Fundamentals of Gas Dynamics

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Week - 06
Lecture – 23
Discussion on Converging Nozzles

In this class, we will do couple of problems related to converging nozzles. So, first let us try to understand the importance of start a critical pressure. So, if I have a tank with the converging nozzle, I have P0 here, P ambient here, and P exit here.

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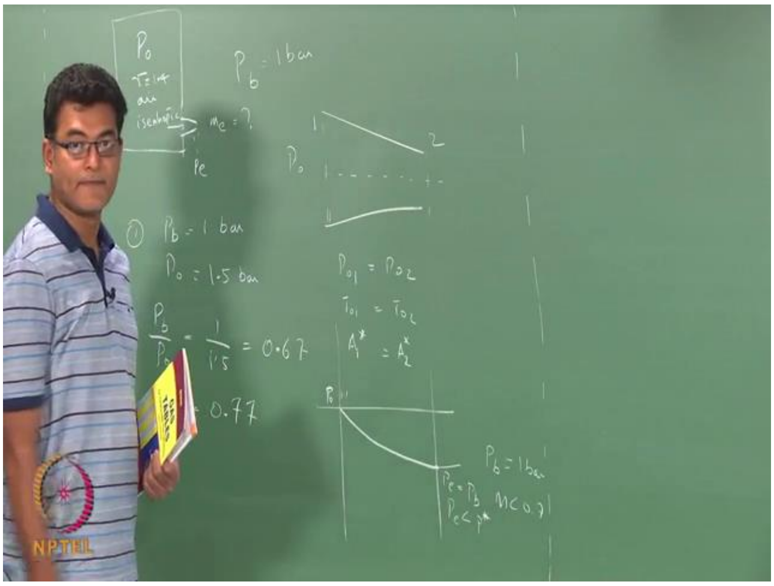

So, the exercise is to show you the importance of P star. So, first case we will assume Pb to be A1 bar. In this problem, Pb is 1 bar. So, I have a high pressure cylinder and it is going to eject through this nozzle. So, I am not talking about the area or area ratio of the nozzle. I am just going to talk about the pressure ratios. Suppose the first question if my P 0 is 1.5 bar, what is the Mach number at the exit. The question is to find the Mach number at the exit. So, I have P0 by P. So, my Pb by P0 is 1 by 1.5 which is 67 and the Mach number associated with this I look at the tables.

So, I take the isentropic tables, look for gamma equals 1 because I assume the fluid to be

1 air inside the cylinder and the process is isentropic. So, what I have here is an isentropic flow. So, this P0 what I have inside the tank is also the P0 throughout the nozzle. So, the P0 here inside the tank is constant here. If this is my section 1 and this is my section 2, P0 1 equals P0 2, T0 1 equals T0 2 and A1 star will be equal to A2 star or any other at any other section. This is true with this assumption.

I have taken the ratio Pb by P0 and found to be 0.671 from which I can find a Mach number either use the tables or use the relations. So, my P by P0 is 0.67. I look at the isentropic tables for gamma equals 1.1.4 and I find P by P0 1.67 is going to be 0.77 which means I plot my pressure. This is my P0. Pb is 1 bar, this is 1.1 bar. Px, it is also going to be sure and this is a subsonic flow which means P exit is less than. So, when it reaches P star, you are going to have this value.

