Acoustics & Noise Control
Dr. Abhijit Sarkar
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module – 27
Lecture – 32
Insertion Loss

So, in the last class, we discussed the electro acoustic analogy and we also had some discussion about the source impedance. So, here was the (Refer Time: 00:28) and circuit that we had derived. So, here we had done the source modeling, the source was this P n plus 1.

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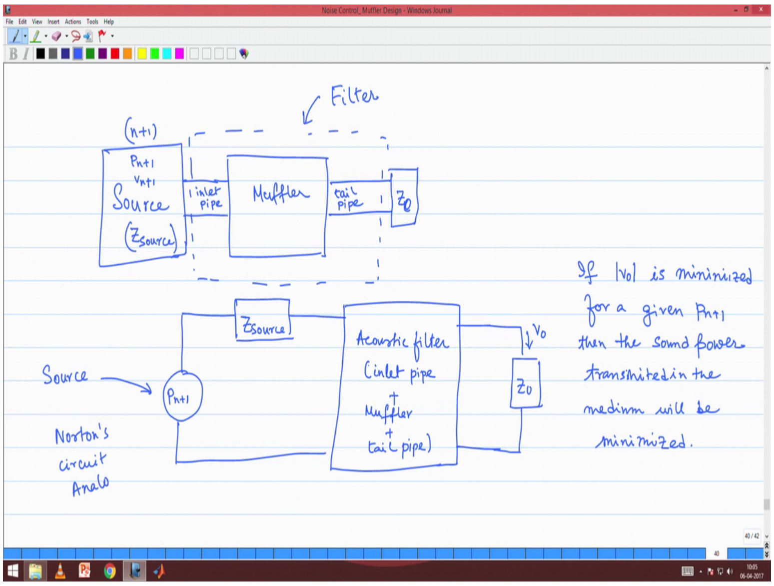

So, this is like saying that the source is a constant voltage source, but then that it goes through a source impedance and then reaches the load, right. So, this is what is called the Norton's circuit analogy. A completely equivalent picture is brought about by the Thevenin circuit analogy which is what will touch upon now.

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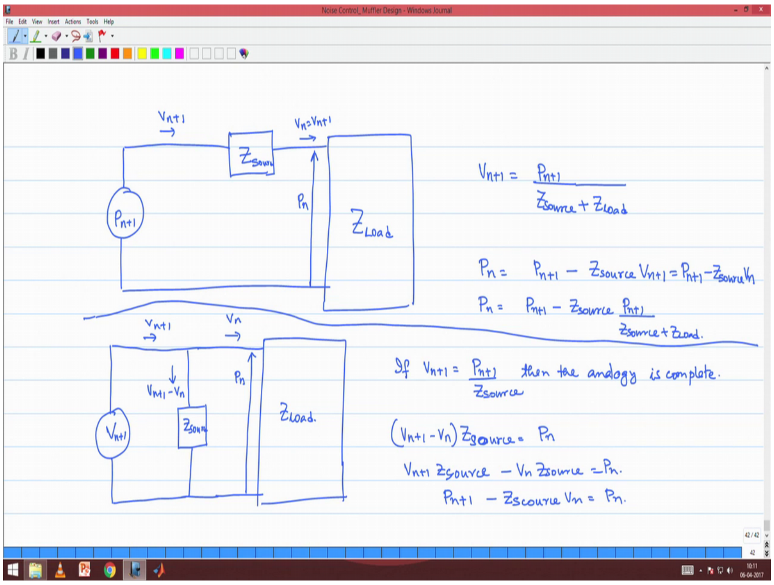

So, this is where in the source is model as a voltage source and then there is a source impedance Z S or Z source if you might call it and then there is a load. So, this load will include both the muffler as well as the radiation impedance. So, this part if you like is the load. So, I am just redrawing this picture by collating the muffler impedance together with the radiation impedance and calling it as the load impedance. So, in here what you will have is that though the source is supposedly working at a voltage of P n plus 1, but what reaches the load is not P n plus 1. This is P n which is different from P n plus 1 because there is a source impedance which is connected to it.

