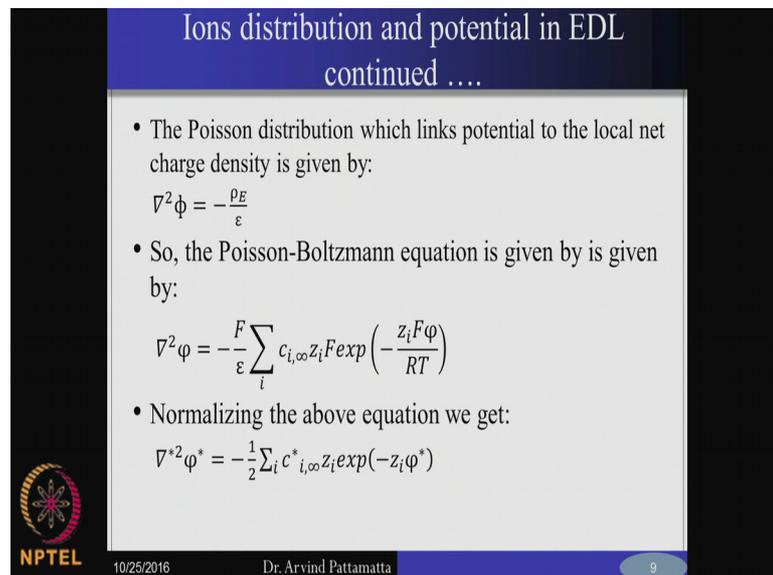


**Micro and Nanoscale Energy Transport**  
**Dr. Arvind Pattamatta**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**

**Lecture – 36**  
**Fundamentals of Electro Kinetics in Micro channels Part 3**

Let us continue our discussion related to electro kinetics.

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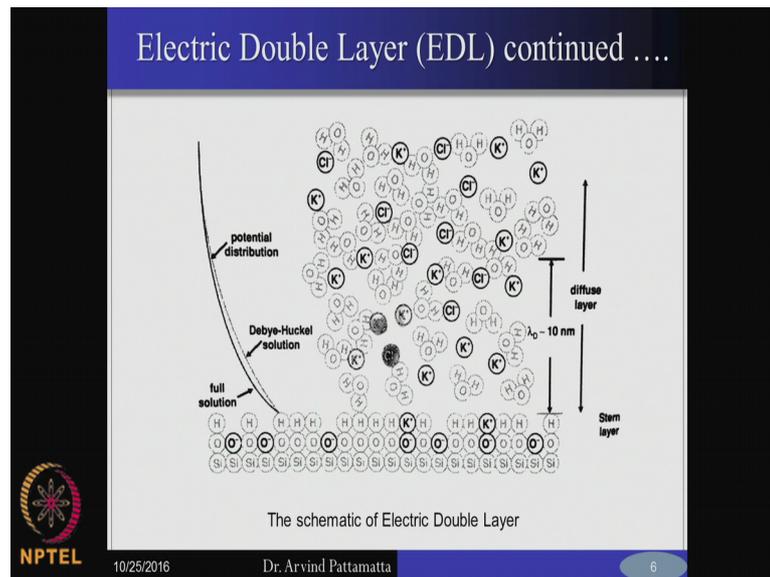
Ions distribution and potential in EDL  
continued ....

- The Poisson distribution which links potential to the local net charge density is given by:  
$$\nabla^2 \phi = -\frac{\rho_E}{\epsilon}$$
- So, the Poisson-Boltzmann equation is given by is given by:  
$$\nabla^2 \varphi = -\frac{F}{\epsilon} \sum_i c_{i,\infty} z_i \exp\left(-\frac{z_i F \varphi}{RT}\right)$$
- Normalizing the above equation we get:  
$$\nabla^{*2} \varphi^* = -\frac{1}{2} \sum_i c_i^* z_i \exp(-z_i \varphi^*)$$

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I think some small mistakes I would like to correct from the last presentation related to the electro osmosis.

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So, I hope you recollect our discussion on the double layer. We talked about, you can also have normal water you know which is polar fluid and also you can have an aqua solution of some salt in such as, sodium fluid, potassium fluid in anyway. You will find that whether you have a polar fluid or you have aqua solution of this, because of the presence of stern layer close to the wall, so you have a migration of all the ions which will go towards the wall and therefore this set up concentration gradient.

So, in this particular example the wall is negatively charged and therefore you have the migration of one layer of positive ions could be hydrogen or it could be the potassium ions as sticking to the wall. And this will also start the migration of the other positive ions closes. So therefore, there is a concentration gradient number one, and this will also result in the electro static potential gradient. So, two things are happening parallel there is a mass transport process there is also setting up of electro static potential and difference.

Therefore, if you apply an external field of course close to the wall where we see the significant concentration gradients this where we have the diffuse layer, so this diffuse layer will be of the order of few tens of nanometers. So, when we solve this problem if you take the case of flat plate and you look at the picture of one dimensional velocity profile due to an external electric field. You can look at this problem like a case where we saw the slip flows. So, the slip flows case we apply directly the slip boundary

condition at the wall and here you can assume the wall is shifted up by the order of 10 nanometers. And we solve for solution for the inner layer that is for the electric double layer. Get the solution at the edge of electric double layer for the velocity profile, and apply that is slip boundary condition for the bulk flow.

So, this is how usually the electro osmosis problem is dealt with, because all the forces are acting close to the wall and this is a very small region, does not matter if you assume that there is a wall shifted up by 10 nanometers so it is not going to disturb the bulk flow profiles as long as you can give the right boundary condition.

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**Ions distribution and potential in EDL**  
continued ...

- Double layer potential is given by:  $\phi = \phi - \phi_{\text{bulk}}$ .
- The value of ' $\phi$ ' specifies how the electrical potential at point differs from that in the bulk far from walls.
- From the Boltzmann statistics, we can write in general that:  

$$c_i = c_{i,\infty} \exp\left(-\frac{z_i F \phi}{RT}\right)$$
 where ' $z$ ' is ion valence, ' $F$ ' is Faraday constant and its value is 96485 C/mol.
- The local net charge density ' $\rho_E$ ' as the function of the local potential is given by:  

$$\rho_E = \sum_i c_i z_i F$$
- Therefore, the Boltzmann solution is given by:  

$$\rho_E = \sum_i c_{i,\infty} z_i F \exp\left(-\frac{z_i F \phi}{RT}\right)$$

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Therefore, the inner solution; before go into the inner solution, I just want to talk about the concentration gradients. So, what we have assumed is a Boltzmann statistic type distribution of the concentration from the wall to the bulk value which is your  $C_i$  infinity. So, we are assuming this Boltzmann statistic. Here this is the function of phi, the electro static potential this is the excess potential. So, this is the difference between the local potential minus the bulk value. This is indirectly a function of  $y$ , the vertical position through the dependence of phi on  $y$ .

Therefore, we can relate your concentration gradient to the net local charged density at some location which is  $\rho_e$ , as summation of your concentration multiplied by the charge of that particular concentration  $f$  is times your Faraday constant. This gives you the net local charge at that particular point. And this is again a function of  $y$ . So therefore,

to solve the overall the electro static potential field phi you need to solve the Poisson's distribution; Poisson equation where the right hand side is the local net charge density given by this particular expression.

