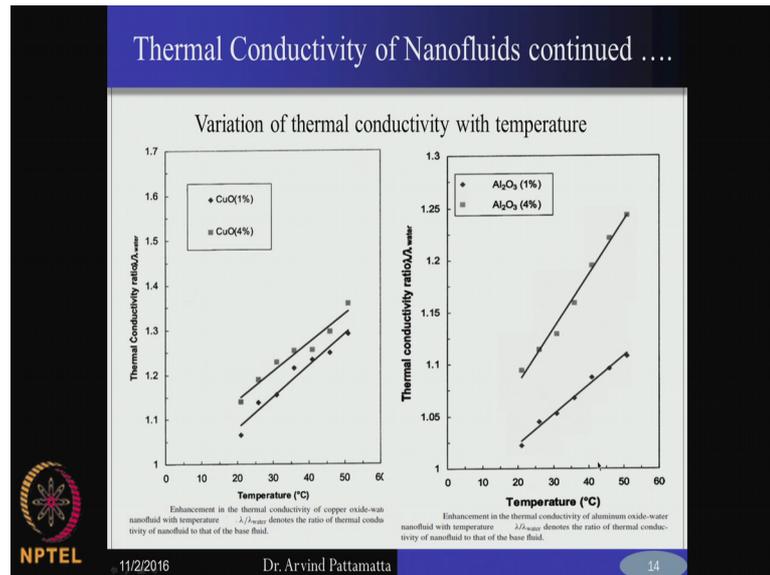


Micro and Nanoscale Energy Transport.
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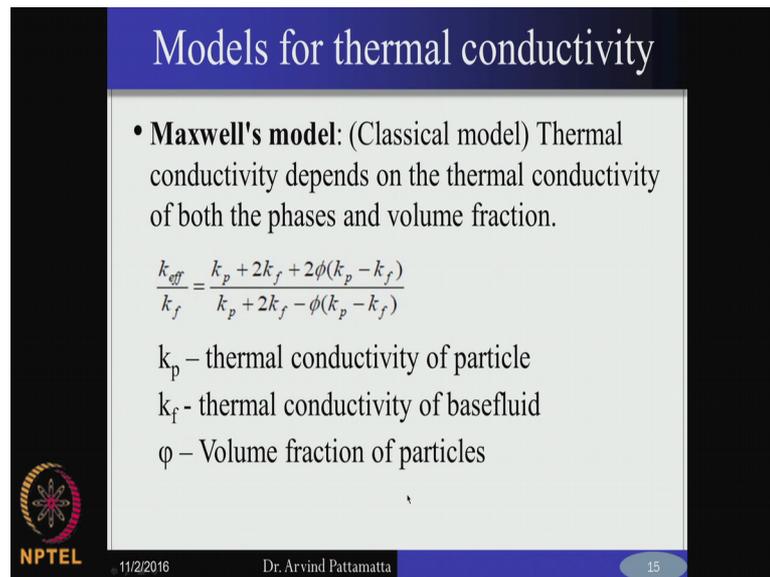
Lecture – 40
Nano Fluid Heat transfer Part 2

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Good morning, we will look at the different models that can be used to predict thermal conductivity, effective thermal conductivity of Nano fluid as a function of the Nano particle volume fraction and most importantly also the dependence on temperature.

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Models for thermal conductivity

- **Maxwell's model:** (Classical model) Thermal conductivity depends on the thermal conductivity of both the phases and volume fraction.

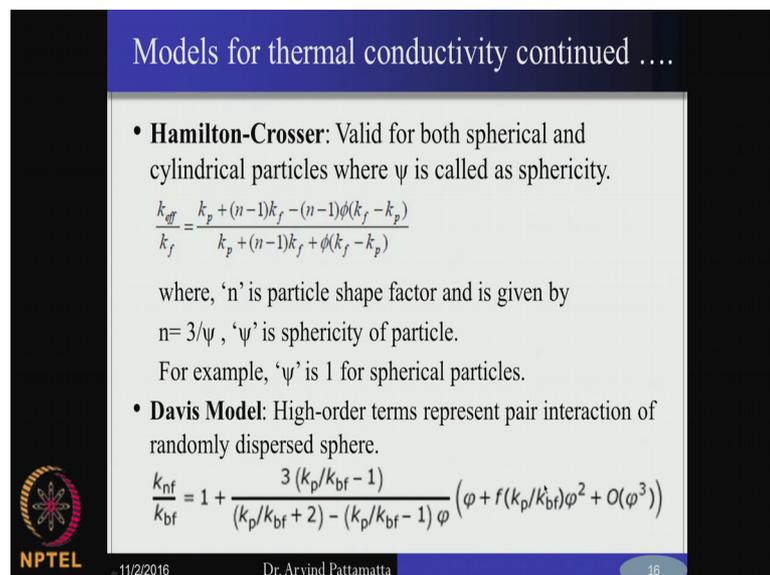
$$\frac{k_{eff}}{k_f} = \frac{k_p + 2k_f + 2\phi(k_p - k_f)}{k_p + 2k_f - \phi(k_p - k_f)}$$

k_p – thermal conductivity of particle
 k_f – thermal conductivity of basefluid
 ϕ – Volume fraction of particles

NPTEL 11/2/2016 Dr. Arvind Pattamatta 15

The dependence on the volume fraction can be accounted by models such as the Maxwell's model. Depending on the value of phi whether, it is 0 or 1, takes the case of for example, phi equal to 0 it takes the case of 0 base fluid, phi equal to 1 the Nano particle and some kind of weighted value of K in between K P and K F.

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Models for thermal conductivity continued

- **Hamilton-Crosser:** Valid for both spherical and cylindrical particles where ψ is called as sphericity.

$$\frac{k_{nf}}{k_f} = \frac{k_p + (n-1)k_f - (n-1)\phi(k_f - k_p)}{k_p + (n-1)k_f + \phi(k_f - k_p)}$$

where, 'n' is particle shape factor and is given by
 $n = 3/\psi$, ' ψ ' is sphericity of particle.
For example, ' ψ ' is 1 for spherical particles.

- **Davis Model:** High-order terms represent pair interaction of randomly dispersed sphere.

$$\frac{k_{nf}}{k_{bf}} = 1 + \frac{3(k_p/k_{bf} - 1)}{(k_p/k_{bf} + 2) - (k_p/k_{bf} - 1)\phi} (\phi + f(k_p/k_{bf})\phi^2 + O(\phi^3))$$

NPTEL 11/2/2016 Dr. Arvind Pattamatta 16

There are also models which account for the shape of the particle because the Maxwell model is actually for spherical particle. Modification to that is called the Hamilton crosser model.

This is quite popular when you want to apply this to non-spherical particle. We have parameter called the shape factor. This parameter is denoted by the notion N and this is equal to 3 by ψ . Where ψ is nothing, but is call the spheriity of the particle. It denotes how spherical the particle shape is. If sphericity is equal to 1 , this is the perfectly spherical particle. If you substitute therefore, N equal to 3 into this expression, you will get the same expression as the Maxwell's model.

Therefore, for the limiting case of spherical particle, the Hamilton crosser model collapses into the Maxwell's model and for values of ψ which is dot equal to 1 , you can look at particle of different shapes, but nevertheless and there is also something called as Davis model which also includes some of the higher order terms, may be important if you want to look at the interaction between the Nano particles and if you do not include the second order and third order terms, then it becomes again equal to the Maxwell's model.

You see that these are the fundamental models used widely to predict the effective thermal conductivity of the Nano fluid as the function of the volume fraction, but very important behavior is the variation with temperature, which none of these models can account for. The variation in temperature is very clear you can see that although the base fluid shows reduction in thermal conductivity temperature; however, the Nano fluid on the other hand shows the continuous increase. This can be predicted or this can be modeled only if you account for the Brownian motion. The Brownian motion is the most important mechanism which can say why the thermal conductivity should increase to temperature. None of these models are actually accounting any temperature dependents into them.

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Models for thermal conductivity continued ...