So, to calculate; what is the value of this P n, we are doing this simple calculation. So, firstly, we understand that the current that is flowing through the circuit V n plus 1 is P n plus 1 divided by plus Z load, right. So, therefore, P n is P n plus 1 minus Z source into V n plus 1 which is also equals to V n because between the input and the output terminals of the source impedance, there is no fall in current; that source impedance is in line, right, it is not in the shunt position. So, therefore, you could as well write this as P n equals to P n plus 1 minus Z source into P n plus 1 divided by Z source plus Z load. Now we want to have another equivalent picture which will assume that we have a constant current source V n plus 1 to denote the current source. This is a constant current source and this current source is acting over the load, but this time since I am modeling the source as the current source rather than a voltage source, I have to give the source impedance in parallel.

So, let us see what this reveals and again we have the load impedance here, right. So, here the V n plus 1 will get spilt into 2 parts 1 is V n. So, V n plus 1 and V n will not be same in this situation because apart of the current will move in this shunt line which will be of the form V n plus 1 minus V n, right. So, if this is again P n because we want this P n to be the same we want to have an analogy between these 2 circuits and what we will show that if V n plus 1 is chosen as P n plus 1 divided by Z source, then we will have then the analogy is complete. The analogy between the 2 source models; one in which we are modelling the source as a voltage source or in acoustic terms as a pressure source and the other in which we are modelling the source as a velocity source or in electrical terms as a current source.

So, let us see what our electrical modeling this time will give us, it will give us that V n plus 1 minus V n into Z source must be equals to P n, right. So, in other words V n plus 1 into Z source minus V n into Z source must be equals to P n or the terminal voltage across these 2 ends. So, here what we have is V n plus 1 Z source will be gain P n plus 1 as per the definition. So, P n plus 1 minus Z source into V n should be equals to P n and here in this circuit, we had V n plus 1 to be the same as V n itself. This is about; please understand the 2 circuits are different. So, in this circuit, we had V n plus 1 is equals to V n. So, we could replace V n plus 1 here as minus Z source into V n. Now note that these 2 equations are exactly reading out to be the same with the condition that I have chosen my current source to be equal to the voltage source divided by the source impedance.

So, these 2 forms of source modeling are perfectly equivalent both in the electrical community as well as now that we are talking about the electro mechanical acoustic community, we will exploit this analogy of source modelling to give us better insights into how a muffler can be better designed. So, the point is this that if this condition V n plus 1 is equals to P n plus 1 divided by Z source is invoked then we get to see that the current by applying the circuit loss in this second circuit, we are getting the same equation. These 2 equations are identical right which means that the 2 equations; the 2 circuits are completely equivalent.

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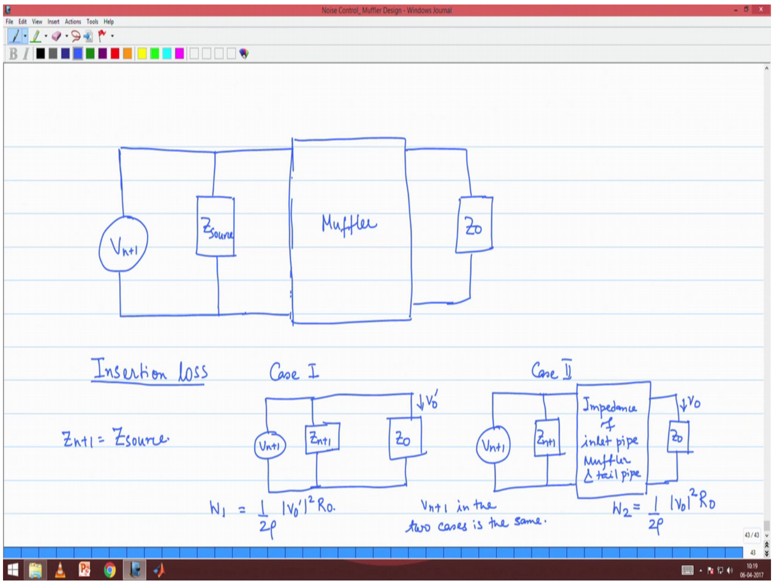

So, therefore, the velocity modelling of the source in our electro mechanical system would read in this following form, you will have a velocity source V n plus 1 and then you will have a source impedance which is in shunt position, then you will have the muffler which could have elements in line position which could have the elements in the shunt position and finally, you will have the radiation impedance Z 0, right. So, this is the circuit when you do the modelling in terms of velocity source, whereas, what was shown earlier here is the circuit in terms of the source modeling taken as a pressure source, but both has its advantage. For example, if the source impedance is near 0, then what is happening is that the entire source voltage or the source pressure is getting communicated downstream where as if the source. So, in other words, if the source impedance is approaching 0, then this circuit is getting shorted, right.