So, here you substitute for  $C_i$  which is the local concentration as a function of the  $C_i$  infinity from the Boltzmann statistics. And therefore, you get what is called the Poisson-Boltzmann equation. This is an equation this is a field now for which you have to solve for because you have everything as a function of phi on the left hand right hand side. Now what we do is this is the non-linear equation and first thing we do is non-dimensionalize it.

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**Ions distribution and potential in EDL**  
continued ...

- Where,  $\varphi^* = \frac{F\varphi}{RT}$  and length is normalized by Debye length  $\lambda_D$ .

$$\lambda_D = \left( \frac{\epsilon RT}{2F^2 I_c} \right)^{0.5}$$

- The Debye length gives a measure of the characteristic length over which the over potential at a wall decays to the bulk.
- 1D form of non-linear Poisson-Boltzmann equation is:

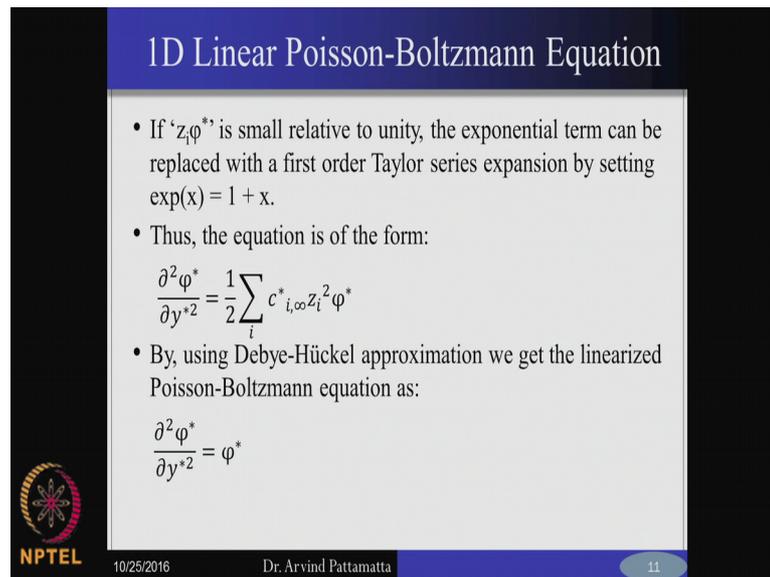
$$\frac{\partial^2 \varphi^*}{\partial y^{*2}} = -\frac{1}{2} \sum_i c_{i,\infty}^* z_i \exp(-z_i \varphi^*)$$

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So, we non-dimensionalize it with the following variables; one is a non dimensional electro static potential phi star, and the coordinates on non-dimensionalized with the d by length lambda d that is the extent of the defuse layer.

So, with these two the Faradays constant f is actually absorbed. So, in the non dimensional form you do not have the Faradays constant, I think this was a mistake the last PPT the Faradays constant was shown now please correct that. So, the Faradays constant is absorbed the non dimensional form. Now, we write down the non-linear Poisson-Boltzmann equation in one direction that is along why we want to solve to get a profile along y.

(Refer Slide Time: 06:21)



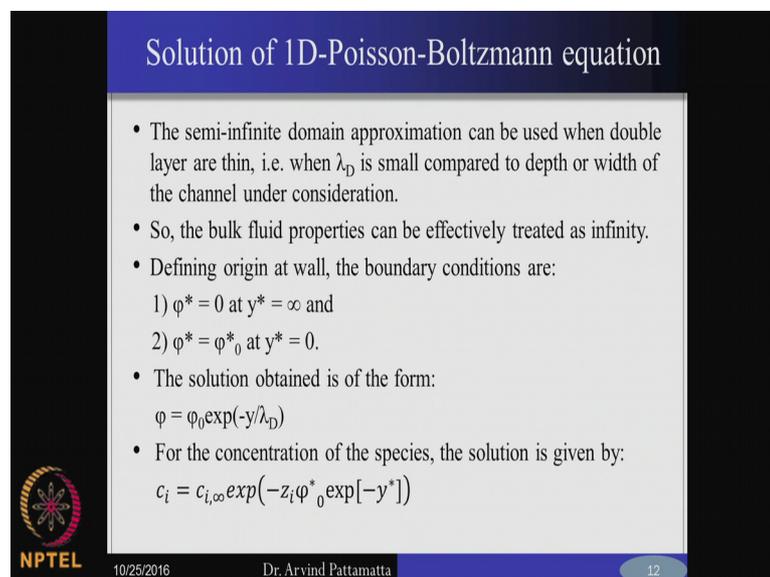
1D Linear Poisson-Boltzmann Equation

- If ' $z_i\phi^*$ ' is small relative to unity, the exponential term can be replaced with a first order Taylor series expansion by setting  $\exp(x) = 1 + x$ .
- Thus, the equation is of the form:
$$\frac{\partial^2 \phi^*}{\partial y^{*2}} = \frac{1}{2} \sum_i c_{i,\infty}^* z_i^2 \phi^*$$
- By, using Debye-Hückel approximation we get the linearized Poisson-Boltzmann equation as:
$$\frac{\partial^2 \phi^*}{\partial y^{*2}} = \phi^*$$

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Now, what we do is to get an analytical solution we can linearize it. So, how do we linearize it, is we replace using Taylor series, your exponential function can be replaced as 1 plus x plus x square plus x cube and so on.

(Refer Slide Time: 06:47)



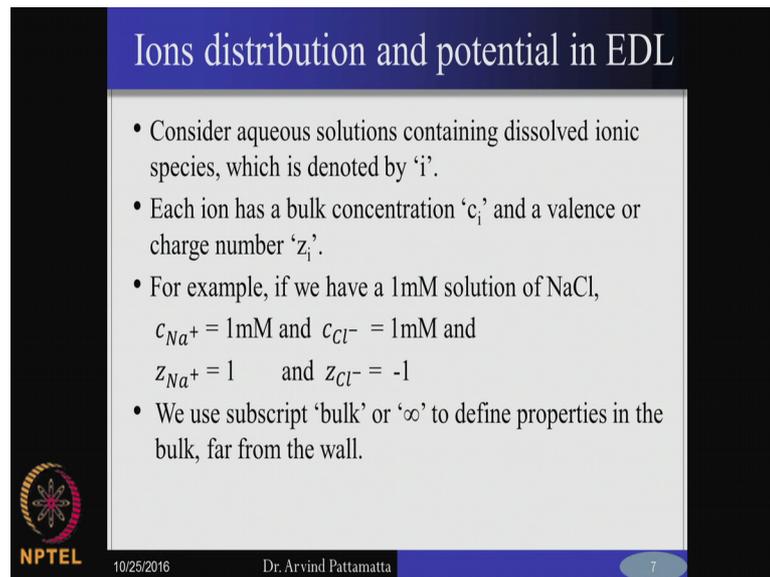
Solution of 1D-Poisson-Boltzmann equation

- The semi-infinite domain approximation can be used when double layer are thin, i.e. when  $\lambda_D$  is small compared to depth or width of the channel under consideration.
- So, the bulk fluid properties can be effectively treated as infinity.
- Defining origin at wall, the boundary conditions are:
  - 1)  $\phi^* = 0$  at  $y^* = \infty$  and
  - 2)  $\phi^* = \phi_0^*$  at  $y^* = 0$ .
- The solution obtained is of the form:
$$\phi = \phi_0 \exp(-y/\lambda_D)$$
- For the concentration of the species, the solution is given by:
$$c_i = c_{i,\infty} \exp(-z_i \phi_0^* \exp[-y^*])$$

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So, if your value of y star for example or just let me go back, if your value of Z i phi star so within the exponential term is reasonably small therefore you can neglect all the higher order terms of Z i phi star. So, you can only retain 1 plus Z i phi star.