- **Dhar et al.(2013):**
$$k_{gmf} = k_{sd}^{\alpha} \cdot k_{perc}^{(1-\alpha)}$$

a) k_{sd} is the component due to temperature dependent sheet dynamics. It is given by:

$$k_{sd} = k_{medium} + k_{EMT} + k_{dynamics}$$
 - k_{medium} is the thermal conductivity of basefluid.
 - k_{EMT} is thermal conductivity of the fluid solely due to presence of nanosheets (Effective medium theory).
$$k_{EMT} = \frac{k_p \phi d_m}{(1-\phi)L_g}$$

k_p is in plane thermal conductivity of graphene, d_m is molecular diameter of basefluid, ' ϕ ' is volume fraction and L_g is effective face size of nanosheets.



11/2/2016 Dr. Arvind Pattamatta 17

Therefore, 1 of the more recent models is our own work in our lab here. This is published in 2013 with the Ph.D. student in our lab.

This model is more comprehensive model, which we have built that accounts for 2 components. 1 is the random motion due to the Brownian effects. That is called the sheet dynamics. The sheet dynamic part consists of 3 components. 1 is the thermal conductivity of the base, the overall conductivity of Nano fluid. Now, if you break this into 2 components, 1 is sheet dynamic component, the other is the percolation component the sheet dynamic component includes the thermal conductivity of the base fluid, to which we add the Nano particle. Therefore, we also account for the thermal conductivity only due to Nano particles.

That expression is given separately. This is coming from what we call as effective medium theory, if you put here ϕ equal to 0, this part will be 0 and ϕ tending to 1. ϕ can never be directly to 1, but ϕ tending to 1 the $D M$, the ratio of you know $D M$ by $L G$ will be approaching 1 for that case because $D M$ is nothing, but the molecular diameter of the base fluid which is of the order of the close to angstrom, where as if you are talking about $L G$, $L G$ is the face size of the Nano sheets.

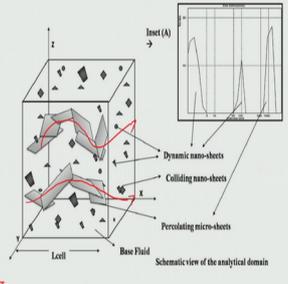
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Models for thermal conductivity continued ...

b) k_{perc} is obtained from percolation theory.
The total number of parallel graphene chains in the domain is given by:

$$M = \frac{\phi L_{cell}^3}{(n_g - 1)d_g L_g^2 N}$$

where, ' ϕ ' is volume fraction,
 n_g is the average number of layers for the graphene nanosheets, d_g is inter-sheet distance for graphene sample, ' N ' is number of nanosheets in one single percolation chain.



NPTEL 11/2/2016 Dr. Arvind Pattamatta 19

That means if you look at these the construction of a unit cell. We are looking at 1 unit cube of the entire Nano fluid in which we have these Nano particle dispersed.

The Nano particle can be generic, they can be of any shape you know any cross section. If you look at the generic case, they can be like sheets which are linked to each other. These are forming a certain links, if you have pure particle then you may not find this linkage structure to be dominant. It will be just disperse, but if you look at Nano sheet for example, carbon Nano tube, These more likely form chains. These chains consist of the lot of these micro sheets which are held together as linkages and there are several parallel layers like this. If you therefore, take the length of any particular micro sheet, that length is basically the face size of these sheets.

Once you put this, therefore, the value of $L G$ will be of the order of few 100 Nanometers whereas; the value of $D M$ will be few angstroms. Therefore, if you ϕ is going towards 1, something like 0.99. This $1 - \phi$ will be a very small term. Therefore, that multiplied by $L G$ will go to the value of $D M$. That the denominator and numerator the $D M$ by $1 - \phi L G$ will be approaching unity and therefore, the value of $K E M T$ will approach the value of $K P$, for limiting case ϕ approaching 1. You have therefore, contribution from the base fluid, you have contribution from the Nano particles plus you have contribution from the Brownian motion. These are the 3 components which are included in the sheet dynamic part. The contribution from the Brownian dynamics is

simply given by an expression which is a function of the Brownian velocity U_B . It is also a function of other things like volumetric capacity Nano particle volume fraction ϕ and D by temperature and so on.

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Models for thermal conductivity continued ...

- k_{dynamic} is dynamic thermal conductivity due to Brownian motion.

$$k_{\text{dynamic}} = \bar{U}_B \lambda C_v \phi \theta$$

where, ' λ ' is mean free path which is assumed to be 10^{-6} m, C_v is volumetric specific heat given by:

$$C_v = \frac{36\pi^4 k_B}{15} \left(\frac{N}{V}\right) \left(\frac{T}{\theta_d}\right)^3$$

(N/V) is number of atoms per unit volume and θ_d is Debye temperature for graphene for planar modes of phonon transport.

$\bar{U}_B = 3U_B$ Brownian velocity given by kinetic theory of gases. Brownian velocity is given by: $U_B = \frac{2k_B T}{\pi \mu_{\text{eff}} L_G^2}$

11/2/2016 Dr. Arvind Pattamatta 18

And also the mean free path, but most important is strong function of the Brownian velocity. The Brownian velocity is given by the Einstein expression. This is also call the Einstein scope expression. This also tells you that the Brownian velocity is directly proportional to the temperature and inversely proportional to the size of the particle; the smaller the size, you larger your random velocity and so, is dependents with temperature therefore, if you are applying this particular expression. This now accounts for dependents of thermal conductivity both on volume fraction as well as temperature through the Brownian motion. Therefore, if you are able to use this model, you can accurately predict the variation of the thermal conductivity temperature. We have checked this model with different kinds of Nano fluid including particle type Nano fluids, including sheet type of Nano fluids and they seem to work very well and good prediction.

Now the other component which is very important for the sheet type of Nano fluids like carbon Nano tubes. This is called the percolation part. As I said, if you have carbon Nano tubes, they will form linkages with several of these Nano sheets which are linked together in a chain and several of these chains in parallel. Therefore, we have to also look

at the conduction through these chains. There is the conduction of heat from for example, left to right. There will be a conduction of heat through these chains because they form a conduction network similarly through them.

Not only it is a random motion, but also they form a chain which will promote network for transport of heat. This is called the percolation component and to understand to this percolation component we have to first estimate how many number of such parallel chains can be formed. There is 1 empirical expression we have used to calculate this capital M which denotes the number of parallel chain and within each chain, you have several Nano sheets. You can say capital N here represents the number of Nano sheets in 1 chain. That is basically given by this N here. Once you know the number of Nano sheets in 1 chain, you can use this empirical correlation to find out how many number of chains that can be actually formed.

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Models for thermal conductivity continued

- This model utilizes net resistance approach to determine effective thermal conductivity due to percolation theory.

Therefore, R_{net} is given by:

$$R_{net} = \sum_{i=1}^N \left[\frac{\left(\frac{R_{FL,P} + 2R_c}{M} \right) \times R_{FL,P}}{\left(\frac{R_{FL,P} + 2R_c}{M} \right) + R_{FL,P}} \right]$$

NPTEL 11/2/2016 Dr. Arvind Pattamatta 20

Now you can draw resistance network analogy for the thermal resistance; that means, what are all the resistance is acting? You have if you are talking about heat conduction through this is all in series. Through 1 chain this is in series. Through the Nano sheet you have conduction and between the Nano 2 Nano sheets you have interfacial thermal resistance. Actually these 2 Nano sheets are not exactly touching each other. There is a small gap, I mean you have actually the fluid separating these 2. There is actually contact resistance between the fluid and the solid.

Therefore, in effect if you take 1 Nano sheet you have conduction resistance plus twice the contact resistance on either ends. That is the serial series component of the resistance; you also have a parallel component of resistance which is heat conduction through the base fluid itself and therefore, these M links are all in parallel. Therefore, if you draw the resistance network analogy, essentially you have in series the contact resistance on either ends and you have the conduction resistance to the Nano particle. These are all in series and you know you can have this for N number of chains. This is for 1 Nano film or Nano particle.

Since you have capital N number of Nano particles in 1 chain, they can all be in series and in parallel you have for M number of chains Nano particle chains. You can have all these in parallel. You have M parallel chains and N serial Nano particles forming particular chain not only that, you also have the fluid resistance which is acting in parallel. For each unit cell if you assume this is 1 unit that is 1 Nano particle. You also have the base fluid which is conducting the heat parallel. Therefore, for 1 Nano sheet you have the base fluid in parallel and you will have such M parallel linkages acting. Then you have to repeat this for capital N number of Nano sheets acting in 1 link.

