Course Name: Turbulence Modelling

Professor Name: Dr. Vagesh D. Narasimhamurthy

Department Name: Department of Applied Mechanics

Institute Name: Indian Institute of Technology, Madras

Week - 4

Lecture – Lec21

21. Production rate of TKE and Mean TKE - II

Chapter 3, we call it the turbulent boundary layer. Interesting? Turbulent boundary layers. So, of course, we can take up any flow problem to analyze. I am taking this because the boundary layer is also a shear layer, ok? There is, of course, there is an effect of the wall here. So, that is similar to any shear layer, and turbulent flows are shear flows.

There is no shear means you do not get that mean strain, right? $\frac{\partial \overline{u_i}}{\partial x_j}$, that is your, let us say $\frac{\partial \bar{u}}{\partial y}$ term in the boundary layer. This term does not exist means there is no boundary layer. So, you need those terms. So, turbulent flows are also shear flows and boundary layer is also a shear layer.

So, before we begin a turbulent boundary layer, so let us just look into a question Saketh asked, right? So, for the presence of turbulence or for turbulent has to be there in your system, walls must be there. So, walls must be there. If you have no walls, then you must have stratification. Without walls without stratification you cannot have turbulence. So, in the absence of stratification, that is, temperature gradients, you must have a wall.

What does a wall do? In the context of turbulence, wall does many things. When you think of a turbulent flow, what comes to your mind? vorticity, right? vorticity. All eddies going around. So, what does wall do to this vorticity? Walls generate vorticity. Without walls, how do you generate vorticity? It provides that shearing, that rolling motion of that fluid, right? So without walls here, you can take a note.

Without walls or density changes. that we call stratification. So, without walls or density changes turbulence cannot exist in your system. So, what does walls do? I am more interested in the walls than the density changes in this course. So, what walls do is walls generate vorticity.

The presence of walls where fluid interacts with the wall leads to the generation of vorticity. Okay, so this is important vorticity, right? I defined turbulence, so if anybody gives you a flow, just a data is given to you, and the question is, is it turbulent or laminar? That is the question to you. Data is given. I told you all oscillating, fluctuating system doesn't mean that it has to be turbulent. We saw that the stocks, financial stocks are also exhibiting randomness and randomness is established in many signals.

Just because it is random doesn't mean that it is turbulent. Turbulence is also a random process, but all random processes are not turbulent. So, how do you find out the flow is turbulent or laminar? I told you in one of the classes. 3D vorticity fluctuation, the presence of three-dimensional vorticity fluctuation is important. All you have to do is if the data is given to you, that is the three velocities components are given to you in a three-dimensional space and time, you compute whether three-dimensional vorticity fluctuation exists here.

If it is non-zero, yes, it is a turbulent flow field. So, vorticity is very important. in the context of a turbulent flow. And now you may question me: there are flows which does not have a boundary layer like jets, If you have studied free shear flows, you would have what is called a free shear. So, what is it free from? Free share means what? Free from the boundary, it is boundary-free; that term is dropped, right? boundary-free shear flows, or people simply say free shear flows; what are the examples of a free shear flow? Oceanic flows, which part of it? The surface, only if you take the wave and liquid, a mixing layer, a free shear mixing layer, correct? So, if you can have a mixing layer flow, let us say I have, I have two different mixing effects here.

Let us say this is at a lower velocity and this is coming at a higher velocity and this is your surface or the interaction wherever it is occurring. So, you can have a high speed, low speed flow or of different density. It could be air or liquid, or it could be two liquids or the same fluid at different velocities or different densities, a mixing layer, a mixing layer here. So, in the absence of density, since I am not considering the density effects, let us take that both the fluids are the same, both are liquid, let us say, or both are air, and then you have two different velocities that you set, and then you let them interact so this is your wall so I have a wall right on either side I am letting the same fluid flow at different velocities this is let's say u_1 okay and then let this be u_2 velocity two different velocities Then this leads to the generation of a mixing layer downstream, and it may break down into a turbulent flow. So this is a mixing layer is one of the example of a free shear layer.

But if you see even for that you need a wall up front. A wall is required where it is separating it out, and these two fluids are coming out, ok? You can think of this as not like a mixing layer, but like let us say we are inhaling and exhaling. So, our nostrils or the mouth that is the surface we have, the wall surface. So, that is generating the vorticity for you. If I am blowing out air right, the fluid coming out of the ambient fluid is at a lower velocity.

If I am blowing out that is a high speed flow coming out. So, the surface the lips or the nostrils act like walls. So, there is a what is it a generation at its origin before it actually mixes and forms a mixing layer. Similarly, you have a jet, right, so you have a jet flow, which is, and then you will get a turbulent jet for that again; the jet has to come from an orifice, right? So again you have the presence of a wall, so there is vorticity generation also coming inside the pipe inside the orifice here and if you take a wake so this is your jet Or you can take a wake, let us say you take a ball, and then you have flow over it, flow over a bluff body, a bluff ball. Again you have wall surface.

The entire surface is solid surface. So, you have a presence of walls. Even though these are classified as a free shear flows, there is a boundary present. We remove that boundary, and let us say this is an isothermal flow, no stratification; turbulence cannot exist here. The boundary is important even in a free shear flow and obviously, it is much more important in a boundary layer, the surface generates vorticity.

So, now let us look at quickly a review of two important boundary conditions that are important that you all know. Right? Boundary conditions that are important in a boundary layer flow, turbulent boundary layer flow. You already know this is just a recall. You have this kinematic boundary condition, kinematic boundary condition and then you have a no-slip boundary condition, ok. So, the kinematic boundary condition is for the wall normal direction or the what we call no penetration.