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**Ions distribution and potential in EDL**

- Consider aqueous solutions containing dissolved ionic species, which is denoted by 'i'.
- Each ion has a bulk concentration ' $c_i$ ' and a valence or charge number ' $z_i$ '.
- For example, if we have a 1mM solution of NaCl,  
 $c_{Na^+} = 1\text{mM}$  and  $c_{Cl^-} = 1\text{mM}$  and  
 $z_{Na^+} = 1$  and  $z_{Cl^-} = -1$
- We use subscript 'bulk' or ' $\infty$ ' to define properties in the bulk, far from the wall.

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And when you substitute this for the case of Na plus Cl minus or k plus Cl minus; so where for example, you have the values of Z for Na plus same as that of Cl minus you know but only the opposite signs so you have plus 1 and minus 1 and also the concentration is similar for both of them.

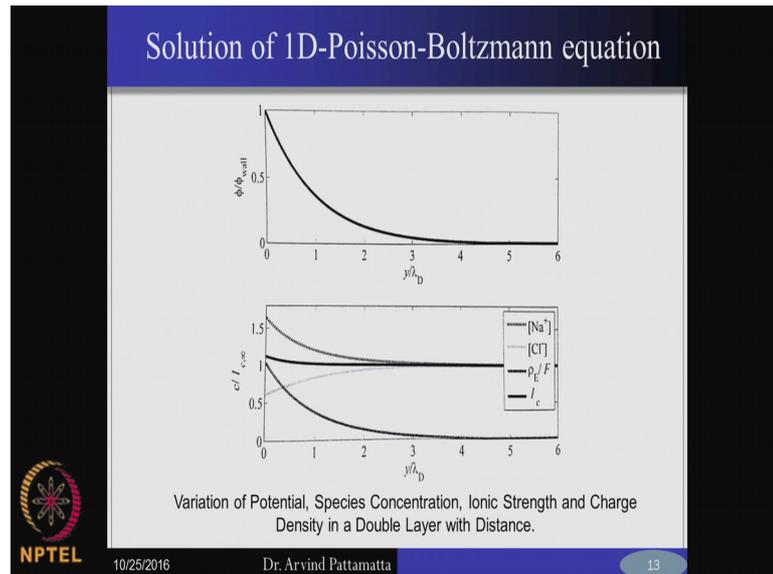
In that case this term related to 1 will cancel off, so you have basically this value will be 1 and this is 1 square, so the other case it will be 1 and 1 the other one will be one and minus 1. Therefore, these two terms will cancel off. Therefore the term involving 1 will disappear only the term involving  $Z_i \phi^*$  will be there. So, that should come to this particular form here, right here. And from there if you apply this for Na plus Cl minus or k plus Cl minus so that should also simplify to the following form, is that clear.

So, that is how we reduce the non-linear Poisson-Boltzmann equation to a linear Poisson-Boltzmann equation, now for which we can find an analytical solution in a straight forward manner. Any doubts on this. Therefore, now the generic solution will be in terms of exponential plus  $y^*$  and the other is exponential minus  $y^*$  so we know that in order to satisfy the case that  $\phi^*$  should be 0 at  $y^*$  equal to infinity so the term containing exponential  $y^*$  should disappear. That means, we have only an exponential decaying function with the respect to  $y$ , which we can find out the constant by applying the other condition that at  $y$  equal to 0 your value of  $\phi$  should be equal to

phi naught star; that is the maximum value at the wall, maximum value of the excess potential.

So from this, therefore we can also substitute this into the expression for concentration and find out how the concentration profile varies with the respect to y.

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Therefore, this if you plot it you can see that phi is exponentially decaying function from the wall till it is about five times the d by length. So, you should see that exactly at the d by length your phi does not become equal to 0, so take some time beyond the d by length for the potential to completely excess potential to d k. And similarly with the concentration profile also. For example, the Na plus concentration in this case will be very high right at the wall because of the negatively charged wall; whereas the value of Cl minus will be deficient will be very less compare to the bulk values. And these two shows the Boltzmann type distribution and exponential d k from the wall to the bulk values.

Now we therefore, have the solution for phi and we know how the phi actually varies with respect to y, and we also can find out the case where we apply an electric field.

(Refer Slide Time: 10:50)

## Electroosmosis

- When electric field is applied to the microchannels, bulk fluid motion is observed.
- The velocity of the motion is proportional to the applied electric field and depends on the material of the microchannel and solution in contact with wall.
- This motion is called as electroosmosis and is due to electrical forces on ions in the EDL.

$\vec{f}_{\text{Coulomb}} = \rho_E \vec{E}$

$\phi = 0$

$\phi = \phi_0$

$u(y)$

$z = \phi(y)$

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So, far we not applied electric field everything was statics, we just solve for the static field. Now, once you apply the electric field you should have a velocity profile.

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## Electroosmosis continued ....

- For the analysis of the electroosmosis flow, we find two asymptotic solutions:
  - 1) Inner solution: It is corresponding to EDL, in which we keep track of Coulomb forces and the resulting velocity gradient and vorticity but we assume external electric field uniform.
  - 2) Outer solution: The fluid is assumed to be electroneutral and irrotational, but external electric field is varied spatially.

inner solution      outer solution

location of Van Dyke condition

effective slip boundary condition

$\lambda_D$

$E_{\infty}$  is assumed constant here

here,  $\lambda$  varies because  $E_{\infty}$  is varying. Coulomb forces are ignored

here,  $\lambda$  varies because of viscous drag at the wall and Coulomb forces on the ions in the electrical double layer

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And how this velocity profile develops is now divided into an inner problem and the outer problem.

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### Inner Solution

- Navier-Stokes equation for thin region near wall is given by:  

$$\rho \frac{\partial \bar{u}}{\partial t} + \rho \bar{u} \cdot \nabla \bar{u} = -\nabla p + \eta \nabla^2 \bar{u} + \rho_E \bar{E}_{ext}$$
- $\bar{E}_{ext,wall}$  is caused by the external power supply and is uniform within EDL.

$\phi = 0$  if  $y \gg \lambda_D$

domain of integration

$\lambda_D$

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So, the inner problem consists only of the EDL where we solve the Navier-Stokes equation plus the Coulomb force which consist of the external field. And therefore, the Coulomb force here has the local charge density  $\rho_e$  time's  $e$  external.

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### Inner Solution continued ....

- Considering steady isobaric flow along x-direction N-S equation reduces to:  

$$0 = \eta \frac{\partial^2 u}{\partial y^2} + \rho_E E_{ext,wall}$$
- The uniform permittivity Poisson equation:  

$$-\epsilon \nabla^2 \phi = \rho_E$$
- Substituting for  $\rho_E$ , we get:  

$$0 = \eta \frac{\partial^2 u}{\partial y^2} - \epsilon \frac{\partial^2 \phi}{\partial y^2} E_{ext,wall}$$
- Integrating from wall to a point outside the EDL, we get:  

$$\eta u = \epsilon E_{ext,wall} \phi + C_1 y + C_2$$

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We can substitute for therefore,  $\rho_e$  through the relation between  $\rho$  and the electrostatic potential from the Poisson equation. Therefore, for a simple case where we want to consider a fully developed steady state solution without pressure gradient, just of flow passed a flat plate with the driven by an external electric field. So, all the other gradients

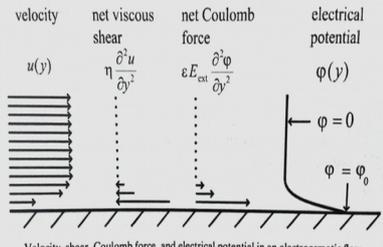
become 0 only you have the diffusion in the vertical direction balanced by the Coulomb force. So, the other convective terms are 0 pressure gradient terms as 0.

