Introduction to Boundary Layers

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Module – 03
Lecture – 23
Similarity solutions to the BL equations (other than flat plate)-II

Hi, so welcome back. So, where we stop last time was that we were trying to do you know solved the boundary equations, for a set of bodies other than the flat plate. So, we were using transformed co-ordinates, which is localized co-ordinates basically. So, which is zeta and eta? So, this is how we said zeta and eta.

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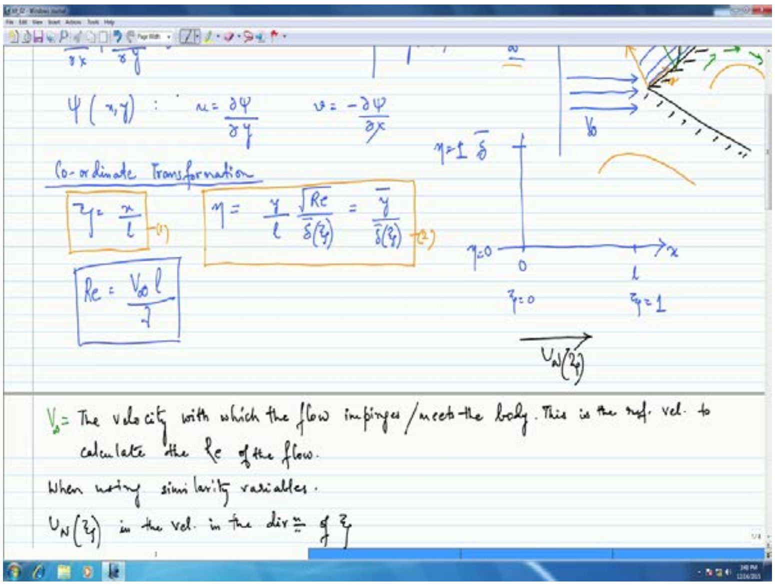

And using that and we came up with a trail solution which is given here in 3. Right?

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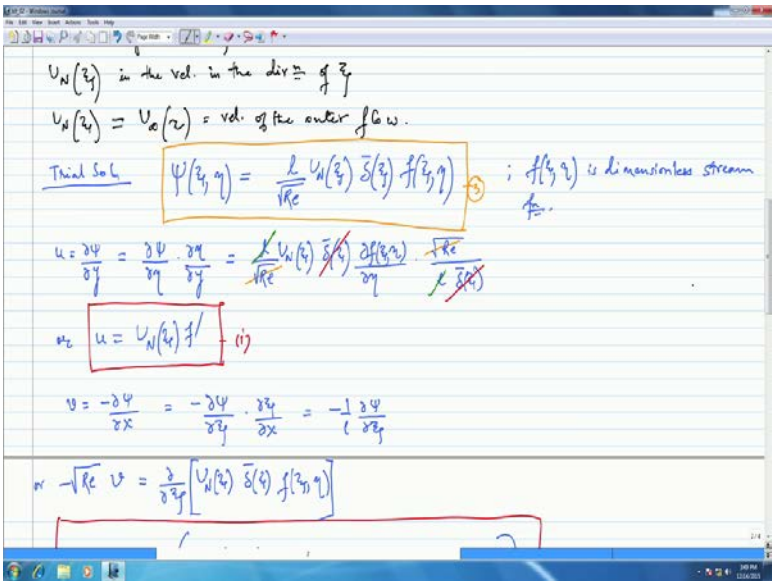

Using that we came up with a solution we basically wrote out the expressions for u and v in terms of the zeta and eta. When we input this into the boundary layer equations and what we get is this 3.

