements of micro-robots. The micro-robots may require an external energy source that should mostly be non-contact based, which would be helpful for powering such kinds of micro-robots. Further, as far as manufacturing methods are concerned, microfabrication techniques need to be employed that would cater to the overall micro robotics systems. When we observe the scaling laws, the scaling laws describe the physical property changes as the size of the system changes. These laws are based on the mathematical relationship between the different physical quantities. There are two major scaling laws used in the micro-mechanical system design of microrobots. The first type of scaling law is strictly dependent on the size of the physical object, i.e., scaling a geometry.

Examples of this law include scaling of rigid body dynamics, electrostatic, and electromagnetic forces. The second type of scaling law involves the scaling of a phenomenological behavior of microsystems. Now let us see the basic mathematical form of a scaling law. Let us consider a function describing a system with some parameters that are dimensionless. Let us consider this W term and the other geometry-dependent parameters as v terms. Now, v_0, w_n is f of v_0 to v_n and w_0 to w_n equals y. So, if we now assume that we scale all geometrical dimensions by a factor k, we get $k v_0$, $k v$, w_0, w_n, $f(v_0, v_n, w_0, w_n)$, which is nothing but $k s y$. The parameter s is the scaling factor. So, by convention, we can use Ls to describe the dimensional scaling effect on the properties. Further discussing the scaling effect on the overall surface and volume ratio.

Scaling effect Surface and volume

surface can be represented in terms of double integral summation over a given domain Ω.

If we scale down the integration domain we can show that the surface scales by L^2
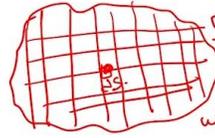
$$f(x,y) \propto \iint_\Omega ds = \iint_\Omega dx\,dy$$

$$\Rightarrow f(kx, ky) \propto \iint_{k\Omega} ds = \iint_{k\Omega} d(kx)\,d(ky)$$

$$= k^2 \iint_\Omega dx\,dy = k^2 f(x,y)$$

The same method can be applied on a volume and we can show that the volume scales by L^3

length = L^1	Volume =
Surface = L^2	L^3

So, the surface can be expressed in terms of a double integral summation over a given domain. So let us consider this as a domain. So in this domain, we are representing this domain as omega, and we are representing this domain as ohm, and there is a point ds. So, the surface can be represented in terms of a double integral summation over a given domain. If we scale down the integration domain, we can show the surface scale by L squared, which is f of x and y, is directly proportional to the integral, which is equal to the integral dx dy. So, f(x, y) that is kx, ky, f(kx, ky) is directly proportional to k ω kΩ ds, kΩ dkx dky which is equal to k² ∫Ω dx dy which is k² f(x, y). So, the same method can be applied on a volume, and we can show that the volume scales by L cube. Now, if we try to nomenclate it, length equals L1, surface equals L2, and volume equals L3. So, from this simple analysis, the physical properties that are proportional to volume will become much less significant than surface properties. Now, let us examine some of the other various physical properties, like mass, moment of inertia, spring force, and the oscillatory frequency.

So, the scaling effect on the mass can be calculated using a definition of mass as density is an intensive variable and it is independent of the geometrical dimension.

Influence of various physical properties (Mass, Moment of Inertia, Spring force and Oscillatory frequency)

density is an intensive variable and it is independent of the geometrical dimension.

mass scales down as a volume (L^3)

$$m = \rho v \xrightarrow{k} \left\{ \begin{array}{l} \rho \xrightarrow{k} \rho' = k^0 \rho \\ v \xrightarrow{k} v' = k^3 v \end{array} \right\} \Rightarrow m' = k^3 m.$$

Now, the mass scales down as a volume or L cubed that is m equals p v k, p k p' equals k0 p and v k v d' equals k3 v that is m' equals k3 m. Similarly, when we consider the moment of inertia, by definition, the moment of inertia is the distribution of points of a solid around a given axis. Now, with reference to a mathematical perspective, it can be represented as I del equals M epsilon s h m square d m. The M is a point that is being represented here, and then the M is the point that belongs to the surface.

moment of Inertia :- The distribution of points of a solid around a given axis

mathematically $I_\Delta = \int H r^2 \, dm.$: M is the point belonging

to the surface S.   H is the point on the axis $m \in (s)$
from which the moment of inertia is calculated.

$$I_\Delta = \int_{ay}^{by} \int_{az}^{bz} f(x,y) \, dx \, dy \xrightarrow{k} I_\Delta' = \int_{kay}^{kby} \int_{kaz}^{kbz} f(x,y) \, dx \, dy.$$

Scaling factor:

$$I_\Delta' = \int_{ay}^{by} \int_{a}^{b} f(x',y') \, dx' \, dy' = k^4 I_\Delta.$$

Belonging to the surfaces S and H is the point of axis from which the moment of inertia is calculated. Now, when we try to investigate the overall scaling factor. we can estimate the scaling factor as I del dash equals by ay bs as f of x dash y dash dx' dy', which is nothing but k^4 I del. Now a similarity can be established for the spring force and the oscillating frequency. In the case of spring force, the stiffness of the spring is constant, and the spring force scales like the length we have, or f equals k del L.

