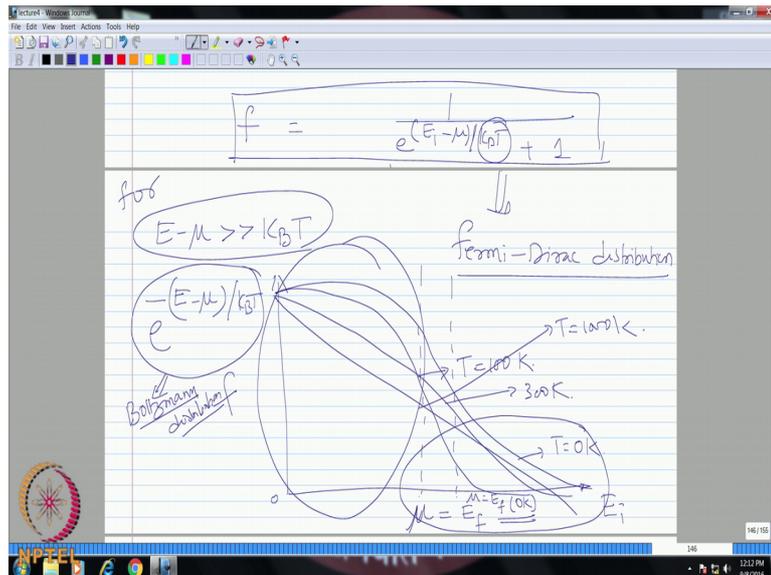


**Micro and Nanoscale Energy Transport**  
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**Lecture – 18**  
**Fundamentals of Statistical Thermodynamics Part 3**

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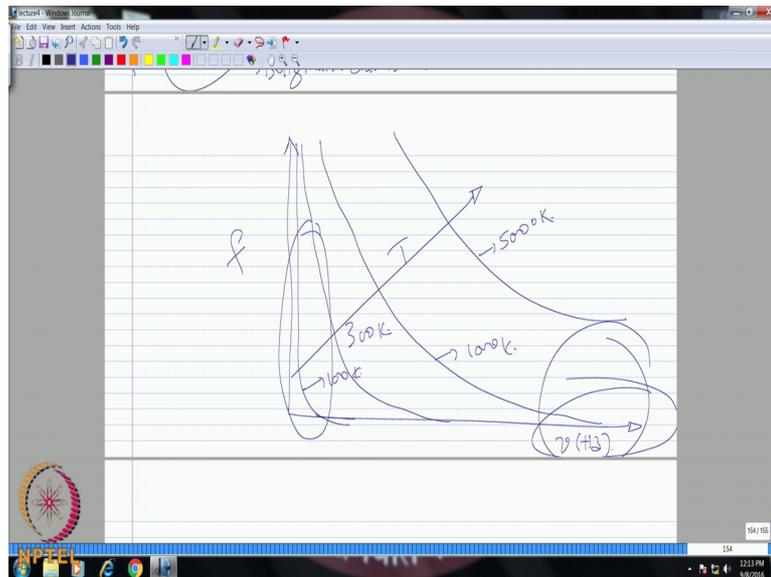
We were looking at deriving the distribution functions. So, depending on the ensemble whether it is micro canonical or grand canonical, we saw that we can fit for example, the electrons in the regime of canonical ensemble and the phonons and photons and molecules in the grand canonical ensemble. So, therefore, if you look at the distribution functions that we have derived the electrons fall under what we call as the Fermi Dirac distribution function and, similarly the phonons and photons. Photon is just parallel extension of what we did for phonons.

So, they also fall under what we call as the Bose Einstein distribution function. The molecules on the other hand fall under the classical distribution function which is referred to as the max well Boltzmann distribution. So, this is classical because we do not have any quantization of the energy levels. So, you have therefore, this is a continuous distribution function. So, therefore, if you are looking at certain asymptotic limits for example, if you focus on the Fermi Dirac distribution function and look at extremely large values of energy or you are looking at values of temperature, which are

quite small in these cases you have the distribution function collapsing to the Boltzmann distribution function.

So, nevertheless what it gives you is the occupancy of electrons at different energy levels. So, it tells that the probability of finding the electrons at lower energy levels are much higher. So, therefore, the number distribution is always towards the lower energy levels. So, this is why we focus on this portion. So, this is the portion which is dominating the electron population. Whereas, if you are looking at higher and higher energy states the chances of filling them with electrons are lower and lesser and lesser and finally, the highest energy states are completely empty the distribution function will be going to 0.