No penetration, impermeability that is also a term which is used. No penetration boundary condition. No slip is for the tangential velocity components. So, the tangential velocity sticks to the wall. So, these two conditions you know and then let us see if I take Navier-Stokes equations.

Let us consider a laminar boundary layer first and I hope everybody understands this before we jump into a turbulent boundary layer. So, let us consider a laminar boundary layer. So, if I consider the Navier-Stokes equation here. The instantaneous Navier-Stokes equation, not the RANS equation. So, I get here what I do is I also non-dimensionalize.

I take a non-dimensionalized Navier-Stokes equation, non-dimensional Navier-Stokes equation. So, I non-dimensionalize using some velocity scale, some time scale. So, I can use, for example, I can define the scales as u_i by some u infinity, ok? Some velocity scale I am using to non-dimensionalize u. Similarly, non-dimensionalize using some length scale x_i, D is some length scale. This you would have done in your fluid mechanics courses in non-dimensional Navier-Stokes equation.

Then I have a non-dimensional time scale which is and for the pressure p by ρ. So, if I do this, I can write my non-dimensional form of Navier Stokes equation as $\frac{\partial \hat{u_i}}{\partial t}$. I am writing in terms of the material derivative term equal to minus dou p by dou xi plus 1 by Reynolds number dou square ui cap by dou xj cap square. So, you get your Navier-Stokes in a non-dimensional form, and we are looking into a laminar boundary layer flow.

$$\frac{D\hat{u}_i}{D\hat{t}} = -\frac{\partial \hat{p}}{\partial \hat{x}_i} + \frac{1}{Re}\frac{\partial^2 \hat{u}_i}{\partial \hat{x}_j^2}$$

So, what would happen at when the Reynolds number is very large? Let us say Re tends to infinity. What would happen to this equation? Any term you will drop? This term, right? You would drop this term. Re tending to infinity. So, the viscous term is gone. This is your viscous term. So, this entire equation falls back to the known Euler's equation.

Can you solve a laminar boundary layer flow using Euler's equation? No. Then how do I save the viscous term at high Reynolds numbers? Right? So, this is what was the D'Alembert's paradox that you would have studied in your fluid mechanics courses, right? So, the engineers stopped believing in fluid mechanics some 100 years back because of this. We are saying at high Reynolds number this term is going away, viscous term is negligible and I need to solve only Euler's equation and the engineers measure what is called drag and they are saying no your equations are not helping me. I don't believe in fluid mechanics. So how to save viscous term at high Reynolds numbers? At low Reynolds numbers, obvious, this term will survive.

At high Reynolds numbers, I want to save this. How do I save? It's already saved. Somebody saved it. Who? Prandtl. How did he save this? So, if you see this Euler's equation, this satisfies kinematic condition, but not the no slip.

Euler's equation, this satisfies kinematic condition. but not no slip. So, the kinematic condition is purely kinematic; there is no dynamic component of it like flow cannot penetrate inside the body. So, you have that kind of a condition. So, Euler's equation satisfies kinematic boundary condition but not no slip here.

So, how do I save the viscous term? You would have already studied this. How did he save? You are all experts of fluid mechanics. This is a very important thing, right? He saved fluid mechanics. Otherwise, we would not be having studying fluid mechanics. But historically, the RANS equations were derived even before all this.

Right? Osborne Reynolds, I think if I am correct, all those equations, whatever we are doing, turbulence kinetic energy equations, Reynolds stress, this were all done much earlier. His timelines are much earlier than Prandtl's. He lived in a, he was very, very ahead of his time, Osborne Reynolds anyway, now we come to this: how do I save this term this is very important before we proceed to a turbulent boundary layer. It is much more complicated than a laminar boundary layer very close to the wall.

Ok, think of the boundary layer first. Very close to the wall, the gradients are sharp. So, very close to the wall, this neighbouring term ok $\frac{\partial^2 \overline{u_i}}{\partial x_j^2}$ must be so large that it should compensate for the 1 by Re effect. So, he said there is a thin region close to the wall. Where even though your Reynolds numbers are extremely large, this particular term becomes so important that it balances it out so that the term survives.

That is how we saved fluid mechanics. Right? It is very important to know before we proceed into the turbulent boundary layer, which is much more complicated. So, what Prandtl did is or I can write down here is. Prandtl postulated that I am quoting, in his own words, in a thin region close to the wall, we call of some delta. There is a region close to the wall whose length is delta, boundary layer thickness. Strain rate that is your $\frac{\partial^2 \overline{u_i}}{\partial x_j^2}$, this strain rate term will become large.

to compensate for low 1 by Re values. So, he defined that there is a region which is today we call it boundary layer. and delta is the boundary layer thickness. So, he said there has to be a region close to the wall, so tiny, it does not matter, where the strain rate must become extremely large so that it can compensate the very low 1 by Re values for a finite Reynolds number. We are not looking into infinite Reynolds number. Even for very high Reynolds numbers, the viscous effect has to be there.

Even if the flow is, let us say, 1 million Reynolds number, inside that boundary layer, thin boundary layer, viscous effects have to be present. So, the term has to survive because of extremely large strain and strain I told you is in the production rate of turbulence. Reynolds stresses interacting with the mean strain rate. So, the strain rate is an important term. So, at least now everybody is clear on what is a boundary layer.

Now we go deeper into what is a turbulent boundary layer. Is it the same or different in the next class? Thank you.