

Micro and Nanoscale Energy Transport
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Lecture - 33
Single phase Liquid Flow and Heat
Transport in Micro Channels Part 2

Good morning all of you. We have looked into the aspects of liquid flows in a mini and micro channels.

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Developing Laminar Flow continued

- Difference between the incremental pressure and the fully developed friction factor over the length 'x' is given by incremental pressure:

$$K(x) = (f_{app} - f) \frac{4x}{D_h}$$
- For $x > L_{h1}$, incremental pressure attains the constant value known as Hagen Bach's factor $K(\infty)$.
- So, the pressure drop in term of incremental pressure is:

$$\Delta p = \frac{2(f_{app}Re)\mu u_m x}{D_h^2} = \frac{2(fRe)\mu u_m x}{D_h^2} + K(x) \frac{\rho u_m^2}{2}$$
- Shah and London(1972) given the equation for the pressure drop as:

$$\frac{\Delta p}{(1/2)\rho u_m^2} = 13.74(x^+)^{1/2} + \frac{1.25 + 64x^+ - 13.74(x^+)^{1/2}}{1 + 0.00021(x^+)^{-2}}$$



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The major difference from the macro channel flow case is that some of the effects like the developing length become very important for the liquid flow in micro channels. Therefore, when you are calculating, for example the overall pressure drop, if you did not jump into taking the fully developed condition and estimating the pressure drop. So, you also have to account for the developing section and this is therefore obtained through the way we defined the apparent friction factor. So, the apparent friction factor now has 2 contributions one from the fully developed friction factor and the other from the pressure drop in the developing regime.

So, if you are talking about lengths which are greater than the developing length. So, this factor K affects becomes the Hagen Bachs factor k of infinity that becomes a constant if your X is greater than l h and having a said that. So, this how do we estimate the Hagen

Bachs factor? So, that again depends on different cross sections different aspect ratios depending on that the Hagen Bach factor will be different.

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Developing Laminar Flow continued

- x^+ is non-dimensional length given by:

$$x^+ = \frac{x/D_h}{Re}$$
- Frictional pressure drop for circular duct is given by:

$$\frac{\Delta p}{(1/2)\rho u_m^2} = 13.74(x^+)^{1/2} + \frac{1.25 + 64x^+ - 13.74(x^+)^{1/2}}{1 + 0.00021(x^+)^{-2}}$$
- Steinke and Kandlikar (2005) obtained the curve fit for the Hagenbach's factor for rectangular channels as:

$$K(\infty) = 0.6796 + 1.2197\alpha_c + 3.3089\alpha_c^2 - 9.5921\alpha_c^3 + 8.9089\alpha_c^4 - 2.9959\alpha_c^5$$

forward

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So, I have given you very commonly used correlation by Steinke and Kandlikar to estimate the Hagen Bach factor this is a function of the ratio of a by b. So, I think if you go back.

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Fully Developed Laminar Flow continued

Fanning Factor and Nusselt number for Fully developed Laminar flow in Ducts Kakac et al. (1987)

Duct shape	Nu_i	Nu_f	$Po = f/Re$
 Circular	4.36	3.66	16
 Flat channel	8.24	7.54	24
 Rectangular, aspect ratio, $b/a =$	1	3.61	2.98
	2	4.13	3.39
	3	4.79	3.96
	4	5.33	4.44
	6	6.05	5.14
	∞	8.24	7.54
 Hexagon	4.00	3.34	15.05
 Isosceles Triangle, Apex angle $\theta =$	10°	2.45	1.61
	30°	2.91	2.26
	60°	3.11	2.47
	90°	2.98	2.34
	120°	2.68	2.00
 Ellipse, Major/Minor axis $a/b =$	1	4.36	3.66
	2	4.56	3.74
	4	4.88	3.79
	8	5.09	3.72
	16	5.18	3.65

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So, that is the smaller dimension to the larger dimension a by b. So, if you take a by b as this is the inverse of aspect ratio and if you put that into this correlation they should give

you the corresponding value of Hagen Bach factor. And once you estimate the Hagen Bach factor therefore everything is known. So, we know K of infinity we will know what is friction factor for fully developed case. So, depending on again the different aspect ratios that you consider the Poiseuille numbers are different, so based on that you can estimate the friction factors.

Therefore, this will you give you the overall pressure drop for the micro channel including the developing affects. So, apart from that there are also some correlations like this form which is available like shah and London for example, have given a more explicit correlation in terms of x you do not have to specifically a take fully developed developing and add them separately. So, you directly have everything in terms of X. So, if you substitute the corresponding value of X if your X is less than l h, it will give you that certain value your x is greater than l h it will give you certain value.

So, if your x is greater than l h and if you are calculating the delta p for entire tube length for example, that will become a constant value. So, you substitute your X as l by d basically if you want to calculate the pressure drop for the entire length then that becomes l by d by re which becomes a constant value and therefore, this will give you the overall pressure drop for the entire micro channel. So, these are some of the commonly used methods of estimating the pressure drop for that is with the different cross sectional shapes.

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Fully developed and developing turbulent flow

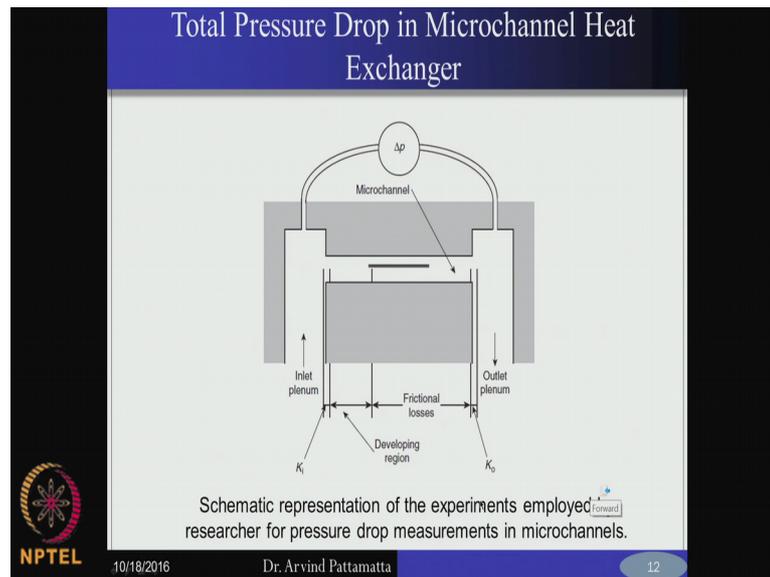
- Blasius developed the correlation: $f = 0.0791 \text{Re}^{-0.25}$
- Phillips (1987) developed the expression for developing and developed regions.
- He presented Fanning friction factor for a circular tube as:

$$f_{app} = A \text{Re}^B$$
 where, $A = 0.09290 + \frac{1.01612}{x/D_h}$ $B = -0.26800 - \frac{0.32930}{x/D_h}$
- For rectangular geometries Re is replaced by laminar-equivalent Reynolds number given by:

$$\text{Re}^* = \frac{\rho u_m D_{le}}{\mu} = \frac{\rho u_m [(2/3) + (11/24)(1/\alpha_c)(2 - 1/\alpha_c)] D_h}{\mu}$$
 where, D_{le} is the laminar-equivalent diameter.


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And those are for the laminar flow, now if you move ahead to the turbulent flow case again I talked about the correlation by Philips. So, he has correlated the f apparent as a function of the parameters a and b functions of X by d . So, all we you have to do is put in the value of X by d if your estimating the overall friction factor for the entire length of the tube substitute X equal to l and you get the fixed value. Assuming that you have turbulent flow throughout the tube this accounts for both the developing and developed regimes of turbulent flow.

So, these are the 2 common methods for estimating the f apparent. Now the other important difference when it comes to micro channel this is the case of inlet and exit losses, I mean this is already there in macro channel case, but it becomes more apparent because the compared to the overall pressure drop within the channel this losses can also be significant, because you generally have very large plenums bits are usually larger compared to the channel diameter. The channel diameters are the order of microns plenum diameter could be of the order of millimeters. And therefore, there is a large amount of constriction when it enters in to the small diameter channels.