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Similarly, if you look at the nature of the distribution of phonons or bosons phonons and photons, they are called bosons which obey the Bose Einstein distribution function. Once again it tells you about the nature of the occupancy of these energy carriers following this particular distribution function. And again for the limiting case where you are looking at values of large frequencies or large energy values energy is  $h \nu$  corresponding to the frequency  $\nu$ . So, this again collapses to the Boltzmann distribution function. So, what we will do is quickly summarize the distributions and the corresponding energy carriers they represented.

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Maxwell-Boltzmann Statistics (distribution)	Bose-Einstein statistics	Fermi-Dirac statistics
Particles are distinguishable there is no limit to the no. of particles on each energy level	Particles are indistinguishable	Particles are indistinguishable Each quantum state is occupied by an fermion.
ideal gases (molecules)	Photons & Phonons	electrons

So, we will look at Maxwell Boltzmann Statistics. So, I mean statistics distribution they are interchangeably used, and you have the Bose Einstein Statistics, and have the Fermi Dirac.

So, already many of your familiar with Bose Einstein because our guest house is also named after is to people right. So, s n Bose fundamentally derived this distribution function and he send this work to Einstein, who could really appreciate the magnitude of that distribution function and how it impacted during the time condensed matter physics condensed matter physics is mostly dealing with this kind of Bose Einstein bosons Bose Einstein distribution function and so on. So, this condensed matter physics itself means evolved out of basically this work by s n Bose. So, this therefore, has being credited this particles have been credited with the name of Bose as bosons.

So, even the so called particle they there hicks particle is also boson in condense matter physics. So, it is called hits boson right and of course, Fermi Dirac both are pioneers in the field of you know looking at the distribution and so on, electronic theory of metals materials and max Boltzmann by himself was looking at the nature of the atom stick models and relating them to macro scales. So, therefore, he did similar contribution to field of statistical thermodynamics. So, therefore, these are all you know pioneers in the respective areas, and if you are looking at Maxwell Boltzmann distribution function. So, these are applied to particles which are distinguishable. I think I have already mentioned

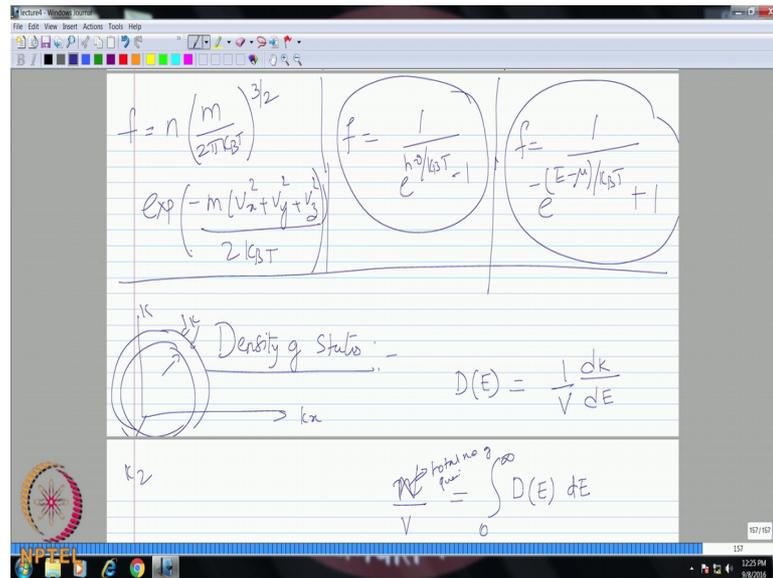
this once before, but I am reiterating again and also there is no limit to the number of particles occupying a particular energy level. Quite contrast to that is the Fermi Diracs statistics where particles are indistinguishable.

So, electrons look alike where as molecules you can distinguish them. So, the indistinguishable particles, and again how bolt the filling of quantum states. So, only one electron with a particular spin can occupy a described quantum state. So, this is therefore, according to the Pauli exclusion principle. So, each quantum state is occupied by an electron. So, in general fermions did not be just only electrons, but you can say by any fermion for electrons you have the spin. So, therefore, plus half you will have 1 particular quantum state minus half another quantum state, but again they will exhibit degeneracy. So, all other quantum states or plus half and minus half. So, you have the same value of energy.

Now, coming to the Bose Einstein, these particles like phonons and photons they are also indistinguishable, they all by you cannot even therefore, visual I mean we cannot even spot phonon or photon they are all virtual particles. So, they are indistinguishable and again, similar to the Maxwell Boltzmann that is no limit to the number of particles and occupying each and every energy level. So, that is the way when we therefore, try to derived the statistics. So, when we calculated the partition function is summary that for these cases over all the possible energy states. So, any particle any number of particles can occupy a given energy state.

So, where as for the Fermi Dirac we sum them either a particle which can occupy a given quantum state or it can be empty. So, no particle can exist. So, this is basically the kind of particles and for example. So, these Maxwell Boltzmann will be applied quite easily to ideal gases. So, these are gas molecules for example. So, while you are talking about Fermi Dirac in our context these are electrons and Bose Einstein these are phonons in solids and for the radiation heat transport you can also include photons.