So, if we wish to study the situation where source impedance is very low, then you can have this circuit to your help because that will be easier to interpret whereas, the interpretation for a very large source impedance can possibly be better with this modelling because when the source impedance is very large, this will be made an open circuit, see this will be a current source with an open circuit in the shunt position and that will have a very interesting special cases, but special cases apart; let us now try to define insertion loss which is a very important aspect in muffler design.

So, insertion loss; you consider 2 cases; case one when you have only the source and I am going to draw only the circuit, here you have only the source, you have the source impedance and you are having the radiation impedance for sure, Z 0 right and in case 2; I will call Z source as Z n plus 1 because that will simplify my nomenclature Z n plus 1 is the source impedance. So, Z n plus 1 equals to Z source and case 2 wherein, we will have a current source V n plus 1 a source impedance which is in the shunt position and then you have the impedance associated with the muffler and the inlet and exhaust pipes.

So, impedance of inlet pipe muffler and tail pipe; all of it goes in this box and finally, it comes out through the radiation impedance. In this 2 cases, we need to evaluate the 2 sound powers W 1 and W 2 sound power; in case 1 and sound power in case 2 has to be evaluated and then the difference between the 2 needs to be calculated for the insertion loss idea because this as I said is the real bench mark of how effective your muffler arrangement is. If you do not give any muffler or anything, it will have the value of the sound radiated to be W 1, if you give a muffler, it has the value of the sound radiated to be sound power radiated to be W 2, right the difference between the 2 is the insertion loss which is basically the benefit that you are getting by inserting the muffler together with its inlet and exhaust.

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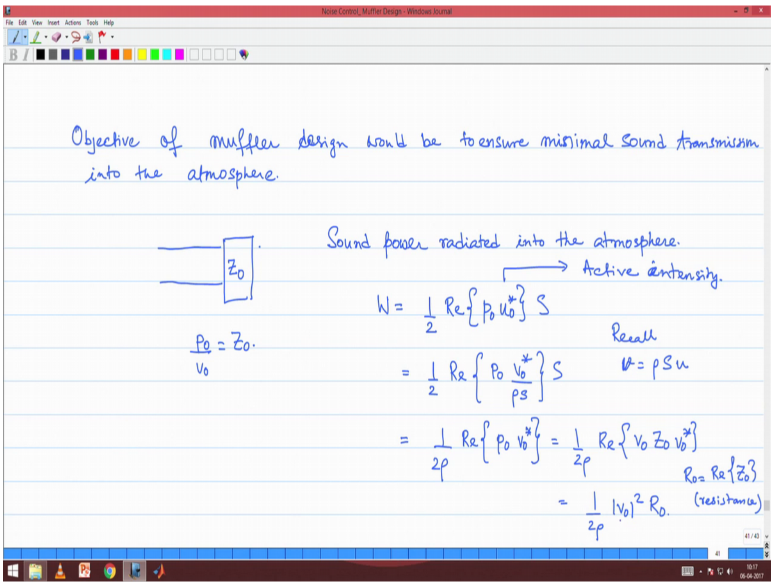

So, that is the measure of the effectiveness of the exhaust system in terms of acoustic performance. So, here for W 2; we know that if V 0 is the current that was flowing through the final load resistance we have already derived this expression that W 2; W will be 1 by 2 rho V 0 square into R 0, right. So, this expression was already derived one by 2 rho mod of V 0 square R 0, R 0 is the resistance associated with the radiation impedance value radiation impedance has reactants as well as resistance, we only care about the resistance, we do not care about the reactants while calculating the sound power because it is the time averaged power that is radiated out. Here also it is just the same, but the point is the V 0, here even if the V n plus 1, in both these cases is the same because the source is; obviously, the same even if the source is the same the amount of V 0 which goes through the impedance block Z 0 is going to be different. So, here it will be V 0 prime, right.

So, W 1 will be 1 by 2 rho V 0 prime m square R 0 please understand that V n plus 1 between the 2 system is same in the 2 cases is the same because it represents the same source Z n plus is the same Z 0 is the same right between case 1 and case 2, but V 0; obviously, will be different because the amount of current which actually passes through the Z 0 impedance will be different depending upon what circuit you have in this plot.