You have simultaneous resistances in series and parallel. Therefore, the net effective resistance, you can actually very quickly we can you can derive this, because first consider only the series the parallel resistances here may be you can all try this out now. You consider only this resistance of 1 Nano sheet all in parallel. Find out what is effect resistance of this. That is $\frac{1}{R}$ effective parallel is equal to $\frac{1}{R}$ by. You have in series R_C , 2 times R_C plus R_{IP} and there are M such linkage M M such linkages in parallel. Therefore, this should be plus $\frac{1}{2 \text{ times } R_C \text{ plus } R_{IP}}$. M number of time; therefore, you can replace this by M by this. Now plus you have the base fluid which is acting in parallel for a particular Nano particle.

Therefore, your R effective parallel will be nothing, but R_{IP} plus 2 2 times R_{IC} times R_{FIP} by this is M divided by plus. If you if you take the reciprocal of this, this is basically product in the denominator summation in the numerator if you take reciprocal product moves to the numerator, reciprocal the summation moves to the denominator. And then finally, what do you have? This is 1 set of effective resistance. Now you have series of them so capital N number of them. Therefore, the effective resistance should be

summation of this over N number of these Nano particles. Therefore, finally, from that you get this expression. All of you can quickly check if you are able to get this.

Basically we start with 1 Nano sheet, everything in parallel and then for extending in this for N number of Nano sheets in all over all the linkages. Basically this R I P is nothing, but the conduction resistance through each Nano particle and R C is the contact resistance between successive Nano particles.

Student: (Refer Time: 18:16)

We are con considering together as 1 fluid resistance.

Student: upper resistance as well as lower resistance is the single.

Correct. What we say is the total fluid resistance is included as 1 resistance in parallel and each of these links, we are considering the resistance is separately in parallel. This is the approach we follow therefore, from once you estimate the net resistance. We can calculate thermal conductivity component due to percolation.

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Models for thermal conductivity continued ...

- $R_{F,l,p}$ is the resistance in the parallel liquid path.
- R_c is the interfacial contact resistance between two sheets given by:

$$R_c = \frac{1}{A_{\text{contact}} G} = \frac{1}{(n_s - 1) d_s L_s G}$$
 where, G is thermal conductance per unit area.
 $G = 25 \text{ MWm}^{-2}\text{K}^{-1}$ for Graphene/Water interface
 $G = 12 \text{ MWm}^{-2}\text{K}^{-1}$ for CNT/Water interface
- $R_{i,p}$ is the thermal resistance of the Graphene nanosheet.
- Therefore, the thermal conductivity based on percolation theory is given by:

$$k_{\text{perc}} = L_{\text{cell}} / (R_{\text{net}} A_{\text{cell}})$$

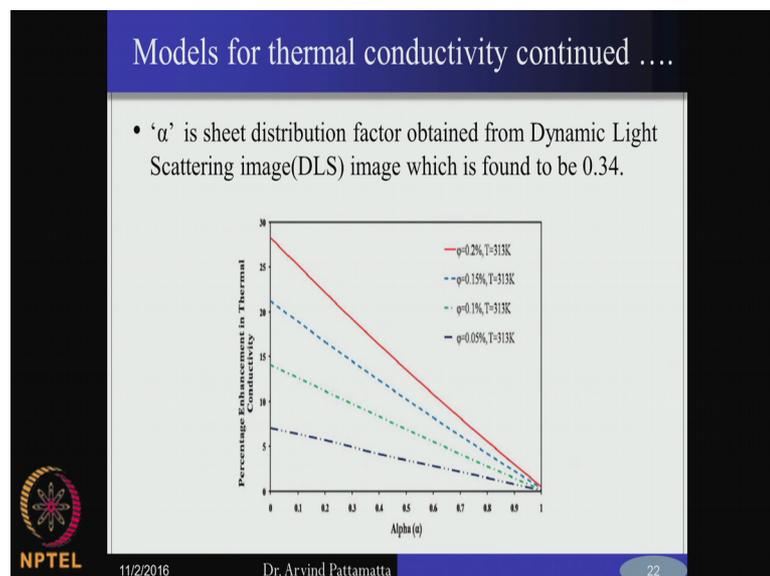
NPTEL 11/2/2016 Dr. Arvind Pattamatta 21

How do you calculate by using this expression finally, L by R net A cell. You remember you furrier's equation where we express your resistance conduction resistance as L by K A. From which we can calculate K S L by R A. Same way now we our L is nothing, but the length of the cell; that means, this cell is a unit cell with the particular length L and

this as total net resistance R_{net} and the corresponding cross sectional area of the cell K_{cell} . Understand? From this we get the K percolation component and we can also estimate the conduction resistance from another empirical correlation which depends on the thermal conductance per unit area and depending on the type of Nano fluid, you have to use the value of G appropriate.

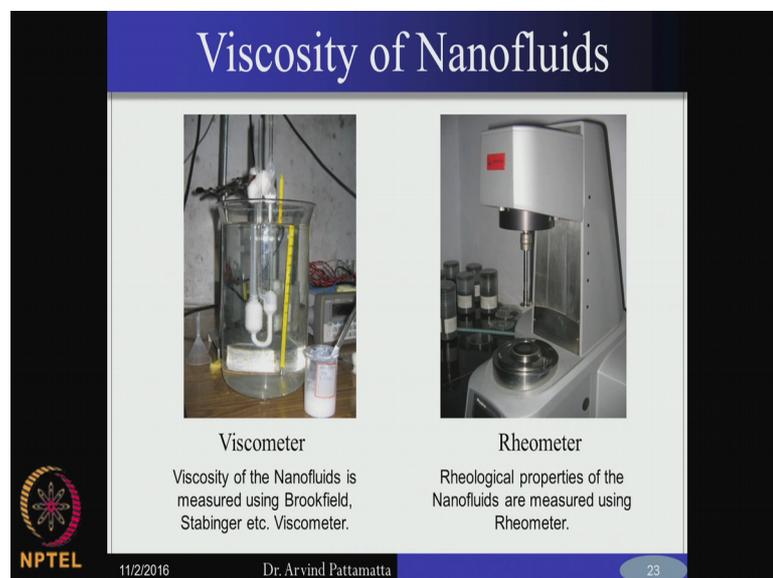
From the components of K percolation and sheet dynamics, we use a power law rule to calculate the total thermal conductivity of the Nano fluid. We use a factor α and $1 - \alpha$ to take the individual contribution. Suppose if there was it was purely spherical particles then what will happen to the percolation component? Percolation component should not be there. If your value of α equal to 1 then K Nano fluid will be only K sheet dynamics so; that means, only Brownian motion is responsible for increase in thermal conductivity, but if you take Nano fluid which is purely carbon Nano tubes, then these carbon Nano tubes form these flakes structures, linkages and your Brownian component will be almost 0 because they cannot move as fast like the spherical particles. In that case your α will be 0. Your total effective conductivity will be only coming from the percolation conductivity. For Nano fluid like grapheme, this has somewhat intermediate it is neither to be big form huge linkages not too small to just simply move around. It will have contribution from both components Brownian components as well as the conduction component. In that case we use a value of α may be 0.7 or 0.8 and it will take into both these components.

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Therefore, the alpha is called the sheet distribution factor. It tells you how much weightage you have to use for the percolation component versus the Brownian motion. See for example, graphene, we use the value 0.34. We give more weightage to percolation, for the graphene compared to sheet dynamics. For purely spherical Nano particle like alumina, copper, metallic, Nano particles then we put alpha equal to 1, pure sheet dynamics component. This is a very very comprehensive model, which accounts for explanation on γ and how the thermal conductivity increases with temperature with volume fraction not only for 1 type of Nano particle, but different shapes of Nano particles.