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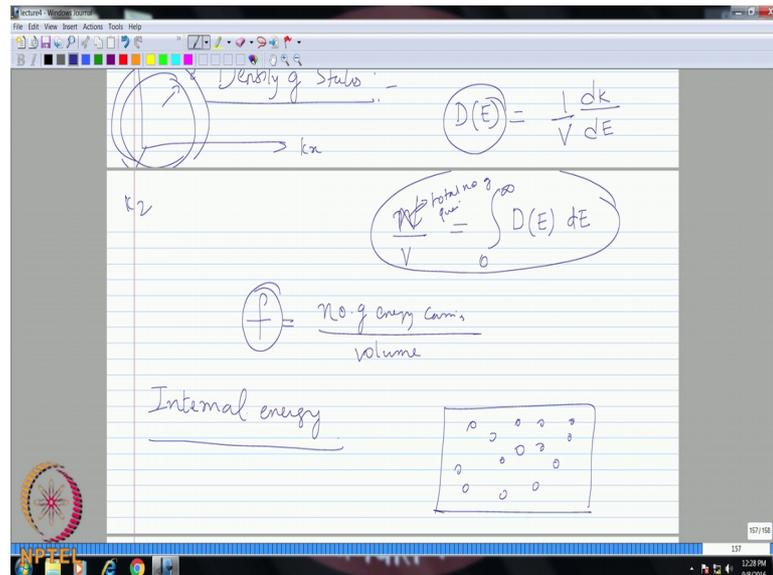
So, in summary their distribution functions look like for example, in case of Maxwell Boltzmann you have  $n$  number of particles per unit volume. So, you have  $m$  by  $2\pi k B T$  over  $3$  by  $2$  where  $m$  is the mass of the molecule right. So, exponential minus  $m V x$  square plus  $V y$  square plus  $V z$  square by  $2 K B T$ . So, this gives you the distribution function density, distribution of particle density of this per unit volume. Similarly, for the phonons and photons you have  $1$  by  $e$  power  $h \nu$  by  $K B T$  minus  $1$  for electrons you have  $1$  by exponential minus  $e$  minus  $\mu$  by  $K B T$  plus  $1$ .

So, if you therefore, look at the difference between the Bose Einstein and Fermi Dirac; if you assume that this  $1$  is energy by  $K B T$ , their main differentiated is you have plus  $1$  here and you have a minus  $1$  here. So, this makes all the difference in the distribution nature of the distribution function. So, I think with this now you should be able to have a better grasp about the overall statistical distributions of the different energy carriers. So, we will now use this concept of distribution to go ahead, and now define some macro scale property. So, if I mean from what we did from quantum mechanics and solid state physics was to introduce the concept of what density of states. I hope you can remember and recall how we defined density of states.

So, it can be something like  $D$  of  $V$  or  $D$  of  $\omega$  depending on and talking about electrons or phonons. So, these are basically number of quantum states. So, what we have is you have a quantum space quantum wave vectors space  $K x K y K z$  and now

you are just plotting all the possible quantum states, that are available which are actually discrete points, now you basically slice them into you know nice spheres and within a spherical band you are counting how many quantum states exist. So, therefore, these are basically number of quantum states per unit energy interval and these are quantum states per unit volume of the solid. So, therefore, you have also  $1/V$ .

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So, this is how we have defined. So, therefore, how do what we get if you simply integrate  $D$  of  $E$   $dE$ .

So, that means, in the  $K_x \times K_y \times K_z$  space. So, you have now some band. So, this is your  $D$   $K$ . So, like this I cut out several bands and now I want to integrate this over the entire energy spectrum. So, you have integral  $D$  of  $E$   $dE$ . So, what should this give me? So, the total number of quantum states per unit volume, of the entire crystal correct. So, this is I would say put your capital  $N$ . So, this is your total number of quantum states. If I integrate this all the way from say zero to infinity or if I want for example, the for case in case of metals, I want to find out how many quantum states exist between the Fermi level and the end of a conduction band or the starting of conduction band to where the Fermi level is.