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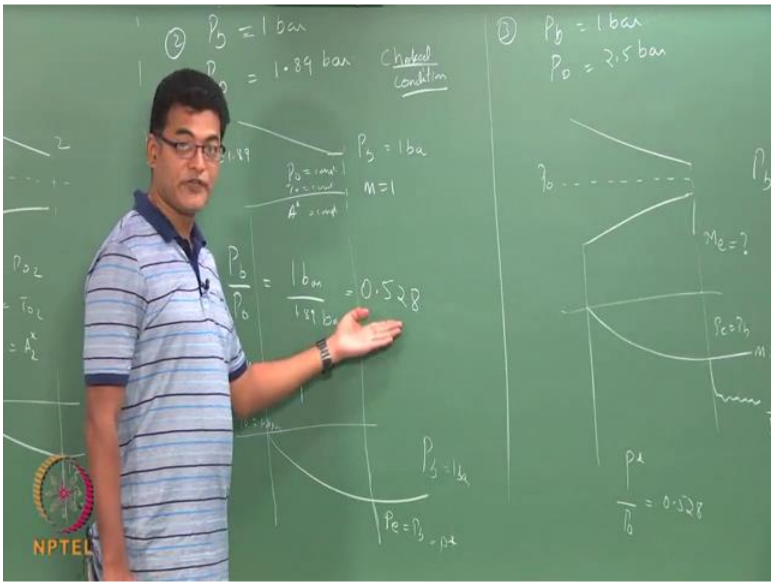

Now, if in the second case I keep Pb same and I increase my P0, I have a tank with a larger pressure ratio. So, let me put it a as 1.89 bar. So, I have a tank with gammA1.4 air and the process is isentropic, I want to find the exit Mach number P0 1.89, Pb 1 bar isentropic flow. So, P0 is constant, T0 is constant, A star is constant with this assumption I can write Pb by P0 which is 1 bar by 1.89 bar which is 0.528 and the Mach number at the exit associated with this pressure ratio is 1. So, I have a sonic condition here m equals

1 and if I have a Pb equals 1 bar p exit equals Pb which is also equal to your P star and your P0 is 1.189.

Now, in my third exercise have my Pb 1 bar P0 2.5 bar. We have isentropic flow P0 is all up to this, I have Pb 1 bar outside the nozzle in a converging nozzle. Question is to find m exit. So, in the previous case since m equals 1, this is also your choked condition. So, any pressure higher than this or any pressure smaller than this ratio is going to be choked condition. So, that is what we had seen in the previous lectures. So, I have already reached my choked condition at the exit, where your P exit is equal to Pb, any condition smaller than this would give you this. So, your P is P by P0 is P exit by P0 is smaller than your P star by P0 for the choked condition, ok.

So, instead of reducing the Pb, what we have d1 is, we have increased our P0. So, P star by P0 we know it is 0.528 and now, the Pb by P0 your P exit is same as your P star which Pb by P0 should be less than your P star by P0. P exit is same as your P star because that is not going to change even if you decrease your Pb from node. Your p exit is not going to change. What is happening outside the nozzle is going to change.

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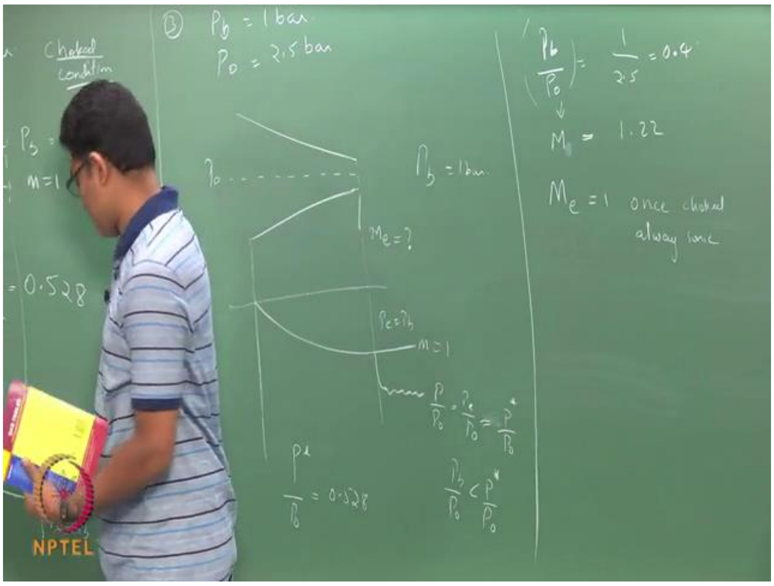

So, this pressure Pb keeps going down. So, your Pb by P0 is now 1 by 2.5 is 0.4. So, the

Mach number associated with this is if you look at the gas tables 0.4 Mach number associated with this should be supersonic 1.22. Since there is no diverging section, you cannot get this value.