Similarly, the exits losses are also significant there is a lot of lot of divergence of the flow and not only that you also can account for the pressure loss as it flows through the plenum inlet and outlet plenum, if you put have differential transmitter some were no at this between this point and you know this point. So, these 2 points it should give you the

overall pressure difference that includes also the exit and entry losses and also the bend that is a sharp 90 degree bend in these micro channels it is not smooth or rounded off. So, in these cases you also have account for the bend losses.

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**Total Pressure Drop in Microchannel Heat Exchanger
continued**

- The pressure drop measurement represents the combined effects of the losses in the bend, entrance and exit losses, developing region effects, and core frictional losses.
- So, the pressure drop is given by:

$$\Delta p = \frac{\rho u_m^2}{2} \left[(A_c/A_p)^2 (2K_{90}) + (K_c + K_e) + \frac{4f_{app}L}{D_h} \right]$$

where, A_c and A_p are the total channel area and the total plenum cross-sectional area, K_{90} is the loss coefficient at the 90° bends, K_c and K_e represents the contraction and expansion loss coefficient due to area changes, and f_{app} includes the combined effects of frictional losses $f_{friction}$ and additional losses in developing flow.



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Therefore, the overall pressure drop is apart from what you have already estimated for a single micro channel. Now if you consider the plenums have into picture you also have consider the inlet losses the entry losses exit co losses. These are the contraction and expansion pressure losses and also the pressure loss due to the bend 90 degree bend. So, therefore, we have the contribution of all of this in determining the overall delta p for full micro channel assembly that is with the plenum into picture.

So, how do we estimate the entry losses exit losses and bend losses? So, there some standard charts available for circular tubes for example, this losses are all plotted as charts you can just refer to them depending on the contraction ratios contraction area ratio and expansion area ratio you can estimate the values of K_c , K_e and you can substitute them. So, there are standard tables available. So, that is why I am not giving them here you can take any fluid mechanic text book you should be able find them out.

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Total Pressure Drop in Microchannel Heat Exchanger continued

- In terms of the fully developed friction factor 'f' and pressure drop defect $K(x)$:

$$\Delta p = \frac{\rho u_m^2}{2} \left[(A_c/A_p)^2 (2K_{90}) + (K_c + K_e) + \frac{4fL}{D_h} + K(x) \right]$$

- For $L > L_h$, $K(x)$ is replaced by the Hagenbach's factor $K(\infty)$



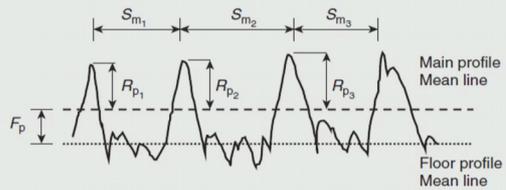
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Therefore, if you are now substituting for a apparent in terms of f and the Hagen Bach factor you get a complete expression right therefore, if you are asked to calculate the overall pressure drop for a micro channel system single micro channel system you have to also account for the other contribution.

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Roughness Effects

- Parameters based on various roughness characterization schemes are investigated by Kandlikar (2005):



- **Average maximum profile peak height (R_{pm}):** The distance between the average of the individual highest points of the profile ($R_{p,i}$) and the mean line within the evaluation length. The mean line represents the conventional average roughness value (R_{forward})



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So, we have I think done all this last time then we looked into the other important affect in micro channels which is the roughness effect. So, all the previous discussion is valid only for the smooth channels perfectly smooth channels no roughness. So, you use 1

same correlations for the friction factor as the micro channel same possibly number and so on, but any practical micro channel if you look at the surface profile it is not smooth and definitely has considerable roughness and the roughness to diameter ratio which is very important will be quite significant in micro channels the roughness itself will be order of the few microns compared to the channel diameter of the order of few hundred microns.

Therefore, in the year 2005 Kandlikar defined certain parameters a R_p is basically what he defines as the average maximum profile peak height that is we draw a floor base line and then we estimate what is the average height of the roughness. So, that is the mean profile or mean line from which we calculate what is the distance from the p and we take a simple arithmetic average of all these respective values R_{p1} , R_{p2} , R_{p3} and. So, on and that gives a kind of an average of the maximum profile peak height. So, that is R_{pm} and the value of F_p which is the displacement of this mean line from the floor is fixed. So, if you sum this value of R_{pm} with F_p . So, that gives you the equivalent roughness.

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Roughness Effects continued ...

- **Mean spacing of profile irregularities (RS_m):** consists of the mean value of the spacing between profile irregularities within the evaluation length. The irregularities of interest are the peaks, so this is equivalent to the Pitch.

$$RS_m = \frac{1}{n} \sum_{i=1}^n S_{m_i}$$

- **Floor distance to mean line (F_p):** Consists of the distance between the main profile mean line (determined by R_a) and the floor profile mean line. The floor profile is the portion of the main profile that lies below the main profile mean line.
- **Equivalent roughness:**
 $\varepsilon = R_{pm} + F_p$

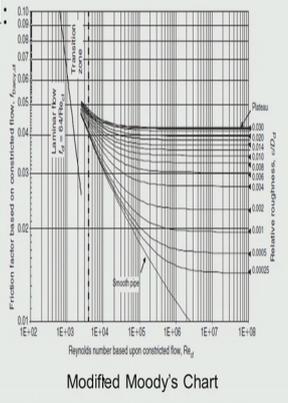
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So, the overall roughness ε since it is varying it is not a constant value it is varying its position. So, it is represented now by means of this equivalent roughness. Which is basically the summation of the average maximum profile peak height and the distance of the mean profile from the base.

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Roughness Effects on friction factor

- Constricted flow model:
 - Kandlikar et al. (2005) considered the effect of cross-sectional area reduction due to protruding roughness elements and recommended using the constricted flow area in calculating the friction factor.
 - So a modified Moody diagram was formulated based on the constricted diameter.
$$D_{cf} = D - 2\varepsilon$$



The Modified Moody's Chart plots the friction factor f on the y-axis (ranging from 0.01 to 0.10) against the Reynolds number based on constricted flow, Re_d , on the x-axis (ranging from 10^{-12} to 10^{+8}). The chart includes curves for relative roughness values ε/D ranging from 0.0005 to 0.0200. Key regions are labeled: Laminar flow ($Re_d < 2300$), Fully rough flow, and Smooth pipe. The chart shows that for a given Reynolds number, the friction factor increases as the relative roughness increases.

Modified Moody's Chart

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So, now the good thing about defining this equivalent and roughness is what Kandlikar did was? He simply replaced the diameter which is there in all the correlations for friction factor Reynolds number and so on, by what is called as the constricted diameter; that means, the flow is now see not the full diameter, but because of the roughness it is seeing reduced diameter that is d minus 2 times epsilon this equivalent roughness is useful in defining this constricted diameter and now if you re plot the Moody's diagram.

So, that is if you take the circular tube for example, for the laminar flow your f is equal to 64 by re now in terms of instead of diameter you replace everything in constricted flow diameter. So, it become 64 by re based on $c f$; that means, your velocity also gets modified your velocity has to increase in order to satisfy mass conservation and the diameter also is come down.

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Constricted flow model

- The flow and geometrical parameter based on the constricted flow diameter is given by:
$$\Delta p = \frac{2f_{cf} \rho u_{m,cf}^2 L}{D_{h,cf}}, \quad u_{m,cf} = \dot{m}/A_{cf} \quad \text{and} \quad Re_{cf} = \frac{\rho u_{m,cf} D_{h,cf}}{\mu}$$
- For fully developed laminar flow and $0 < \epsilon/D_{cf} < 0.15$ we have:
$$f_{cf} = \frac{Po}{Re_{cf}} \quad \text{where, Po is Poiseuille number}$$

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So, therefore, the definition of Reynolds number based on constricted flow will be $\rho u_{m,cf} D_{h,cf}$, where $u_{m,cf}$ is calculated as \dot{m}/A_{cf} . So, based on this you plot the Moody's diagram and all the micro channel the liquid flows in micro channels basically collapse on to this lines. So, very simple change in the definition of the diameter is sufficient to account for the roughness effects. So, all though we are not really modeling all the individual roughness and how they interact with the flow we are saying that the effect of the average roughness is to reduce the flow path way.