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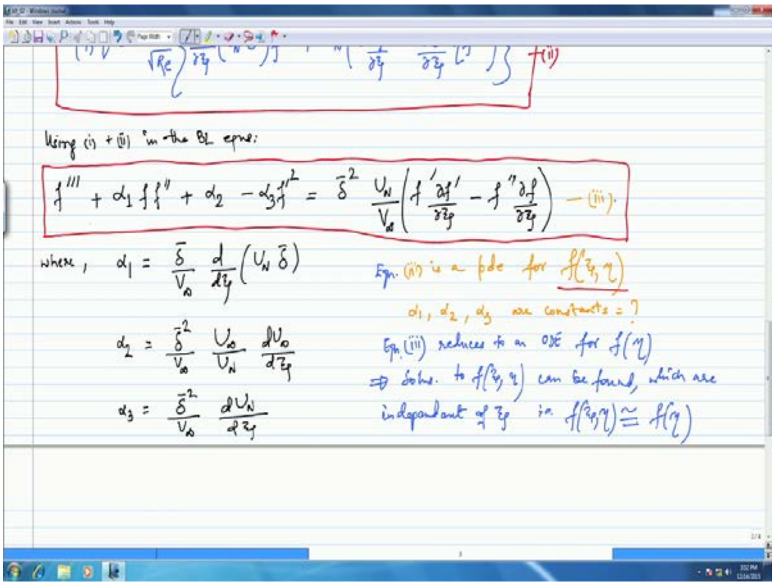

So, this was the equation that we get where alpha 1, alpha 2, and alpha 3 look like this. Now, clearly like what we said before stopping last time is that this equation. So this equation here is a PDE, is a partial differential equation is you can see for f zeta and eta. Now the question I have posted to you last time is that what happens if alpha 1, alpha 2 and alpha 3 are constants? So, if you have some basic idea about differential equations, you will see that if alpha 1, alpha 2 and alpha 3 are constants, then equation three basically reduces to an ODE an ordinary differential equations for the function f of eta that is del f del zeta is 0. So, which means that if alpha 1, alpha 2, alpha 3 are constants, equation 3, let us put it this way equation 3.

So, let us say equation 3 is a PDE for this thing but if alpha 1, alpha 2, alpha 3 are constants, equation three reduces to an ODE-ordinary differential equation for f of eta. That is that solutions to f of zeta, eta can be found, there are independent of zeta. So, essentially what that means is that, solutions to f of zeta, eta can be found there are independent of zeta, so which are independent of zeta. That is like we just now wrote f of eta. So; now, it is basically a function f is basically a function of just eta one variable.

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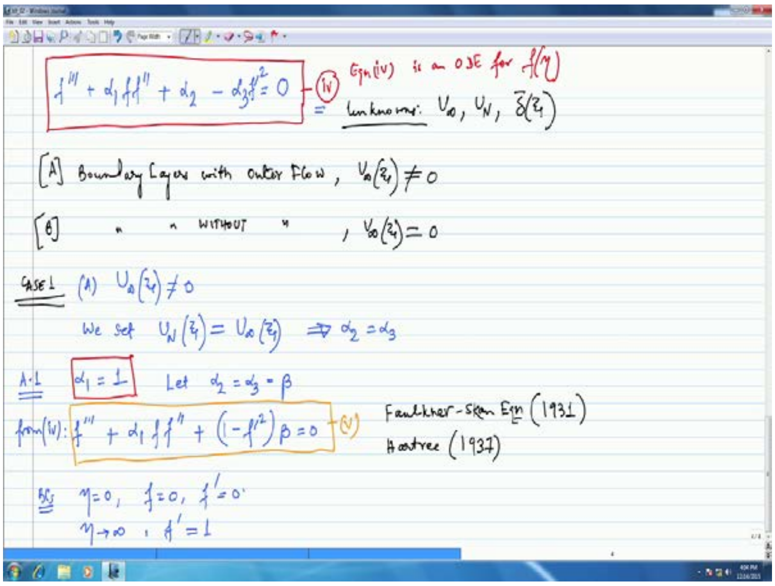

So, therefore, so we rewrite equation 3 to be the ODE. So, transform to ODE and that looks like f of plus this is a constant. f of f double dash plus alpha 2 minus alpha 3. And since now this is a function of only eta, so you can see on the right hand side, f del zeta will be 0. del f dash del zeta will also be 0, because it is a function of only eta. So, what we can on the right hand side, so the right hand side completely goes.