So, this is not exit Mach number, this is the Mach number for associated with this pressure ratio, but after the choked condition whatever you do the exit Mach number is not going to change. Exit Mach number is always going to be 1 after once if you reached your choked condition. So, once your pressure ratio has reached your choked condition, whatever you do after that to decrease your pressure ratio, your exit Mach number is still what, but this is technically if you had a diverging section, you would have got of this pressure ratio. This is the Mach number which you can attain.

We will discuss about this is what happens, but this is not the exit Mach number that is what we are trying to give. So, the P star this ratio or the critical pressure ratio for air with 0.528 is very important and that is where you demarcate the possibility of having a supersonic flow or a subsonic flow.

So, if I have a tube say for example, bicycle a tire which is punctured, you are going to get the pressure ratio is very small. You are going to get only subsonic flow where as if you have a puncture in your say big tyre in a truck or a bus, then you are going to lightly to get this pressure ratio, but the exit Mach number is still 1.

So, you may get this pr Mach number. After the exit if you appropriately do something, but this is the exit Mach, this supersonic flow that you are going to see in the bus tire puncturing something which you are going to discuss after discussing the converging diverging nozzles. So, the jet is going to do an area change and that jet you would see the supersonic shocks and other thing which is everyday life example for a supersonic flow which is just a puncturing of your bus tire.

So, the exit Mach number is still 1. It is not going to increase because it is already choked. Now, we go to the next question. So, you could do this Mach number identification from this pressure ratio using gas tables. So, the equations which you have already written which is p by P0 equals 1 plus gamma minus 1 by 2 to the power gamma minus 1 by gamma. So, you could use this equation or the gas tables to get this formulation, the Mach number associated with the pressure ratio.

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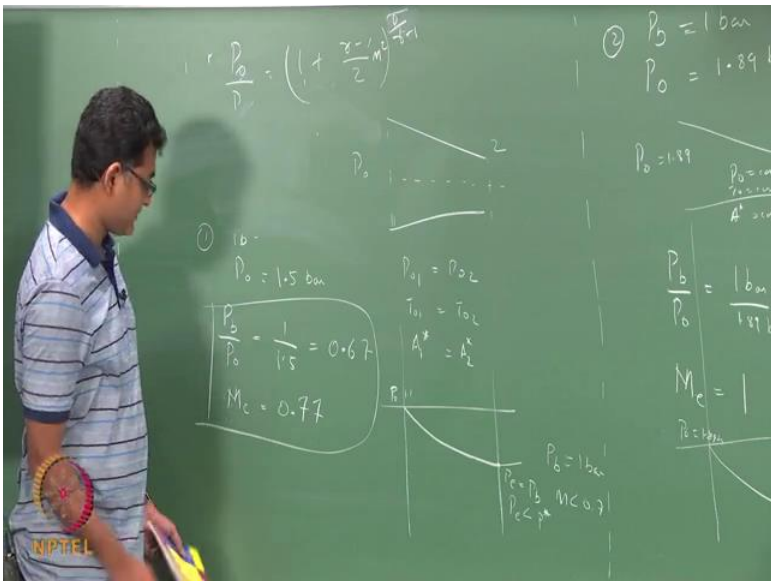

So, next question is I have the nozzle inlet area is 0.1 meter square and the contraction ratio is A1 by A2 is 5 V1 is 5 meter per second, P1 is 100 kilo pascal m at exit is 1 find v 2 p 2 and m dot assume isentropic.