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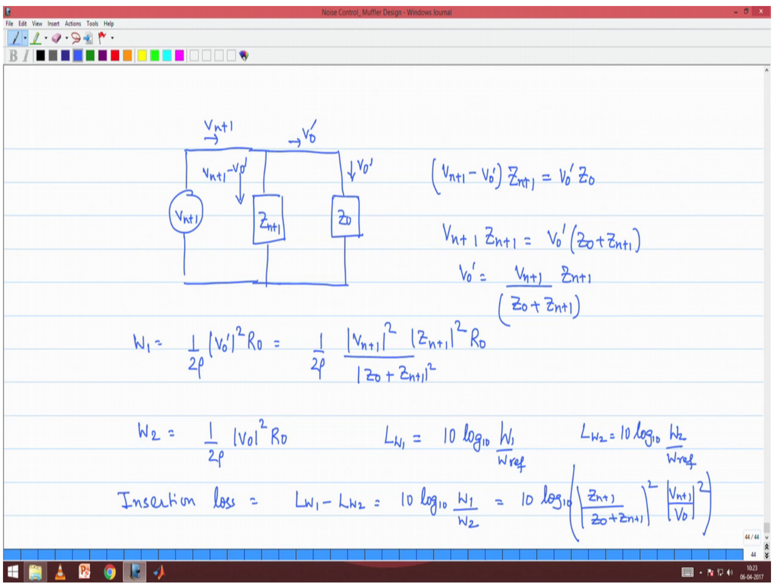

So, therefore, V Z prime; we have to keep track of it. So, now, what we will do as a final reduction process is that we will try to relate V 0 prime in terms of V n prime that should be trivial from the circuit analysis perspective. This is V n plus 1, this is Z n plus 1 and this is Z 0 and we know that this is V 0 prime; this is V n plus 1. So, what goes here is also V 0 prime. So, what goes here is V n plus 1 minus V 0 prime right. So, V n plus 1 minus V 0 prime into Z n plus 1 is the potential drop across the first shunt across the impedance Z n plus 1 and that potential drop has to be messed with V 0 prime Z 0. So, this is the potential drop across this position right across the impedance Z 0.

So, therefore, what you have is V n plus 1 Z n plus 1 is equals to V 0 prime Z 0 plus Z n plus 1 and finally, V 0 prime is equals to V n plus 1 divided by Z 0 plus Z plus 1 into Z n plus 1 right. So, therefore, W one which was the sound power which is radiated in the situation where there is no muffler no inlet pipe no tail pipe nothing of that sort. So, W one was given as one by 2 rho V 0 prime square R 0 and V 0 prime; we wish to now write it as 1 by 2 rho V n plus 1 whole square divided by mod of Z 0 plus Z n plus 1 whole square mod of Z n plus 1 whole square. So, this is the expression for the sound power that is radiated this is the sound power radiated in the linear scale will come to the log scale soon enough, but this is just the sound power that is radiated or power that is lost in the circuit.

Student: Into R.

Into R 0 yeah into R 0; I have missed out. So, this is the sound power that is radiated in the case where there is no muffler and W 2 is going to be any way 1 by 2 rho mod V 0 square R 0 then what we have to do is we have to find the sound power in the logarithmic scale. So, that should be easy sound power in the logarithmic scale as we know is 10 log base 10 W 1 by W reference value and similarly L W 2 is going to be 10 times log base 10 W 2 by W ref.

So, therefore, the insertion loss is going to be L W 1 minus L W 2 which is 10 log 10 W 1 by W 2 and then if we put the appropriate values now which we have calculated for W 1 and W 2, it reads in the following fashion the R 0 term. Now you see it actually does not affect the insertion loss the R 0 term you see is going to get cancel from both the numerator and the denominator in that ratio; what we have will be left with is Z n plus 1 divided by Z 0 plus Z n plus 1 magnitude square into V n plus 1 divided by V 0 magnitude square this is the insertion loss formula, right.

Now; obviously, Z n plus 1 and Z 0 are sort of something for which you do not have any control Z 0 is the radiation impedance it is god given, we have to just accept it, similarly source impedance depends upon the mechanical or the fluid dynamic properties of the source you cannot change it. So, the only way in which you can change the insertion loss to your benefit is to look at this ratio of V n plus 1 V 0.

