Fundamentals of Gas Dynamics
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Week – 06
Lecture – 22
Area ration & Pressure ratio in Converging Nozzles

So, we will continue with converging nozzle. We will do some problems as well and then we will go on to CD nozzles in the next class.

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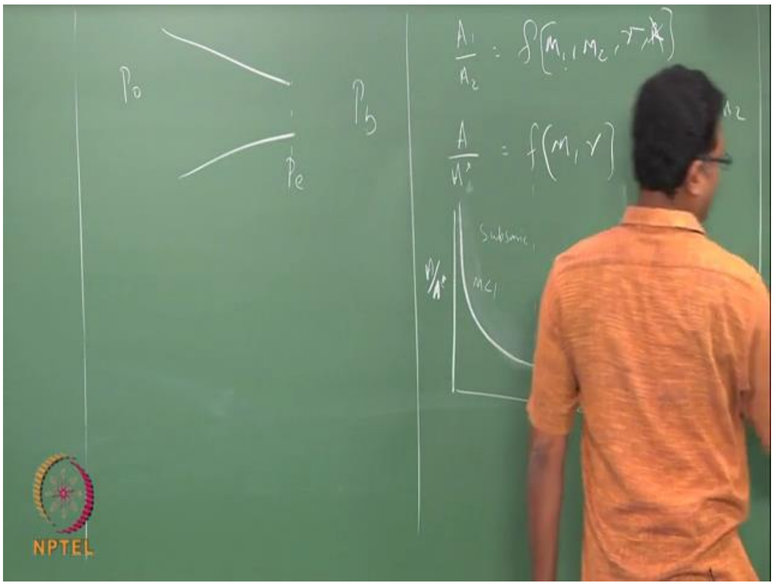

So, what we were discussing the other day was the area ratios, the important for the area ratios, and the pressure ratio between the exit and the inlet pressures. So I have a P b here, I have P exit here, I have a P 0 here we know a relation. A 1 by A 2 in terms of mach number at 1 mach number is 2 also depends on R and gamma. We also know relation A by A star in terms of Mach number and R mach number and gamma.

We have seen the plot A by A star versus Mach number, it goes to a minimum and that minimum always happens at M equals 1, and this actually is a skewed, it is not a symmetric curve, so it goes like this. So, this is M greater than 1, this is M less than 1. So I will have for a given value of A by A star, I will have two mach numbers, one is a subsonic, another one is supersonic. So, this is my subsonic this is my supersonic regime. The plot of this also we have seen the other day for M 2 and M 1.

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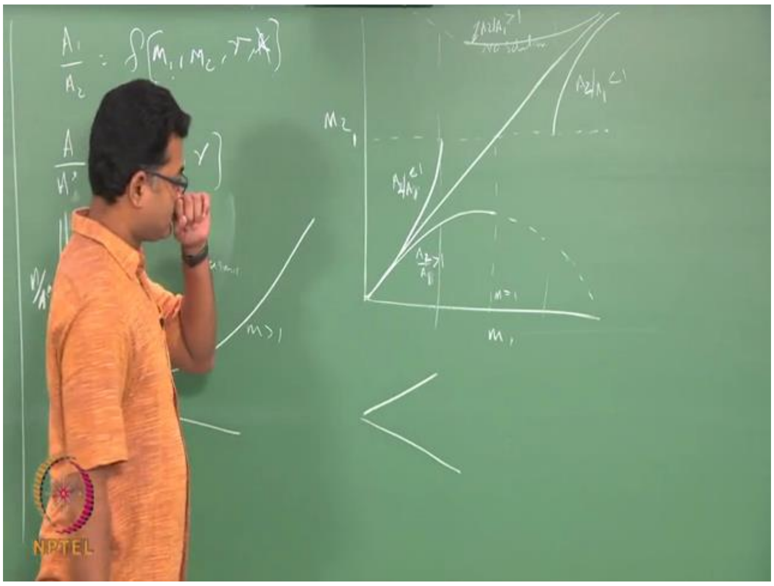

If A 1 by A 2 is 1, we have this 45 degrees lines; it is a constant area duct. Or if it is something less than 1, we have curves like this depending on your inlet mach number, I would get mach number at 2, So I have A 1 by A 2 or something around something less than 1 to which always reaches one mach number 1 at some where somewhere in the curve. So, it depending on my inlet mach number, I will have a value M 2 depending on my area ratio and there is a region in between where you do not have a solution for this particular area ratio. So, this is the curve for area ratios A 2 by A 1 less than 1.