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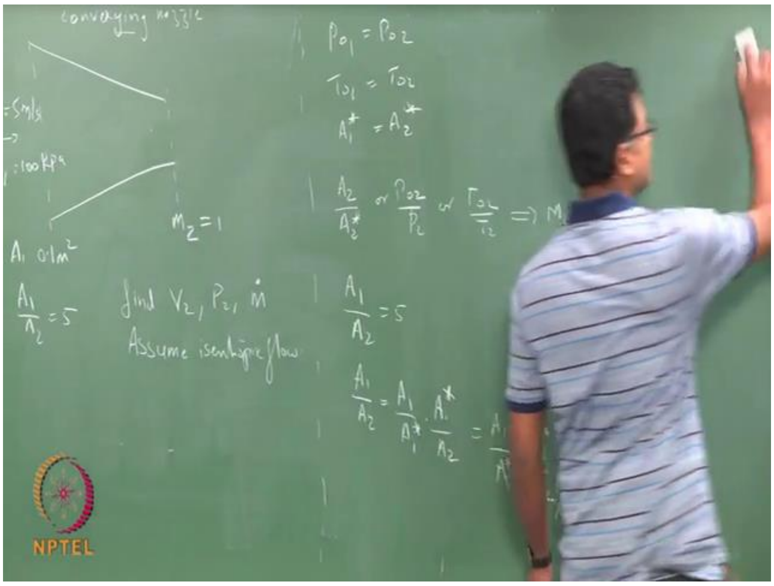

If you assume isentropic flow, we can use the equation which we have derived otherwise we cannot. So, we will assume the isentropically. So, the moment you assume isentropic flow from section 1 to 2, there are three things which you need to write. First thing is P0 1 is equal to P0 2, T0 1 equals to T0 2 and A1 star equals A2 star. So, if I know my A2 by A2 star or P0 2 by P 2 or T0 2 by T 2, I can get my M 2.

So, we are given A1 and A1 by A2 star. So, let us try if we can get A2 by A2 star from this ratio relation. So, A1 by A2, I can re-write it as A1 by A1 star into A1 star by A2 since A1 star is also equal to A2 star, I can write this further as A1 by A1 star into A2 star by A2. Wait a minute. We are already given M 2 equals 1. So, I can get the appropriate A2 by A2 star from where M 2 is 1 which is 1. So, I would go further.

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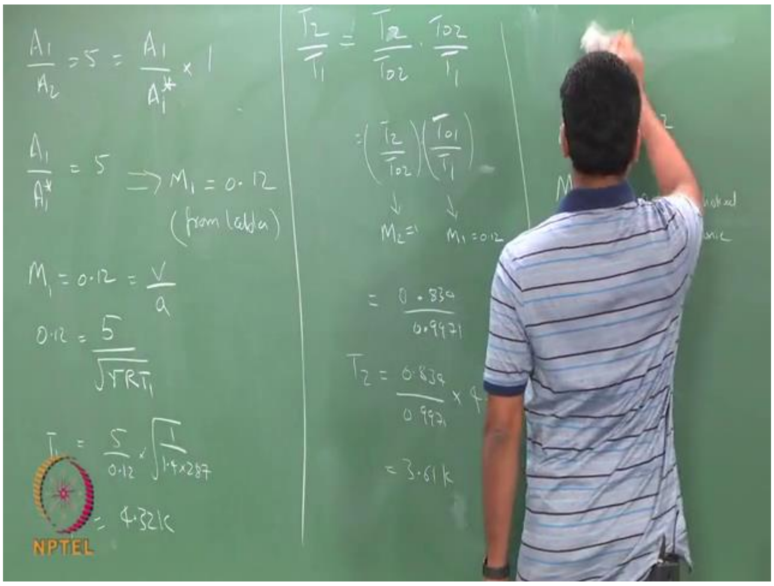

So, A1 by A2 is 5 equals A1 by A1 star into 1, correct. So, my A1 by A1 star is 5 from which I can get M 1 which if you look at the tables, you would get it to be from tables or from the equation. If you can remember the equations, ideally you should get it from the table, isentropic table when M 1 is 2. So, if the question is to find V 2 and P 2, but let us go ahead, do the temperature calculation. So, if I have M 1 equals 0.12 which is my v by a 0.12 equals v is 5 meters per second into root gamma r T1. So, I can get T1 as 5 by 0.12 root of 1 by 1.4 into 287 for air which is approximately 4.32 kelvin. Is this correct?