Therefore, we just reduce the diameter using this particular simple relation and then we find that this can predict the flow through rough micro channels very nicely therefore, depending on the amount roughness that you have you basically calculate the constricted diameter and for different values of roughness you can actually plot the modified Moody's chart. So, this is simple modification that the constructor flow model. So, your relation become it is still the same the Poiseuille number still the same only the definition of Reynolds number is now based on the diameter of the constricted flow and velocity of the constricted.

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Constricted flow model continued

- In the fully developed turbulent region for $0 < \epsilon/D_{cf} < 0.03$ Haaland (1983) gave the relation for friction factor as:

$$f_{cf} = \frac{f_{Darcy,cf}}{4} = \frac{1}{4} \left\{ -1.8 \log_{10} \left[\left(\frac{1}{3.7(D_{cf}/\epsilon + 2)} \right)^{1.11} + \frac{6.9}{Re_{cf} \left(\frac{D_{cf}}{D_{cf} + 2\epsilon} \right)} \right] \right\}^{-2} \left[\frac{1}{1 + \frac{2\epsilon}{D_{cf}}} \right]^5$$
- Schmitt and Kandlikar (2005) studied the effect of relative roughness in artificially and roughened rectangular microchannel.
- They concluded that for shorter pitches, with pitch to roughness ratios less than 5 the constricted model is able to predict friction factor well. For higher, it approaches the smooth channel value

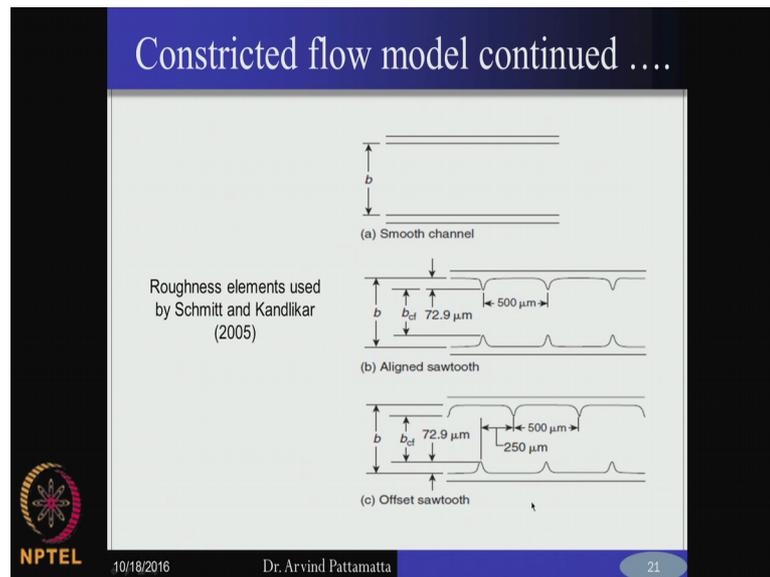


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This is for the laminar case similarly the constricted flow model there is a popular correlation. So, you can also use the standard correlation for turbulent flow and just use the constricted flow model diameter into that, but the 1 which is more commonly used was given by Haaland. This is a function of obviously the roughness to the constricted flow diameter. So, this correlation if plug in the value of epsilon by d c f should give you the value of the constricted flow friction factor in the turbulent regime. So, now, the kind of modified Moody's diagram has been found to be pretty good you also consider look into the pitch between the roughnesses.

So, that is how we are defining in an average pitch that is s we have s m 1, s m 2, s m 3. So, we take an a arithmetic average and call the mean spacing of profile area gravities and it has been found that if you have a pitch to roughness ratio less than five the constricted model is able to predict well. This is usually used if this satisfies this particular condition for higher values you go back to the smooth channel condition. So, there based on the lot of experimental studies which have been done this kind of an empirical formulation has been arrived at. Generally you first check what is the ratio of your pitch to the roughness height, if it is less than 5 than you use the constricted flow model are greater than 5 you use the smooth flow model.

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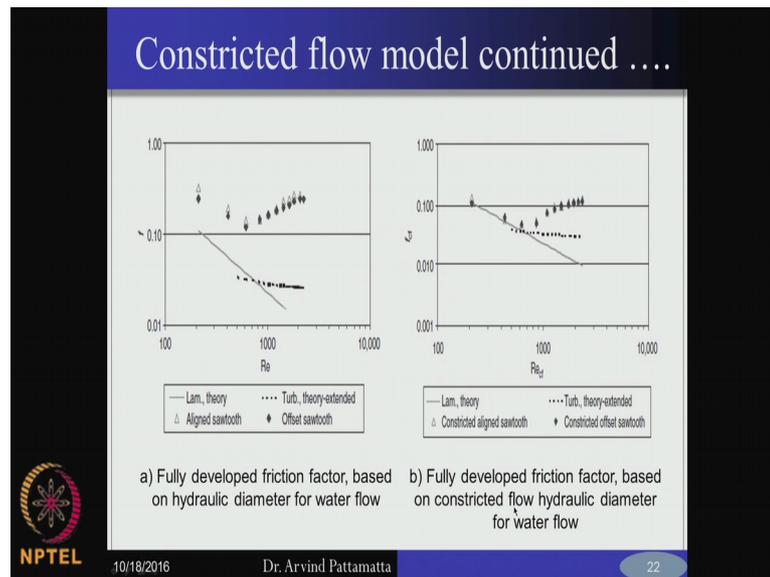


Yes, therefore you can have different kinds of the roughness elements and also the separation between them might also look different for example, this is the smooth channel now you can have roughness elements which are nicely spaced and the top and bottom uniformly spaced and if you calculate for example, the constricted flow width it may come down for example,. So, you might have a roughness element which is equivalent roughness seventy two point nine micron.

Accordingly your constricted flow width might come down there are also cases where there is an offset the top roughness is aligned in a different way from the bottom roughness. So, in that case the constricted flow width is calculated based on the 1 at the bottom for example, and you can calculate the width the constricted flow width as $b_c a b$ minus the 72.9 macro meter. So, in the aligned case you are calculating $b_c a b$ minus 2 times 72.9 because they are both aligned.

So, you just a use the 2 times the roughness height you calculate the constricted to diameter in the case were you have an offset, you consider only 1 side one roughness on 1 of the sides whether it is bottom or the top and then say $b_c a$ of is equal to b minus seventy two point nine. So, based on this the definition of constricted flow the diameter changes and the other correlations are used as it is.

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Now, just to show how they are constricted flow theory predicts well with the experiments now you have the fully developed friction factor you have the laminar theory which is given by this solid line and you have the different kinds of profiles you have aligned you have the offset and you use the constricted flow model. The constricted flow model shows completely a different profile compared to using the standard in a smooth diameter curve it shows like this, and now when you use the constricted flow theory.

So, the Moody's chart now get modified and it matches very well with, the experiment this line here is basically coming from $64 \text{ by } r e$ if you use a your normal diameter in that you see there is a considerable offset between the experiment and theory now if you use $64 \text{ by } r e c f$ you find that this line moves up and it agrees very closely with your particular experiments. So, this set is for the laminar the other set is for Turbuler you know once you go to turbulent flow your friction factor increases then you have to use the turbulent correlations appropriately. So, maybe you can use a correlation like this and if you plot it should kind of come like this dotted lines here. So, this is the turbulent theory extended.

Therefore, the constricted flow model is been fairly accurate we cannot say it is perfect model, but then a say very simple model, but just by changing the $r e$ definition of the diameter your able to match it very well with experimental data yes.

Student: (Refer Time: 19:50).

Correction factor in the sense.

Student: (Refer Time: 20:02).

This epsilon is usually of the order of few microns because this is the roughness height itself.

Student: (Refer Time: 20:11).

