Design of Mechanical Transmission Systems

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Week – 02

Lecture – 05

Lecture 05_Machine Tool Gearbox: Centre Distance and Teeth Calculation

So, very good morning to all. So, we will continue to do the problem in today's lecture. So, the today's lecture learning outcomes are using ray diagram, kinematic diagram, both ray and kinematic diagrams to calculate number of gear teeth or each gear in the gearbox. So, we are going to learn about how to calculate number of teeth using both ray as well as kinematic diagram. And this is the problem. So, as above we have a 12-speed gearbox, and minimum speed is given and maximum speed given, and this was the structural formula we have used.

So, initially what we did, we completed the ray diagram aspect, then constructed kinematic diagram. Now, the third part what we are going to do is to calculate number of teeth on each gear, that is what we are going to do that. Assumption, so in the problem statement, it just mentioned about the gearbox power and the speed range, and no other information. We know that only 12-speed gearbox, maximum speed, minimum speed, and each level, no other information.

So, how are we going to proceed? So, what we have to understand by we know that this is a sliding mesh, it is a sliding mesh. The moment you have sliding mesh by default we always prefer to use spur gear. So, the gear type spur, do you have any other information? No. What about gear profile? Gear profile, gear profile, involute profile, it is involute, full depth. How about pressure angle? You can use a standard pressure angle. So, we will choose here 20° as a pressure angle. So, these are the standard thing. So, spur gear type, gear profile is a involute, full depth profile, the pressure angle we assume as 20°. Any other information do we need? Of course, we need to know your basic formula. Though, model not given, but we are not going to calculate, but using that we will derive other information. So, the model equation m equal to your pitch diameter by number of teeth, pitch diameter d suffix p is number of teeth z.

$$m = \frac{d_p}{z}$$

So, I will use z is the number of teeth, z is the number of teeth. One more thing is N we use as a speed rpm. These are the rotation we are going to use in the entire course work. How about center distance? See in gear box we always use a called a line diagram.

See assuming this is the shaft, this is the shaft, one gear is here, we will do like this. It is messing with another gear. Then we need to have a one more shaft for missing aspect. So, this would be mess with another gear. So, the distance between these two shafts is called your center distance a. a is your center distance. In fact, we are going to use center distance method in our calculation. Assume that we have one more gear on the same shaft. This is the smaller one, right, messing with the bigger one. So, what happen to the center distance here? Would you

expect different or same? Should be same, otherwise cannot have a transmission. So, that is the reason center distance is a very critical. So, one more information usually the center distance a is equal to the number of teeth between two gears. You can have $a = \left(\frac{z_1+z_2}{2}\right)m$. Assuming this is gear 1, gear 2 and this is gear 3 and this is gear 4. So, this is for a 1 assuming. So, center distance constant. So, another equation you would expect $a = \left(\frac{z_3+z_4}{2}\right)m$. So, if we equate both center distance a right and b, we can clearly say that number of teeth. Since, the module is same as all the time whenever the two gears are missing, it should have a it must have the same module. Then,

$$z_1 + z_2 = z_3 + z_4.$$

So, this is the center distance. This is what we are going to use it in our calculations. And one more information whenever we do the gear box aspect, we always start minimum number of teeth. This minimum number of teeth to avoid interference and under getting right. So, that should be the teeth minimum less than or get less than greater than or equal to 17. This is the way we have to start minimum thing. So, this is the these are the assumption. So, as we go on, I will introduce more other parameters other information also. Let us move on to the at solving the problem. So, this is the kinematic diagram.

So, this already we constructed and we have a kinematic diagram of the entire thing. So, in fact we are going to start with the stage 3. This supposed to be 3. This is supposed to be 3. Stage 3, if you look at the gears of the teeth are missing G_{11} mesh G_{12}. Then, G_{13} mesh with G_{14} which is the smallest gear among this is a 13 or 11. Remember, whenever we calculate the number of teeth, we will should focus for the drive gears not the driven gears. Obviously, the drive gear which is the transmitting power or torque. So, that is the one it is creating a stage to the driven one. So, obviously you have to focus for your driving gears rather driven gears.