So, in that case I can do this integral all the way from conduction band beginning to Fermi level, and that will give me how many therefore, electrons exist at 0 Kelvin which are free to move in the conduction band in metals, how many quantum states exist

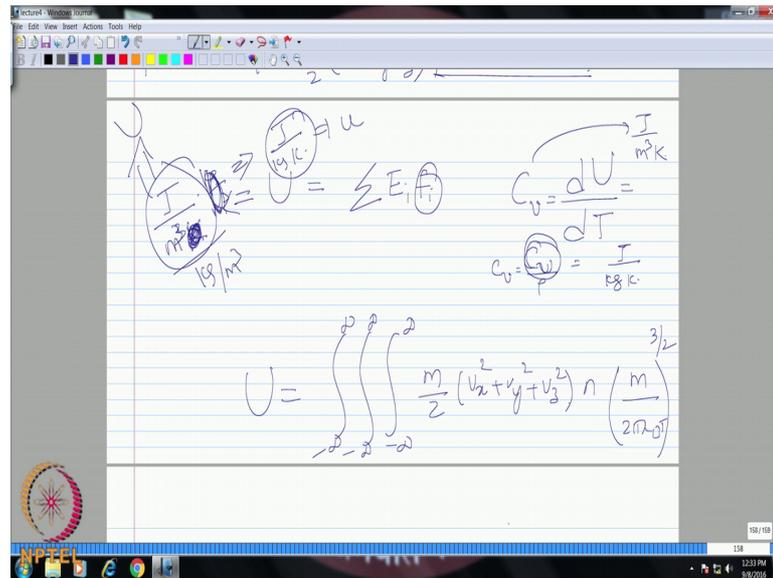
basically. So, this gives you information about only the quantum states. So, now, apart from that we have defined our number density that is number of energy carriers per unit volume occupying a particular energy level.

So, therefore, if you take the product of  $f$  and  $D$  of  $E$ , density of states give you what are the available quantum states. Now distribution function tells you how the energy carriers are distributed across the quantum states. So, if you take convolution of  $f$  and  $D$ . So, that will directly give you the number of particles distributed in each quantum state and therefore, with that we can therefore, count if you take product of  $f$  and  $D$  and integrate it over the entire energy spectrum. We can find out how many energy carriers are actually present, which are occupying all these states, and with that we can also calculate other macro scale properties such as internal energy.

So, this is the way we will now go ahead. So, let us first calculate the internal energy. So, now, we are you see how statistical thermodynamics is connecting us from the micro states to the macro states. So, now, internal energy is now the macroscopic property for the entire system. So, if you are assuming say a box filled with gas molecules you have all this molecules present. So, for each molecule you know the kinetic energy of translation let us assume these are monatomic. So, therefore, only translation kinetic energy is and you also know what are the possible quantum states for these.

These are this is the classical distribution functions. So, you do not have to worry about the quantum state here, but the Maxwell Boltzmann distribution tells you at each and every given now velocity interval how many particles can actually fit in this.

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So, therefore, from this information how can we go ahead and calculate the internal energy for the entire system per unit volume. How do we think we can do that you know the energy for each and every molecule  $E_i$  for example, which is half  $m v^2$  square 3 dimensional momentum space. So, this is your internal energy per molecule. So, now, from this information, how do we calculate the internal energy for the entire system?

We also know the distribution function for the molecule. So, you multiply  $E_i f_i$ . So, this will be for 1 molecule. So, therefore, this has to be summed over all the molecules this is also for 1 state. So, this will tell you particular distribution function that means, number of particles occupying the particular state. So, this is a generic formulation for a classical distribution function it is an integral this continuous, but if you look at any distribution like phonon or electron. So, it tells you how many numbers of electrons or phonons can occupy given state. So, therefore, we know the energy of that state. So, therefore, this has to be summed over all the states.

So, if you sum this over all the states. So, now, distribution function is per unit volume now number distribution per unit volume. So, this resulting internal energy will be volumetric internal energy. So, the units of this will be what? Meter cube Kelvin therefore, if you multiply this by density kg per meter cube. So, you get your specific internal energy that is Joule. So, you have divided it by density. So, you have kg per meter cube then you get joule per kg Kelvin. So, this will be your specific internal energy

usually use the small  $u$  to denote that, and this is your total internal energy, but per unit volume. So, this is volumetric.

So, based on this you can go 1 step further and calculate what is your heat capacity, if you are using the volumetric internal energy, and you differentiate it with respect to temperature. So, this gives your volumetric heat capacity. So, that should have the units of Joule per meter cube Kelvin. So, this is just Joule per meter cube here. So, therefore, if you differentiate it this should give you Joule per meter cube Kelvin. So, therefore, if you now divide this by density you will get your specific heat capacity which will be Joule per k g Kelvin. So, basically all you need to know. So, for macro scale system you will know the density so obviously, that is the known information.

So, all you need to know is the volumetric heat capacity is this is a very important property of this system. So, let us apply our Maxwell Boltzmann distribution. So, in the in that case the summation now becomes an integral because it is a continuous distribution. So, you have a triple integral over all the 3 velocities minus infinity to infinity. So, and the energy itself is  $n$  by  $\frac{1}{2} m (v_x^2 + v_y^2 + v_z^2)$  plus  $V$  y square plus  $V$  z square into  $n$  is the number density of molecules that is number of molecules per unit volume of that entire box. So,  $m$  by  $2 \pi K_B T$  the whole power  $3$  by  $2$  times the distribution function.