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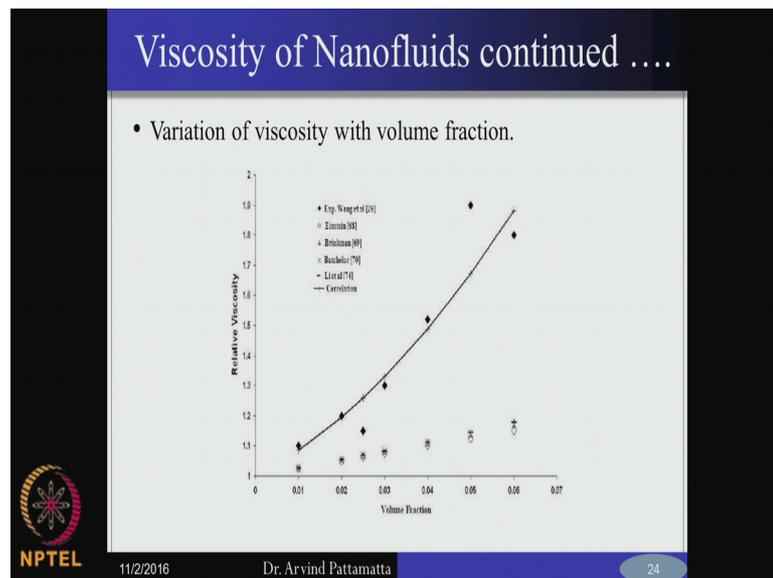


This is probably 1 of the best model that are available, for in literature which can be used for a wide different kinds of Nano fluids and you can get satisfactory agreement. This is 1 component, the other is to estimate the viscosity of this Nano fluids. Now, when you add the Nano particles to base fluid, your viscosity also will increase and therefore, you do not know 2 things. 1 what is the effective viscosity, you cannot take a simple volume average for effective viscosity here you have to determine them or we have to have a good model which can predict and you do not even know whether you will always get Newtonian fluid. That is the second question.

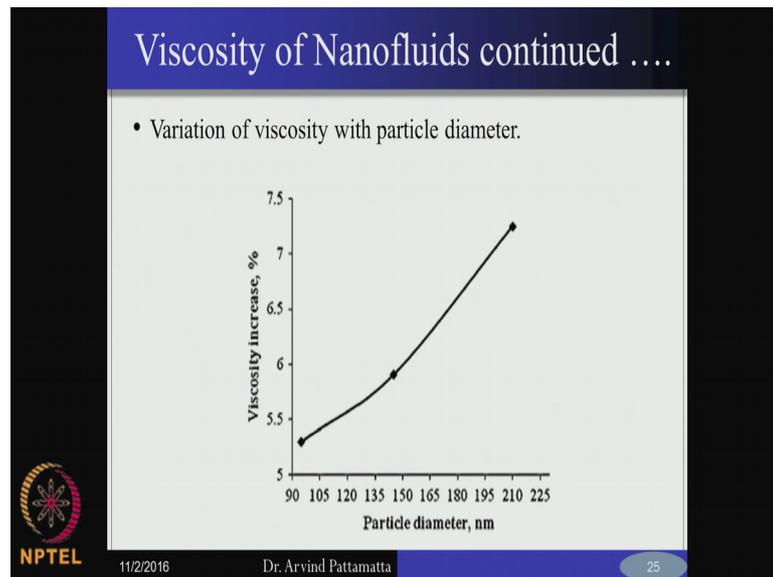
Therefore, if you want to simply estimate only the viscosity, you can use a viscometer, which gives you directly what is the dynamic viscosity of this fluid, but if you want to

really know if this is a Newtonian fluid or not we cannot use a viscometer, but we to have to use a Rheometer. Rheometer, it gives you, what is the, for a given shear stress? It gives you what is the corresponding strain rate. So, you plots stress verses strain verses stress and you will yourself know whether this is linear curve or it is changing somewhere so that we can approximate this as a Newtonian fluid or non-Newtonian fluid. Therefore, the Rheometer in that sense is even better to assess the fundamental Newtonian behavior of these Nano fluids, but most of the basic Nano fluids like alumina, copper, what are even carbons Nano tubes graphite or very small volume fractions. Mostly they behave like a Newtonian fluid. We can use this like the way you measure thermal conductive with hot wire technique, we can measure viscosity of any fluid need not be Nano fluid say whether using a viscometer or Rheometer.

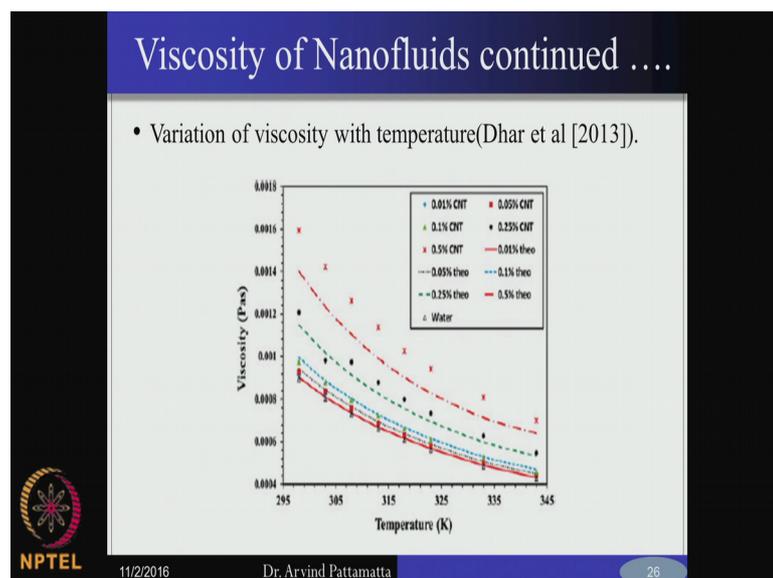
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And we can get the effective viscosity and we can see that this is a very strong function of once again the volume fraction. The volume fraction increases considerably and also with the temperature, the volume fraction drops. For the fluids again with increasing temperature the viscosity actually drops. Overall the Nano fluid also shows this behavior;

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Models of Viscosity of Nanofluids

- **Einstein Model (1906):**
$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5\phi$$
- **Batchelor Model (1972):**
$$\mu_{nf} = (1 + 2.5\phi + 6.5\phi^2)\mu_f$$
- **Chandrashekar model :**
$$\frac{\mu_{nf}}{\mu_f} = 1 + b \left(\frac{\phi}{1 - \phi_n} \right)^n$$
- **Abu-Nada Model :**
$$\mu_{nf} = -0.155 - \frac{19.582}{T} + 0.794\phi + \frac{2094.47}{T^2} - 0.192\phi^2 - 8.11 \frac{\phi}{T} - \frac{27463.863}{T^3} + 0.127\phi^3 + 1.6044 \frac{\phi^2}{T} + 2.1754 \frac{\phi}{T^2}$$



11/2/2016 Dr. Arvind Pattamatta 27

However, you have to have again accurate models which can show, predict the behavior of a variation of the Nano fluid viscosity with respect to both Nano particle volume fraction and temperature, the model such as the Einstein model, bachelor model and Chandrasekhar model. They are used for suspension, they have been used for a long time, but there are all only functions of phi and there not comprehensively built in temperature relationships. The temperature relationship affectively comes from only the base fluid viscosity as a function of temperature, the model by Abu Nada for example, as explicitly dependence on temperature for the Nano fluid also. These are some of the common models which can be used, some of them mostly have been a obtained from experiments and some from simple theory.

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Models of Viscosity of Nanofluids continued

- Dhar et al. model (2013):
 - a) For Graphene Nanosheets:
$$\mu_{GNS} = \mu_{perc} + \mu_{sd}$$
$$\mu_{perc}$$
 is viscosity due to sheet percolation given by:
$$\mu_{perc} = \mu_{bf}(1 + L^* d\phi\alpha)$$
where, μ_{bf} is viscosity of the basefluid, L^* is non-dimensional length given by $L^* = L_G / L_{crit}$ L_G is the average sheet face size of graphene and L_{crit} is critical sheet size, 'd' is percolation network dynamicity factor, ' ϕ ' is particle volume fraction, ' α ' is sheet distribution.



NPTEL 11/2/2016 Dr. Arvind Pattamatta 28

We have also proposed a symbol or model, using our percolation a component and sheet dynamics component. It is not that the percolation and sheet dynamic affects only the thermal conductivity, but also they play a very important role in the viscosity as well. Your sheet dynamics component and percolation components for viscosity are separately calculated using these expressions. For example, here L^* is nothing, but the non-dimensional length which is the size of the Nano sheet divided by some critical length and D is the D is called the percolation network dynamicity factor; ϕ is the volume fraction α is the sheet distribution. If suppose, you take the case of pure spherical particles, percolation components should be 0, in that case you just only have the sheet dynamic components.