Now, I will go ahead and write to find t 2 from this particular ratio because the stagnation temperatures are same. I can write T0 T 2 by T0 2 into T0 2 by T1. Now T1 T0 2 and T1 are the same. So, I write T0 1 t 2 by T0 2 into T0 1 by T1. So, this I should get from Mach number 2, this I should get from Mach number 1 which I again look at the isentropic tables T0 2, this ratio for Mach number 2 which is 1 and this is for 0.12 which is 0.834 by 0.9971.

So, I look at the tables for gamma equals 1.4 and get these ratios which is a, and I get T2 as this ratio 0.834 by 0.9971 into T1 which we have just now found 4.32 kelvin. So, your T2 is 3.61 kelvin. Now, we know the exit Mach number is 1 which means the nozzle is choked. So, M 2 is 1. So, your v 2 is A2 which is root of gamma r t 2 which is root of 1.4

into 287 multiplied by the temperature.

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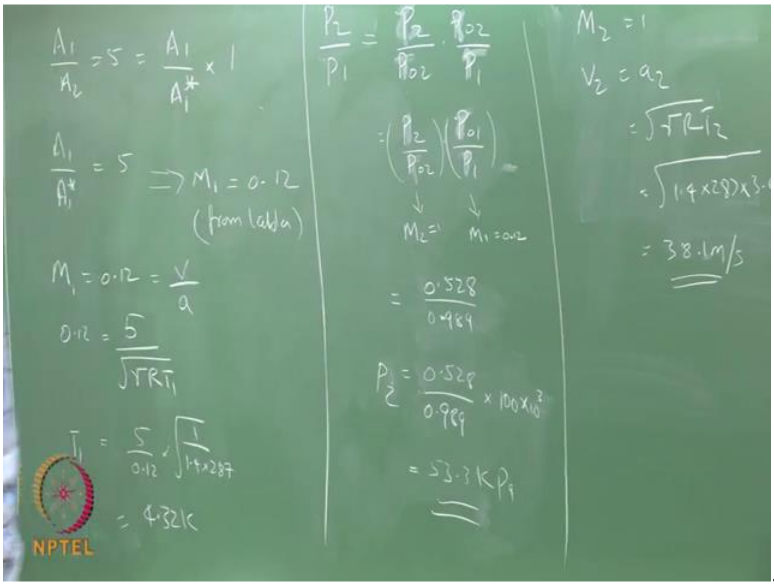

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Now, we found and your v 2 is going to be approximately 38 meters per second 38.1 meters per second. Now, similarly you can find P 2 by P1 and substitute the values that we know. So, I will just rub t and write P2 by P1. So, this is P 2 by P0 2 multiplied by P0 2 P. Since P0 2 is same as P0 1, I replace P0 2 by P0 1 by P1. So, I should get this value from m equals to this value m equals 2.

Remember this 2 is at the exit, not the ambient pressure that is happening at the exit. So, P 2 is this. What we have is something different. So, what we are calculating is p 2 at the exit of the nozzle, there is no where we can find this. We only know that should be less than p exit. So, I do this process and I compute similar value which is Mach number 8, Mach number 1 is 0.528 divided by 0.989 for M equals 0.12.

Again look at the isentropic tables where gamma equals 1, look for m equals 1, get the pressure ratio look for m1 equals 0.12, and take the pressure ratio. This is inverse. That is why we divide this here and here, the P2 would be 0.528 divided by 0.989 into P1 is 100 into 10 power 3 pascal. So, your P2 is 53.3 kilo pascal. Essentially we can get all the properties that are happening inside the nozzle.

If your T and area ratios are given and some of these quantities are given, using the tables or from the equations for example, this you could actually compute from equations also if you remember it, otherwise look at the tables and get the Mach number for the given gamma value of 1.4. Now, it has become more easy to find your m dot. M dot is rho a v. So, 1 area we know is the inlet area and this has to be constant.

