Design for Quality, Manufacturing and Assembly
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Lecture – 06
Review of Six Sigma and Quality Loss Function (QLF)

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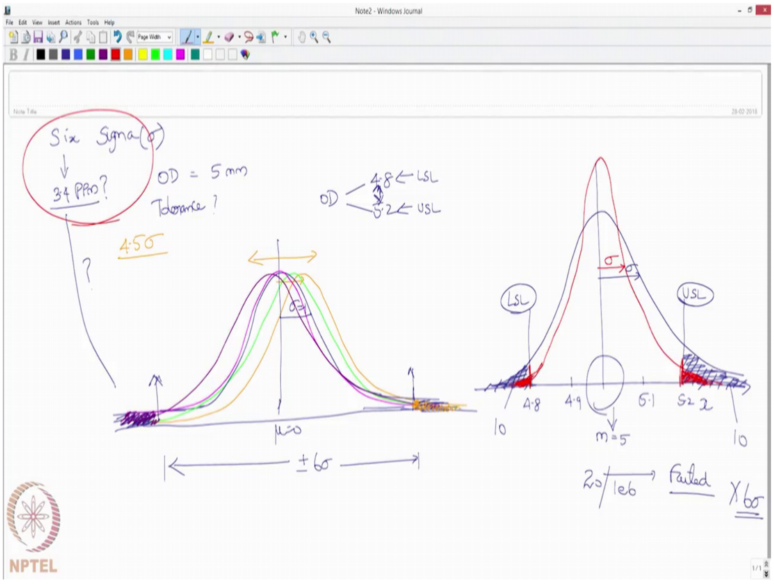

In order to discuss Six Sigma we wanted to have what this sigma means. That is why we spoke about standard deviation. So, what this one says is, you take a distribution, a normal distribution with a mean equal to 0, irrespective of what your process is. So, you can go and manufacture in microns for an integrated chip, or you can go and manufacture a bridge which is few meters, that does not matter, but I can convert my data to mu equal to 0 that is all you wanted to know, right. Now let us take a manufacturing example of the washer whose outer diameter is 5 mm.

But as we mentioned yesterday, the moment you go to a vendor, the vendor is going to ask what is my tolerance. So, you say my OD, I preferably would like to have at 5 mm, but I can accept anywhere between 4.8 or it should be no, this is, sorry, this is another way to write it, ok. So, the OD should be between 4.8 and 5.2, it should be between these 2 guys, ok. This is something that we discussed yesterday. So, this is usually called the lower specification limit, and this is called the upper specification limit, ok.

So, now let us say that this is equivalent to the m equal to 5, but I have converted it to mu equal to 0. So, this is equal to 5, which is that diameter, ok. So, let me just, this has a mean of 5, which is what is my outer diameter, ok. And then I have some values here. 4.9 4.8, 5.1, 5.2 right. So now, these are my specification limits. This is my lower specification limit, and this is my upper specification limit. What it means is any sample that is going to fall, because we discussed yesterday how you get this graph. This graph is not magic it is nothing but the frequency of the products that are falling in this area, right?

So, the area that is below this curve, and the area that is below this side, are failures, ok. Now this area together will define; what is the process of your sigma, ok. What is your sigma? Sorry, what is the sigma of your process? Whether it is a 3 sigma, whether it is a 4 sigma, whether it is a six sigma process; this will define, now we will go back and see where that number 5-point sorry, 3.4 parts per million come. Because the six sigma means it is 3.4 parts per million. So, we will see where this number 3.4 parts per million comes from.

But let us say, right now you just take it granted from me, that six sigma means it is 3.4 parts per million, ok. Let us imagine that you are doing million parts. And then I had the means to measure whether the dimensions were intact. And then I figured out that you have about 10 products fail here and you have about 10 products fail there, about 20 components in the million components that you did, 20 over 1 is 6, failed so, 20 of them failed.

So, this is not a six sigma process, because I can fail only 3.4 parts. So, it should have been 3 parts, right? Now the question is if I were to make this process a six sigma compliant process, do not worry about the design part of it, how you are going to do is a different question, ok. But from a pictorial sense, from a pictorial sense on this picture, what will you have to do? You have to shrink the distribution, why shrink the distribution? So that I will have this area reduced. Now imagine this situation.