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Models of Viscosity of Nanofluids continued

Value of L^* for graphene is approximately equal to 1.25
 μ_{sd} is viscosity due to sheet dynamics given by:
$$\mu_{sd} = \mu_0 \phi (1 - \alpha)$$
where, μ_0 is dynamic viscosity term given by:
$$\mu_0 = \rho_G \lambda U_B \theta$$
 ρ_G is density of graphene, ' λ ' is the mean free path, ' U_B ' is Brownian velocity, ' θ ' is collision cross-section.

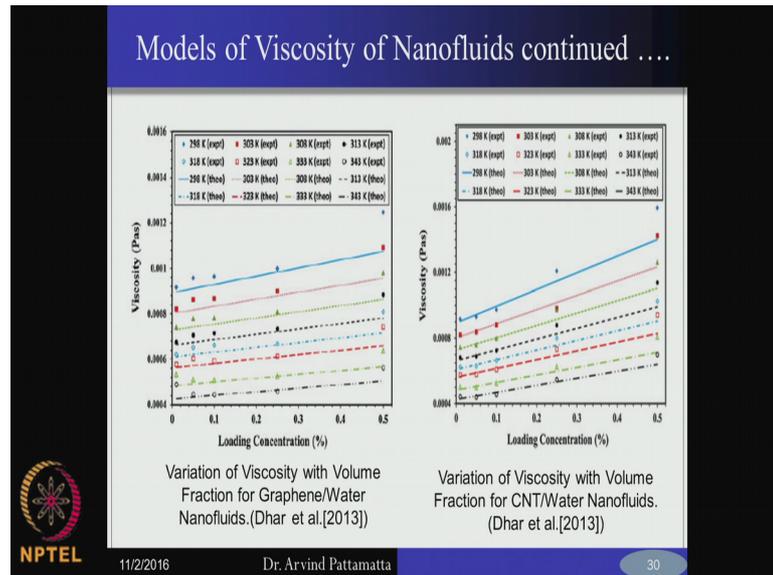
b) For CNT's:
$$\mu_{CNTS} = \mu_{bf} (1 + L^* d \phi).$$
Value of L^* for CNT is approximately equal to 10



11/2/2016 Dr. Arvind Pattamatta 29

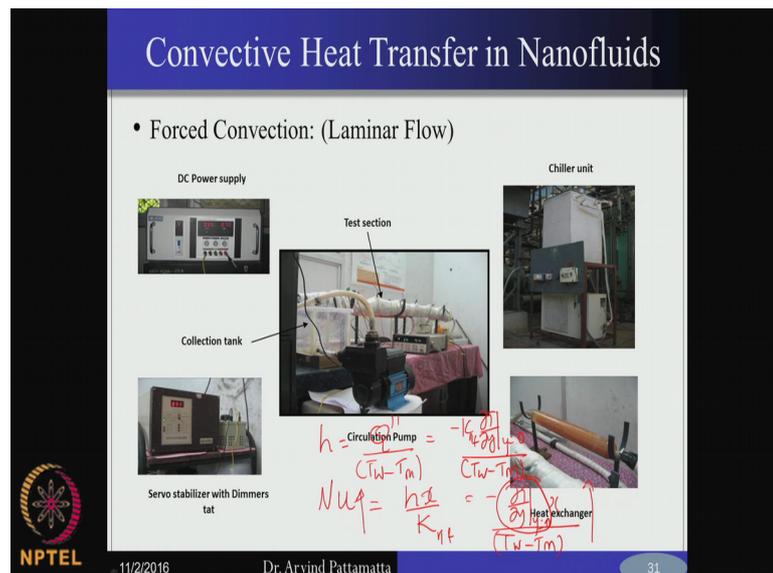
If you substitute, for example, alpha equal to 0, you have only the sheet dynamic component and the percolation component will disappear. You have new percolation will be only mu base fluid. Mu base fluid plus the sheet dynamics component will be there. Mu not here from the dynamic sheet dynamics point of view, it is again a function of Brownian velocity. What it says the Brownian velocity also affects the viscosity, not only thermal conductivity, but also it describes the viscosity increase. Higher the Brownian motion, more is the increase in the viscosity and, this is and now again the dependence on temperature is also brought in through the Brownian velocity. We have used this model, for example, if you take the case of carbon Nanotube, the value of this non dimensional length L^* is approximately equal to above 10.

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And if you use this, we have found that this model can also very accurately predict the effective viscosity of the grapheme water Nano fluids, carbon Nano tube water Nano fluids. 1 on the left is grapheme water; the other on the right is carbon Nano tube water. All functions of different volume fractions and at different temperature also, not only the volume fraction dependence, but the temperature dependence is also properly predicted by these models.

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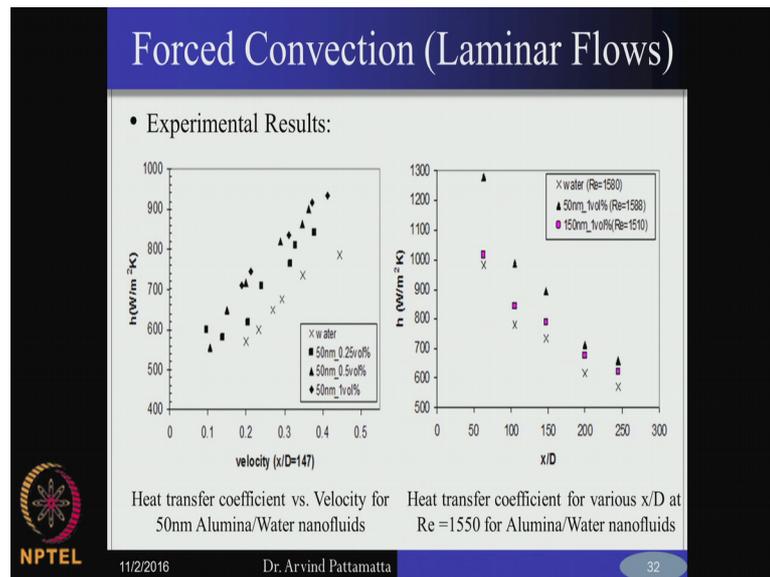


We will finally, move now in to the heat transfer characteristics. We have whatever we have talked about till now, it is a only the case were it is static, static fluid were we measure the thermo physical properties now when you start motion, bulk motion what happens if you have Nano fluids replacing the bulk fluids? For that several experiments had been conducted on the forced convection, single phase forced convection, natural convection, then phase change. Quickly I will run over all of these. The first convection experiment also we had in our lab before students who were doing this.

This is a kind of a setup where we heat the tube using a D C power supply and we pass fluid. First the base fluid and look at the Nusselt number, calculate the Nusselt number of the base fluid by measuring the heat flux and also the temperature distribution along the wall of the tube, then we replace this with the Nano fluid and again they do the same thing and then we plot the ratio of the Nusselt number Nano fluid by Nusselt number of the base fluid, and we check what kind of enhancement we get using Nano fluid; that means, it is not just enhancement coming from thermal conductivity itself, but that coming from the convection component because if you define Nusselt number, the say if you say H . H is nothing, but heat flux divided by $T_{wall} - T_M$. This is a bulk temperature or bulk mean temperature.