This is when the inlet Mach number is subsonic; this is when the inlet Mach number is supersonic. We have M equals 1 condition here before larger Mach number there would be curves this. This would be A 1 by A 2 greater than 1, so that would be a case where A 1 by A 2 is A 2 by A 1, this is A 2 by A 1, so your A 2 by A 1 is larger here something similar would happen here. So, this is my A 2 by A 1 greater than 1. Here again you would see so this is a diverging channel or duct we will discuss this later. For the time being we will stick with A 2 by A 1 less than 1 and this is something we had seen in the other day. Now the problem that we see now is what we find, how we find V exit. Can we write v exit in terms of P 0 by P b.

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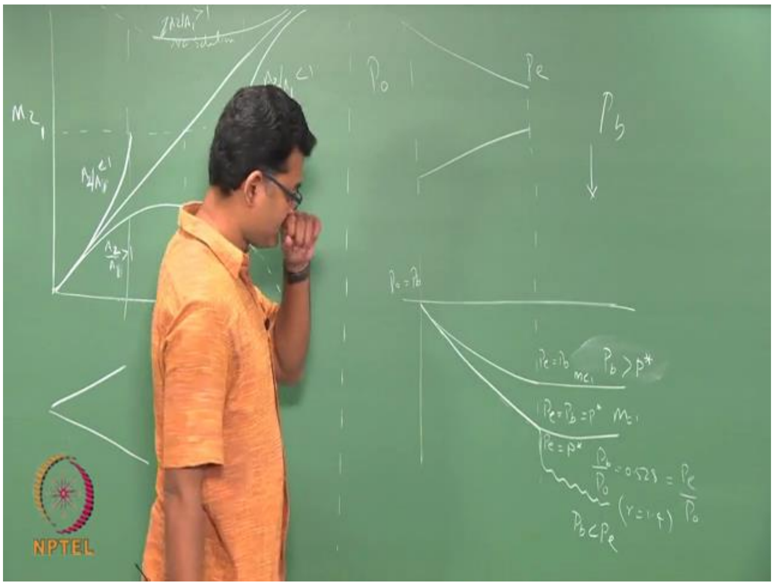

So, we go back to what we have done. Let us make sure what we are trying to do is correct. So, what did I do, I decrease my P b. When I do this, when P 0 equals P b, I have no flow, now I decrease it to some value, I would get a P exit since it is subsonic and the pressure has the no change between your ambient pressure or the backpressure with the P exit. Your P exit is same as your P b, M is less than 1. You decrease P b further M equals 1, P exit reaches P b. When this happens your P by P 0 reached that critical value 0.528 or the P b by P 0 is this value since P b by P b equals P e P exit by P b is also this value.

So, when P exit when this ratio reaches this particular value for gamma equals 1.4 then you have M equals 1 condition at the exit. After this, if I decrease it further, you have a P exit which is equal to your say called P star would be equal to your critical value P star, but your P b is less than your P exit. The P star here would be, so if I compare my P star value here the P star is this is for M less than 1, so your P b is greater than your P star, which is also greater than your P exit. Once you bring it down to P star, you would get M equals 1; and if you decrease it further, your P b would decrease, but your P exit is going to remain the critical value P star.

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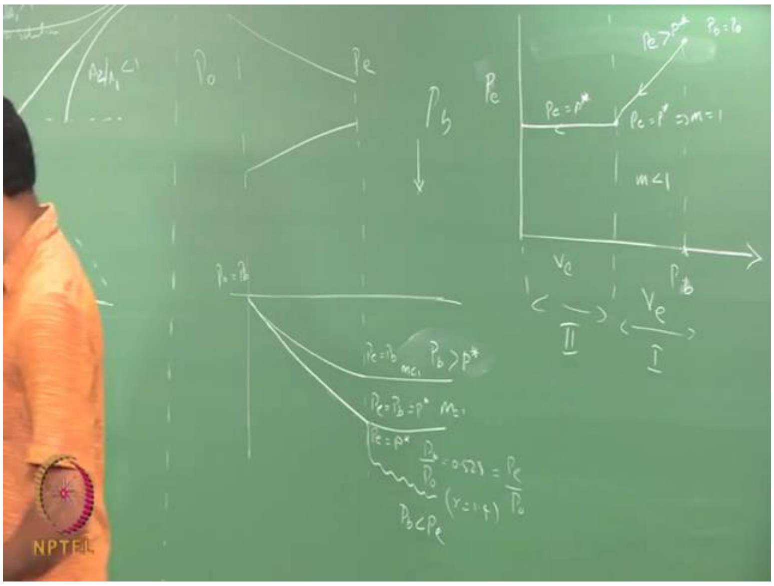