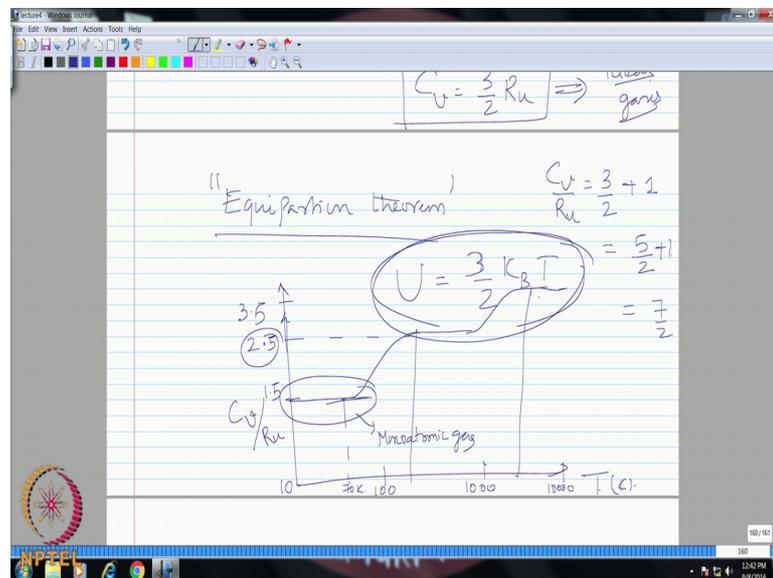
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The image shows a screenshot of a presentation slide with handwritten mathematical derivations. At the top right, the expression  $e^{-2K_B T}$  is written. Below it, the internal energy  $U$  is given as  $U = \frac{3}{2} n K_B T$ . The heat capacity  $C_v$  is derived as  $C_v = \frac{dU}{dT} = \frac{3}{2} n k_B$ . An arrow points from  $n k_B$  to  $R_u$ , which is labeled as the universal gas constant. Below this, the equation  $C_v = \frac{3}{2} R_u$  is boxed, with an arrow pointing to the text "ideal gas". The slide is displayed in a software window titled "Lecture4 - Windows Journal".

Now, which is  $e$  power minus  $m V_x$  square plus  $V_y$  square plus  $V_z$  square divided by  $K_B$  times  $K_B T$ , and the integral is over  $D V_x D V_y D V_z$  is that. So, therefore, once you know your distribution function you can directly calculate your internal energy like this. So, if you evaluate this integral I will only give you the final result for this case. So, this should come out to be  $3/2$  times  $n$  times  $K_B T$ . So, finally you will get the volumetric internal energy as a function of a number of molecules per unit volume the Boltzmann constant and the temperature.

So, therefore, if you differentiate it to get the volumetric heat capacity, this is simply  $3/2$  times  $n$  times  $K_B$ . So, this  $n$  times  $K_B$ . What is this? This is your, universal gas constant or subscript  $u$ . So, if you divided by the molecular mass then you get your real gas constant. So, this is your  $R$  subscript  $u$  this is your universal gas constant. And therefore, we get a resulting expression for a volumetric heat capacity, which is simply  $3/2 R_u$  and these are applicable for ideal gasses. Ideal gasses monotonic gasses because we have neglected the rotational and vibrational energies we have considered only the translational kinetic energy.

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So, what it tells you is that if you know look at a theorem very famous theorem in statistical thermodynamics this is called the equi partition theorem.

In fact, you can also write your  $u$  as simply  $3/2 R_u T$ . So, your equi partition theorem here actually says that. So, in each direction the total energy therefore, the internal

energy contribution comes to  $\frac{1}{2} k_B T$ . So, therefore if you account for all the 3 directions 3 degrees of freedom therefore, you end up with  $\frac{3}{2} k_B T$ . So, therefore, each degree of freedom contributes to half  $k_B T$  amounts of units of energy. So, this is what the equipartition theorem says and when you plot this particular variation of specific heat. So, you can convert this into specific heat also or you can retain this as volumetric heat capacity  $C_v$  as a function of temperature. So, what we have is just  $\frac{3}{2} R_u$  which is just if you therefore, normalize it with  $R_u$  the value will be 1.5.

So, for actual ideal gas when you look at ideal gas which has 2 atoms for example, and which has also rotational and vibrational degrees of freedom at lower temperatures say something from 10 Kelvin to about. So, I am just going to plot this in a log scale on the x axis I have 1000 Kelvin and then I have 10000. So, on the vertical axis I can go all the way up to say 3 or 3.5. So, this is a linear scale and I use a log scale on the x axis for any diatomic ideal gas. So, you will find that typically at low temperature this will follow this particular equipartition theorem, and you will find that the value will be equal to 1.5 till about 70, 80 Kelvin. So, all this is in Kelvin.