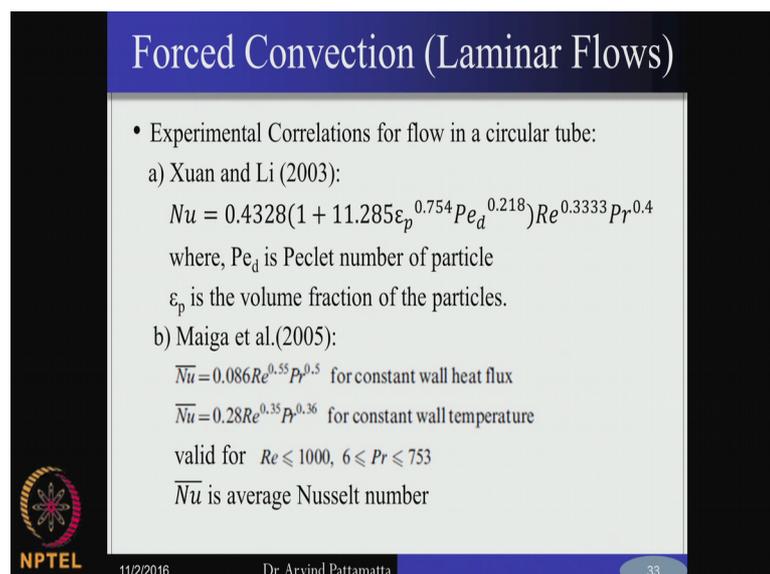
Your Nusselt number now becomes $H \times D$ by K and this Q and the wall is nothing, but minus $K D T$ by $D Y$ say at Y equal to 0. Now, for the Nano fluid, this becomes K_{Nano} fluid. If you use Nano fluid in the definition of Nusselt number, what happens if you substitute this? This will become minus $D T$ by $D Y$. Y equal to 0 X by $T_{wall} - T_M$. The thermal conductivity gets cancelled out so; that means, any increase in the Nusselt number, if you replace the base fluid with Nano fluid, should be purely because of increase in the temperature gradient and not because of increase in the thermal conductivity, whereas if we look at H and if you and you look at values of H , H can be higher, but Nusselt number, if you divide it by thermal conductive heat of the Nano fluid, the effect of thermal conductivity increase will be compensated. That gets cancelled out. Essentially you are only relating Nusselt number to the temperature gradient. If a temperature gradient increases; that means, you are saying the Nano fluids have a better heat transfer characteristics.

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Most of the literature, what they do is they mainly plot heat transfer coefficient, but of course, you know when they are doing this, they sometimes have this problem where for the base fluid you use K of base fluid and for the Nano fluid we use K of Nano fluid. You do not know whether the increase in H is due to the higher value of thermal conductivity or it is actually due to increase in the temperature gradient. This is actually where some of the literature becomes confusing.

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And the correct way of plotting it will be to actually plot the Nusselt numbers. The Nusselt number does not have the enhancement due to thermal conductivity. There are some standard correlations for laminar flows.

Equation 1 is proposed by Xuan and Li, which states that even in these are average Nusselt numbers; that means, you are not looking at either developing region or fully developed region, but overall for the entire tube which has both these components, developing and developed you are calculating the average Nusselt number and this average Nusselt number is expressed as a function of Reynolds number and Prandtl number. Now also as a function of the Nano particle volume fraction and for example, Xuan and Li have used Peclet number, Peclet number is nothing, but product of Reynolds and Prandtl number and, but how are this is defined for the Nano particle so; that means, we use the diameter of the Nano particle in defining the Peclet number these are correlations built from lot of experiments.

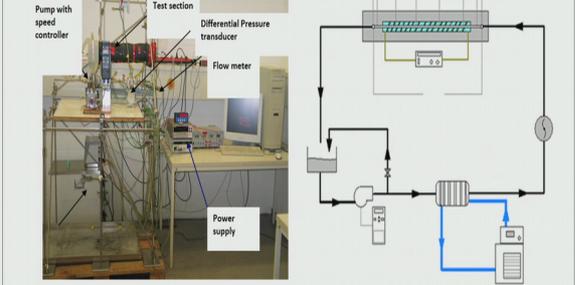
So, experiments like this for heat transfer coefficient Nusselt number have been done for different Reynolds numbers, Prandtl numbers different volume fractions and they have built a very detailed correlation and similarly there is Equation 1 by Maigaoui et al in 2005. Depending on the boundary condition whether it is a constant wall temperature or constant wall heat flux and they have given 2 different expressions as a function of Reynolds and Prandtl number. Overall what it says that if you replace your base fluid with Nano fluid here? For example, if you put ϕ is the volume fraction if you put this to 0 you get the base fluid average Nusselt number and if you add Nano particle to this the Nusselt number is increasing. Therefore, most of the experiments say that the addition of Nano particles lead to an improvement in the convective heat transfer rate.

This is a Nusselt number; you can say that this is not due to thermal conductivity increase alone, but also convective heat transfer enhancement. The thermal conductivity enhancement will already contribute to conduction heat transfer, but we are only looking at temperature gradient which is the measure of convective heat transfer increase. Clearly most of the experimental studies agree that there is an increase in the Nusselt number with the addition of Nano particles, but only the percentage increase is not very consistent. Some people claim it can be 10 percent, some people claim it can be 15 percent or some 20-25 percent. It is, there is not a very consistent number.

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Forced Convection (Turbulent Flows)

- Experimental Setup:



The slide shows a photograph of the experimental setup on the left and a schematic diagram on the right. The photograph labels include: Pump with speed controller, Test section, Differential Pressure transducer, Flow meter, and Power supply. The schematic diagram shows a closed-loop system with a pump, a test section containing a tube with a heat exchanger, a flow meter, and a differential pressure transducer, all connected to a power supply.

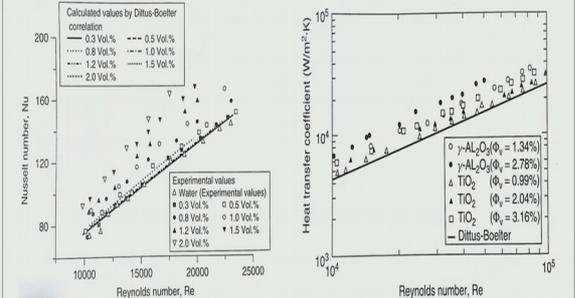
Photographic representation Schematic of experimental Setup

NPTEL 11/2/2016 Dr. Arvind Pattamatta 34

(Refer Slide Time: 36:55)

Forced Convection (Turbulent Flows) continued

- Experimental Results:



The slide contains two graphs. The left graph plots Nusselt number (Nu) on the y-axis (80 to 200) against Reynolds number (Re) on the x-axis (10000 to 25000). It compares experimental values for water and various volume percentages of Cu/Water nanofluids (0.3, 0.5, 0.8, 1.0, 1.2, 1.5, 2.0 Vol.%) with calculated values from the Dittus-Boelter correlation. The right graph plots Heat transfer coefficient (W/m²·K) on the y-axis (10³ to 10⁵) against Reynolds number (Re) on the x-axis (10⁴ to 10⁵). It compares experimental values for various nanofluids (γ-Al₂O₃ at 1.34%, 2.78%, and 3.16% volume fractions; TiO₂ at 0.99% and 2.04% volume fractions) with the Dittus-Boelter correlation.

Nusselt number vs. Reynolds Number for Cu/Water nanofluids(Xuan and Li[2003]) Heat transfer coefficient vs. Reynolds Number for various nanofluids(Pak and Cho[1998])

NPTEL 11/2/2016 Dr. Arvind Pattamatta 35

Similarly in turbulent flows, also they have seen that there is possible enhancement with addition of Nano particles. Same Xuan and Li themselves have conducted experiments in the turbulent regime.

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Forced Convection (Turbulent Flows) continued

- Experimental Correlations for flow in a circular tube:
 - a) Pak and Cho(1998):
Modified Dittus-Boelter Equation:
$$Nu = 0.021Re^{0.8}Pr^{0.5}$$
 - b) Xuan and Li (2003):
$$Nu = 0.0059(1 + 7.6286\varepsilon_p^{0.6886}Pe_d^{0.001})Re^{0.9238}Pr^{0.4}$$

where, Pe_d is Peclet number of particle
 ε_p is the volume fraction of the particles



11/2/2016 Dr. Arvind Pattamatta 36

And they have again given a similar expression, similar to the laminar flow regimes. The turbulent flows also have a function of the Nano particle volume fraction and Peclet number. Only the constants are little different, some people have also used the Dittus Boelter equation which says that, there is a dependency on Reynolds number to the power 0.8 and prrenal number to the power either 0.3, 0.4. They have modified that little bit for the Nano fluids by changing the constants for the prrenal number and also the constant in front that is 0.021 and this was given by Pak and Cho. This is a very simple expression does not have present account for the volume fraction whereas, the expression by Xuan and Li explicitly states that for different volume fractions you have different values of Nusselt number.