So, among 11 and 13, 13 is the smallest gear because the number of teeth also depends on the size. So, that is the lowest among the 14 gears in the gear box. The 13 is the smallest one. Gear 13 i put G which indicate of gear 13 is smallest. So, now we know that similar also this should be stage 3. Now, look at this. So, we have stage 3 C moves to B and A. So, C what is the input speed for C? So, what is the input speed for C? So, C is the input speed is 125 rpm. Am I right? Yeah, it is 125 rpm. For B, it is a output speed 250 rpm and A which again output speed is 250 rpm. So, C is the output speed for C. So, that is your 31.5 rpm. You can see clearly when gear right. So, when you have a C to A or in terms of, we talk about the gear aspect, gear meshing right. It is a gear 11 meshing with gear 12 right. That is the one will give you the lowest speed.

So, if you again refer to your ray diagram, if you look at this, you will see that the gear 11 is meshing with the gear 12 and the gear 13 meshing with gear 14. So, if you want to have a lowest speed, the gear 13 meshing with the gear 14 will give you 13.5 right. Gear 13, gear 14 is it speed reduction or speed up? Speed reduction yes correct. So, it is speed reduction. Now, we talk about C to B. Its gear number 11 meshes with the gear number 12 is speed up. So, you will understand. Now, you can see that why we have to use both ray as well as kinematic diagram to understand the aspects. That is fine. Now, we can move to the next information. So, number of teeth calculation we are going to have a. So, as I said we are going to use a center distance method. That is the one. So, I am sure we can start straight away. I have done

the trial-and-error method. You can start as you knew 17, 18, 19. Since, I have done that I will straight away start with the gear number 20. Let use your gear. I am talking about gear 13. z_{13} should be 20. That is there. So, I am sure you know the formula.

$$\frac{z_{13}}{z_{14}} = \frac{N_{14}}{N_{13}}$$

z_{13}, z_{14} is a number of teeth. Inverse with proportional to your speed right. N_{14}, N_{13}. Tell me what is the speed for N 14 and N 15. That is what we need to know that. So, N_{14} speed would be yes 31.5 right. 31.5 rpm. Your N_{13} is your 125. So, that is your N_{13} is your 125 rpm. So, when you substitute, I want to understand because of already I fixed N_{13}. So, I need to find out z_{14}. When you want to find out $z_{14} = z_{13} * \left(\frac{N_{13}}{N_{14}}\right)$ right. You substitute the values $20 * \left(\frac{125}{31.5}\right)$. So, I would expect some fractions. So, I am getting 79.36 as a z_{14}, but looks you know it is a fraction. Gear supposed to be whole number. It cannot have a fraction. So, what are we going to do right. Either we can round up to 79 or 80. Either round up to 79 or 80. So, assuming that we will have a case A. Case A when you have a rounding of z_{14} as a 80 and case B will do both. So, that you will understand what is the way to do that z_{14} as a 79. Please understand earlier we discussed step ratio. This step ratio can have 10 percent of variation. So, that will take care of that. You can ask the question we have to make the step ratio by rounding of are we not affecting the step right. Yes, we are, but within the permissible limit right.79.36 is just one number. So, it is a very very fraction not even 1 percent will be even fraction of 1 percent. So, that is fine it is acceptable.

So, we will do the case A. We will do that based on that. Case A we assume that the z_{14} as a 80. Now, what we are going to do? We are going to use the center distance method using center distance method because the center distance constant. So, we have z. So, you can see that

$$z_{13} + z_{14} = z_{11} + z_{12}$$

So, this equation you can have a 1. So, now we our known data what is the known data? We know the only data is the only speed we do not know the number of things. So, we have to refer again number of speeds aspect your N_{11} would be what is the speed and N_{12} please look at your radii graph that will help you 125 rpm and this should be 250 rpm 125 and 250 rpm. So,