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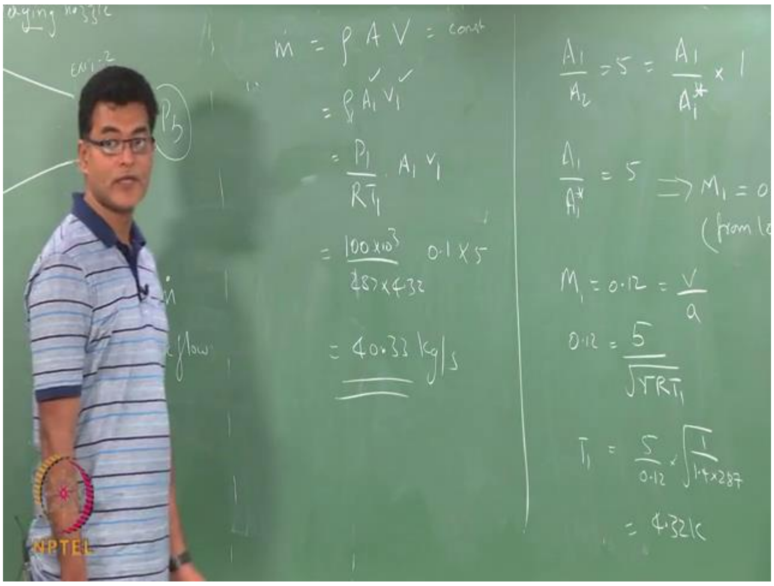

So, we will use that area. So, it would be rho 1 A1 v 1. So, we know A1 we know v 1. So, only thing that is missing is rho 1 which we can write it as P1 by r T1 into A1 v 1 assuming ideal gas equation to be valid here. So, this is 100 into 10 power 3 divided by 287 into T1, we have found out to be 4.32 into area is 0.1 meter square into 5 meters per second. This will give me a value of 40.33 kilo gram per second.

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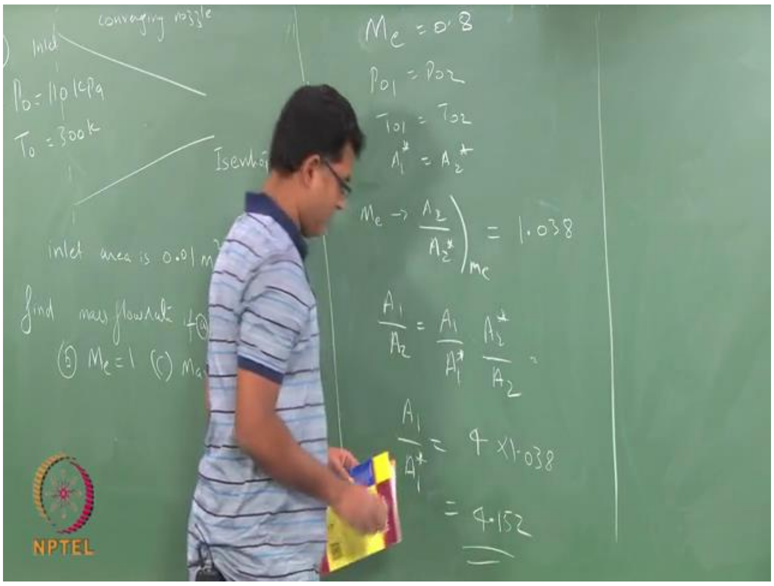

So, we will do the third question again a converging nozzle. So, the inlet stagnation pressure is 110 kilo pascal, T0 is 300 kelvin, the inlet area is 0.01 meter square, find mass flow rate? If mass flow rate if m at exit is 0.8, that is your first question. Second question if at m exit is 1 c, find the maximum mass flow rate. The area ratio also we would need the area ratio A1 by A2 is 4. So, if your m exit is 0.8, assume isentropic as before isentropic flow the moment I write isentropic flow, I write these three equalities.