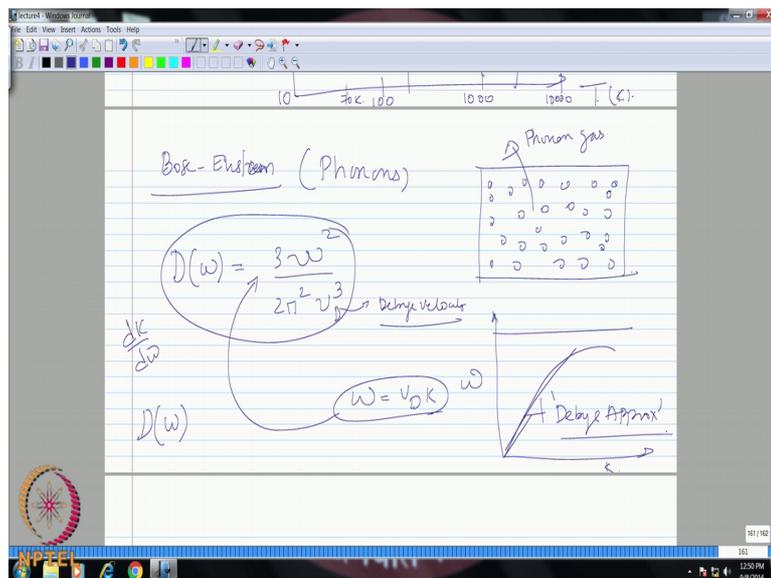
Now, as the temperature increases the contributions from the rotational motion now will become very significant, it will affect your heat capacity we have not considered that into the simplistic picture, but for the diatomic case that will become more important at higher temperatures and that will kick in and you will find that it will slowly transition to a value of  $C_v$  by  $R_u$ . So, now, you have to add contributions from the rotational. So, that is another half  $k_B T$ . So, therefore, your  $C_v$  by  $R_u$  from the translational degrees of freedom is  $\frac{3}{2}$ . So, if you add plus another degree of freedom due to rotation. So, you will have basically  $\frac{5}{2}$ . So, this will reach a value of 2.5 and if you keep increasing the temperature say beyond 1000 Kelvin. So, from room temperature till about thousand both the translational and rotational modes will contribute to the heat capacity or to the internal energy.

So, if you keep increasing your temperature further your vibrational mode will also kick in, and you have to add from equipartition another plus 1. So, this will be result in  $\frac{7}{2}$ . So, this value now will reach 3.5 at higher temperatures, and then it will stay that. So, this is the actual plot of normalized heat capacity as a function of temperature for diatomic gas. So, if you are looking at monatomic gas then you have all the time this value and constants across the entire temperature range. So, this is for monatomic gas

which we derived just now from the Maxwell Boltzmann distribution. So, if you are looking at room temperature ranges, then both the rotational and translation modes are equally important and therefore, you will have a value about 2.5, if you are looking at very high temperatures. So, then the vibration mode also will contribute.

So, the temperatures are quite high enough you can imagine for supporting the vibrational energy levels and therefore, the final value will be 3.5. So, now, you understand how these values are coming. So, if you are looking at studying thermo dynamics and looking at ideal gasses generally they are giving you for monatomic gasses  $C_v$  by  $R$  is equal to 1.5 diatomic gases till this. So, this is plotted, but you should understand this is coming as a consequence of using the Maxwell Boltzmann distribution function. So, similarly, this is a classical distribution function and therefore, the outcome is like this, what about using the Bose Einstein distribution function.