$$\frac{z_{11}}{z_{12}} = \frac{N_{12}}{N_{11}}$$

So, but one more thing interesting thing is what happen you are increasing the speed from 125 rpm to 250 rpm that is already known from the speed if you look at this. So, in the second pair of gears. So, then equation what happen my center distance equation. So, since speed up 125 rpm to 250 rpm in this stage your $z_{11} = 2z_{12}$ that is assuming as a equation number 2. So, then substitute you will have a $z_{13} + z_{14} = z_{11} + z_{12}$ and if you rewrite this is 20 and this is 80 instead of z_{11} I can have 2 times z_{12} plus another z_{12}. So, ultimately, I would expect 3 times z_{12} must be equal to 100. So, now the z_{12} approximately right it is came as a come as a 33. 33 either you can take it as a 33 or 34 that is the thing 33 or 33 or 34, but we know that your z_{11} must be 2 times z_{12} already you know that the number of teeth of each pair should be equal on the both sides right can we do the quick calculation. So, by doing that if you choose as a 34 right if you choose as a 34 z_{12} if you choose z_{12} as a 34 then z_{11} should be 2 times of 34 will be 68. So, this is the z_{11} and z_{12}. So, as aware of that already in this side your z_{13}, z_{14} must be

20 plus 80 we should expect 100 if you calculate z_{11} plus z_{12}. If you do that what happen z_{11} would be 34 plus 68 what is the answer you are getting 102. So, do you think is it correct does not match does its no. So, they are not equal they are not equal. So, because if you choose 34 is not making any sense, I mean it is not making any sense. So, what is the way to do that right let me choose instead of 34 if I choose z_{12} as a 33 what happen if you use z_{12} 33 my z_{11} would be 2 times 33 is good 66, 66. But earlier what we have done. So, z_{11} plus z_{12} 33 plus 66 should give 99, but the calculation we got 100 right if you look at this 100. So, they are not equal. So, how are we going to do that absolutely correct. So, the case B right in the case B what we have assumed at 79 right what is that number 14 right the 79 we have choose we choose z_{14} as a 79 right then z_{13} plus z_{14} equal to right it is 20 plus 79 right that should give us the same would come right if you choose your z_{11} which is 33 and 66 and that is coming as how much 99. So, we are satisfied right you can clearly see that if you choose z_{14}, as a 79. So, you can see that if you choose z_{14} as a 79. Then you are able to satisfy the center distance you are able to satisfy and maintain center distance as a constant for all four gears. So, by we can establish z_{11} equal to how much z_{11}. Now, z_{11} should be equal to z_{12}. So, z_{12} is should be 33 your z_{12} 66 z_{13} 20 and z_{14} as a 79 these are the speed, we obtained right for stage 3. Now, we will move to stage 2. So, we have will move to stage 2.

So, in the stage 2 we are going to have three gears right. So, the gear is meshing G_5 mesh with G_6 that is one thing then you have a G_7 meshing with G_8 gear 8 then G_9 meshing with G_{10}. So, you have a gear 8 mesh with G_9 mesh with G_{10}. So, you have three pair of gear meshing. Now, we need to know what are the corresponding the speed also. So, if you refer the diagram the kinematic diagram. So, you could see that you are getting three speeds right F is split into E right next one is D and then you have other one is C. In fact, F speed from F to E the speed is 500 rpm then you have a F to D 250 rpm and C F to C is 125 rpm 125 rpm whereas, F speed what is F speed 355 right. 355 in fact, F to E what is the corresponding gear meshing what is the corresponding gear meshing right. So, in stage number 2 stage number 2 you have to look at your ray diagram also right.

So, tell me what is the gears are meshing for F to E yes. So, what is the corresponding gear meshing? So, what is the corresponding gear meshing for F to D 7 8 G_7 to G_8 and also look at this your input speed will be 355 rpm right your input F input 355 and your output E output let us do that. E output what is the rpm 500 rpm and D output 250 rpm and C output as a 125 rpm. So, clearly says here the observation would be we are going to speed up. So, it is start 355 to 500. So, this is speed up that is information we are observing. Now, we talking about this is one aspect let us move on to the other aspect F to D. What gears are meshing from your ray diagram please what gear 5 6 gear 5 gear 6 right. So, gear 5 and gear 6. So, you can see clearly see it is 355 to right. You are going to have a 1 second is supposed to be D right 250 rpm which is speed reduction speed reduction that is observation we can see that.

Now, finally, we look at F to C. what gears are meshing the gear is missing. So, what is the corresponding gear 9 to gear 10. So, the speed would be 355 rpm to 125 rpm. So, clearly this also speed reduction. So, from this we can able to clearly understand when the gear 7 and 8 meshing you will have a speed up when gear 5 and 6 meshing you will have a speed reduction and gear 9 and 10 meshing you will you are having another speed reduction. So, you will have a two reductions one gear up. Now, we will let us move on to the number of teeth calculations.