In this case we substitute for rho in terms of phi and when we integrate it we get the final solution for you in terms of phi.

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**Inner Solution continued ....**

- Applying the no-slip boundary condition and forcing the velocity to be bounded at  $y = 0$ , we obtain:

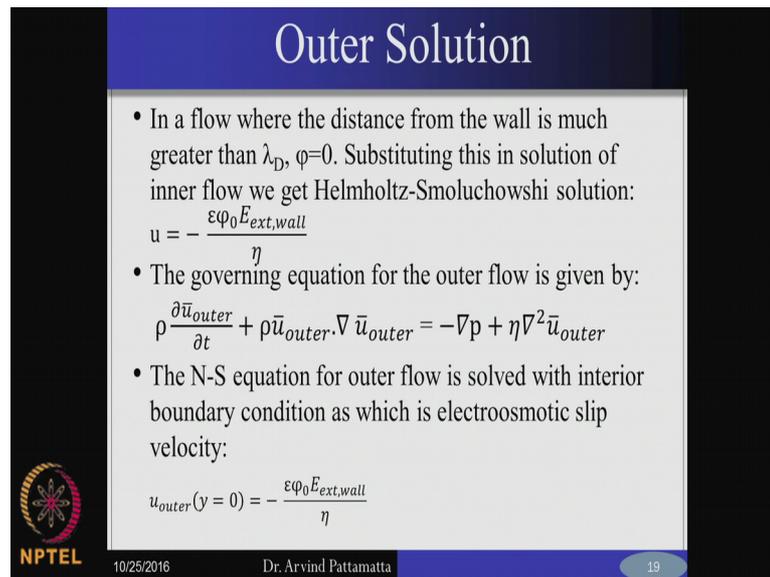
$$u_{inner} = \frac{\epsilon E_{ext,wall}}{\eta} (\phi - \phi_0)$$


Velocity, shear, Coulomb force, and electrical potential in an electroosmotic flow.

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And therefore, when we apply the condition that at  $y$  equal to 0 your phi should be equal to phi naught we get the velocity profile finally. You know this is the inner velocity profile. So therefore, now this is the condition, this is the velocity profile in the inner layer or within the EDL, now if you want to find out the value of velocity at the edge of the EDL.

(Refer Slide Time: 12:43)



**Outer Solution**

- In a flow where the distance from the wall is much greater than  $\lambda_D$ ,  $\phi=0$ . Substituting this in solution of inner flow we get Helmholtz-Smoluchowshi solution:  
$$u = -\frac{\epsilon\phi_0 E_{ext,wall}}{\eta}$$
- The governing equation for the outer flow is given by:  
$$\rho \frac{\partial \bar{u}_{outer}}{\partial t} + \rho \bar{u}_{outer} \cdot \nabla \bar{u}_{outer} = -\nabla p + \eta \nabla^2 \bar{u}_{outer}$$
- The N-S equation for outer flow is solved with interior boundary condition as which is electroosmotic slip velocity:  
$$u_{outer}(y=0) = -\frac{\epsilon\phi_0 E_{ext,wall}}{\eta}$$

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So, at the edge of the EDL you substitute the value of phi equal to 0. Therefore, the profile at the edge of the EDL becomes minus epsilon phi naught into e divided by eta.

So, all of you follow till here, I think we covered this in the last class I am clarifying again. This will now become the boundary condition or the slip condition for the outer flow. In the outer flow its electro statically neutral, so you do not have the direct effect of the electric field on the outer flow. However, at the edge of the EDL you have an induced motion due to the field and that is given as a provided as a slip boundary condition to the outer flow. So, the outer flow we just solve the protein Navier-Stokes without any Coulomb forces, but provide this as the boundary condition at y equal to 0. So, we assume now that y is shifted from the actual wall to the edge of the EDL by the order of few tense of nanometers and the velocity profile at that location is given as the slip velocity.

Now for the simple case of quad flow that is between two parallel plates which are both experiencing the same value of Coulomb force. So, what will be the limiting solution the entire profile will be same as the boundary profile. That means, we are talking about quad flow which is steady state and also fully developed without any pressure gradient. In that case you are velocity profile will have to satisfy the value at the boundaries. So, the application of such electro osmotic phenomena is used in one of the ways of using this in microfluidics is through what are called as electro kinetic pumps.

So, in the conventional sense you use only pressure gradient to pump the fluid as a mechanical pump, now you also have the electric field which is the driving the flow. Therefore, we can write to expressions; one is the electro osmotic contribution which is driven by the electric field and that is coming from the velocity expression here. So, you have the corresponding flow rate by integrating this velocity profile across the channel, channel cross section. The other is you can consider is a purely pressure gradient flow.

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**Electrokinetic Pumps**

- Electrokinetic Pump are ones in which an electric field is applied along a capillary to generate flow and pressure.
- Consider a system in which an electric field  $E=\Delta V/L$  applied across an open rectangular microchannel with depth  $2d$ , width  $w$ , length  $L$ , and cross-sectional area  $A=wd$ .
- Assume no pressure gradient and  $\lambda_p \ll d \ll w \ll L$ .
- For a 2D system, the electroosmotic and pressure driven flow is given by:

$$Q_{EOF} = w \int_{-d}^d u_{wall} dy$$

$$Q_{PDF} = w \int_{-d}^d \frac{1}{2\eta} \left( \frac{\partial p}{\partial x} \right) (d^2 - y^2) dy$$

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So, the actual system could have both the pressure gradient terms and also the Coulomb force driving the flow. If you do not have any Coulomb force then your pure pressure gradient flow is your conventional flow between two parallel plates or flow in a channel; that is a completely pressure gradient flow. So, that is your pressure driven flow expression in the second equation here.

So, your actual flow total flow in the electro kinetic pumps which have both the electro osmotic flow as well as pressure gradient flow is a some of these two terms.

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### Electrokinetic Pumps continued ...

- Integrating and setting the net flow rate equal to zero, we get:  
$$\frac{\Delta p}{\Delta V} = -\frac{3\mu_{EO}\eta}{d^2}$$
- This are the limiting cases: an open capillary with maximum flow rate and closed microchannel for which flow rate is zero.
- Therefore,  $Q = 2wu_{wall}d + \frac{2w}{3\eta} \left(-\frac{dp}{dx}\right) d^3$
- Rearranging this terms we get:  
$$Q = Q_{max} \left(\frac{\Delta p_{max} - \Delta p}{\Delta p_{max}}\right)$$



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Therefore if you integrate this across the channel cross section you will be able to get the net flow rate which is the first term is driven by the electro kinetic, second term is driven by the pressure gradient. So therefore, from this expression you can get the case where your pressure gradient can be maximum and the case where your flow can be maximum. For example, the pressure gradient can be maximum for a case where you block the flow at the exit, so you do not have an outlet, so the outlet flow is 0.