So, the oscillating frequencies can be represented as, if f equals 1 by 2 pi times the square root of k by m, which is directly proportional to. Now, when scaling the spring dimensions down, assuming its stiffness is constant, the natural frequency goes up. Now, let us understand the influence of the scaling effect with reference to the electrical and inductive properties. The same methodology can be applied to any property that involves physical dimensions. To evaluate the electrical resistance, we can mention it as R equals rho L by S kr' rho k plus 1 L di k plus 2 s equals k power minus 1 rho l by s equals k minus 1 r, where l is the length of the conductor element being considered, s is the surface cross-section, and rho is the resistivity. In particular, when we try to consider the scaling law L minus 1, which basically indicates the resistivity increase with scale reduction, it should be stressed that we are making the assumption that electrical resistance can still be described by the law of formulation. A nanometer-scale quantum effect starts to be observable, and

therefore the law above is no longer suitable. Similarly, we can also estimate the scaling loss of the capacitance, which is the resistance from the perspective of capacitance. ε_s / d k c - k + 2 s / k + 1 d, which is nothing but k_1 ε_s / d = k_1 c, where s is the surface considered, d is the distance separating the two surfaces on which the potential is applied, and ε is the permittivity of the medium. Now, in a similar fashion, when we consider inductance, the magnetic field is ideally due to inductance.

So, when we consider a solenoid with n turns, a wire of length L, and current I through the coil. The coil is given by B equals mu ni by L, and the magnetic energy stored in the solenoid is equal to p square by 2 mu. So the magnetic energy in the coil scales as, assuming the current density remains constant and keeping the number of turns constant, the magnetic energy in a coil scale as L power 5. Now, as far as the scaling laws are concerned, they ideally follow the square-cube law. This law states that as an object grows, its volume increases faster than its surface area.

Normally, the surface area is directly proportional to L squared, and volume is directly proportional to L cubed. So, S by V is normally represented as L minus 1. For example, in the case of an elephant, the S by V is 10 to the power of minus 4, and in the case of a dragonfly, the S by V is 10 to the power of minus 1. So, when we observe the overall effect of geometric scaling in a micro robot, at a small scale, a higher surface area to volume ratio is considered, faster heat dissipation, and stronger adhesion forces are considered. Whereas a fluid dynamic shift is due to the motion that is basically dominated by viscosity due to a low Reynolds number.

Influence of various physical properties (Mass, Moment of Inertia, Spring force and Oscillatory frequency)

Spring force & stiffness of the spring is considered to be constant. and the spring force scal. like the length we have

$$F = k \Delta L.$$

The oscillating freque is

$$f = \frac{1}{2T}\sqrt{\frac{k}{m}} \Rightarrow f \propto L^{-3/2}$$

when scaling the spring dimensions down, assuming its stiffness is constant the natural frequency goes up.

When we consider electromagnetic forces, when they become dominant, gravity is negligible at the micro scale. However, at larger scales, the volume grows faster than the surface area, though structural integrity must be considered. And when we consider mass and inertia, the motion realizes momentum and an external force thus exhibited. Now, let's

discuss the scaling loss in rigid body dynamics. As far as rigid body dynamics is concerned, let us consider a rigid body with a mass m traveling from one place to another over a distance s. Like there are two masses, the mass that is moving from one end to the other end where S is being considered. So S is directly proportional to L, and the L stands for the linear scale. Similarly, S is equal to ut plus half at squared since it is the movement of the particle when u is equal to 0 and a is 2s of t squared. Also, the dynamic force is considered m a, where m is the particle mass, a is the acceleration due to gravity, and f is the acceleration, and f is the 2s by mt squared, l cubed by t power 2. When we see through a rigid solid traveling from one position to another.

Influence of electrical properties and inductance

$$\text{Resistor} \quad R = P\frac{l}{s} \xrightarrow{k} R' = P\frac{k^{+1}l}{k^{+2}s} = k^{-1}P\frac{l}{s} = k^{-1}R$$

$l =$ length of the conductor elem.

$S =$ Surface cross section.

P is the required

$$\text{Capacitor} \quad C = \varepsilon\frac{S}{d} \xrightarrow{k} C' = \varepsilon\frac{k^{+2}S}{k^{+1}d} = k^{1}\varepsilon\frac{S}{d} = k^{1}c.$$

$S =$ Surface Consider , d is the distance separating the two Surface on which the potential is applied.

$\varepsilon =$ permitivity of the medium.

Let us consider the solid traverse S, which is directly proportional to L, where L stands for the linear scale. The velocity v is v equals s by t, and hence v is directionally proportional to L t to the power of minus 1, in which t is the time required for the travel. So, overall, when we consider S equals v naught t plus half a t squared. Here, v0 is the initial velocity, and alpha is the acceleration. V0 equals 0, alpha is 2s by T squared. The dynamic force F for Newton's second law can be expressed as F equals ma, where m is mass in kilograms, a is acceleration in meters per second squared, and T is time in seconds, giving the dimensions L T^2 M. So, appropriately, it has a kind of deviation in the tremor force scaling mixture, which we will be discussing in the upcoming classes.

$\varepsilon =$ permitivity of the medium. Surface on whic. applied.

Inductor : Solenoid with n turns of wire of length L and current I through the coil is give by

$$B = \frac{\mu n I}{L}$$

= The magnetic energy stored in the solenoi is

$$E = \frac{B^{2}}{2\mu}V.$$

The magnetic Energy in the coil scales as L⁵.