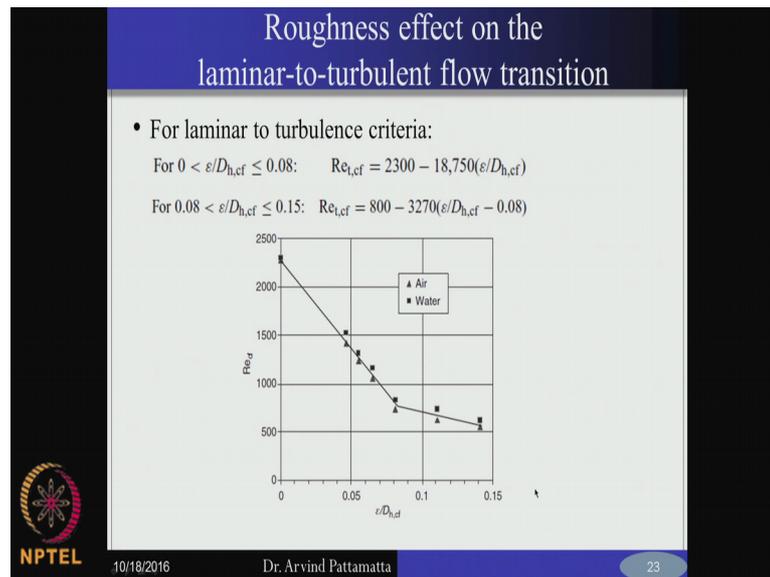
You mean here.

Student: (Refer Time: 20:16).

Yes, this is in the large scalar right. So, if you are now looking at the definition of Re_{cf} 2 things are changing not only the diameter, but also the velocity. So, you have to calculate; what is the mass flow rate with the smooth channel now with that you divide it by the constricted flow cross sectional area and you get the velocity. So, your velocity changes and your diameter also changes. So, then your Re_{cf} will change and stable.

Yes I mean you can do this as a calculation, I will give you problem were you will calculate the f based on smooth diameter normal case and then you use constricted flow diameter and you see how much difference it comes out to be, and it again depends on the extent of roughness you know for example, in this case you talking about roughness height which is 72 micron which is considerably very if your actual channel diameter is 200 microns and your constriction diameter now will be substantially small right. So, therefore, this might make a big difference in the friction factor. So, it depends on purely the amount of surface roughness.

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So, the other important effect is that you know there is a transition that is happening from laminar to turbulent regime and this roughness plays very important role in the transition right. So, if you have a smooth surface then the transition will happen when your Reynolds number is of the order of 2,300 that is 1 which is quoted in literature for smooth channel, but if you have a considerable channel roughness this Reynolds number can be reduced. So, for example, you look at the 1st correlation.

So, if you put your roughness height as 0 you get your transition as 2,300 which is your classical case, but for values of epsilon by d greater than 0 you have values which are less than 2,300. So, it will keep reducing progressively and this is a kind of linear reduction. So, therefore, the larger the value of epsilon you have a considerable reduction in the Reynolds number becomes turbulent and again there are 2 correlations here depending on the roughness height.

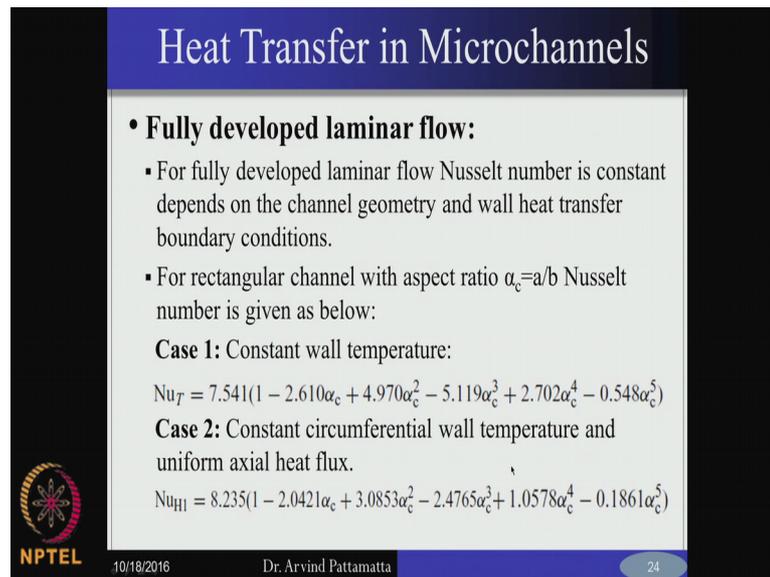
If your roughness height is very large again you do not want to use the 1st correlation that might actually over predict the reduction in the Reynolds number for transition and therefore, you switch to another correlation which will be more meaningful and then if you use this criteria for and you plot the transition Reynolds number as a function of the roughness height, it has been found that this correlation matches very well with experiments for conducted for both air and water.

So, whether it is again air or water you know. So, you have considerable effect of roughness on the transition right and therefore, we have to be careful how you define your transition criteria. So, you if your roughness is quite large you might actually talk about turbulent flows for channel size the Reynolds number of the order of even 700 or 800. So, it is not very surprising that your micro channels you start seeing turbulent flows for small Reynolds numbers even of the order of 700, 800 macro channels that is not common right. So, these are some of the important differences.

So, 1 is the effect of developing length the entrance and exit losses the other very important effect is the roughness which is often over looked, for macro channels we do not even bother roughness effects we directly use the standard correlations for smooth channel credit friction factor nasult number and more or less it matches well with experimental data. So, whenever we do experiments for tubes circular tubes you want to validate them. So, validate only with the standard smooth tube correlation, but if you do same experiment with micro channel you will find the values are quite off your fiction factor will be under predicted your fiction factor from experiment will be much higher than what you are predicting with your correlation and the other is your nasult number the other is your transition.

So, all these will be quite difficult different because of the roughness of it right. So, unless you account for the roughness properly you know; that means, you make sure that you have take in to account the restricted flow constricted flow model and you use a appropriate correlation based on the constricted flow model then only if you match with your experimental data otherwise your experimental will on the higher side as you see here compared to the theory. Similarly the transition many times we think that we are operating in the laminar regime. So, 700, 800, but your roughness is considerably large we might be actually seeing turbulent flow in the micro channels.

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Heat Transfer in Microchannels

- **Fully developed laminar flow:**
 - For fully developed laminar flow Nusselt number is constant depends on the channel geometry and wall heat transfer boundary conditions.
 - For rectangular channel with aspect ratio $\alpha_c = a/b$ Nusselt number is given as below:
 - Case 1:** Constant wall temperature:
$$Nu_T = 7.541(1 - 2.610\alpha_c + 4.970\alpha_c^2 - 5.119\alpha_c^3 + 2.702\alpha_c^4 - 0.548\alpha_c^5)$$
 - Case 2:** Constant circumferential wall temperature and uniform axial heat flux.
$$Nu_{HI} = 8.235(1 - 2.0421\alpha_c + 3.0853\alpha_c^2 - 2.4765\alpha_c^3 + 1.0578\alpha_c^4 - 0.1861\alpha_c^5)$$

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Therefore, these are some important aspects you know although these are common to macro and micro channels, but these effects are more significant if you look at the micro channel case.

Now, coming to the heat transfer so far we are all focusing on pressure drop friction factor these are the hydro dynamic characteristics now your heat transfer characteristics are also similarly effected strictly speaking if you have a smooth channel then you can go head and use your standard heat transfer nasult number correlation what do have derived for macro case. So, for the case of for example, constant wall flux in the laminar regime what is the fully developed nasult number 4.36 if you look at constant wall temperature 3.66. So, you can directly use that for micro channel also if it is hundred percent smooth laminar.

Similarly, the turbulent correlations also are valid, but what happens if you have roughness effect then again you have to look at difference. So, for fully developed laminar flow, now we are talking about when you when you are talking about these micro channels another important thing is most of them do not have a circular cross section compared to the macro in the case of macro ducts you are talking about circular ducts which are commonly available machine and you use these ducts with a circular cross section, but when you machine the micro channels usually they are etched from a normal surface.