So, please understand still my specification limits are the same. I am not changing my specification limits it is still 4.8 below 4.8 is failure greater than 5.2 is failure. However, you can see this shaded regions of red is much smaller than the blue, ok. So, how can I achieve this? By only shrinking the distribution; what does this shrinking mean? It means that I am working on my sigma, the red sigma is smaller than the blue sigma, but please also remember that my mean is the same for both the process, ok. If I had moved the mean, if there was a mean shift, this discussion might not hold good, right?

So, in this case we are assuming I mean my graph is not very correct here, but assuming that the mean is the same for both of them. I am only reducing my standard deviation, which means that I am stringing the distribution. As a result of it the number of failures is less compared to the other situation, ok. So, one way in which you can reduce the number of failures is to shrink the process variation, ok. So, you have to bring down the process variation. And this is what we discussed in the clean example. If you remember the tile example, ok. Because the initial blue curve mapped to the larger variation of your tile size, and the red curve maps to the tile size with more or less 1 by 1 foot, that is what you want. You get the point?

So, this is something that I wanted to discuss, wanted to make sure, because your goal pose these are your goal pose, right your LSL and USL your goal pose remain the same, ok. Now the blue is a manufacturer like maybe like 3 sigma or lesser than are slightly more than that, whereas, the red is six sigma, because this will be lesser than or equal to 3.4. This is only conceptually I am discussing you know.

Now, the question is where did you get this 3.4 parts per million, ok. If you take a statistical distribution, a standard normal statistical distribution, what it says is, if you walk 6 units in this which is six sigma, I am going in this side six sigma, and in this side six sigma. So, let us assume that it is somewhere here and here, ok. So, this means it is plus or minus six sigma, ok. So, what you can do is, is you can go and find out what is the area here; which means, imagine that I replace these 2, these 2 guys I replaced by my lower specification limit and upper specification limit. That is for my manufacturing process, but it can be for anything any design it can be for any performance it can be, ok.

So, in that case I just need to know what, I am sorry, this is I am just yet to get used to this, this is a lower specification limit right. So, I need to find this area, and I need to find this area. If I find these 2 areas which means the failure, I find these 2 areas which is the failure. What do you think the value will be? The value will not be 3.4. It will be much lesser than that. I do not know how many of you saw the picture that I showed you yesterday, ok. If you remember, I would have shown 1 3 5 0 4 3 sigma, whereas for six sigma it would have been it would not have been 3.4. It would have been something of the order of 0.001.

So, then why do you say six sigma is equal to 3.4 parts per million. If actually statistically you take a distribution and then you take the areas that are beyond the six sigma. It is going to be lesser than 3.4, the area wise of course, you divided by 1e6, then why do we say? Six sigma is 3.4 parts per million, it is actually a lesser sigma, ok. The deal is there is going to be variation between the samples that you are going to take.

So, let us imagine that you are going to a shop floor today, and today morning you are drawing 1-6 samples, one million samples you are drawing of the washer. And you are let us let us say that you have the capability to measure all the 1e6. You would have got a distribution like this in the morning. Then you go in the afternoon, for whatever reason. You might get a distribution like this; there is a small mean shift, ok.

So, that night you go back, and then you might get this. The next day morning you come back, you might get this. The next day afternoon, you might actually get this. So, there is a lightly chance that the variation might more or less remain the same. But my mean itself could vary there is a mean shift possibility. But is that going to make a difference certainly yes, certainly yes, if your mean gets shifted this side the area of my failure is going to be larger on this side, ok.

So, is the case this side also? It depends on a colour that I have chosen it might be larger yes it might be compensated on the other side, but you never know ok. So, in order to account for this variation in the mean what people suggested is, you should put a factor of safety. So, this is a deterministic factor of safety for a probabilistic concept. So, the factor of safety that is used is 1.5, ok. That is why it is actually not six sigma it is 4.5 sigma, ok. Now do not go outside and say this six sigma is actually not six sigma, but it is 4.5 sigma, ok.