For this case if you put  $q$  equal to 0 you get an expression for  $dp$  by  $dx$  in terms of the electro osmotic velocity, and therefore you get the expression which is given here, where  $\Delta v$  is the voltage difference that is your  $e$  nothing but your  $e$ . So, your  $e$  is nothing, but  $\Delta v$  by  $l$ . This is the case where you can get your maximum pressure rise.

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### Electrokinetic Pumps continued ....

- Where,  $Q_{max} = 2wd\mu_{EO} \frac{\Delta V}{L}$  and  $\Delta p_{max} = -\frac{3\mu_{EO}\eta\Delta V}{d^2}$
- Thermodynamic efficiency can be defined as:  $\xi = \frac{\Delta p Q}{\Delta VI}$

Flow field inside an electrokinetic pump

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So, the other case where you can get your maximum flow rate is the case where your  $dp$  by  $dx$  is 0, because these two are in the opposite directions. Your pressure gradient is actually driving your flow in this direction, as you can see the profile here, whereas the electric field is driving the flow in the opposite direction.

So, if you want maximum flow so you have to set a  $dp$  by  $dx$  is equal to 0 and it will be completely driven by only electro kinetics. So, in that case the expression for  $q$  max is given like this. Therefore, you have expression for  $q$  max; you have expression for  $\Delta p$  max depending on what kind of mode you want to operate. If you want to get the maximum pressure rise you operate it in that mode, if you want to get maximum flow rate you operate it pure retro osmotic mode. And you can also define the efficiency of this pump as the pumping power divided by what is the power input. The power input is your electrical power. So, that is your applied potential difference times your current. And your pumping power is your product of pressure drop and your volumetric flow rate.

Usually, if you calculate with these values with these expressions for  $\Delta p$  and  $q$ , so you can estimate your  $\Delta p$  from this expression - for example and  $q$  from this expression and you can substitute and check. These will be of the order of 4 percent or 5 percent maximum. If you use the value of  $\Delta p$  max and  $q$  max, you will be able to predict what is the maximum thermo dynamic efficiency you can get. So, that will be of

the order 4 to 3 percent. So therefore, they are not very high, but never the less instead of using make conventional mechanical pump at very small scale, it is very difficult to design a conventional mechanical pump at very small devise microfluidic devise scale. In that case you can the electro osmotic phenomena to build an electro kinetic pump. Even with lower efficiencies it can you know basically move the fluid and also cause a pressure difference.

Student: Sir, (Refer Time: 19:44).

Concentration where we are neglecting.

Student: That finally, it was before the detailed (Refer Time: 20:07)  $\sigma_c h^*$  (Refer Time: 20:09).

So, in this case for Na plus or Cl minus so your  $C_i^*$  in infinity of Na plus and Cl minus are same, equimolar solutions.

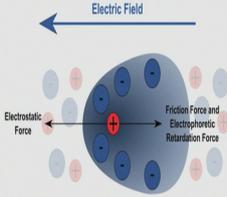
Student: Same.

Same; and then the value of  $Z_i^2$  is also same you have 1 and 1. So therefore that part will simplify to this.

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## Electrophoresis

- Chemical species are transported through fluids due to both diffusion and convection.
- Species diffusion refers to the migration of species owing to Brownian motion in the system.
- Species convection is due to the fluid flow and motion of the chemical species due to electric field.
- In the presence of electric fields, charged ions move in response to the Coulomb force they feel in that electric field, this process is termed as electrophoresis.
- The force exerted by electric field is given by:  
$$\vec{F} = ze\vec{E}$$



The diagram shows a central blue sphere with a red '+' sign, representing a charged particle. It is surrounded by smaller blue and red spheres, representing ions in the fluid. An arrow labeled 'Electric Field' points to the left. A red arrow labeled 'Electrostatic Force' points from the central sphere to the left. A black arrow labeled 'Friction Force and Electrophoretic Retardation Force' points from the central sphere to the right.

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Therefore, this is one phenomena in electro kinetic that is the electro osmotic flow. In which you are able to cause the motion of the bulk fluid due to the charge concentration close to the wall by an applied external field.

Now, the other phenomena is where the ions themselves move with the bulk fluid being stationary or you can also imagine that if you have small particles, these particles can gather some charge so they will never be perfectly electrically neutral they will be either having a possible or negative charge. So, these will form an electric double layer the same way that we have shown for the previous case. So, they will attract the counter charges from the bulk fluid and they will form the electric double layer around this particular particle.

And, now if you apply an electric field in a stationary fluid, so you will find these particles are actually migrating and moving. So, this is a different from the electro osmosis. In the electro osmosis with the applied external field the entire bulk flow seems to move, but in this case only the particles will be moving and the bulk flow appears to be stationary. So, this is mostly related to particles and sometimes people refer this to as particle electro forces.

And also there is a because of the motion of these particles there is a concentration gradient. So, there is a if you have species different, kinds of species then again you have concentration gradient and the field being set up. Now, how do we analyze the case of forces on the particles and similarly how do we get the velocity of these particles. So, this is now done in a very similar fashion as the electro osmosis. So, only you can assume the field to act in our opposite direction as in the previous case, because in the earlier case we were looking at the case where you are plate is stationary, the wall is stationary and then the ions are actually moving because of the applied external field. Now in this field you can make an approximation that you are bulk fluid is stationary and the wall is now moving.

So, in this case we can impose an electric field in the opposite direction as the previous case. So, you will get the same solution here only the signs will be reversed, you will have  $\phi$  naught minus  $\phi$ , because in this particular expression you will have a plus here because of the opposites sign of the electric field. So, in the previous case you are

viscous diffusion is happening in the opposite direction as the applied Coulomb force now this will be both in the same direction so you will have a positive sign.

So, it is just changing the coordinate frame of reference. In the previous case we had a fixed coordinate frame of reference; the wall is also fixed and then the bulk fluid is moving. Now, in this case the particle is moving, the bulk fluid is stationary. So now we fix the coordinate to the particle so it becomes similar to the earlier case; where the wall is stationary bulk fluid is moving but the electric field is acting in the opposite direction.

So, the same expression can be used for velocity only we replace the potential; I mean with minus sign. Same expression, but instead of using epsilon phi naught eta e we have a minus there and we replace that with the plus sign here. This was your earlier expression we just replace that with the positive sign in this case. So, is that clear.

(Refer Slide Time: 24:25)

The slide is titled "Electrophoresis continued ...". It contains two bullet points and a diagram. The first bullet point states: "The steady-state response of the ion occurs at equilibrium by equating Coulomb and drag forces, where drag force is given by:  $\bar{F}_{drag} = -6\pi\bar{u}_i\eta r_i$ ". The second bullet point states: "For the particle with thin EDL, electrophoretic velocity is given by:  $\bar{u}_{EP,i} = \mu_{EP,i}\bar{E}$  where,  $\mu_{EP} = -\mu_{EO} = \frac{\epsilon\phi_0}{\eta}$ ". Below the text is a diagram of a circular particle with a thin electric double layer (EDL) shown as a layer of small circles on its surface. Below the diagram is the caption: "A particle with a thin EDL ( $a \gg \lambda_D$ ).". The slide footer includes the NPTEL logo, the date 10/25/2016, the name Dr. Arvind Pattamatta, and the page number 24.

This is the very simplistic approximation, and this is particularly valid for a case where you have a thin EDL. If you have a thin EDL case where the radius of the particle or diameter of the particle is much larger than the Debye length, so you can apply the same velocity, because locally compare to the curvature; the curvature appears to the very small for the bulk fluid.

