Course Name: Turbulence Modelling

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Week - 12

Lecture – Lec68

68. Large Eddy Simulations: Filters and its types – I

So, let us get started again. So, we were looking into Large eddy simulations in the last class, right? So, we managed to get a governing equation for LES, which is Filtered Navier-Stokes Equations. and we briefly discussed the difference between result component, residual component, SGS component and all these things. So, today we will see what are the factors that affect LES result ok. So, factors that affect LES.

So, the first one is the filter type ok, the one you choose filter type and obviously, the filter size just like the grid size the filter size and what is the third one? This is turbulence modeling are we modeling anything in LES? SGS model. So, the choice of filter type and choice of SGS model can give you a slightly different results ok. Of course, it depends on the filter size that is also your choice. So, we will see what are the choices of filter type first for you ok.

So, filter type, essentially, there are two popular filters that we use one is called a box filter, the other one is a spectral filter, some may use a Gaussian filter and so on. So, there is a general definition for a filter that is given by Leonard. So, let us take that one and then that will tell us what different types we can use ok. So, a generic definition, generic filter definition, one-dimensional filter by Leonard, 1974. So, any quantity $\bar{\phi}$ of x filtered over bar is filtering is equal to minus infinity to infinity the integration applies over the entire flow domain, and then you get a filter kernel or a filter function $G(r)$ and the variable that you are filtering x minus r dr.

$$\bar{\phi}(x) = \int_{-\infty}^{\infty} G(r)\ \phi(x-r)\, dr$$

Here r is a spatial variable as I said earlier r is a spatial variable ok. And here the first thing is the integration applies over the entire flow domain. So, integration is over the entire flow domain and this $G(r)$ the filter kernel that decides what filter you are using box filter or spectral filter. So, the $G(r)$ is the filter kernel or filter function ok. So, now we can see the examples of different filters by choosing a different $G(r)$ or the filter kernel.

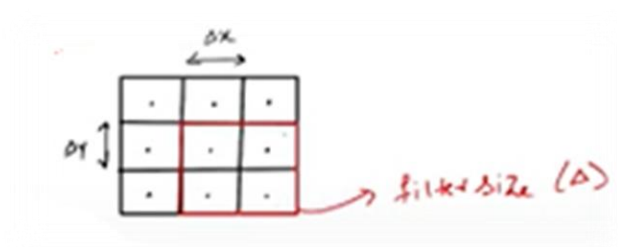
So, the first option as I said is this box filter ok, box filter. I can call this, let us say, equation 1. So, in a box filter, the $G(r)$ will take the value it takes the value one by delta delta is the filter size it takes $G(r)$ will be one by delta for all values of r less than or equal to delta by 2 ok. So, where delta is your filter size. So, what does this mean? Let us give an illustration. So, 0 otherwise ok 0 otherwise let us call this let me just move this a bit let us equation 2.

$$G(r) = \begin{cases} 1/\Delta & |r| \le \Delta/2 \\ 0 & \text{otherwise} \end{cases} \longrightarrow (2)$$

Δ *is the filter size*

So, what this implies is if I give an illustration let us say I take let us say I take a grid, let us say this is your mesh, these are the node points. So, this is the mesh on which you are computing your LES calculation. Then the filter is, let us say I am using delta. The filter size delta is equal to 2 delta x by 2 delta y, let us say.

So, then the filter size I can say the filter size is this much it is twice that of your grid size ok. So, I can say this is your filter size delta, and this is your delta x delta y. You understand what it means? Now r is in between delta by 2 on either side here. Then only it will be delta will be equal to 2 delta x by 2 delta y in this particular case let us say equidistant. So, it takes the filter function takes value 1 by delta, otherwise it takes 0 value.



So, this you can see that it is in a physical space that is in terms of your $\Delta x, \Delta y$ being in millimeters or micrometers. In physical space this is sharp, but not in a spectral space because sometimes turbulence data is taken in a spectral space when we look into energy spectrum. So, you measure from LES let us say you are computing the velocity signals velocity fluctuations this fluctuation velocity fluctuation signals can be you can use a fast Fourier transform to take it into a Fourier space or a spectral space where instead of this millimeters micrometers distances will be in wave number space. So, you will plot energy at a particular wave number versus wave number in that x and y coordinate system. Instead of eddy size versus energy, how much energy is associated with each eddy size versus let us say its length, you would have energy as a function of wave number versus wave number in a wave number space.

So, sometimes you would also do that. You compute data in real space and then use a Fourier transform to take the data into spectral space. So, box filter advantages it is sharp in physical space. you know what is the length at which or the size at which your filtering operation is being performed what is its length width and everything right. So, you can take a note here.