So, already we know that we know that the minimum number of teeth already we found from z_{13}. So, no need to do it again because earlier all the gear that are unknown. So, we made one gear as unknown and using center distance method try to find out the number of teeth of each gear right. So, since already we know that information using that information, we are moving to the stage 2 and try to continue for stage 2 gears also for number of teeth.

So, here look at the speed reduction speed reduction from we start from G_9 to G_{10} reduction from G_9 to G_{10} right which is coming 355 rpm to 125 rpm. So, this is the speed reduction that is what is happening. So, from here,

$$\frac{z_9}{z_{10}} = \frac{N_{10}}{N_9}$$

whereas, N_9 is given from your ray diagram 355 rpm your N_{10} 125 rpm clearly given. So, this is the speed reduction so I think we need to again start earlier statement what I gave may not because of each stage you need to have a assumption again because that is cannot carry. So, now I am assuming my z_9 as a 20 I am assuming z_9 as a 20 then z_{11} sorry z_9 from z_9 we need to find out $z_{10} = \left(\frac{355}{125}\right) * 20$. I would expect 56.8. So, if you do approximation because already right, we know the number of teeth also. So, when you do the approximation z_{10} should be equal to 57 and my z_9 is already 20 that is one thing. Now, we will talk about other next one speed increase speed up speed increase from gear 9 sorry should be gear 7 to gear 8 right from 355 rpm to 500 rpm 500 rpm that is clearly given. So, $\frac{z_7}{z_8} = \frac{N_8}{N_7}$ 500 by 355. So, if you want to identify z_7 right in terms of z_8, z_8. I will keep it just like that and I know the value that is coming as a 1.408. And if you refer your center distance the

$$z_9 + z_{10} = z_7 + z_8$$

substitute that 20 plus 57 right yeah 57 must be equal to z_8. I can replace z_7 equal to z_8 right yeah 1.408 plus z_8 and if you find out what is that value you are getting 31.97 approximately 32 approximately 32. So, what happened my z_7 then z_7 it is coming as a 45.06 should be 45. Look at earlier we got z_{10} as a 57 and z_9 as a 20. So, my total number of teeth would be 97.

So, let check cross check if I do z_8 and z_7 that should come 32 plus 45 what is coming 70 77. So, does it match now is right it has it does. So, they are and one more thing. So, in the beginning of this gear box was particularly machine tool gear box what we said the minimum number of teeth difference between the two adjacent gears must be 4. So, that the sliding can be taken without any trouble without any problem. So, if you check the difference between z_7 and z_9 to, they are getting minimum difference of 4 correct is not it. So, we know that your z_7 is 45 your then z_9 is 20 obviously, you have big difference similarly, z_{10}, z_8 it should go greater than 4. So, here also satisfying that. So, now, we are able to do within the second state for four gears there is one more pair is left now.

The final one within the second stage right. So, here you can see the speed reduction is happening speed reduction G_5 to G_6 that is 355 rpm reduce to 250 rpm this is already we are able to see that $\frac{z_5}{z_6} = \frac{N_6}{N_5}$ which is 250/355. So, I want right in terms of because of both are unknown to me. So, $z_5 = z_6 * 0.70423$. I am taking as a equation within this equation number 3 and if you refer your center distance aspect the number of teeth is $z_7 + z_8 = z_5 + z_6$ you can take it as a equation number 4. So, but we know the speeds also your z_7 is 45 right and z_8 is 32