So, if I plot P 0 and P b, I will plot P b here. So, when P b equals P 0 that is P b equals P 0 somewhere here, P b equals P 0, so that would be a square, let us put somewhere here; P b equals P 0, there is no flow. Now I am decreasing my P b. So, I am decreasing my P b. This slowly decreases to P e till P e is P star. So till P e is my P star that is when my Mach number equals 1; after that if I keep decreasing my P b there is no change in my P exit so the P exit would remain the same in this. So, your P exit is this so I have two regimes here one is this condition, where P exit is greater than P star or your P b is greater than P star, and after this?

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Vertical axis is let us put it as P exit. So, these are the two regimes here if you keep decreasing your P b. So, I am decreasing my P b from some given value if that value is same as the reservoir pressure which is my P 0, there is no flow, and I am slowly decreasing it, so that their flow starts. The flow here is less than 1 inside the converging nozzle. Once the P b reaches the critical value P e equals P star equals P b, I get mach number equals 1 that is also our chocked condition. We have shown that to be our M star condition to the minimum area that is happening in the nozzle which is the A star that happens here when you reach that critical pressure ratio.

And further decrease in P b will not change your exit pressure; it is going to remain the same. And the mach number at the exit also 1, whatever happens is outside the nozzle which we will discuss it later. For the time being nothing happens inside the nozzle what from wherever you have done. Now, our aim is to find v exit in these two regimes. So, I have two regimes regime one and this regime two, and I have to find the V exit of these two in terms of P b and P 0 or P b and P exit. So, P b and P 0 is the ideal quantity that which we can use.

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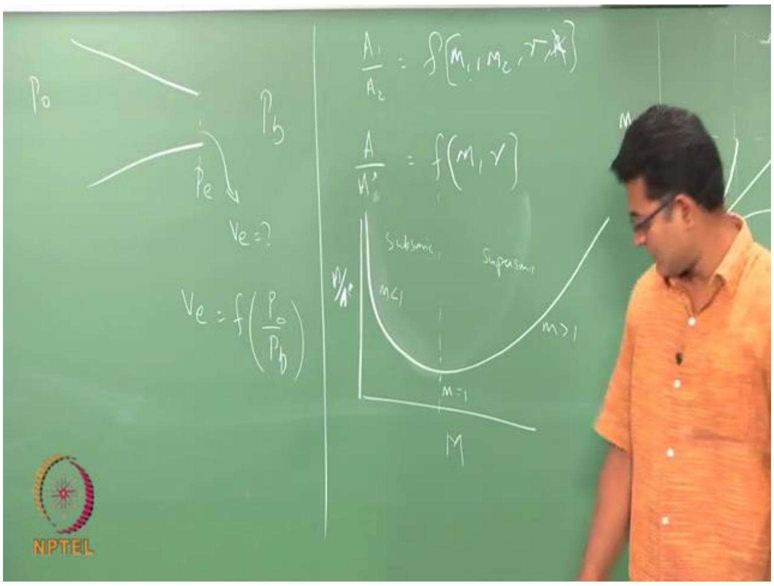

So we will find V as a function of P b and P 0 which a very simple exercise. All you need to do is the find the write the relation between velocity and Mach number; convert Mach number in terms of pressure ratio.