Let us understand; what is this ratio V n plus 1 by V 0. So, if you look at this circuit diagram, it is very clear that from the current at the source which is V n plus 1 only a part of it of this will be able to reach the load which is of the value V 0. So, if you have some lines which are in the parallel position and divert the current not towards 8 0, but towards the other parallel bypass lines then you will achieve a very large reduction of V 0 with respect to V n plus 1.

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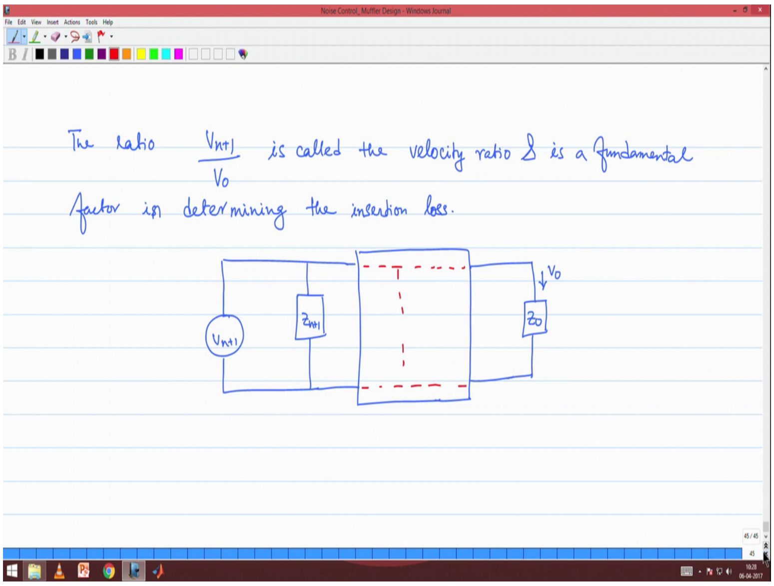

So, this ratio is of critical importance the ratio V n plus 1 divided by V 0 is called the velocity ratio and is a fundamental factor in determining the insertion loss and insertion loss as I said it is just the benefit that you get by addition of your exhaust system, if you do not have an exhaust system and you led the source, directly radiate out into the atmosphere then you will get a sound power radiated as W 1 and on the other side it is W 2, if you have an exhaust system, if you just take a difference between these 2 sound powers in the logarithmic scale this is what is insertion loss. So, insertion loss is the true benchmark for the performance of the acoustic performance of your exhaust system.

So, what it turns out here; from here what it turns out is the following you have to have some bypass paths in this block such that most of the current does not flow through this path, right, you have to reduce V 0 for a certain value of V n plus 1, how can you do that? If you have a certain within this block; if you have a short circuit path; let us say that will be ideal if you a short circuit path of this sort; what will happen, then the entire current will go; throw this short circuit path and minimal current will achieve, go throw the radiation impedance Z 0 and therefore, V 0 will be drastically less and V n plus 1 and V 0 that ratio will be very high insertion loss will very high.

So, that is how we should go about trying to design for good insertion loss for a muffler, right, sometimes the problem is you will not have to; in your table the knowledge of the source impedance because let us say you are a vendor for just the muffler component and there is no way that you know the source impedance in which case transmission loss may be sort of the best approximation that you can do to bench mark your mufflers. But once you know that this muffler has to be fitted with a particular source, then you must always calculate the insertion loss and to choose the best muffler for a given source you must do the insertion loss calculation unless you do the insertion loss calculation based on transmission loss the exercise could go completely futile.

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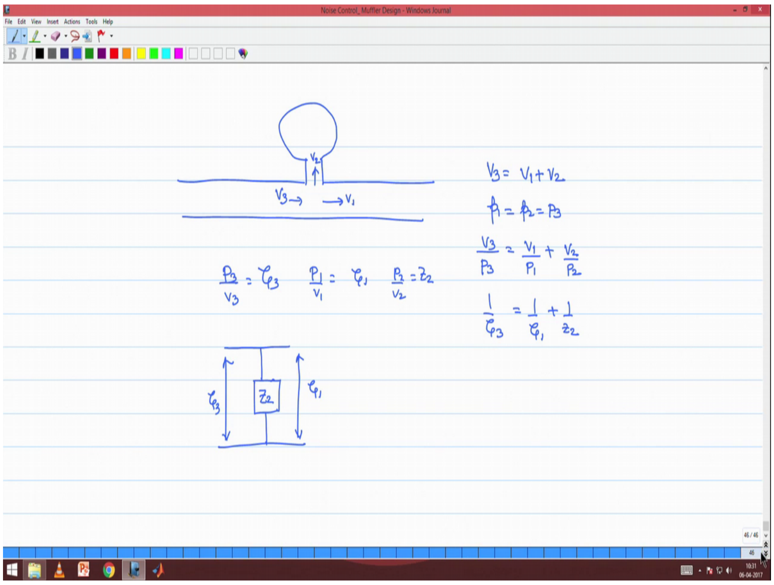