Not majority of the people appreciate this concept, ok. This is the math behind it, if you are just reading only what six sigma is this is all you need to know, ok. It means 3.4 parts per million, but statistically if you look at it will not turn out to be 3.4 divided by one million. It will be much lesser than that, but instead if you put 4.5 sigma, you will get 3.4 divided by one million, ok. In order to account for this mean shift, they proposed 4.5 should be called as six sigma.

So, this is a conservative design from that perspective. And if I am not wrong Motorola was the first one to propose this, ok. So, though it originated the idea originated with Taguchi and those ideas, but it was heavily used by companies in the west in the US I think, the major players were IBM Motorola, they used this in their manufacturing and they were able to make better computers and robust printers and things like that; so they in order to account for the variation of the probability distribution itself. So, see this is a very interesting concept. The probability distribution gives you the variation of the parameter or the quantity that is of interest

But that probability distribution itself can vary. So, this leads if you are going to work in the area of reliability. There is a question that always arises what is the reliability of your reliability estimate. Because you are only estimating your reliability, ok. How far it, how far can I rely on your reliability estimate? Similar to that, there is a variation in the in the description of the variability. What is the description of the variability? The probability distribution, and there could be a very it could the probability distribution itself could vary and you need to account for that variation that is where you are using that factor of 1.5. So, you are offsetting six sigma to 4.5 sigma that is what, ok.

So, but your take home information in this one is if it will mean six sigma, I am going to walk 6 units from my mean, plus sigma plus six sigma minus six sigma, and then the it is it is actually 12 sigma distance if you look at it, ok. And the area that is beyond those 2 curves should add up to 3.4 divided by 1e6. What does this? Roughly translate to if you are going to manufacture one million components, the same type of component, then about 3.4 can fail. There is no meaning to 3.4 if it is 10 million, then 34 samples can fail. This is what it means.

So, based on the failures that you present as a vendor, you will be rated as six sigma, 5 sigma 3 sigma manufacturer. So, usually if you are a higher sigma manufacturer, then the cost is higher than a lower sigma manufacturer. And any OEM today want to have a higher sigma manufacturer, because if you imagine for a particular car. Any car that you talked about not the OEM itself manufactures all of that, ok. The OEM practically manufactures their engine and the most important components.

The remaining components come from vendors. And it is it becomes very important the vendors also practice these sigma practices such that, the variability of the components that they supply to the OEM is very less, because all these variations will interact with each other, and they should not blow up at the system level. So, it becomes very important these sigma practices become very important in the manufacturing sector, ok.

So, this is something that I wanted to discuss before we go ahead, is that clear? That is also one thing that I discuss yesterday. So, that 4.5 sigma this is where it comes from, ok.

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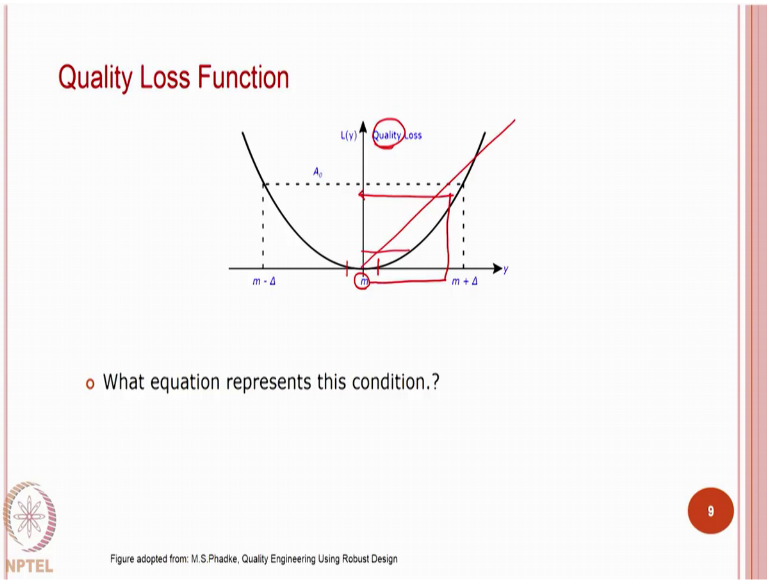

The next question that we wanted to ask our self is, we spoke about quality, right. So, we spoke about quality we spoke about robustness a little bit, ok. Now we will go back and visit quality, ok. Then we will see know how this robustness concepts will match with quality. The one thing that we spoke about quality is we only verbally described what qualities is, we gave you the example we had the discussion about the car breakdown at the middle of the road early in the morning, and how the different people get affected. Now how do I measure this loss in quality?