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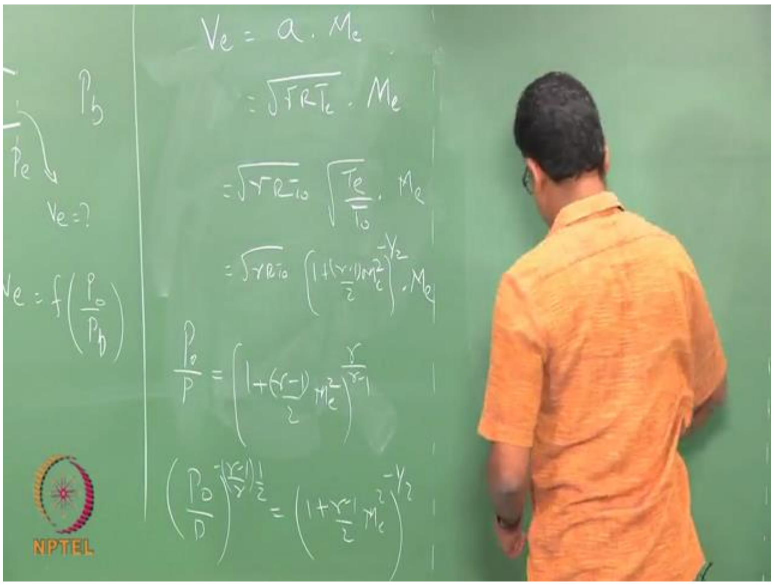

So, first let us do V e is the velocity of sound into Mach number at the exit; velocity of sound is gamma R T at the exit into M e. If write it in this form T e by T 0 into M e, I assume an isentropic from inlet to outlet, so my P exit is related to the stagnation temperature at this point which is related to the stagnation temperature at the inlet which is the same. So, the inlet stagnation temperature and the outlet stagnation temperature is same, so I can do this. So, once I have this, I can write the relation in terms of mach number which is 1 plus gamma minus 1 by 2 M e square to the power minus 1, there is a root here, so there will be 1 by 2 into M e. Now P 0 by P is 1 plus gamma minus 1 by 2 M e square to the power gamma by gamma minus 1.

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M square; here also there is an M square, and there is a bracket here. So, what I have here is 1 by 2. So I would write P 0 by P to the power gamma minus 1 by gamma into minus 1 by 2 equals 1 plus gamma minus 1 by 2 M e square to the power minus 1 by 2.

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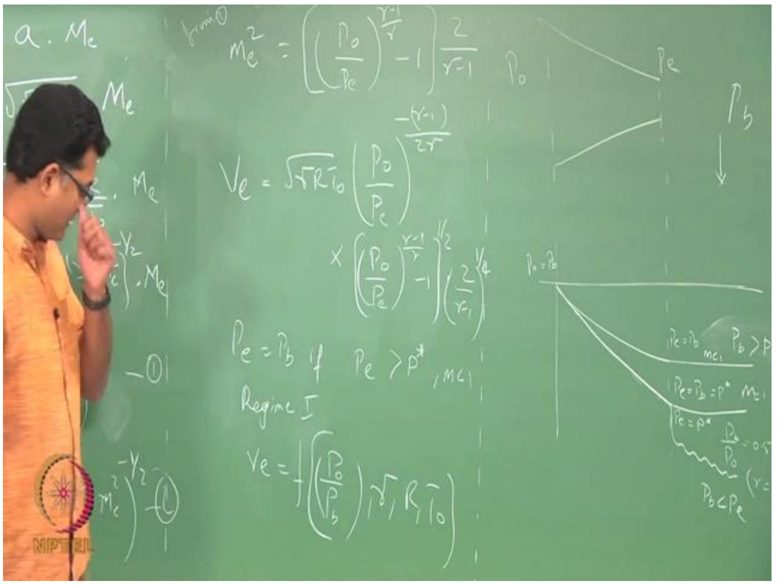

I will also get M e square from this equation. So, from 1, 2, so M e from equation 1, M e square is P 0 by P to the power gamma minus 1 by gamma minus 1 multiplied by 2 by gamma minus 1. So, I substitute these two quantities to the velocity relation V e is root of gamma R T 0; this is nothing but P 0 by P to the power minus gamma minus 1 divided by 2 gamma into M e is this or M e is M e square, so it would be P 0 by P to the power gamma minus 1 by gamma minus 1 by 1 by 2 into 2 by gamma minus 1 to the power 1 by 2.

Now, what is P? P is we are talking about the exit pressure. So, your P is your exit pressure, e, e, e. So, your P exit equals P b if your P e is greater than your P star or your M is less than 1. So, if you are talking about regime 1, your P e is always greater than your P star, hence P e is equal to your P b. So, for regime 1, I can write this equation, the same set of equation in terms of P by P naught by P b and gamma, R and T naught.