So, let us quickly illustrate how you can create a bypass path. So, suppose now we have an exhaust system which is this form. So, what I have done is I have put a Helmholtz resonator on my exhaust pipe. So, what is going to happen here? So, here I have an incoming velocity, I mean acoustic velocity V 3 and there here there is an outgoing acoustic velocity V 1 and here there is also an outgoing acoustic velocity V 2, right. So, applying conservation of mass flow at these junction point we get V 3 is equals to V 1 plus V 2, right, what about the pressures because all these points are continuous all the points will have the same pressures P 1 equals to P 2 is equals to P 3, right. So, now, if this is so then what we can write is V 3 by P 3 has to be equals to V 1 by P 1 plus V 2 by P 2 if we call the impedance P 3 by V 3 as zeta 3 then this can be written as 1 by zeta 3; similarly if we call P 1 by V 1 impedance as zeta 1 then we could write this as zeta 1 and finally, if we call P 2 by V 2 as the impedance Z 2 then we could write this as Z 2 1 by Z 2.

So, in this situation we are going to have if you just look at what happens upstream point of the Helmholtz resonator and what happens at the downstream point of the Helmholtz resonator you are going to find that V 3 is going to get bifurcated or sub divided into 2 parts right and V 3 stands for the current the velocity stands for the current in our analogy. So, the current is going to get divided the voltages are going to remain same which is the pressures and the impedance that you are now seeing is effectively that of the equivalent circuit of in which you have parallel impedances in parallel, right.

So, the equivalence circuit for this case would be as simple as this, you have 1 impedance Z 2 which is in parallel and this impedance is zeta 3 and this impedance is zeta 1. So, zeta 3 impedance can be recovered from zeta 1 impedance and from Z 2 by simply appealing to the impedance of 2 parallel circuits, right and with this again what you can find out is that if the Z 2 impedance and what is the Z 2 impedance Z 2 impedance is exactly the impedance of the Helmholtz resonator because V 2 is exactly the impedance at the inlet of the Helmholtz resonator, right.

So, if the impedance of the Helmholtz resonator goes to 0, what will happen this path will be short circuited, right? So, this is the circuit of only this portion, now if I add a source. So, in the next picture I will add a source.

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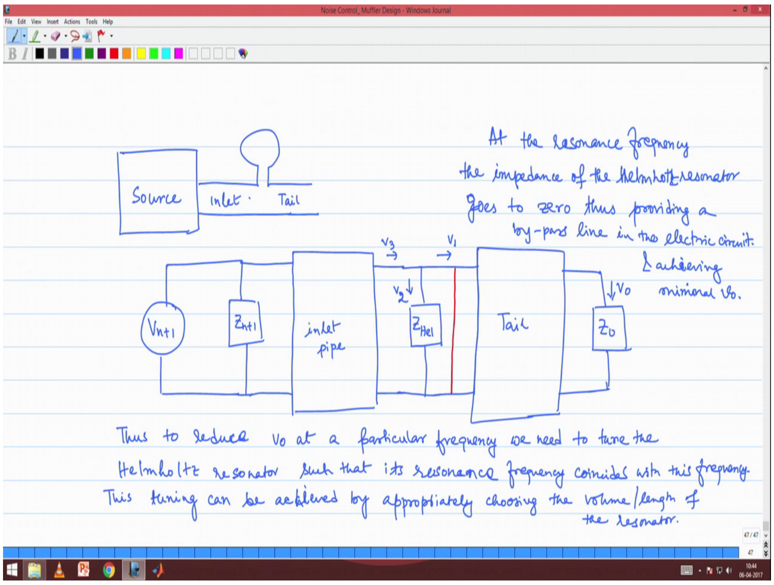

And I will add a Helmholtz resonator and a tail pipe which opens into the open atmosphere right. So, this is the source and this is the Helmholtz resonator. So, what will happen here the equivalent electrical circuit will read in the following fashion you have a V n plus 1 associated with the source you have a Z n plus 1 associated with the source impedance, you have a certain tail pipe sorry, an inlet pipe. So, this is the impedance associated with the inlet pipe and remember the pipe impedance we have discussed in the form of sorry the pipe modelling we have done it in the form of transfer matrix in general it is neither in shunt position nor in inline position it is a distributed impedance.