Therefore, this will be almost like a flow fast of flat late case. And you also have the EDL which is very thin and we can assume that both the EDL as well as the particle is moving with the same velocity.

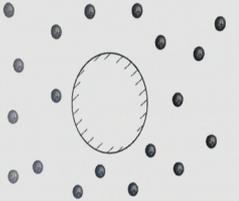
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## Electrophoresis continued ....

- To describe the particle whose radius is not necessarily greater than  $\lambda_D$ , we multiply with factor 'f' known as multiplicative factor.

$$\bar{u}_{EP,i} = f \frac{\epsilon \phi_0}{\eta} \bar{E}$$

- This factor 'f' accounts for the variation of the local electric field throughout the EDL.
- The total velocity of the ion is give by:  $\bar{u}_i = \bar{u} + \bar{u}_{EP,i}$   
where,  $\bar{u}$  is fluid velocity.



A particle with a thick EDL ( $\lambda_D > a$ ).

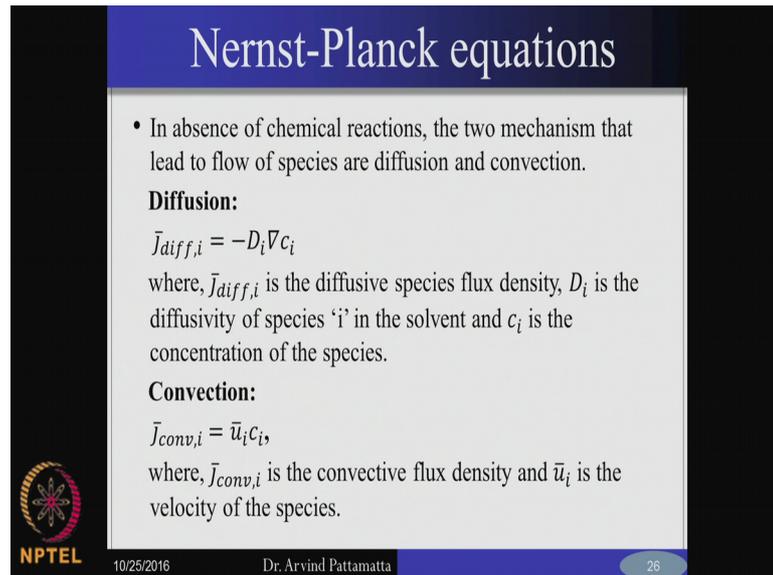

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If you have much thicker EDL, so that is the thick EDL case. So, first of all the electric field itself will not be uniform, because now the d by layer is quid thick and also there will be a variation in the velocity of the particle, so there will be a drag force between the particle and the actual EDL. Therefore, in this case we multiply the previous expression for the thick EDL case with the correction factor f. So, the earlier case is the same expression as what we have derived for electro osmotic flow for flow fast of plate the same expression is used only we change the sign, but if you have a thick EDL there will be a constable slip effect between the EDL and the particle itself. So therefore, you have correct, the velocity by means of a correction factor, multiplicative factor.

Therefore, if you also have a bulk motion that may be it could be due to the pumping power or whatever externally. So therefore, the total velocity of the ion or the particle can be equal to both the bulk velocity plus, so you have do a vector sum depends on what direction your electric field is applied and what direction in the bulk fluid is moving. You cannot always do a plus you have to be careful it should be a vector summation. Therefore, you have a vector summation of both the particle motion as well as the bulk fluid motion.

Now, if you want to solve for the concentration in this case. The earlier case the static case we just assume an exponential decay in the concentration from the wall, because there the static is there is no fluid motion. Now, if you are talking about the particle which is also migrating, therefore it will also have advection component as well as diffusion component, so you have to solve for both of them.

(Refer Slide Time: 27:39)



The slide features a blue header with the title "Nernst-Planck equations" in white serif font. The main content is on a white background with a black border. It includes a bullet point, two bolded section headers, and their respective equations and descriptions. The NPTEL logo is in the bottom left, and the date, name, and slide number are in the bottom footer.

## Nernst-Planck equations

- In absence of chemical reactions, the two mechanisms that lead to flow of species are diffusion and convection.

**Diffusion:**

$$\bar{J}_{diff,i} = -D_i \nabla c_i$$

where,  $\bar{J}_{diff,i}$  is the diffusive species flux density,  $D_i$  is the diffusivity of species 'i' in the solvent and  $c_i$  is the concentration of the species.

**Convection:**

$$\bar{J}_{conv,i} = \bar{u}_i c_i,$$

where,  $\bar{J}_{conv,i}$  is the convective flux density and  $\bar{u}_i$  is the velocity of the species.

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And in the advection component your velocity now will have both the velocity of the bulk fluid, sum of the bulk fluid velocity plus your particle migration velocity due to the electrophoresis.

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### Nernst-Planck equations continued....

- Thus, the transport of the species 'i' in the absence of chemical reactions can be described by Nernst-Planck equation:

$$\frac{\partial c_i}{\partial t} = -\nabla \cdot [-D_i \nabla c_i + \bar{u}_i c_i]$$

Species fluxes for Cartesian control volume

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So, both of this will contribute and therefore we write the equation for the advection diffusion equation for the concentration of the species and this equation is called the Nernst-Planck equation. So, very commonly in microfluidics, when people want to solve for concentration distribution and also where you have the electro kinetic effects; so they have to calculate both the electrophoretic velocity as well as the bulk velocity and do a vector sum and use that in the advection part. And the other part it is diffusion part which is the routine diffusivity of the species, and they solve this partial differentially equation to describe the evaluation of the concentration of this particular species over time.

So, this is very popular equation, this does not have any chemical reaction. If you also have chemical reactions then you have appropriate source and symptoms. So, this equation is called the Nernst-Planck equation and people generally solve this. Apart from your electrophoretic velocities you also solve the Nernst-Planck equation to look at the distribution of sum ions or particles or some chemicals species in a bulk fluid.

So, these two whatever we have covered; the electrophoresis as well as the electro osmosis, these two are the most fundamental electro kinetic phenomena. Although they appear to be similar, so one we have a frame of reference in which your wall is fixed and fluid is moving the other the fluid is moving on the wall is fixed. But it can be apply it to two different things; one where we have only fluid with setting up electrical double layer and the complete motion of this fluid, the other where we have particle diffusion and the

motion of these particles due to an applied electric field. So, one example could be if you suspend say nano particles or micron size particles, so you actually observe the electrophoresis phenomena more than electroosmosis. Although the base fluid will set up the EDL and there could be some motion, but these particles will migrate more rapidly than the base fluid. And therefore, in such cases where you have suspensions in liquids you will observe the electrophoresis more commonly than electroosmosis, but these two can exist together.

Therefore, I will stop our discussion on the electrokinetics here. There are also some other parts of electrokinetics, as I talked about the other effect where we move and then this motion can create an electric field and then that is called the streaming and zeta sedimentation potential.

So, we will start the next topic. So, so far we have micro scale heat transfer we started with the single phase gas flows, then we talked about single phase liquid flows, and associated with the liquid flows we also looked at the electrokinetics. Now, what is again very important in heat transfer is the phase change. Phase change process, it could be we can also call this is two phase fluids to be very generic, because two phase can be without phase change right you can have a liquid air system which is a common two phase flow in mini or micro channels. Or you can also heat this system it is a non isothermal or non adiabatic system which could also result in change of phase from the liquid to vapor.