So, when you etch them you usually make a rectangular cross section you do not etch into a circular cross section unless you talking about capillary tubes. Now, if you are talking about capillary tubes they are glass tubes again. So, they are not very good for heat transfer. So, then you have to find copper tubes which are of the order of 100 microns which is very difficult to make with the even profile and all that. So, usually whether you etch them with using lithography photo lithography is one option. So, that is also done on silicon wafer and usually they are all rectangular cross section with different aspect ratio usually the aspect ratio is not 1.

So, that is why all of these correlations are particularly emphasizing the use of different values of a by b because most 90 percent of these micro channels have rectangular cross section with large aspect ratios. So, otherwise we could simply use the standard circular cross sectional you know Darcy number and Nusselt number, but in this case since the emphasis on you know large aspect ratio rectangular cross section. So, we rewrite everything. So, may not be very familiar with all these correlations, but they have been developed for macro channel case from lot of experimental data they have been derived as a function of the aspect ratio.

For example, if you take the constant wall temperature case once again depending on the value of a if you are αc . So, if you put αc is equal to 1. So, you will get your limiting case of square cross section fully developed square cross section. So, this you may not have done this in theory course because we generally cover either flow between 2 parallel plates. So, 2 d case or flow through a duct which is circular cross section, but rectangular cross section also have an analytical solution possible and if you do this for different values of aspect ratios you can actually build a correlation like this. So, this gives you the fully developed Nusselt number correlation as the function of αc and similarly you have one for uniform heat flux.

So, uniform heat flux case should be always higher than the uniform wall temperature case. So, the same thing can be used for micro channel provided it is smooth, and you also have correlations for thermally developing flows now these are fully developed both hydro dynamically and thermally. So therefore, these are constant values of Nusselt numbers right for a given cross section aspect ratio and so on.

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Heat Transfer in Microchannels

Case3: Constant wall heat flux both circumferentially and axially

$$\text{Nu}_{H2} = 8.235(1 - 10.6044\alpha_c + 61.1755\alpha_c^2 - 155.1803\alpha_c^3 + 176.9203\alpha_c^4 - 72.9236\alpha_c^5)$$

- **Thermally developing flow:**
 - Thermal entry length is given by:
$$\frac{L_t}{D_h} = c \text{Re Pr}$$
 - For circular tube $c = 0.05$ and for rectangular tubes $c = 0.1$
 - The local Nusselt number in the developing region of a circular tube is given by:
$$\text{Nu}_x = 4.363 + 8.68(10^3 x^*)^{-0.506} e^{-41x^*} \quad \text{where, } x^* = \frac{x/D_h}{\text{Re Pr}}$$

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But if you are talking about thermally developing flows; that means, thermal developing length has to be calculated. So, you calculated the hydro dynamic developing length as .05 times Re Pr right. So, now, for calculating thermally developing length it has to be .05 times Re Pr if you have frontal equal to 1 both the hydro dynamic boundary layers in thermal boundary layers grow simultaneously at the same rate. So, there is no problem. So, both developing lengths will be same if your frontal number is larger than 1 a thermal boundary layer will be grow faster and vice versa.

And this .05 again you have to be careful it is usually use for circular tubes. So, for rectangular tubes this value will be slightly larger so; that means, for circular tubes the thermal boundary layer length hydro dynamic boundary layer length developing length is shorter whereas, if you change the cross section to rectangle this now becomes slightly large longer length. So, you have to be also aware of these changes in the constants and then you can use your correlation like the grits correlation grits correlation is one of the standard correlation for developing flows and for circular tube this is given as the function of the non dimensional position. So, at any non dimensional position you can in the developing regime you can estimate the values of nasult number if you put your X star to infinity. So, if you have very large values of X star what happens.

So, this second term would go to 0 then it goes to the nasult number for a fully developed regime that is the constant values of 4.36 close to that 4 point should be close to 4.36.

This is for circular tube again it depends on what kind of channel cross section most likely you may not have a circular tube you may have a rectangular cross sections. So, then you have to use appropriate correlation for the developing flow for a rectangular cross section. So, you have to use some correlation available in the literature where they have given some based on lot of experiments they have given in empirical curve fit for the developing regime for non circular cross section they are not. So, easy to find, but I mean you can make an approximation by assuming an equivalent circular hydraulic diameter and calculating it if you do not find you know suitable correlation for rectangular chance.

But for the developed case you have the analytical equation. So, there is no problem so. So, therefore, I think to quickly summarize this problem let us. So, now, again you have for the turbulent regime.

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Heat transfer in transition and turbulent flow regions

- **For developing turbulent region:**
 - For $0.5 \leq Pr \leq 1.5$

$$Nu = 0.0214[1.0 + (D_h/x)^{2/3}][Re^{0.8} - 100]Pr^{0.4}$$
 - For $1.5 \leq Pr \leq 500$

$$Nu = 0.012[1.0 + (D_h/x)^{2/3}][Re^{0.87} - 280]Pr^{0.4}$$
- **For turbulent region:**

$$Nu_{Gn} = \frac{(f/8)(Re - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)} \quad F = CRe(1 - (D/D_o)^2)$$

$$Nu = Nu_{Gn}(1 + F) \quad f = (1.82 \log(Re) - 1.64)^{-2}$$

where $C = 7.6 \times 10^{-5}$ and $D_o = 1.164$ mm and Nu_{Gn} is given by Gnielinshi's correlation.


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Again similar to the hydro dynamic problem where you had friction factor calculation for laminar and turbulent now you have nusselt number calculation for the fully developed laminar regime developing laminar regime and then you go to the turbulent regime. For developing turbulent regime you use is appropriate correlations and for developed fully develop turbulent regime. So, this is the correlation by Gnielinshi. So, these are standard correlation available for macro channel case macro channel heat transfer for turbulent

flows. So, this can be used for wide range of Reynolds number. So, these this is call the Gnielinshi's correlation.

And good thing is they are plotted as a function of both Reynolds number as well as friction factor. So, depends on the cross section your friction factor could be different. So, if you are talking about circular cross section you have a different pustule number if you have a rectangular cross section. So, depending on the cross sectional shape you can substitute the corresponding value of f and get the value of nasult numbers from this. So, it is not only particularly valid for circular cross section, but can be applied to tubes with different cross sectional areas and shapes.

However, accounts for the increase in heat transfer only through f ; that means, your f should be a function of the roughness height. So, if you know f has a function of roughness height you substitute this and you get the equivalent change in the nasult number also as a function of the roughness. So, the roughness is implicitly there in terms of the friction factor it is not explicitly into this particular correlation, but nevertheless this correlation has been found to be quite for predicting the micro channel case also. So, they have just used it and again you have to be careful if you have lot of roughness you have to use replace all these r_e with $r_{e,c}$ constricted Reynolds number constricted flow Reynolds number.

So therefore, your f will also become f_c . So, f calculated based on constricted flow Reynolds number and the r_e here also will become $r_{e,c}$ and accordingly your nasult number will also get modified is that clear. This is basically looking, I mean in the case of nasult number we have not developed anything specifically for micro channels, all of these correlations are already there for micro channel case that we have directly taken only that you have to be aware that for example, if you are have dealing with roughness then you use the same concept of constricted flow Reynolds number in to this correlation. So, whereas, if you go to the laminar case. So, we have constant values here. So, this does not tell you what the effect of roughness is for example, right. So, whatever Kandlikar's model is only limited to the friction factor prediction the use of constricted flow of Reynolds numbers, but when you go to heat transfer we do not have a very satisfactory theory to account for the roughness on to the nasult number. So, we still use the same nasult number correlations as the macro channel case, but if you really ask the

question how the epsilon by d plays a role on nasult number. So, these correlations does not have any of those effects built in.

But whereas, for the at least the turbulent flow you have the Gnielinkis correlation which implicitly account for the roughness. So, again there are some other effects which could be important like.