right then this is $z_5 + z_6$, but we know the relationship from the equation number 3. So, z_7 is equal to z_8, but we know the relationship from the equation number 3 if you substitute equation in equation 4 right. So, you would expect this is 77. So, this is in terms of 76 right sorry z_6 will be 1.70423 and finally, I would expect my z_6 equal to sorry we will say equal to 45.18 is approximately I will take it as a 45 approximately 45 then the z_5 would be z_5 would be what 32. So, if you look at the summation the 45 plus 32 it should give me 77. So, that is also satisfying in fact, if you summarizing you will have z_5 32, z_6 how much is coming 45 right 45 then z_7 is equal to 45.18. So, this is 45 and z_8 32 and z_9 20 and z_{10} 57 and if you sum it this if you add these two and these two and these two will have a same total number of teeth. So, which is satisfying keeping the center distance constant in that second stage. Now, we will move on to the this is supposed to be stage 1 in the stage 1 the final one gear 1 meshing with gear 2. So, this is supposed to be stage 3 and this is supposed to be stage 4. Now, we will move on to the this is supposed to be stage 1 in the stage 1 the final one gear 1 meshing with gear 2 you can see that right then gear 3 meshing with gear 4 that is fine. Now, again if you refer to the ray diagram what is happening it should be reduction right by default. So, in stage 2, 1 gear up and 2 reductions in stage 3, 1 gear up 1 reduction, but in stage 1 most of the time should be gear reduction. So, what is happening H to G right then H to F. So, from H what is the input speed for H 1000 rpm if you refer with your ray diagram then what is output speed for G which is 500 rpm F output speed 355 rpm. So, in fact H to G can give me corresponding to gear aspect gear meshing gear 1 to 1 to 2 right gear 1 to 2 is speed reduction. Then H to F gear 3 meshing with gear 4 is again speed reduction. So, now we will do the calculation right.

So, speed reduction is happening two speeds aspect G_3 to G_4 which is again I am repeating 1000 rpm to 355 rpm. So, if you do that z_3. I will rewrite $\frac{z_3}{z_4} = \frac{N_4}{N_3}$, again we need to calculate the speed of the gear assume let assume the smallest one z_3 as a 20. So, if you calculate z_4 you would expect right approximately 56 then next pair speed reduction from gear 1 to gear 2 which is 1000 rpm reduced to 500 rpm. So, $\frac{z_1}{z_2} = \frac{N_2}{N_1}$. So, then which obviously double speed reduction by half. So, obviously your $z_2 = 2z_1$.So, this is the speed reduction by half. So, this is the speed reduction by half. If again follow the what you call it centre distance aspect center distance aspect $z_1 + z_2 = z_3 + z_4$. So, this one should be 3 z_1 straight away I am doing and this one 20 plus 56 right. So, $3z_1$ should give me 76, z_1 approximately I should choose as a it is coming around 25.33 either I can choose as a 25.33 or I can choose as a 25 or 26. So, according to number of teeth you suppose get how much earlier we found out 20 right 20 for z_3 right 20 for z_3 then z_4 is 56 that is what we did it because we take it as a approximation.

So, what is your understanding do you think that is correct by choosing z_3 as a 20. Now, let us first we will discuss this is 25 and you are getting what happen to the z_1 is 25 if it if assuming that if z_1 is 25 right z_2 would be 50, but our total is here is 76 right 76 it is not correct. So, this is if you choose some z_1 is equal to 26, z_2 should be 52. So, if 26 52 what happen is not matching with the 76. So, this is not correct. So, what is the way to achieve it you can change z_3 as a different value you can change z_3 as a different value. In fact, if I let take z_3 as a 21 right straight away I will have my z_2 equal to how much z_3 equal to 21. So, z_4 equal to how much you should expect 60 right it is now coming 81 it is coming 81 it is coming 81. Then you redo the calculation my $3z_1$ right it should come as a since right 3 $z_1$is coming as a 80 then z_1I am approximately considering 27 if I do that my z_2 should come as a 54 look at this the new summation 21 plus 60 I should expect 81. If you recalculating based on this calculation z_1 is

27 and z_2 is 54 a summation you would expect 81. So, that is matching that is matching please understand that you may ask question sir we are talking about center distance, but each case the center distance is different how is different the number of teeth are different that is fine, but as long as you maintain the center distance between the two sets it is fine the third set is again is respect to second state now the previous one. So, that should be all right in fact by summarizing what I want to say that so assuming that this is my center distance between first and second, I am getting some value a right the next one can have even smaller distance or bigger distance also that is fine. So, I can have a one more center distance which is this is a_1 and this is a_2 this is third stage a_1, a_2 need not be equal they can be greater or lesser also that is what you are understanding. So, in fact if you summarize the number of teeth $z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}, z_{13}, z_{14}$ as 27, 54, 21, 60, 32, 45, 45, 32, 20, 57, 66, 33, 20, 79. I think I will stop now. In next class, we are going to do the unconventional structural formula of 14 speed gear box. Only the radiogram we are going to discuss.