This is therefore an ODE, let me call this is four. So, this is an ODE. So, is an ODE for f of eta is that right? So, therefore, here in that case, you know what are the unknowns, when we have alpha 1, alpha 2 and alpha 3 are constants, so as per this equation, what are the unknowns in this case? Well, in particular of course, solutions exist for U infinity zeta for the outer flow for which sigma solution are available.

So, we will you know instead of now elaborate on this a little bit as to again what are we looking at, you know so these are things. So, similar solutions again we will classify this, we will classify this. So, similar solutions are classified as say [A] Boundary layers with outer flow that is U infinity zeta is not 0, and Boundary layers without outer flow that is U infinity zeta equal to 0. So, basically you know, we going to look at also of its combinations.

Usually we do a lot of these enough things because when we get a numerical equation, our main focus is how we can solve it. So, we try a lot of permutations and combinations, but of course, when we finally, get a solution we try to really understand it and try to make sure that it does mean something physically and it is not just a numerical exercise. And if you does not then we not sure to stick with that kind of a we did not called a solution exactly, so that is what we will do. So, Boundary layers with outer flow and without outer flow, so which means that U infinity you know zeta may or may not be 0.

So, let us go and look at this. So, we will look at several cases. So, one of the cases, so say CASE 1, so U infinity zeta it is not 0. So, then what we will set U N zeta is equal to U infinity zeta. So, if we do that, if you see U N zeta is equal to U infinity zeta. So, if I do that, so what happens U infinity U N basically is equal to U infinity zeta. So, basically f alpha 2 becomes equal to alpha 3, if you see from here. So, if I do that, so then what happens to, so let us do that. So, in that case the moment we do that. So, this implies alpha 2 is equal to alpha 3.

Now, I having said that that is there now we will look for alpha 1 being positive, negative or 0. What we will trying to look at is whether alpha 1 is positive, negative or 0. Now so let us say this is CASE A dot 1, where this is alpha 1 is equal to 1. So, basically it is positive and we are setting it equal to 1 without any loss of generality.

So, well you can explain this to yourself, but let me instead of elaborate down on there a little bit. So, now alpha 1 is essentially this, so delta star by v infinity d d z of U N. So, U N is nothing but u alpha zeta, right delta. Now, this delta bar length scale, is only a certain percentage of where it achieves the V infinity, where it is able to recover the complete free stream velocity right. So, therefore, this delta bar - the maximum, it is only a percentage. So, the maximum can only be 1 is not it, the maximum can only be 1. So, now we will just say alpha 1 is equal to 1. So, then a let, we will let, alpha 2 equal to alpha 3, and we going to call this is beta.

Then what happens to equation four, if you see that what happens to equation you know four. So, what happens there is, so we get so this is from four. So, we got that, plus alpha 1 we going to write that alpha 1 f of f double dash plus let us just see what was my equation. I think I set of made a small mistake here, f dash of square sorry about that I miss that term for some reason sorry about that. So, this is the our equation fine. So, once we decide, so then that is our equation and if I come back here. So, then of course, alpha 2 is equal to alpha 3, so one minus f dash of square beta is equal to 0.

So, this is we get another equation which is like this, so that you can see here this is again for a specific you know specific case. So, we have a different equation to solve and anyway alpha 1 we setting it as 1. So, this what I am going to call as 5, and this equation is call the Falkner-Skan equation, is call the equation. And it is kind of first purpose around 1931 and the solution and dependence of beta is also, well; you know when you kind of come up with equation like this, like I mean you get a solution and then you will have to study the solution hard to see you know what implication does it have physically and whether it even means anything.

The solution independence of beta was also examined by Hartree and this is basically accompanied by the following boundary conditions. So, the boundary conditions eta is 0, f is 0, f dash is 0; eta is large, f dash is equal to 1. So, what we get here in this particular case is the Falkner-Skan equation, and let us see what we will right here.