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Axial Conduction Effects

- In microchannels the dimensions of the channel wall are comparable to the channel dimensions and so the heat transfer in the walls cannot be neglected.
- Lin and Kandlikar (2012) considered the effect of axial conduction and derived the expression for the Nusselt number.

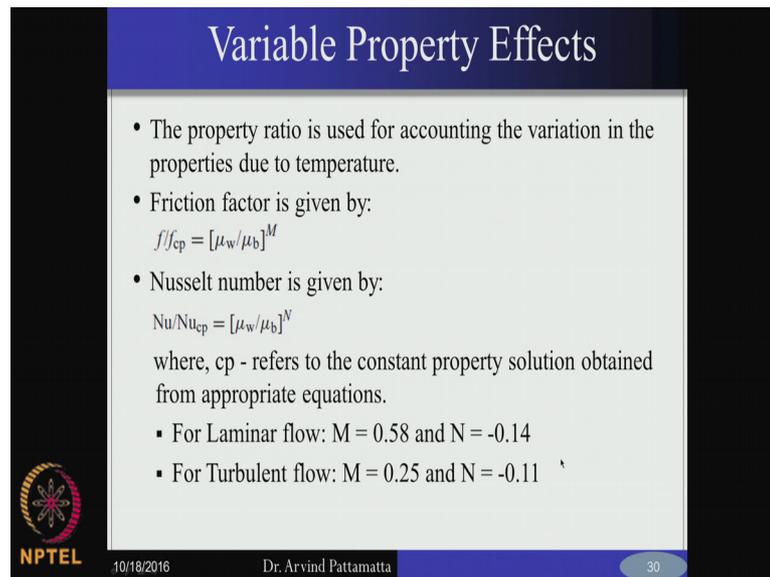
$$\frac{Nu_{ko}}{Nu_{th}} = \frac{1}{1 + 4(k_s A_{h,s} Nu_{th} / k_f A_f (RePr)^2)} \quad \text{where,}$$

Nu_{ko} is Nu neglecting axial conduction effects,
 Nu_{th} is Nu with axial conduction effects,
 k_s, k_f thermal conductivity of solid and liquid respectively,
 $A_{h,s}$ and A_f are heat conduction area in the wall and cross-section area for the fluid flow, respectively.


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For example; the conduction through the walls of the tube, the axial conduction through the walls of the tube; so that is something which could be accounted for the other is called the variable property effects.

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The slide is titled "Variable Property Effects" and contains the following content:

- The property ratio is used for accounting the variation in the properties due to temperature.
- Friction factor is given by:
$$f/f_{cp} = [\mu_w/\mu_b]^M$$
- Nusselt number is given by:
$$Nu/Nu_{cp} = [\mu_w/\mu_b]^N$$

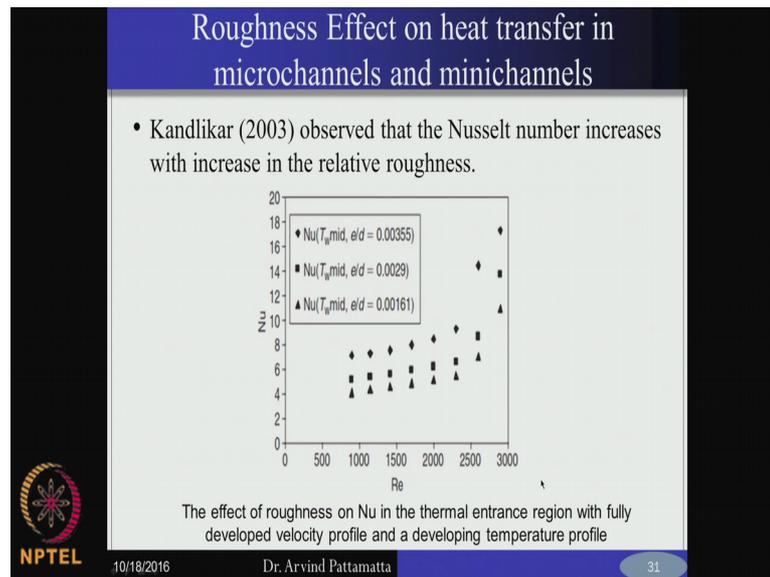
where, cp - refers to the constant property solution obtained from appropriate equations.
- For Laminar flow: $M = 0.58$ and $N = -0.14$
- For Turbulent flow: $M = 0.25$ and $N = -0.11$

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So, if you have variation in viscosity for example, viscosity ratios could be very high because the temperature at the wall could be quite different from the bulk temperature. So, in that case you apply some simple correction factors; that means, you first assume constant property and calculate the friction factor and Gnielinski number and then correct this for variable property by using some simple correction factors like this. So, once you know the constant property value you get the variable property value by multiplying by this correction factor. So, these are again common to macro and micro channel case even in macro channels the assumption of uniform or constant property is not accurate.

So very high; suppose you have very high heat fluxes. So, the wall temperature will be quite large compare to the bulk temperature. So, in that cases you correct your friction factor because there is a variation because of the non uniformity in the correction in the in the viscosity. So, you correct your friction factor and nasult number based on this where these are all just empirical correlations.

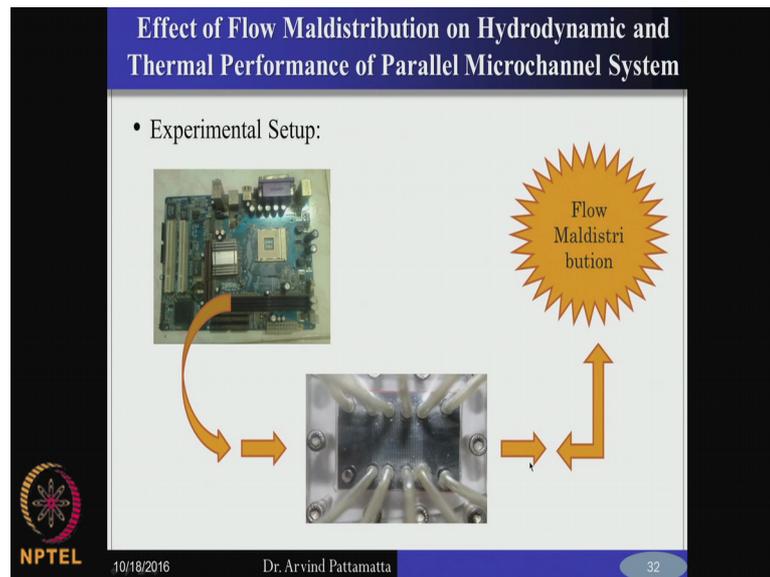
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So, I think coming to the effect of roughness. So, although there has not been properly correlated in text books. So, some experiments have been conducted to understand the relative roughness effect. So, you have three different roughness here for example, $f_{L,0}$ by d varying from .001 to .03 and then you plot the Nusselt number and you will see that for the same Reynolds number you know the larger the roughness. So, your higher the value of Nusselt number right. So, therefore, roughness definitely plays a very important role, but you see all these have been plotted for the turbulent regime.

So, the Gnielinski correlation might be able to predict this well, but if you look at the laminar regime for you know value less than 1000. So, there has not been very satisfactory approach to incorporate the roughness effect into micro channels. So, definitely there is the considerable effect of roughness in the case of micro channels both not only on the friction factor, but also into the Nusselt number. So, experimentally it has been confirmed only how much accurately we can consider this into the theory is not very clear, for at least Nusselt number point of you for friction factor point of you the constricted flow model seems to work reasonably.

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With that I think the theory behind the liquid flows in micro channels is complete I just want to briefly go over some of the experiments we conducted in our lab with the micro channels in the next 5 minutes are. So, so we have an interesting problem of electronic chip cooling. So, we want to dissipate the heat that is arising from these micro processors. So, we have a chip packaging on we want to mount parallel micro channel system. So, this is work of 1 of my ms student.