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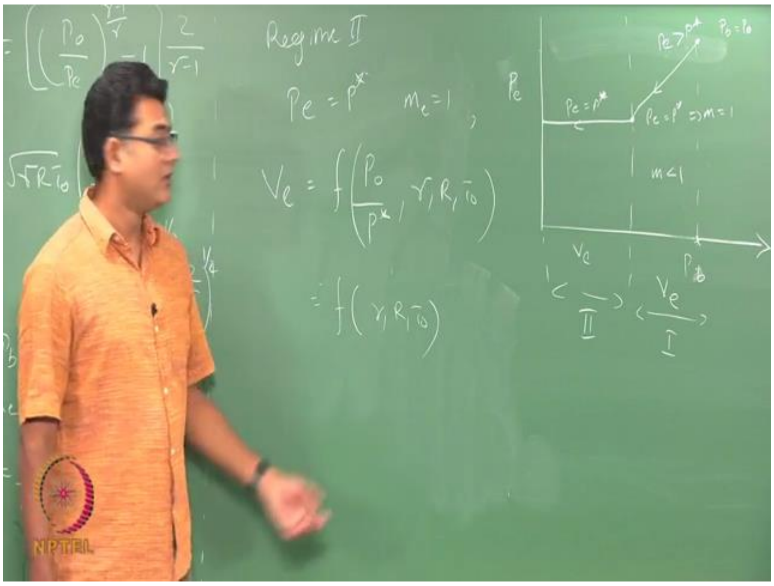

So, regime 2 that is your P e is P star M exit is always 1. I can replace P naught by P as P star. So, my V exit is a function of P naught by P star gamma, R and T. So, your exit velocity if the flow is already chocked whatever value you reduce, the exit velocity is always same which depends only on your P e or which is P star. So, since we already know this is a function of gamma, I can replace this as gamma, R and T naught, because this is a constant for a given gas which we have already seen P 0 by P star is a constant for a given gas, so this depends only on your gamma, R and T naught. So, this velocity is constant for a given T 0. What is T 0? T 0 is the inlet gas inlet gas temperature for this stagnation gas temperature.

Now, all this derivation, we have assumed the process to be steady. So, when we change your P b from one value to the other value, we are assuming a steady process, we are not doing this in time. We are just changing the pressure value and seeing it. So, what do I mean by that I changed P b to P b 1, and watch what happens. Then P b 2 change what happens, then change to P b 3 and watch what happens it is not a function of time, I am not changing in time that is very, very important because the process entire process what we have done here is a steady process.

Whatever analysis we are done whatever conclusion we are making is a steady flow solution, it is not an unsteady flow solution. So, only time when we discuss unsteady flow is when we change the area ratio and some information has passed into the inlet condition and the inlet condition itself was changed for a different mass flow rate that is only time when we assume it was unsteady and wave propagation that is non isentropic etcetera. Otherwise, it is isentropic flow steady flow stimulation.

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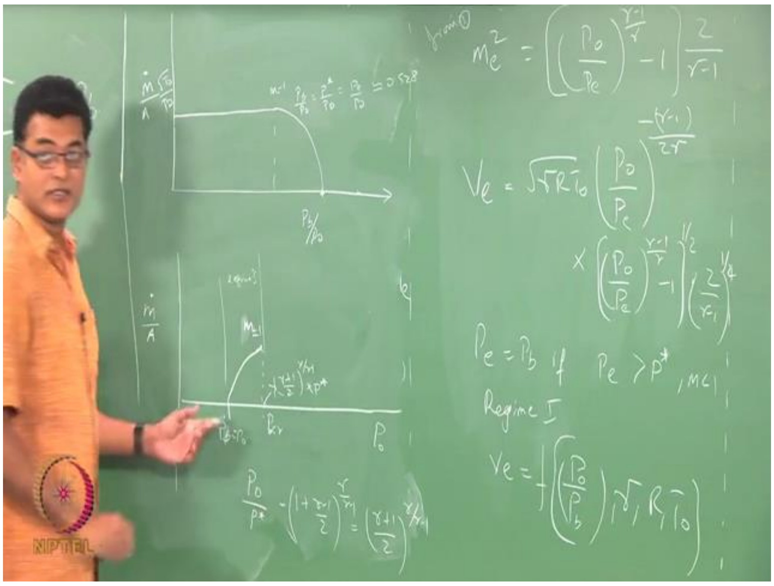

Now, we will do the reverse. I will keep P b constant and change my P 0, let see what happens. So, I have P exit, I have P b. What I am going to do is P b equals constant, progressively change P 0, again not in time, but we change P 0, and see what happens, change to next P 0 then change see what happens.