Since I know m exit, I can get my A1 A2 by A2 star for m exit which is I look at the tables for 0.8. So, I should be careful to pick the chart gamma equals 1.4 m equals 0.8. I will have A by A star to be 1.038 m 8 is 1.038. So, your A star is smaller than your A2 which means that you have to reduce further to get m equals 1 that is the meaning of your A2 star. Now, from A1 by A2 equals A1 by A1 star into A2 star by A2 from which I can get A1 by A1 star which is, so this is 4, this is multiplied by 1.038, correct. A1 by multiplied by this 4.152 from which I can get my M1.

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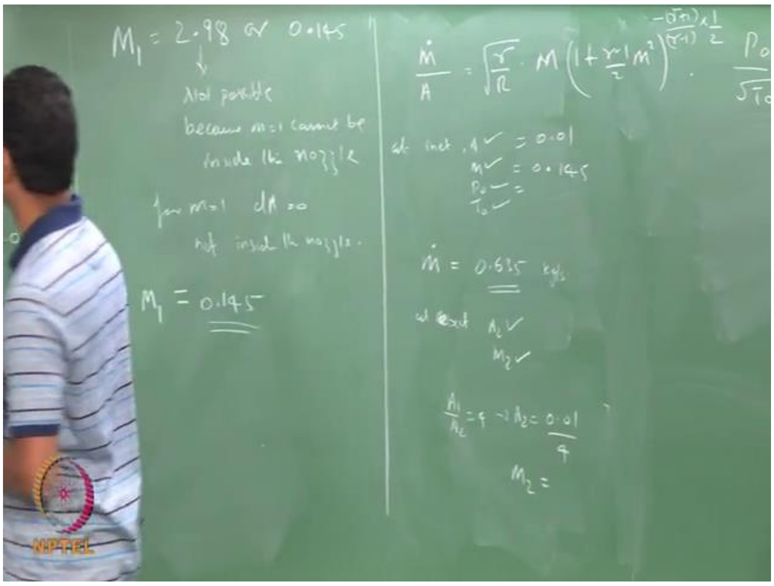

So, if I look the area ratio 4.15 area ratios, I should be careful here because I will have two solutions. One is supersonic; one is subsonic. So, my supersonic value is I will write 4.15. So, that is approximately 2.98 or 4, correct. So, now we have two solutions. Now, we have a decreasing in our velocity here in the converging and increase in our velocity in our converging nozzle. So, if the inlet Mach number is 0.14, it would increase to 0.8. If it is a supersonic value, this will again decrease to 0.8 which is not possible because it has to cross m equals 1 somewhere inside which is not possible.

So, the supersonic flow is not possible because m equals 1 cannot be inside the nozzle. Why is that? For m equals 1 your da has to be 0 which is not inside the nozzle. So, we will stick with the subsonic solution which is m equals 0.145. So, now if I know my Mach number, I can easily find my mass flow rate. So, my mass flow rate by area which is my ma mass flux, it is given as root of gamma by r into m into 1 plus gamma minus 1 by 2 m square to the power minus gamma minus 1 gamma plus 1 divided by gamma minus 1 into 1 by 2 into P0 by root of T0.

So, at the inlet we know all the values is known m is known P0 and T0 anyway is not going to change across along the nozzle. So, I can substitute this value and find my. So, your m is 0.145 and your area is 0.01. So, my m dot is. So, if I know my area at the exit, I can use my M2, but since I do not know my area at exit, I use this area at exit. You can find it from your area ratio. So, if your area ratio is A1 by A2 is 4, I can find my A2 to be A1 which is 0.01 divided by 4. If I know this, I can use my M2. So, for this area and this M2, I can compute my M dot which would be the same.

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What is?

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0.635 kg per second let us assume that is right. So, either you use the exit area and exit Mach number or inlet area and inlet Mach number that has to be constant across the cross section. So, you get this now. The trick is when m equals 1, what do you do remember this mass flow rate will be changing when you change your pressure. So, this is my Pb by P0, this is my m dot. So, when this is 1, there is no flow and then, slowly it goes up and then, it has a constant value.