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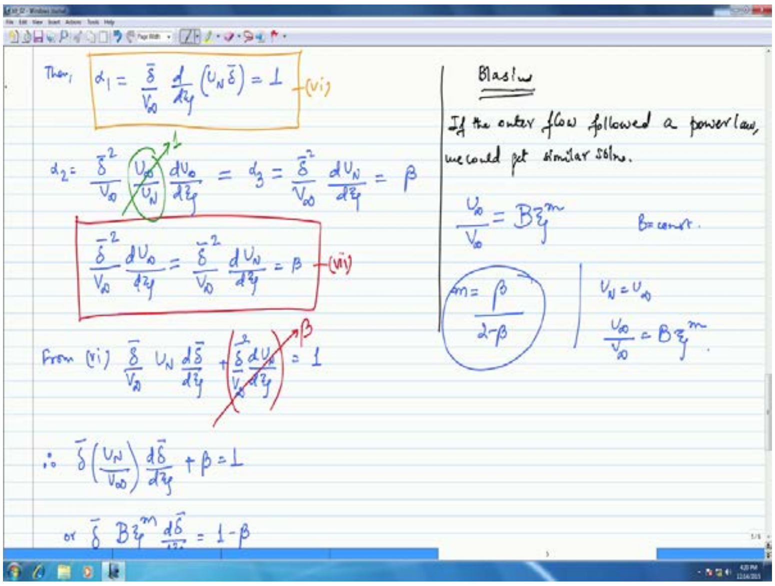

So for this case now, so alpha 1 is equal to 1. So, what do we get, so since we set alpha 1 is equal to 1. So, let us write this here, then alpha 1 is equal to delta bar by v infinity. Yes d d is zeta of U N delta bar right then this is equal to 1, this is 1, and let say this is 6. And again alpha 2 is equal to delta bar square by V infinity U infinity U N d U infinity d zeta is equal to alpha 3 which is equal to delta bar square V infinity d U N d zeta which is equal to beta. Which means, that delta bar square by V infinity yeah well d is equal to beta correct, d u and d zeta.

Now as you can see here, so we can get an expression for beta. So, here it is equal to that, we can see these are equal because U infinity, U N we will set that U N equal to. So, we set that we was set U N equal to U infinity. So, if I do that when this the term becomes 1. So, this term basically is 1, so that makes alpha 2 equal to alpha 3, so that delta bar square by V infinity d U infinity d zeta, well you know so this U N is again equal to U infinity, so that is fine. So, by anyway I will write that down delta bar square again V infinity del U N d zeta is equal to B and I am going to call that seven.

So; now of course, so we got an equation, this is an important equations, so Falkner-Skan equation, so we get this.

This is a particular case, where we are considering a boundary layer with outer flow, and we set the U N zeta to this, and we set alpha 1 equal to 1, is just positive value and so we get something likes you know this expression. So, now the question is that Blasius observed that if the outer flow, observed power law, then we could have basically similar solutions. So, what this is actually from numerical considerations of the numerical solution that Blasius you know he observed that if the outer flow, so if the outer flow followed a power law, we could get similar solutions. So, lets us see whether it is we know we get something interesting or not. If that make any sense to us. Let me see what he means by that.

Now let us say which means that say that is say something like this, I am going to write it here itself. So, what you means is that if, U infinity by V infinity is equal to this is you know some constant. Zeta to the power m is trying to follow a power law. So, the outer flow right which is U and or U infinity by V infinity is B zeta to the power m and B is a constant and this m the power is beta by 2 minus beta. So, if I do that, so beta is something that we get from here. So, if we do that now if you see this equation 6, so now you see now from 6, so we can expand this integral basically. So, what we going to do is delta bar by V infinity, this is going to be U N d delta bar d zeta plus delta bar d U N d 0 is equal to 1.