So, in that case, where we have changed our P 0, changed our backpressure, we had seen how our M dot varies from a value P b by P 0 when P b equals P 0, there was no flow. And then we reduced our P b, so there was a flow till it reaches M equals one condition which is our P b by P 0 equals P star by P 0 which is your P exit by P 0 for air it was something approximately 0.528. So, once this chocked condition, whatever you did, there is no change in your mass flow rate. So, this is something a multiplied by root T 0 by P 0. So, for a given constant T 0 and P 0 your mass flow rate changed accordingly.

Now, we are doing the other problem. We are keeping P b constant and changing P 0, so let see what happens with this. So, I have again mass flux; I have P 0 here when for some value of P 0, which is equal to P b, there is no flow. So, when P b equals P 0, there is no flow. Now, I am increasing my P 0. So, this is my P b, P b equals P 0 there is no flow; and now I am increasing my P 0 so there is some flow till it reaches M equals 1.

So, I can increase my P 0, so I increase my P 0 till it reaches some value where my P 0 gives M equals 1. What will be value here, M equals 1, what would be the value here P 0 by P star equals 1 plus gamma minus 1 by 2 to the power gamma by gamma minus 1. So, this is what gamma plus 1 divided by 2 to the power gamma by gamma minus 1. So, your P 0 here is P 0 in this value into P star. So, this value here is gamma plus 1 by 2 to the power gamma by gamma minus 1 into P star that would be the value here.

So, when the P 0 reaches this value you would get M equals 1 that is your pre critical, this is not P star that is your pre P 0 critical. So, your P 0 has reached a value where you can give you can get M equals 1 which is decided by this value. So, once that is there, after this is your regime 2; so after this is your similar to what you have seen this is your regime 1 then you have regime 2. Once it is chocked, now it has been chocked; now you are increasing your P 0.

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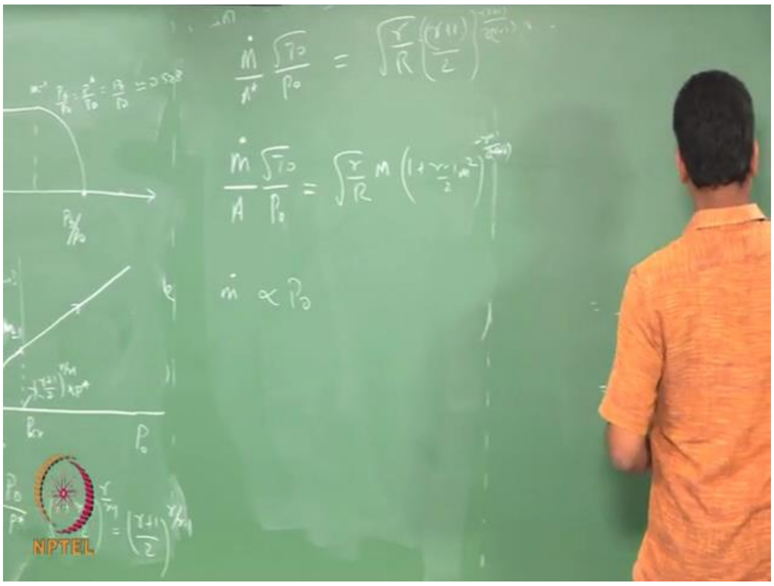

So, what do we have for the mass flow rate M dot is a root of T 0 by P 0 into root of R by gamma by r into M dot by A is?

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Now, yeah, P naught, it is not P, P naught. So, this is the equation you have for mass flow rate which you have derived few classes earlier. Now, I have already reached M equals 1 condition here by increasing my P 0. Now, if I am increasing further my P 0, my M dot is going to increase linearly, so your M dot is proportional to your P 0 from these equations. So, I am increasing my P 0 keeping P b constant. So, what I get here is a increase in mass flow rate as I increased my P 0, I have increased my mass flow rate.