So, you go and buy your TV let us say, ok. Or you go and buy a car; you go and buy a motorbike, ok. So, the guy promises 55 kilometers per liter. But then this bikers motorbike is giving you about 55 kilometers per liter. So, you are very happy about it. You are happy about it. The manufacturer is happy about it or probably you are not even going to report to the manufacturer. So, the point is there is no loss per say. I promise 50 you are getting 55 you are happy. However, I promised 50, you are getting 35.

So, there is a loss, in the promise there is a deviation in the promise that I made there is a deviation in the expectations that you had on the performance of the motorbike. So, what will you do? You will come back and ask the manufacturer or the dealer: hey, come on you said it is 50, but it is giving me 35. So now, the problem is you are disappointed. And I as a supplier or a dealer or manufacturer need to take care of your disappointment. Otherwise I am going to have repercussions, especially with this social media age. You will not keep your hands and mouth shut. What you will go do? You will immediately tweet? You will immediately put it on Facebook, ok.

So, indirectly my business will get affected. So, as a manufacturer I need to be careful. Because I have made a promise and I have to live up to that promise. So, this is going to cost to me also. So, there is a cost repercussions from my side there is a disappointment repercussions from your side right. So, from Taguchis perspective, when you want to quantify quality the idea is to quantify the loss, because the moment you get 55. I am happy, meaning the user is happy as a manufacturer I am happy there is no loss. And I do not have to quantify that profit, meaning the emotional profit, ok. Whereas, from the not living up to the promise that I made or the product, not living up to your expectation I need to necessarily quantify the loss.

So, that is what we are discussing. And what Taguchi proposes is he says that we should use a quality loss function, ok. So, what is says? This here is a variable y, which is the quantity that is of interest it could be the kilometer per letter that we are talking about. And this is the promise m equals 50 kilometers per liter, ok. So, in this case, this is not the correct example or this is not the correct what you call the graph, this is called the nominal the best situation. So, we will talk about an example where plus or minus is a problem, ok.

So, we will say making tea when you are making tea there should be an optimal sugar that goes into it, ok. If I put more sugar, it is like a sugar syrup, if I put less sugar, it is going to be bitter. So, there is an optimal right. So, that is this m, ok. So, do you think whoever makes you tea or you yourself make you will put the correct number of grains of sugar every time. There is going to be some plus or minus right. So, you know that that small plus or minus is, ok. That small you know 5 grains more 5 grains less. There is imagine that there is an optimal number, ok. This so, that optimal number this graph this is your sugar stuff, or your and this is the loss the y axis is your loss. The loss in what?

Here it is the quality loss, but in the example that we are talking about it is the taste the loss in your taste, ok.

So, what happens is for that small 5 grains less or 5 grains more. There is no loss in your taste it more or less tastes the same. But the moment I put half a spoon more, you know that it is it is sweeter than the usual tea you take. Now what happens is my loss grows slowly. Here I have a loss of 0, but the moment I put half a spoon more your loss is slightly, I mean your taste you know you are disappointed ok, but still you will drink it. I put 2 more spoons of sugar, half of the cup is sugar, and the other half is your tea, you are not going to be able to drink. You are going to say oh this is like a [FL], I do not want to drink this tea.

So, the loss is heavy, do not, it is not necessarily a loss in the monetary loss. Monetary loss is also a form of loss, but here what is your quality? The quality is that taste that I have, ok. So, as a seller I am going to lose you probably for the day at least as a buyer ok. So, there is a heavy loss. So, the question is this is the optimal sugar level. But what is happening is I am deviating from that sugar level, and then I am giving you more sugar and it is. So, is the case on the other side also? So, what Taguchi is telling is this loss can be modeled in a quadratic fashion, ok. For a small deviation the loss is 0, but the more you deviate, the loss is not in a linear fashion, it can usually be manufactured it can usually be modeled using a quadratic function, ok.