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So, what we have found now is somewhere here where m equals m exit is 0.8. Now, the question is to find m dot at m equals 1. So, the conditions have changed. So, m dot will also change. So, this is also the choked condition. At choked condition, your A2 is now your A star because A2 by A star for m equals 1 is 1. So, your A2 is also your A star. Now, A2 is A1 by A2 into A2 by A1 into A1, this is 1 by 4 into A1 is 0.01, ok. So, that is my A2 and my m 2 is now 1. So, I will substitute 1 here. So, my A here is 0.0025, m 2 is 1 p naught and t naught we already know which is A1 1 kilo pascal, T0 is 300 kilo pascal substitute. This value I would get my m dot as kilo gram per second.

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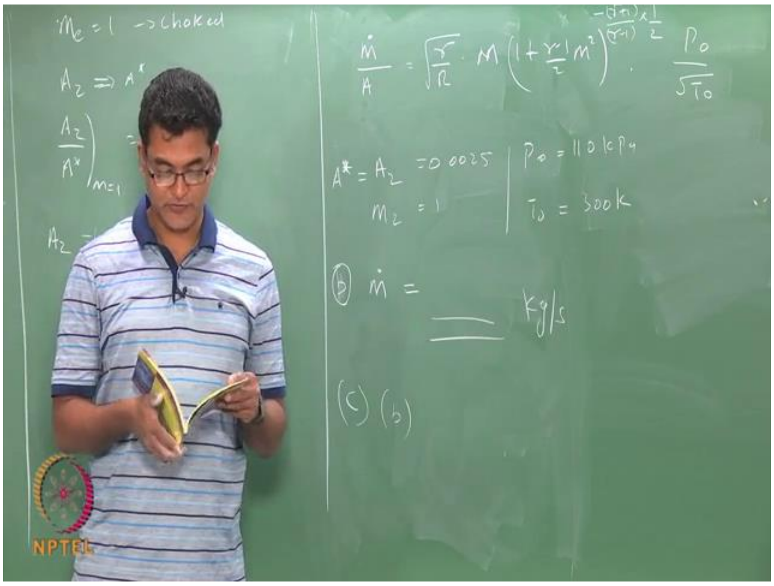

Now, this is the answer to your second question. Now, the third question to answer that all you need to do is look at this particular plot. So, if I reduce my pressure ratio, I would reach m equals 1 at some condition and after that the mass flow rate is going to be constant because we reached the choked condition which is the maximum possible mass flow rate through that particular area ratio. So, for that particular area, you would see the maximum possible mass flow rate.

So, your b answer to the c is also your answer to your b which is what you get from here which will be larger than what the question for a, which is a. So, that brings us to the discussion of maximum flow rate. So, the question is very simple. I can do the same problem by increasing my P0 which is what we have d1 in the previous question here somehow I have attained 0.8. I did not tell you how I have attained 0.8 and somehow we have attained 0.08 keeping P0 and T0 constant and then, computed the mass flow rate. So, depending on your P0 and T0, we can attain your (Refer Time: 44:24).

So, here A2 is also your A star. Once the choked condition is reached, now we can actually do the problem for diverging say channel or duct as well. So, do the same process. So, you can reverse the problem and do whatever it wants. So, with the same numbers, try doing this yourself make a question yourself and put a P0 try to find what is the mass flow rate, how does the area change, how does the velocity change with the area change and all those things you can keep the area ratio same as this and do you know design your own problem and do it.

The important thing to use gas tables to do all this or there are tables which would give you maximum flow rate also included, but this particular book does not have the maximum flow rate included in this chart. The other quantities we will use it appropriately. So, there are other tables as well. At present we have used only the isentropic flow tables for gamma equals 1.4. The procedure is same for other gammas, but we will stick to gamma equals 1.4 for convenience.

Thank you.