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Natural Convection in Nanofluids

- Experimental Setup(Putra et al.):

- Cylindrical block
- End cover as heating surface
- End cover as cooling surface
- Cap
- Resistance heating elements
- The piston shaft
- Cooling water inlet and outlet
- Narrow tube
- Thermocouples

NPTEL 11/2/2016 Dr. Arvind Pattamatta 37

These are average Nusselt numbers.

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Natural Convection in Nanofluids continued

- Experimental Results:

Variation of Nusselt Number with Rayleigh Number for different Aspect ratios

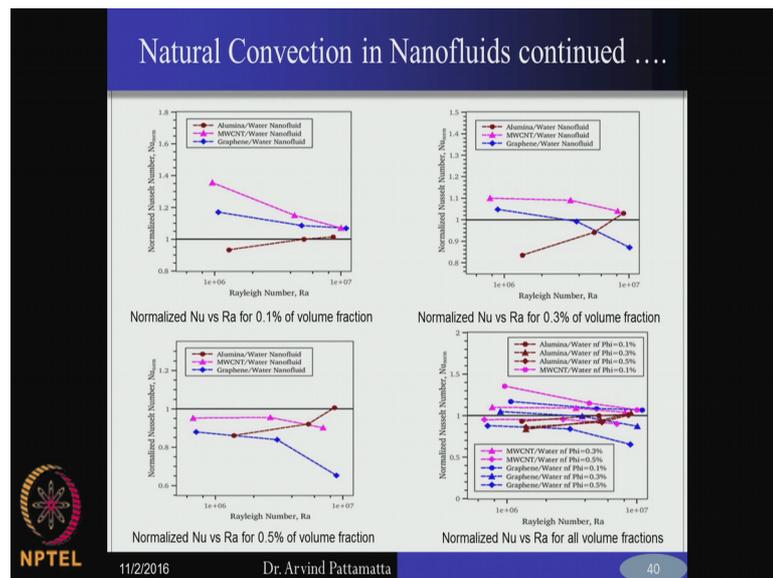
NPTEL 11/2/2016 Dr. Arvind Pattamatta 38

Now coming to natural convection on the other hand, the experiments done by Utra at all, natural convection in a cavity shows that; however, unlike the force convection experiments, there is deterioration in Nusselt number in the natural convection case because the problem here is if you add particles with the density higher than the base fluid. Your effective density is increasing and therefore, what happens to the buoyancy force? It reduces. The higher the effective density, the smaller is the buoyancy force and

therefore, the natural convection which is strong function of density. Lighter fluid will be able to experience natural convection better than heavier fluid. Therefore, there is deterioration with the addition of Nano fluids in terms of natural convection.

Nano fluids are always not giving in a better heat transfer everywhere. Depends on whether you are in the first convection or natural convection.

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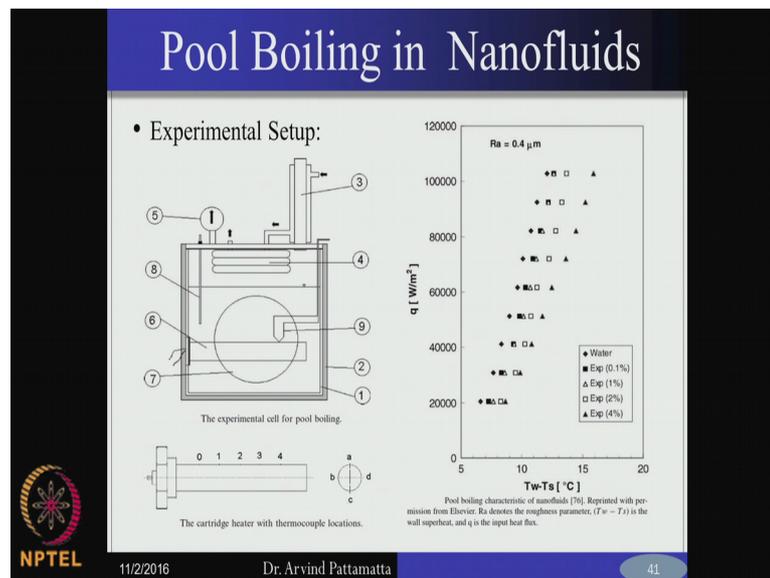


But; however, when we did these experiments in our group, we found that yes, there is a deterioration we find as long as we use the regular spherical Nano particles like alumina, but if you move towards carbon Nano tube, flake type of Nano particles on the other hand we see enhancement. In this particular a figure first figure we have 0.1 percent volume fraction of all Nano fluids. We have plotted the Nusselt number of the Nano fluid by Nusselt number of the base fluid therefore, this value equal to 1 means it is equal to the base fluid value. If you look at carbon alumina, alumina shows that for all Rayleigh numbers, it is always less than the base fluid or at the best it is only equivalent to the base fluid. Whereas, when we add carbon Nano tube on grapheme they show enhancement of up to 35 percent or 30 percent at Rayleigh number of 10 power 6, If any Nano particle addition, which contributes to overall increase in the density, should result in deterioration of buoyancy. Why does carbon Nano tube and grapheme contribute to and enhancement? This was again something which we cannot explain very simplistic ways. Then we found out that there are slip mechanism which play very important role

you know we did a scaling analysis in which we found out that several forces like drag force also play very important role apart from Brownian force so on. What we have obtained from the scaling analysis is the drag is a very important; drag force is a very important contribution especially if you look at flake type of Nano particles like Nano carbon Nano tube and grapheme compared to spherical Nano particles.

That is causing an enhancement even though the overall density is increasing. The heat transfer or the natural convection is augmented by means of this drag force. That is the only way that we can explain why certain Nano particles like carbon Nano tube and grapheme can result in an enhancement while general theory is any heavier Nano particle should contribute to deterioration.

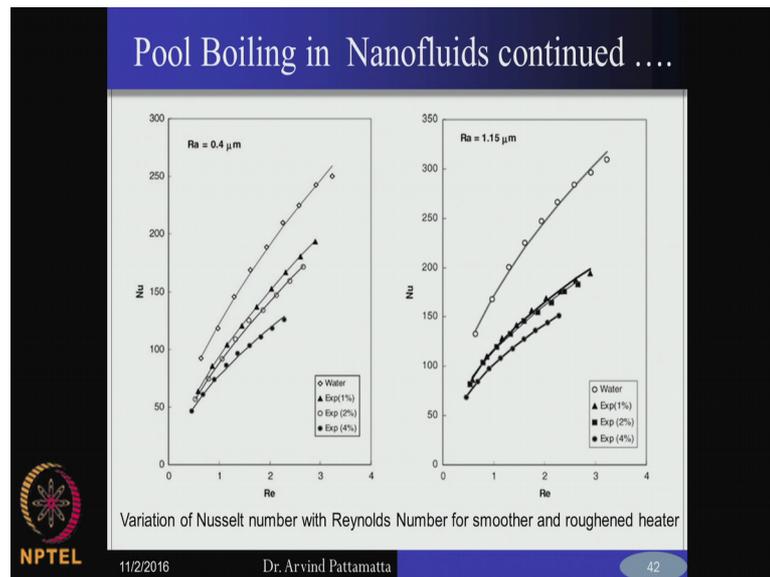
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These are some recent results which we are in the process of publishing. These are with respect to the single face convection when we go to phase change for example, pool boiling that is we just have a cavity filled with Nano fluid and we look at the boiling characteristics. We are finding that generally with increasing volume fraction the heat transfer coefficient decreases with the Nano fluid.