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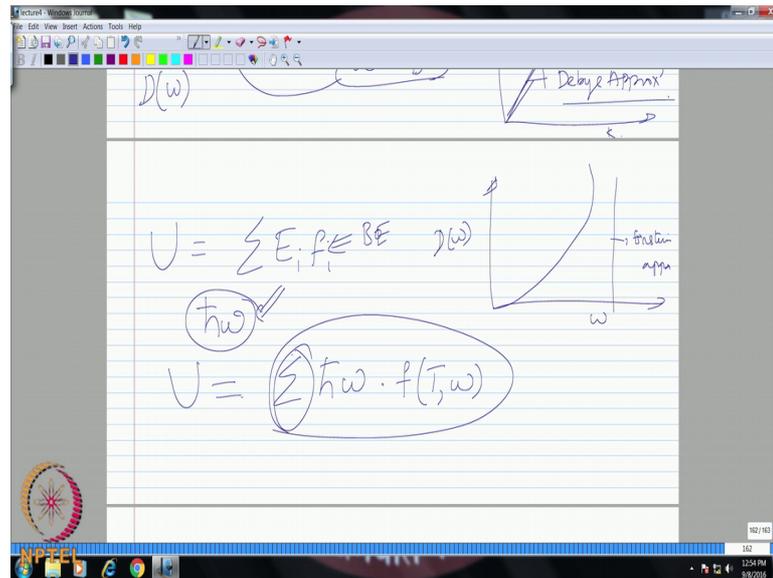
So, therefore, let us look at the case of phonons and consider again solid may be semi conductor or dielectric, where the contribution is mostly from the lattice vibrational energy. So, you can consider this equal to a phonon gas a system which is filled with phonon gas. So, these are phonon particles indistinguishable these are virtual particles just like we talk about, the case where you fill it with gas molecules. So, you can assume the solid is now equivalent to box with filled with phonon gas. So, we can now bring in our Bose Einstein distribution function to calculate the internal energy of this solid.

So, can you try it out now the same way that we did for gas molecules now the states are quantized? So, therefore, you have to still use the summation, now the case of phonons what is the expression for density of states. So,  $3 \omega^2$  by  $2 \pi^2 V D^3$  and what was the definition of  $V D$ ? This was your  $D$  by velocity. So, this expression we derived for which condition, this density of states that you are coating here. So, if you look at the phonon dispersion curves, if you look at also monatomic crystalline structure.

So, there we looked at acoustic phonons you have longitudinal vibration in one direction, and then in the second and third direction you have transverse modes therefore, all these three will contribute to the acoustic phonons, if you looked at two atoms then the out of phase mode will contribute to the optical phonons. So, therefore, if you remember the dispersion curve that is  $\omega$  versus  $K$  for the 1 dimensional monatomic atoms we just add a sinusoidal one which we approximated to be linear. So, we assume that  $\omega$  is equal to  $V D$  into  $K$  right the slope of this will give you the speed of sound. So, therefore, using this linear dispersion we derived this expression for density of states.

So, this is valid only for acoustic phonons with the assumption that this is a  $D$  by approximation. So, you remember is linear approximation is called the  $D$  by, I hope you remember, what was the approximation that can be made to the optical phonons, remember what we made like this. So, in that case what would be  $D$  of  $\omega$  for optical phonons? So, you remember  $D$  of  $\omega$  will involve  $D K$  by  $D \omega$ . So, what would be  $D K$  by  $D \omega$  infinity?

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So, if you draw the density of states now the case of optical phonons  $D$  of  $\omega$  this is  $\omega$  it will be like this. So, these are your Einstein approximation.

So, what we are now dealing with when you put this you should directly now remember what approximation we made, we are dealing with number one acoustic phonons under the  $D$  by approximation. So, you can also make further approximations saying if you extend this to three dimensions, you will have 3 such acoustic branches 1 longitudinal 2 transverse. So, therefore, you can further multiply this by factor of 3 you can assume similar density of states for the other 2 branches and you can multiply this by 3. So, kind of an isotropic assumption transverse modes and the longitudinal mode all look alike. So, density of states will become therefore, 9 times this.

So, therefore, use this how you will use this information now. So, what I am asking you to do is get the internal energy which is what how do you find in this case which is summation of say  $E_i$  into  $f_i$  in this case now, you have the Bose Einstein distribution function. So, how will you substitute into this say you can assume the energy of 1 quantum state to be  $h\omega$ . So, you can assume that this is the energy of 1 quantum state. So, therefore, this will be summation of  $h\omega$  into your distribution function your distribution function is a function of  $\omega$  and temperature.

Now, this is a discrete summation how you will convert this into an integral, how about density of states is this discrete or it is continuous function, say if you are plotting for

acoustic phonons how does it look like this. So, continuous function, therefore, now the hint is you will have to use the density of states to convert this summation into a continuous integral. So, what does it mean you are summing this over so many quantum states or in other words, the equivalent way of imagining is now go back to the definition of this, so that is why he was asking you. So, your number of quantum states per unit volume is nothing, but integral of the density of states. So, therefore, that summation is over all the quantum states. So, therefore, how can you convert that summation by therefore, converting that to an integral multiplied by density of states?

So, the summation over the number of states now is converted to an equivalent integral continuous function.