In this case typically we are talking about; it could be a single component phase change or multi component phase change more complicated, because you have the vapor phase we have also water vapor plus air. So, when we are talking about phase change we are mostly talking about the single component phase change, that means we have only one element water in its element a liquid form or could be in the vapor form. Or if we are generally about two phase it can also include non adiabatic a mixture of air and water. So, that will also be a classical two phase problem.

So, neither case what is the reason to go to study phase change in mini and micro channels. So, like your conventional macro channels or micro ducts. So, you have problems of heat exchanges at the very small scales also. You may not talk about heat exchanges directly in micro fluidics, but in electronic cooling applications for example

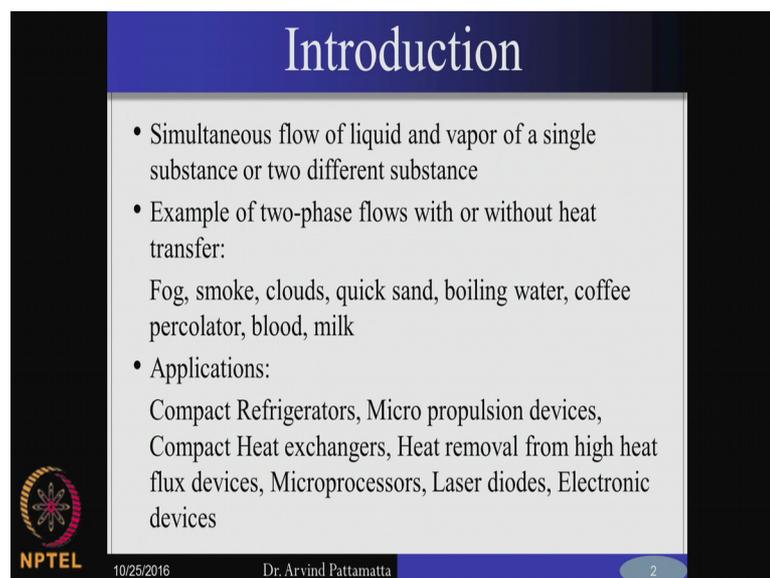
you want to put very small heat exchanges where the diameter of the ducts can approach order of few microns.

So, in that case suppose there is an evaporation process or condensation process, how does the flow regime look? So, it appears that in the flow regimes although there is commonality between your macro channel phase change and then micro channel phase change the regimes are quite distinct. In the case some the regimes are quite unique and distinct to the micro channels which are totally absent in the macro channel case. And these are partly due to the enhanced capillary effects appearing at micro channel case, and also the confinement to the growth of the vapor bubbles due to the smaller diameters.

So, this confinement also brings in the distinct flow regimes. So, let us focus therefore on the flow regimes when you have phase change at micro mini channels, and therefore associated with that what is a consequence on the pressure drop and heat transport. So, these are the two important parameters when we study the phase change problem.

Therefore, when we talk about two phase flows you can have either simultaneous flow of liquid or vapor of single substance that is the single component or it could be two different substances or two different components and this might be general two phase flow.

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The slide is titled "Introduction" and contains the following text:

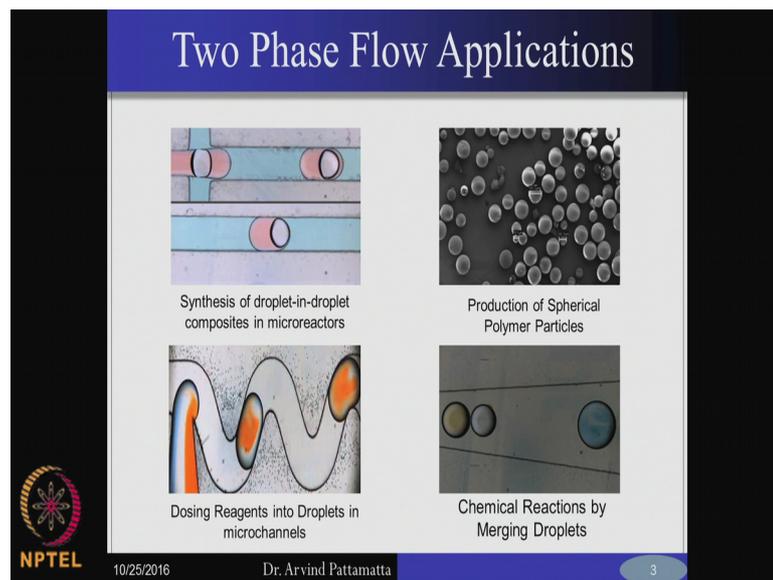
- Simultaneous flow of liquid and vapor of a single substance or two different substance
- Example of two-phase flows with or without heat transfer:  
Fog, smoke, clouds, quick sand, boiling water, coffee percolator, blood, milk
- Applications:  
Compact Refrigerators, Micro propulsion devices, Compact Heat exchangers, Heat removal from high heat flux devices, Microprocessors, Laser diodes, Electronic devices

At the bottom left is the NPTEL logo. At the bottom center is the date 10/25/2016 and the name Dr. Arvind Pattamatta. At the bottom right is a small blue circle containing the number 2.

And this could therefore, exist either in the adiabatic case or with only with heat transfer. So you can see this effect everywhere, in a common two phase flows are every flow in a atmosphere signs what you see in real life problem, everywhere you find two phase flow.

For example; fog smoke, we have clouds, we have quick sand in this case you have solid liquid mixture. So, it is not a air and liquid vapor mixture. You have boiling water; you have coffee percolator blood milk, so all this two phase flow. You have more than one component and more than one phase. So, where all these applications of generic two phase flows they can be applied anywhere as you see from atmosphere sciences to heat exchangers to where ever you need a efficient cooling systems. So, everywhere they can be apply.

(Refer Slide Time: 35:47)



The slide is titled "Two Phase Flow Applications" in a blue header. It features four panels illustrating different applications of two-phase flow in microfluidics:

- Synthesis of droplet-in-droplet composites in microreactors:** A schematic diagram showing a blue channel with a red channel, where a red droplet is being formed inside a blue droplet.
- Production of Spherical Polymer Particles:** A photograph showing a collection of small, spherical, light-colored particles.
- Dosing Reagents into Droplets in microchannels:** A schematic diagram showing a blue channel with a red channel, where a red droplet is being formed inside a blue droplet.
- Chemical Reactions by Merging Droplets:** A photograph showing two blue droplets merging into a single larger blue droplet.