So, what it says is for about; see this is not a linear function. A linear function would look like this, ok. So, what it says is for a smaller variation I will penalize you more as you can see this dotted graph, right. So this and this are not the same, ok. So, he says as you deviate more from what I promised, you will have to be penalized more and more. The more you deviate the more will be your penalization, ok.

So, that is what the quality loss function talks about. Now the question is this is pictorially I am giving you, right? Now the question is we need to be able to quantify this, ok. What do you think what equation represents this plot? I just told it already. What equation represents this plot? If I ask you to give a form to this curve, what is the form? That is one thing yes, can be a parabola, but parabola is what sort of curve it is?

Student: (Refer Time: 25:33).

It is a quadratic, ok.

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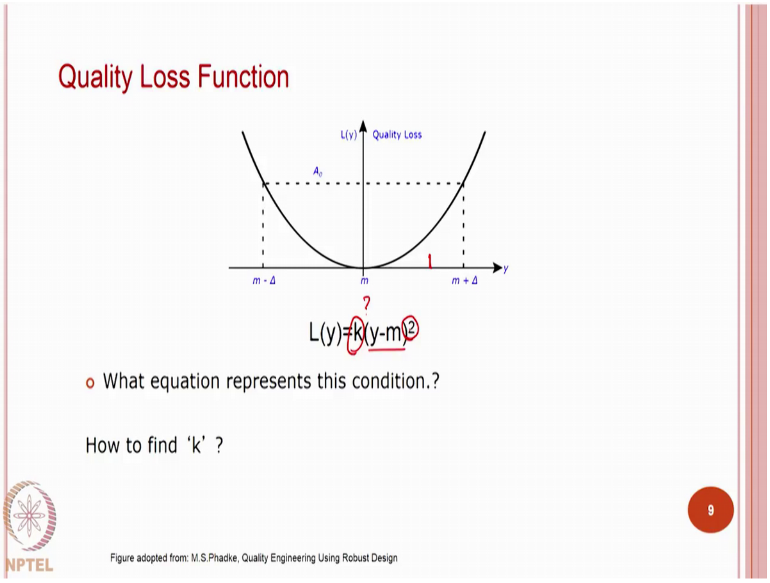

So, a simple quadratic curve is what he is proposing. So, the quadratic curves usually take the y equals mx squared is what they take ok. So, please do this, this is not the usual notation. So, this y minus m is nothing but the deviation, ok. So, y is the value that you are talking about. This is my y ok, minus m, I am normalizing m is my original value that I want I would like to have. And this y minus m is what the deviation is. So, it is again the same story. Deviations squared, and there is some constant k, will give you the loss that is what we are talking about here, ok. So, the question is how do you know this k, because this case seems to influence the value. You understand that it is quadratic that is fine, ok.

So, the quadratic just takes care of this squared, y minus m squared; if you want to penalize even more. See, for instance let us say that you are working with steel, some grade of steel ok, this deviation will mean that you have to dump the steel material. So, you are giving a square penalization. At the same time, your counterpart is working on gold. You think for the same deviation the penalization will be the same? The penalization is huge, it might be cubic it might be 6th order it might be exponential, for I do not know like one tenth of a gram, the cost is so different, for one gram we are talking about 3000 rupees variation.

So, you told someone that I will make you a jewelry of 10 grams, and then you made 9.8, they are not going to just let go. They are going to cut off the price for that. Or you will not give away a 10 point 2 gram jewelry for 10 grams, because the cost is different, whereas if you are making a steel making some steel for 10 grams. And then 10 point 2 grams you think that you are going to spend your time to take off that point to; no you will just give it off that is not a big deal. So, this squared is a general hypothesis that he proposes. So, he says you can use a quadratic function in general.

But now, if you go to any automotive company and all that based on their own legacy for different components they have different loss functions.