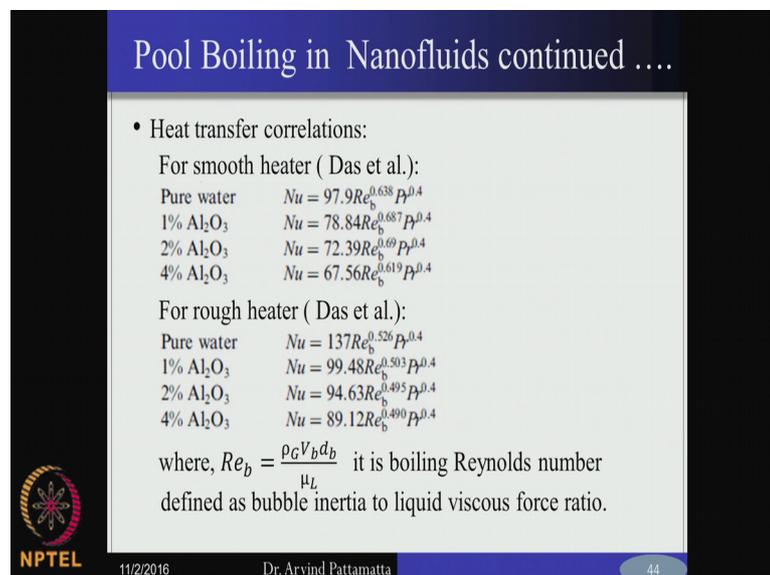
If you therefore, plot Q verses T wall minus T S you take the ratio of at any at any value of T wall minus T S, you calculate Q, the value of Q will be smaller if you add Nano particles; that means, the pool boiling heat transfer coefficient actually comes down with the addition of Nano fluids.

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Therefore, if you plot this as Nusselt number, you can see that the Nusselt number actually deteriorates in the pool boiling case.

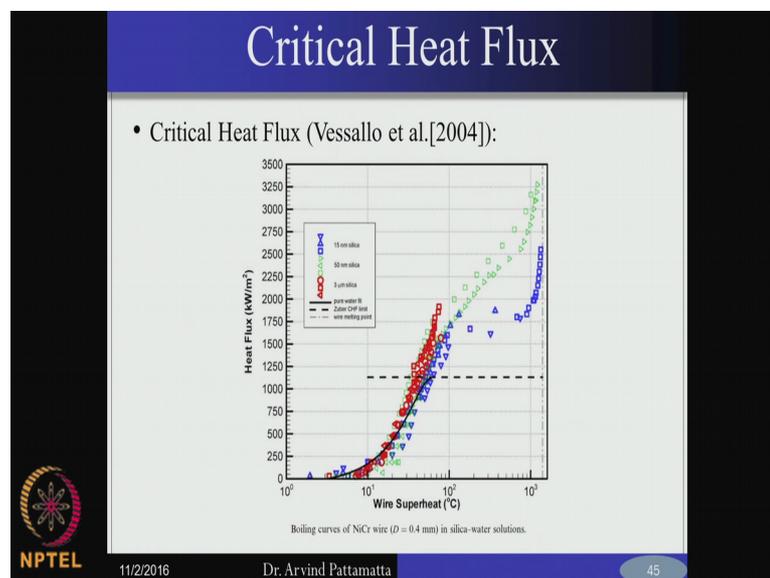
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We have also some correlations developed you know from our lab itself professor Das and their group they have done this for this concentrations and for this heater configuration. Depending on whether you have a smooth heater or rough heater, a rough heater will have a higher value of Nusselt number compared to a smooth heater.

You can see all these constants are multiplying constants are larger in the case of rough heater because of the overall higher heat transfer coefficient because the roughness will promote additional nucleation sites, which will improve the nucleate boiling capability. These are some simple correlations in which there are functions of Reynolds number, here Reynolds number we have to be careful. It is not the flow Reynolds number it is based on the velocity of the nucleating bubbles and the bubble diameter. These bubbles which are just rising from the surface, from the surface of the heater all the way to the top, they have a certain bubble velocity and bubble diameter some average values which you can put in and using that you can predict the Nusselt numbers.

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These are or for the pool boiling case and a very important aspect of Nano fluids is not only the deterioration of heat transfer coefficient, but the critical heat fluxes enhanced by the addition of Nano particles. This is very very important for example, for nuclear safety. If you are talking about boiling water reactors, these boiling water reactors they their boiling at say certain values of critical heat flux. Now if you increase the value of critical heat flux the chance of transition from a nuclear boiling to film boiling is reduced because you are augmenting that cap. If this was lower very quickly you can transition from the nuclear to the film boiling. So, this is very good this is prevent from the dry out you know that can lead to no vapor pressure increase and explosion chances and all that. From cooling point of view this is a very important aspect of Nano fluids.

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Critical Heat Flux continued

- Vessallo et al. (2004) carried out experiments for various volume fractions and different diameters of silica particles.
- They observed increase in the critical heat flux (CHF) for silica/water nanofluids than that of water for both 15 and 50nm nanoparticles. The enhancement was about 200% for both diameters of silica particle.
- While for diameter of about $3\mu\text{m}$ of the silica particles suspended in water, enhancement in CHF was found only to be 100%. For this diameter of silica nanoparticles wire failed before attaining film boiling.

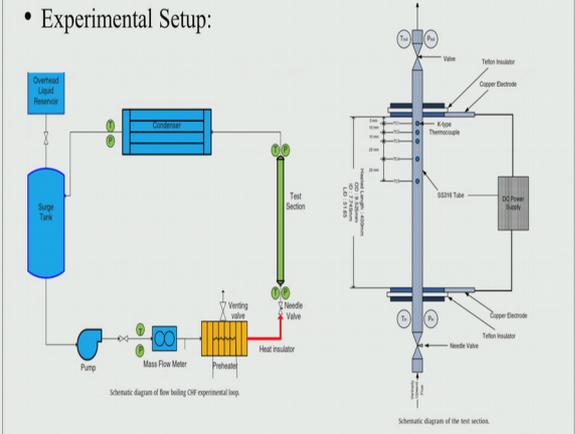


11/2/2016 Dr. Arvind Pattamatta 46

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Flow Boiling in Nanofluids

- Experimental Setup:



Schematic diagram of flow boiling CHF experimental loop

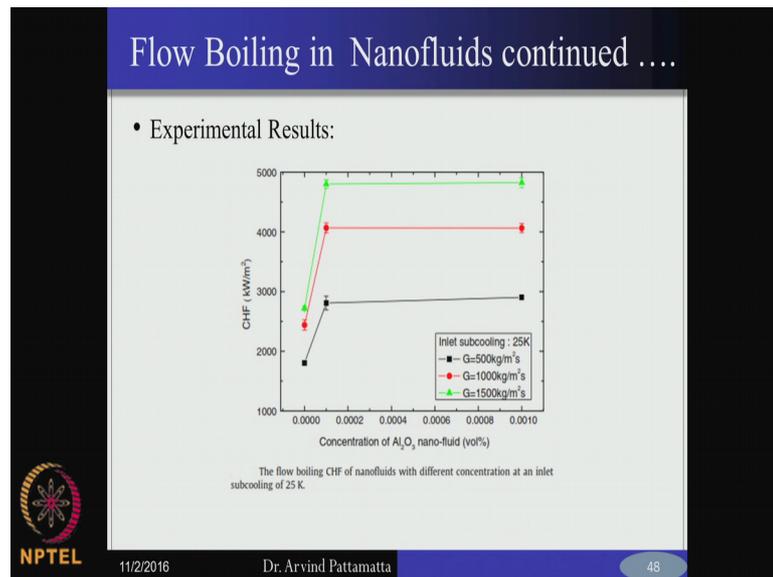
Schematic diagram of the test section.



11/2/2016 Dr. Arvind Pattamatta 47

In the case of pool boiling, you see that the critical heat flux is augmented and very large values, you know sometimes it is of the order of the 200 percent sometimes it is of the order of 100 percent. Finally, coming to flow boiling there have not been many studies, Flow boiling of Nano fluids.

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There also in the flow boiling also they have shown that the critical heat flux can be augmented by the use of you know Nano fluids. This is for different Reynolds numbers. As you change the Reynolds numbers, the critical heat flux its flow boiling increases and also with the addition of Nano particles, there is an augmentation in the value of critical heat flux

I will kind of stop here with this kind of gives you an overall summary about the heat transfer convective heat transfer characteristics because there are still lot of topics which are for in ongoing research is been conducted. This is not a closed ended subject. I am only giving you some important correlations which you can use, but the other things a still being you know developed. With this we will stop our discussion on Nano fluid and from tomorrow, the next couple of classes we will look at our last topic which is measurement techniques, measurement techniques in Nano on micro scale. I will see a how much I can basically cover in these 2 classes; I will try to brush through them.

Thank you.