So, we have several of these micro channels in a parallel assembly as you can see. So, and you have an inlet manifold you have an exit manifold and you have a all these pressure tappings to measure the pressure drop across the micro channels at different location. So, what we want to do in the study is usually when you have a parallel micro channels system the distribution of the flow is not uniform across all the channels, as you know that that is the pressure drop from the inlet to the end of this manifold and this manifold pressure drop will result in non uniformity in the flow of distribution through the channels.

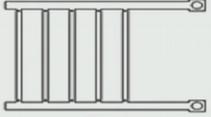
So, we want see how this is present in different types of micro channel configuration and how this effects the heat transfer because the temperature also is going to be non uniform because of risk. This was a study we carried out with three different configurations. So, what you see here we did a very clever design that means.

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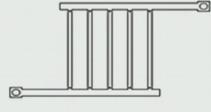
Types of Test Sections

- To study the effect of inlet outlet configurations U, Z, I type of channels are considered.

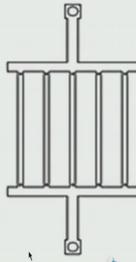
'U' type of microchannel



'Z' type of microchannel



'I' type of microchannel

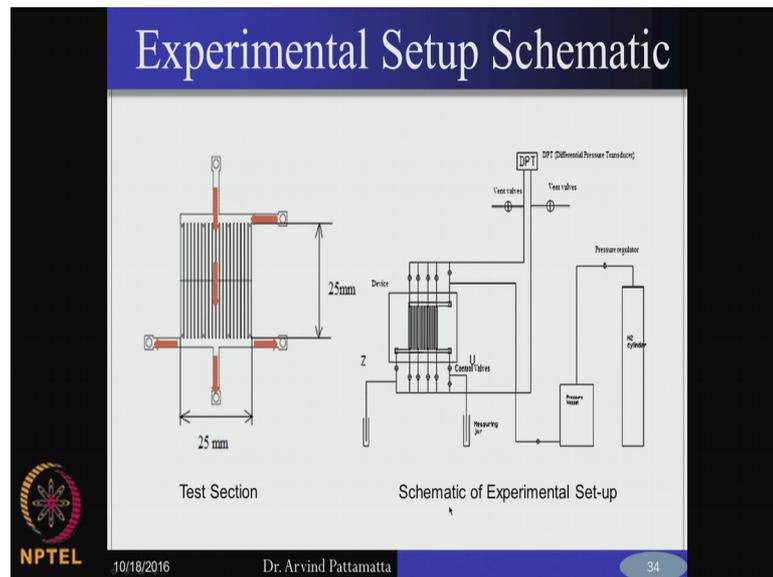


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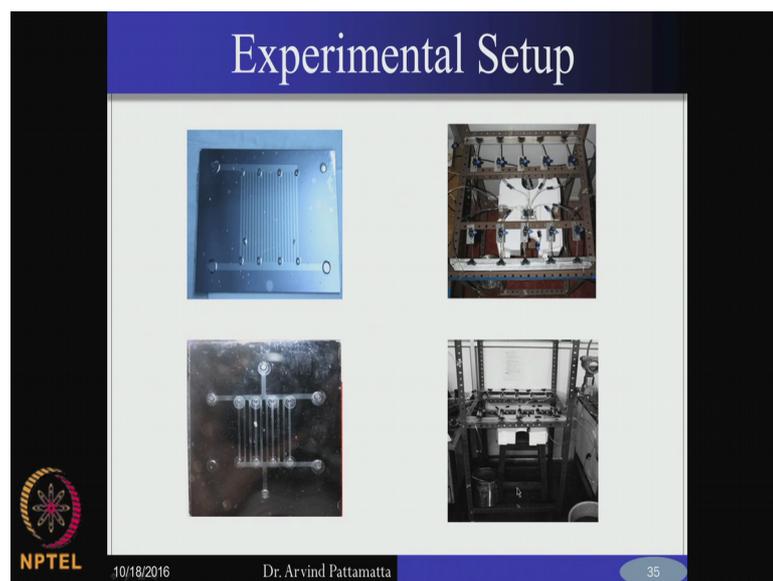
We had only 1 micro channel system which simultaneously incorporates all these 3 designs. So, we had to flow for example, in the case of converting this was the universal design that we had and to convert this into a U type we close the top and bottom and we open these sides we close this side this side this and this becomes U type of micro channel system, the other Z type is we close this end and we close this end we close the top and bottom and this becomes a Z type micro channel if you open the top and bottom you close all the other ends it becomes an I type of micro channel.

That means, the flow comes like this from the inlet and then spreads across through all these. So, I mean you can intuitively observe that in the case of I type the maximum flow will be at the center and the minimum flow. So, it will be a symmetric distribution about center line whereas, if you talk about the type you have there is the maximum pressure drop. So, you have the minimum pressure here and again you have a minimum pressure here. So, you have the minimum pressure drop at this channel, whereas you have maximum pressure here you have the lowest pressure here. Therefore maximum pressure drop will be here. So, you have the maximum flow for this channel and least flow here and vice versa for the Z type.

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Therefore, we considered. So, we had Testric and we measured the pressure difference in the first study. So, this was how etched the micro channel system using photolithography in the Mens lab.

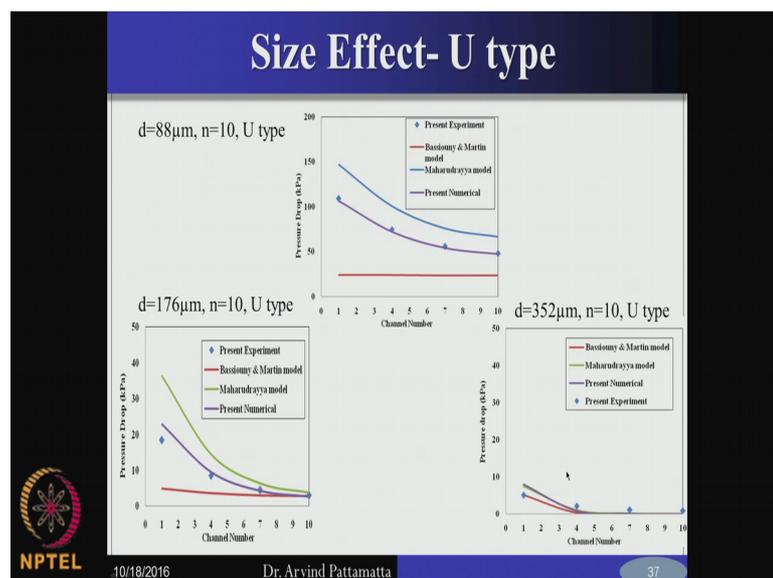
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Method of preparation Micro channel on Silicon wafer

- Cleaned bare Si wafer
- Oxide coating
- PPR is coated and spun over the surface
- Mask is placed over the wafer and the channel portion is exposed to UV radiation
- PPR is removed by dipping the wafer in dipping in KOH solution --- Developing
- Coating PPR on the back side of wafer
- Removing Oxide layer by BHF
- Cleaning with Acetone
- Etching by KOH
- Removing Oxide layer by HF

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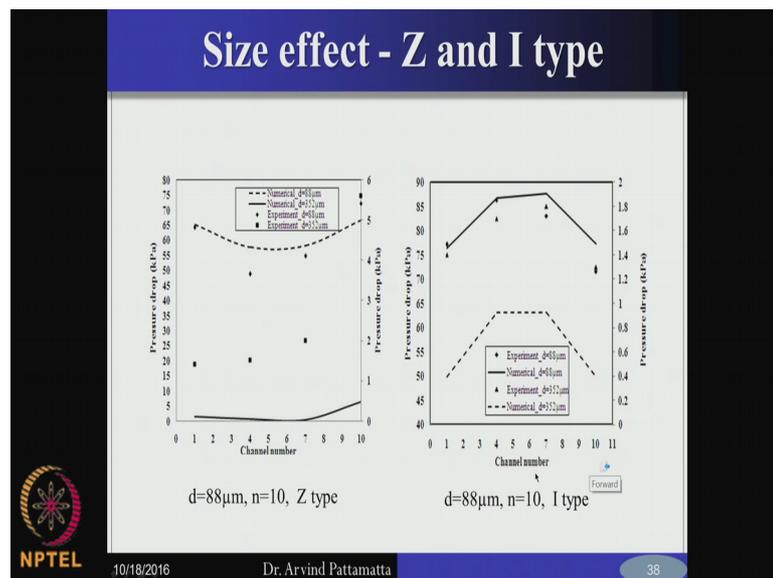
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And this is the procedure for making the mass and then it chin process and then we compared our results for pressure drop and you see that we are able to agree for at least you know 88 micron channel we had some models theoretical model, which are available to estimate the pressure drop for a parallel micro channel system it is not just one single micro channel as a function of the channel number we want to validate and we got good agreement and we also modified the model which was already there and we found that it agrees better.