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$$U = \int_0^{\omega_D} \hbar \omega f(T, \omega) D(\omega) d\omega$$

$$U = \frac{3}{2\pi^2 V_0^3} \int_0^{\omega_D} \frac{\hbar \omega^3}{\exp(\hbar \omega / k_B T) - 1} d\omega$$

So, all we have to therefore, do is re write this has integral  $\hbar \omega f$  which is a function of  $t$  and  $\omega$  just  $d$  of  $\omega$   $d$   $\omega$ . So,  $d$  of  $\omega$   $d$   $\omega$  will give you what the number of quantum states per unit volume. So, this is basically converting the summation into an integral. So, this integral when therefore, in the case of the acoustic phonons under the  $d$  by approximation what will be the minimum and maximum limits for the density of states. This is 0 and what is the maximum value of  $\omega$  we can go  $\omega$   $d$  this is corresponding to the edge of the crystal brillouin zone edge of the brillouin zone because this dispersion is symmetric periodic.

So, all we need to consider is only 1 units cell which forms brillouin zone. So, therefore, it is now going from 0 to omega d right. And now once you calculate this now you substitute for your Bose Einstein distribution function into this and you will be able to you please check this particular expression this is v d cube. So, I am also substituting for the density of states and the Bose Einstein distribution. Please check if you are getting this. Is it they should be h co correct should be h correct is that. So, therefore, now you can define your volumetric heat capacity as d u by d t. So, go ahead and differentiate it with respect to temperature.

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$$U = \int_0^{\omega_0} \frac{h\omega}{\exp(h\omega/k_B T) - 1} \cdot \frac{3}{2\pi^2 v^3} \omega^2 d\omega$$

$$C_v = \frac{dU}{dT} = \frac{3h^2}{2\pi^2 v^3 k_B T^2} \int_0^{\omega_0} \frac{\omega^4 \exp(h\omega/k_B T)}{(\exp(h\omega/k_B T) - 1)^2} d\omega$$

$$\frac{N}{V} = \int_0^{\omega_0} D(\omega) d\omega = \frac{\omega_0^3}{2\pi^2 v^3} \cdot 2$$

So, what you get. So, you will have 3 h bar square juts check if this is 2 pi square v d cube and then you also get a k b t square 0 to omega d omega power four exponential h bar omega by k b t by exponential h bar omega by k b t minus 1 d omega.

Please check this expression. So, now, we are just going to do some manipulation here this is the final expression that we get to make this evaluation of the integral simpler we can now calculate our n by v that is the number of quantum states per volume. So, how do we do that? Simply, 0 to omega d of omega d omega, so this will give you the number of quantum states per unit volume. So, if you substitute of our expression for density of states into this and integrate you will get this number density as this is your quantum number density, quantum state density you can see like this.

So, let us call this as equation 1 here and this is equation 2. So, if you substitute for the fact that we can rewrite this in terms of  $n$  by  $v$  and therefore, we can manipulate it such that we can write this in terms of  $n$  by  $v$  number of quantum states per unit volume.

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The image shows two handwritten equations on a digital notepad. The top equation is:

$$\frac{N}{V} = 3 \times \int_0^{\omega_D} D(\omega) d\omega = 3 \frac{\omega_D^3}{2\pi^2 v_D^3} \quad (2)$$

The bottom equation is:

$$C_V = 9 k_B \left(\frac{N}{V}\right) \left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x dx}{(e^x - 1)^2}$$

where  $x = \hbar\omega/k_B T$

Labels in the image include:  $\theta_D = \frac{\hbar\omega}{k_B}$ , Debye temperature, and  $\theta_D/T$ .

So, therefore, you can rewrite this expression as I just leave it has a homework to you and I also told that this should be 9 times right because we have three acoustic phonons similarly if you are looking at number also they should be 3 times. So, therefore, if you substitute this you should get 9 times  $k_B$  and you can rewrite this in terms of number of quantum states per unit volume  $n$  by  $v$  the whole cube. So, you are changing the limits of integration from  $\omega_D$  to  $\theta_D/T$  I will just define  $\theta_D/T$  the moment just hold on.

So, this  $x^4 e^x dx$  by  $(e^x - 1)^2$ , where  $x$  is defined as  $\hbar\omega/k_B T$ , you can therefore, define what is this  $\theta_D$  as  $\hbar\omega_D/k_B$  this is some kind of equivalent energy that you are defining. So, we know that energy is equal to  $k_B T$  in terms of frequency it is  $\hbar\omega$ . So, we are equating this and therefore, finding out a temperature now this temperature is called the Debye temperature it is just representative temperature corresponding to  $\hbar\omega_D/k_B$ . So, you can check this as homework if you are able to reach this expression.

So, we will stop here I think you are already at 1 o'clock and in the next class, I think we have on Monday we will plot this function and again look at the variation with

temperature how this looks and then we will complete this exercise for electrons also. So, with this you know now we have the most important thermo physical property, which is their heat capacity. So, if you want to solve for conduction the only parameter that you need is your heat capacity. So, this information now comes basically from quantum mechanics and we have used the distribution functions to evaluate this one right.

Thank you.