Box filter is sharp in physical space but not in wavenumber space, ok. So, then we see which one will be sharp in wavenumber space. That is the second option, that is, if you want to choose a spectral filter. That is, sometimes the code itself will be in a spectral code. For example, Navier-Stokes equation you can apply Fourier transform, take the governing equation in Fourier space and then discretize and solve.

This technique is what is usually called a pseudo spectral solution or even in a spectral element solution ok. So, people do that and then you will have the data in that space not in the physical space wave number. So, there it is applicable to use what is called the spectral filter ok, spectral filter. So, here, the filter function $G(k)$, k is the wave number. Usually in spectral space we indicate the parameters with a cap, this cap is given in that tell that we are working in wave number space.

So, $\hat{G}(k)$ wave number takes a value 1 or 0; it takes 1 when wave number k is less than or equal to the cut-off wave number that you choose 0 otherwise. So, where this k, where k is the wave number and k_c is the cutoff wave number.

$$\hat{G}(k) = \begin{cases} 1 & k \leq k_c \\ 0 & \text{otherwise} \end{cases}$$

That is something you choose just like you choose a filter size delta in physical space, cut off wave number is you can choose that up to this wave number you are going to resolve on capture the solution beyond that you are going to model ok. Or same in real physical space it is the delta you can say ok the delta is 1 millimeter up to 1 millimeter I am going to resolve anything like let us say from meters to 1 millimeter down and below 1 millimeter I am going to model. In wave number space you can say the wave number going very large wave numbers to sorry the very small wave numbers it is the other way small wave number indicates large eddy ok.

You know the wave number to wavelength definition right. So, the small wave number indicates large eddy from there to a cut off wave number an intermediate eddy right up to that you are going to resolve and anything larger than the cut off wave number will go into let us say dissipative scale those high wave number dissipative eddies you are not resolving you are going to model. That is the idea this is choice is completely up to you whether you want to use a box filter or a spectral filter these are two popular filter types. Again, take a note here this spectral filter is sharp in wave number space, but not in physical space, ok. So, here I have just a schematic where the y axis shows the energy this is the energy spectrum you can say this entire thing is what I am plotting is called energy spectrum those who do experiments or doing DNS LES can do this one they have access to fluctuating signals velocity signals you can get in get this energy spectrum energy as a function of k, k is the wave number here ok.

So, this is the wave number you have this right k. So, E as a function of k and x axis is the wave number, and $k_c$is your cut-off wave number. So, this is in a logarithmic scale. generally, when you plot the energy spectrum, you would get this in energy, let us say, in a logarithmic scale. So, this this energetic eddies or the large eddies containing most of the energy or the energy injection is occurring here at these scales.

This is again the energy cascade process or the Kolmogorov hypothesis view where the low wave number or the large eddies here and then that intermediate eddies and the tiny tiny eddies this view large wave number indicating tinier eddies. So, you have the dissipative range you have the inertial sub range and then the energetic range three ranges according to that hypothesis and this $\kappa^{-5/3}$ is the slope indicating the inertial range whether your data shows that is immaterial here this is according to the hypothesis. your actual data may not actually collide with a $\kappa^{-5/3}$ or maybe a small fraction of your range wave number range may collide with that one collapse with that ok. But the idea is that as already told you in the Kolmogorov hypothesis the one of the point was that there is a local isotropy at the dissipative range, and Kolmogorov has also quantified it how to get those Kolmogorov microscales, and there is also a definition of what is called an inertial range or a universal sub range where the energy that is being going from the energetic range to the inertial range which is purely inertia driven this is the transfer to down all the way down that is why epsilon SGS. So, SGS, we do not have access to these are sub grid scale motion right.

So, if I say, what is the dissipation rate of the grid scale? I do not have access to this you can get dissipation rate at the resolved at your resolution. not at the scale which is smaller than your resolution. So, that epsilon SGS is according to the hypothesis, must be coming from the P_k SGS. So, basically, whatever dissipation is occurring here must be coming from the production rate of k, right? So, this is the idea and here what this means this box filter and a spectral filter is. So, let us say there is an eddy here.

using that filter size, it is much more clear to define this in a physical space. Its exact wave number is not clear, right for our understanding, and for if you are using a spectral filter, then this cutoff wave number is very sure. Now, you know what this wave number is, but the corresponding eddy is not what you can choose here you can always choose the cutoff wave number. there, you can actually physically you can think ok, this is the eddy size I want to capture right here. You have to work with wave numbers if you are choosing a spectral filter, ok. So, now