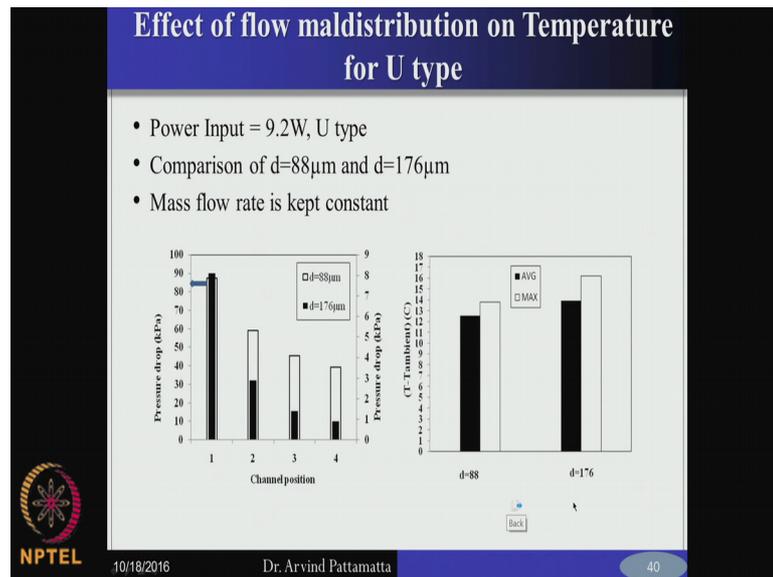
So, thing is there are 2 models one which does not account for the pressure losses or discuss frictional losses to the manifold and that is called the Basavani and Matin model. So, which under predicts the pressure drop and the other which actually does not account for the losses to the micro channel. So, that over predicts. Therefore, it is important to consider both the pressure losses through the micro channel as well as through the manifold.

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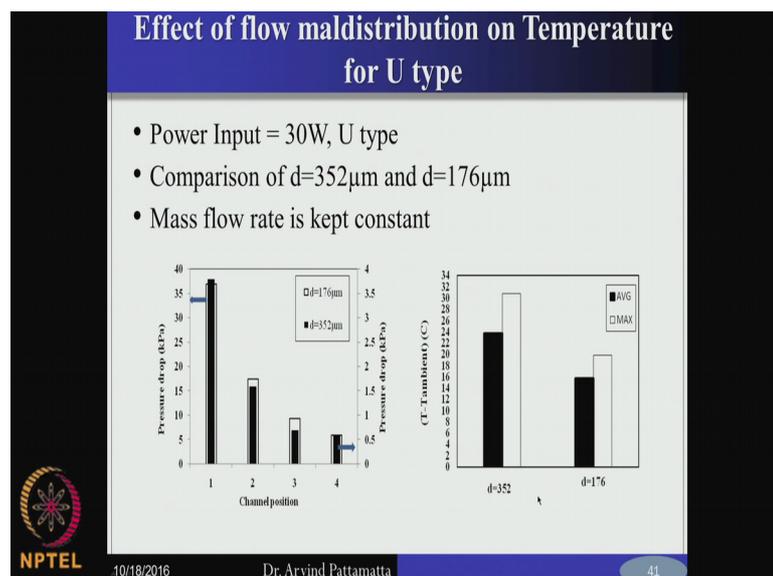
So, we did this for different channel configuration you know hundred microns 178 and 300 and 52.

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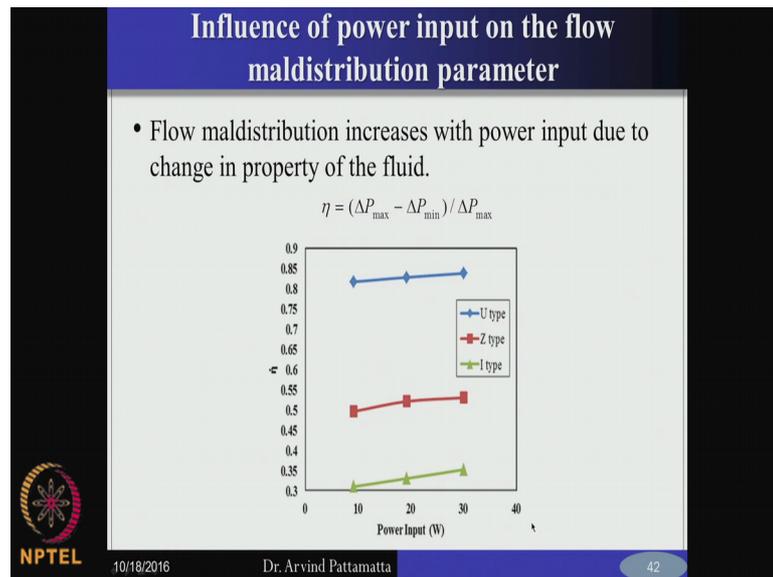


Apart from that we also looked at the keeping the power input constant we wanted to compare 2 different channel configurations 88 micron 176 micron and see what is the effect of the flow Maldistribution on temperature. And we when you measure the average temperature and the maximum temperature we find that between the 88 and 176 case it makes considerable difference in the average and maximum temperature. So, making the channels size smaller is beneficial in this case to reduce these hotspot.

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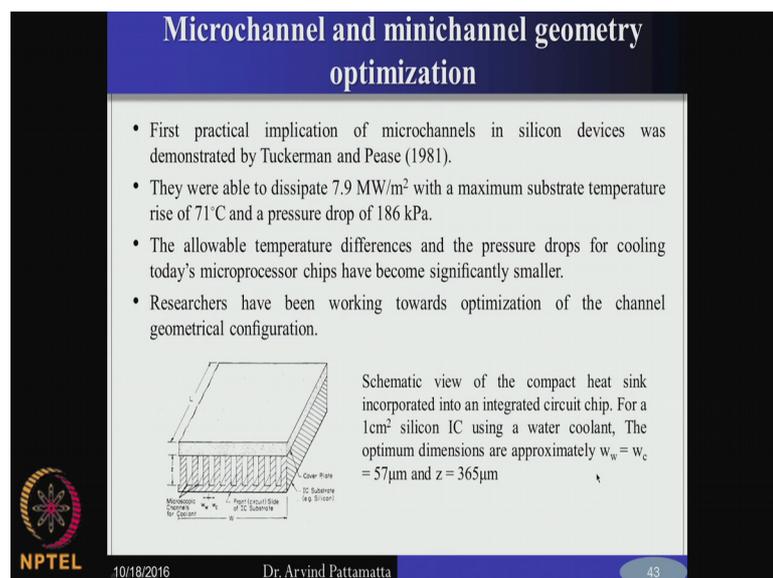


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So, this is the important part of the study that we that we did, and again we looked at the influence of variable property for example, if you have a power input the properties are going to be different and therefore, the flow Maldistribution might be varying. Therefore, we check this as a function of power input and we again see that is reasonably a strong function of input power at very low power the Maldistribution this somewhat reduced.

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All these are generally used for the state of the art literature when you want to incorporate micro channels into electronic cooling. So, you have to look at not just 1

single micro channel, but as parallel system of these channels. And we have to also simultaneously look at both the hydro dynamics and the temperature distribution.

So, people are also looking at what is call the optimization problem. So, you want to optimize how many numbers of channels you need what should be the diameter and these effects in so on. So, with this will conclude this discussion.

Thank you.