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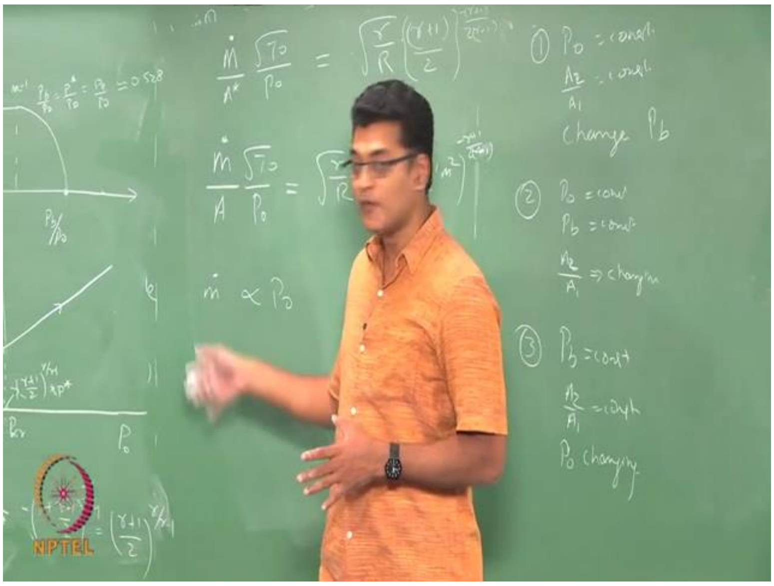

So, for the convergent nozzle, what we have seen is three scenarios; one we kept P 0 equals constant, A 2 by A 1 equals constant, we change P b. In the second case, we have P 0 equals constant, P b equals constant, and we kept changing the exit area or we have reduced the area ratio. And this scenario we are kept P b equals constant, A 2 by A 1 also constant, we changed stagnation pressure. So, all these are three different scenarios which from a steady state analysis isentropic flow whatever derivation we had seen the variation is that. So, for example, in this case, we have changed our P b. This is for a given P 0.

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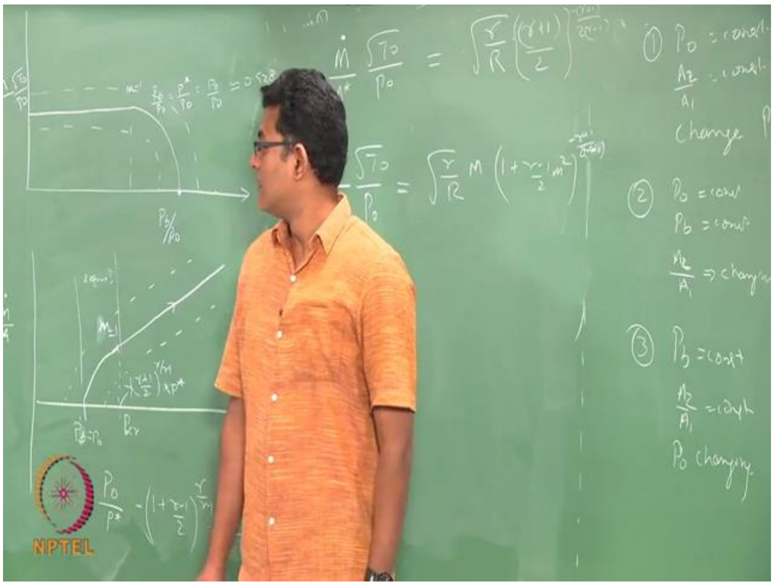

Now, if I increase my P 0 and want to draw the same thing, it would be either here and if I decrease it, it would be here. So, likewise you could think of P b, so instead of one given P b if I increase my P b and want to draw this. I would draw it here. If I decrease my P b, I would draw it here. Again in these two exercises, I have kept A 2 by A 1 constant. Now, you can think the same way when we talk about A 2 by A 1 which is what we have seen in the other class.

So, with this, we will end the discussion on converging nozzle. We will do a set of numerical problems sometime later. So, these three scenarios what we are discussed in converging nozzle; and each of those will give you some different kind of information. You should be clear when you say something is constant on what condition you have said, what is kept constant and what is changing.

Again changing not unsteady change, but steady changes; we change P 0 and watch, change P 0 to the next P 0 and see what happens again steady state solution. Likewise, here I changed area ratio watch what is happened steady state solution again changed to a different area ratio see what has happened so all steady state solution, there is no unsteady scenario in what we have discussed.

With that, I will conclude the converging nozzles.