So, we should not put the impedance associated with the inlet pipe in either inline position or shunt position, but we may put it in this form to denote that it is a distributed impedance something which cannot be put in the 2 extreme cases, then we should have a Helmholtz resonator and the Helmholtz resonator will be in parallel as we have discussed. So, this I will call as Z HCL associated with Z Helmholtz and then you will have the tail pipe, this is the tail pipe, this is the tail pipe, this is the inlet pipe and that is the Helmholtz resonator and finally, it has to open to a radiation impedance set 0, right.

Now, the point is this that out of all these, if you know that you are hearing a large sound at particular frequencies, then what should you do? You should tune the Helmholtz resonator; remember the Helmholtz resonator has resonance frequency, we have described that the mass effect is there, the spring effect is there, one of it is V by c square the other of it is L by S you can tune the frequencies associated with this Helmholtz resonators such that the problematic frequencies are exactly the resonance frequencies of your Helmholtz resonator. In that case, what happens if the frequency happens to be the resonance frequency of the Helmholtz resonator? What will happen this part of the circuit will show 0 impedance, right, 0 impedance is the situation where you have resonance frequency, once you have resonance frequency, this impedance will turn 0 and then it will be replaced by a short circuit line and then in the electrical terminology.

Now it is very easy to understand once this line is shorted, most of the current will flow through this line it will not prefer to go anywhere else right and as a result the V 0 which passes through remember you are ultimate objective is to minimize V 0 that V 0 will get revised.

Student: Here we are during the Helmholtz resonator for only one frequency.

Yes.

Student: So, the rest of the frequency will still pass.

Yes, this is I have now killed one the most problematic frequency in my frequency spectrum. I would see that there are lots of frequencies which are contributing, right, but I can always find out which is the most dominant frequency contributor, right, I will kill 1 with 1 Helmholtz resonator. Now if I have multiple frequencies, I will use multiple Helmholtz resonators, I can give as many Helmholtz resonators as I want in this pipe and start knocking of each of this frequencies, it is a very scientific method that I at least you understand that you understand, how you can knock off one, right, if you know how you can knock of one you can knock off the others also by using a similar strategy for all the other frequencies, but the point I wish to drive home with this electro mechanical analogy which is very difficult to do by staying in mechanical systems or even in acoustic systems is that this idea of shorting the line the Helmholtz resonator is actually providing a short circuit to the equivalent electrical system and thereby saving the current to go into the radiation impedance.

Student: Sir, more enters the Helmholtz resonator is it coming back in to the circuit.

No, all that is taken care of see; what we are saying is that from here on what goes ahead is in that terminology we had said V one and what goes here is V 3 and what goes here is sorry this is V 2 and this is V 3, right. So, from here on what goes from downstream of the Helmholtz resonator is only V 1, right, downstream of the Helmholtz resonator this in the previous diagram, if you see the downstream of the Helmholtz resonator is V 1. See here what is happening is a standing wave is getting generated it is not that it is only a travelling wave and you get reflected a standing wave situation has already create that is why it is a resonator, right otherwise you cannot have a resonator a standing wave situation has created its only that now we are appealing to the impedance of the Helmholtz resonator and we are tuning the resonance frequencies such that you have a 0 impedance in the bypass line.

Thus to reduce V 0 at a particular frequency we need to tune the Helmholtz resonator such that its resonance frequency coincides with this frequency this tuning can be achieved by appropriately choosing the volume or the length of the resonator.

So, I think just one more; note I will do before I go today that is at the resonance frequency the impedance of the Helmholtz resonator goes to 0 thus providing a bypass line in the electric circuit and achieving minimal V 0. So, if your V 0 is minimized velocity ratio will be increased and your insertion loss will increase. So, that is the demonstration of the insertion loss of again a pretty much academic muffler I would say, but quickly in the next class, we will show some practical construction where this idea of a Helmholtz resonator will get nicely approximated.

So, till then good bye.