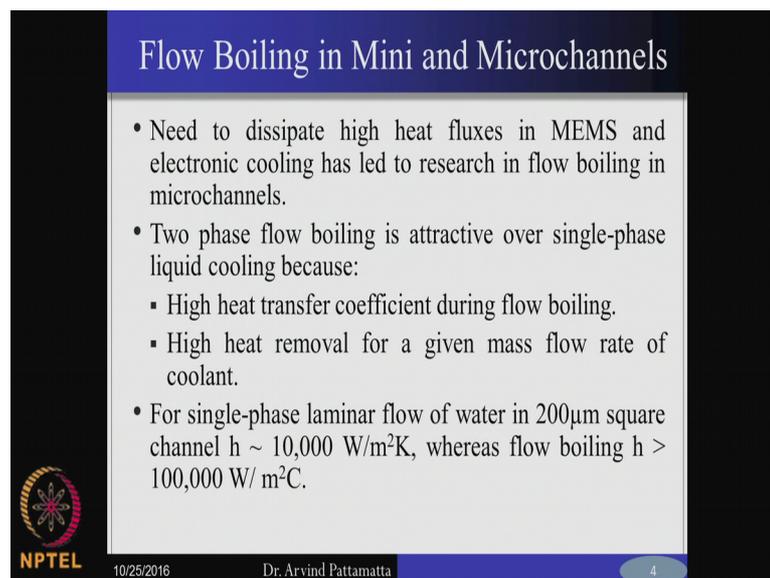
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If you talk about therefore two phase flow application particularly in micro conduits. That means, I am not talking about phase change here just simple two phase flow, two different liquids two different phases or whatever. So, you can talk about micro reactors where you have one drop embedded in the other drop and you can use this for generating some chemical reactions. So, the application could be some small micro chemical reactors or it could be biological or medical application where you want to introduce small drug targeted two a cell and this drug reacts only within that cell and not everywhere.

In that case you synthesize drop which is of a different density and viscosity which is sitting inside larger drop of a different viscosity and density, so you can produce some flow regimes this kind of a distribution. You can also talk about dosing reagents into droplets in micro channels. That means, with some diffusion properties in either chemical reactors or in medical applications you can inject some die or selectively only and look at how the defused then this particular droplet. So, this die could be liquid or a vapor. Again this is a distinct two phase of application.

And you can also have chemical reaction by merging droplets. So, you can have one could be a medicine, so a drug which you inject as a liquid into the a blood stream and then it will be a reacting with the some other component of blood which is again a different liquid and these two will have a chemical reaction and they have can merging of this drop. So, you have two phase flow applications everywhere generically.

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The slide is titled "Flow Boiling in Mini and Microchannels" and contains the following text:

- Need to dissipate high heat fluxes in MEMS and electronic cooling has led to research in flow boiling in microchannels.
- Two phase flow boiling is attractive over single-phase liquid cooling because:
  - High heat transfer coefficient during flow boiling.
  - High heat removal for a given mass flow rate of coolant.
- For single-phase laminar flow of water in 200 $\mu$ m square channel  $h \sim 10,000 \text{ W/m}^2\text{K}$ , whereas flow boiling  $h > 100,000 \text{ W/m}^2\text{C}$ .

The slide also features the NPTEL logo in the bottom left corner, the date 10/25/2016, the name Dr. Arvind Pattamatta, and a small blue circle with the number 4 in the bottom right corner.

But what we are particularly concerned here is the non adiabatic case that is phase change a due to flow boiling or condensation. So, this kind of two phase applications are primarily in the area of electronic cooling, so the electronic cooling applications and some of the MEMS applications also where you need to build small heat exchanges require dissipation of very high heat fluxes and therefore we try to use micro channels.

And why do we again go for micro channel two phase flows, the same justification that we gave for going for micro channels with even the single phase. So, for the same

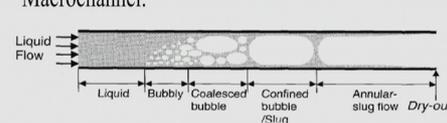
Nusselt number you have a high heat transfer coefficient as you are channel diameter comes down. That is one reason why micro channels are very attractive for heat transfer applications. Now when we look at phase change, you know that even with the macro channel case your phase change heat transfer coefficients are at least two to three orders of magnitude higher than the single phase heat transfer to coefficient. That means, if you are talking about the macro channel where the single phase heat transfer is a coefficient is of the order of 10 or 20 watts per meter Kelvin, your phase change is transfer coefficient if you have boiling can be of the order of 10 power 2 watt per meters square Kelvin in the in the nucleate boiling regime.

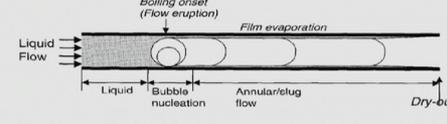
Now, if you go to micro channel this value of single phase h will would have already gone up by 10 times. So, from say 10 or 22 gone to 100 or 200, while you know the boiling heat transfer coefficient can actually go to the order of 10 power 3 or 10 power 4 sometimes even in micro channels. Therefore, compare to your single phase heat transfer always your phase change has a higher value of h, so whether you are constructing micro heat exchanger to dissipate heat from a electronic chip or a MEMS device it is always attractive to operate this in the phase change regime rather than the single phase regime.

Therefore, phase change applications are more in demand then the single phase application. And at the same time the flow and flow physics and heat transfer is also more challenging and complex.

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### Difference between Macro and Microchannels

- **Macrochannel:**


The diagram shows a horizontal channel with liquid flow from left to right. It is divided into five regions: 'Liquid', 'Bubbly', 'Coalesced bubble', 'Confined bubble / Slug', and 'Annular-slug flow'. A 'Dry-out' point is indicated at the end of the channel.
- **Microchannel ( $D_h \leq 100\mu\text{m}$ ):**


The diagram shows a horizontal channel with liquid flow from left to right. It is divided into three regions: 'Liquid', 'Bubble nucleation', and 'Annular/slug flow'. A 'Boiling onset (Flow eruption)' point is marked above the channel, and 'Film evaporation' is labeled above the 'Annular/slug flow' region. A 'Dry-out' point is indicated at the end of the channel.
- Surface tension forces are more dominant and gravity forces are negligible in micro-channels


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So, what is basically fundamental difference between macro and micro channel is that if you look at the effect of surface tension forces. So, they become more dominant in micro channel compared to the gravity which is more dominant in the macro channel case.

So, because of this when you pass liquid flow at inlet of macro channel and you start heating this with the constant heat flux so you would see a the different flow regimes starting from a pure liquid at the inlet and at the exit it would be pure vapor; and intermediate you will find flow such as bubbly flow, coalesced flow, confined bubble slug flow, and annulus slug flow and so on. And provides this also depends on what is the orientation of this tube. So, if it is vertically orientated you see these regimes. If you horizontally oriented then you can see stratification. That means, the denser fluid will settled down lighter fluid will go up, so the effect of gravity becomes very important.

Whereas, in the case of micro channel whether you oriented in the vertical manner or horizontal manner it is relatively insensitive to the orientation, because the gravity forces are very small compare to the surface tension force. And mostly you will find these bubbles cannot keep growing at the same diameter as the macro channel. Therefore, the bubble sizes will be much smaller and also they will be elongated.

So, most of the regimes are confined to the bubble flow regimes, but these are slightly longer than in the macro channel and these are called as Taylor bubble flow or slug flow. Therefore, the dry out happens in the difference manner in the micro channel compare to macro channel case. In the macro channel case you can have all these bubbles, bubbles which are slightly expanded and then finally close to the wall have the film and the film also finally we operates and then you have completely diode.

In the case of micro channel right from close to the inlet all the way have you have the bubbly regime. The bubbles only keep along getting more and more and more, but the thin film will keep sticking for a longer period of time, and finally the film also ruptures evaporates and you have a complete diode. So, the therefore, the mechanism of heat transfer is quite different. So, in the micro channel case heat transfer is mostly through the thin film, because the bubble already structures and elongates much earlier than in the macro channel case.

So therefore, the thin film plays very important role through which the heat is added from the wall in to the bubble. The other parts related to the flow regimes we will

continue tomorrow, also the corresponding expressions for pressure drop and heat transfer coefficient. I hope talk about this for another two classes and complete this by the week so that next week we move on to nano fluids. So, I stop here.