Now, if you see equation seven, delta bar square by V infinity into d U N d z is equal to beta, which means if I were to open the you know remove the brackets here, so what I would get, see if I remove the brackets, this is this by V infinity. But then this term is actually beta, if you see from seven. So, if that is true then what can we write, so then what we can write, therefore, delta bar U N by V infinity d delta bar d zeta plus beta is equal to 1 or we can just say that delta bar yeah fine. So, we got this. Now this U N by v infinity; so now, again from, so we know that U N is equal to U infinity.

I were to so essentially what I have here is U infinity by V infinity which now is equal to b zeta to the power m. So, if I do that. So, therefore, then I can write here, so this bit or so what I can write is or delta bar so yeah so delta bar U N, so U N by V infinity this is nothing but beta zeta to the power m. d delta bar d zeta is equal to 1 minus beta. So, if I write this an again m is equal to beta by 2 minus beta, we got that there. So, if I do this, so now, we kind of got to do this, you know solution which is simple now.

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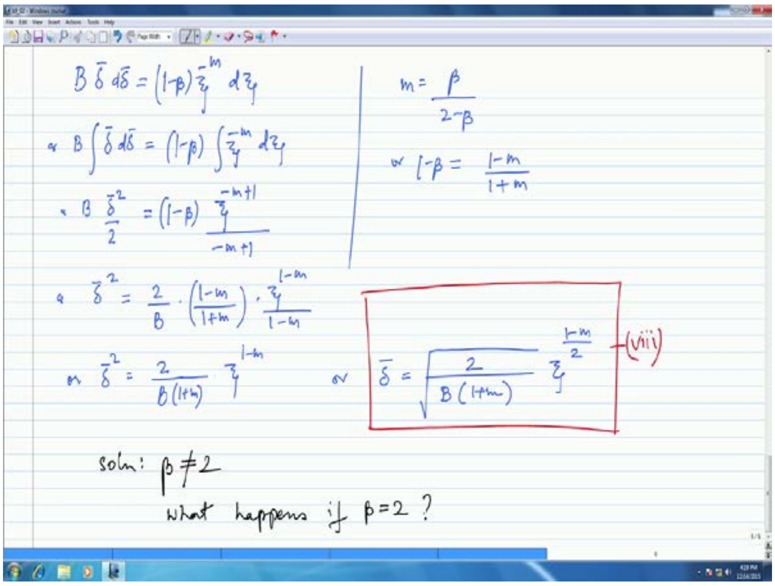

So, let me write it down and let me see if I can do this or I will post it for you. So basically what we get here some instead of rush to this little bit, I cannot spent too much time. So, d delta bar which is please note that, so this is d delta bar is equal to 1 minus beta by zeta to the power minus m d zeta. So, this is essentially integral equation which is coming like this. Now if I then integrate or b integral delta bar, d delta bar is equal to one minus beta integral is z to the power minus m d zeta, so or b delta bar square by 2 is equal to 1 minus beta zeta to the power minus m minus 1 by minus m plus 1. Or delta bar, so well I mean I can just skip a couple of steps, may be not, delta you know square is basically equal to 2 by b into.

Now, you see m, if you see this m is equal to beta by 2 minus beta or 1 minus beta is equal to 1 minus m by 1 plus m that I get just from there. So, you should be able to get that. So, then 1 minus beta is nothing but 1 minus m by 1 plus m into zeta to the power, 1 minus m, I think, I got that wrong. Did I get that wrong? Yes, I mean this should not been plus. So, then I get 1 minus m by 1 minus m. So, if I do that or delta bar square is 2 by b into 1 plus m into zeta to the power 1 minus m, or finally, or I take the square root delta is equal to 2 by b 1 plus m zeta, so zeta to the power 1 minus m by 2. So, that is essentially, therefore, this is my equation that is seven, so this is eight. This is a solution, the above solution is, so if I do, if I carry out the solution. So, the solution is the beta is not equal to 2. That something it is important right beta is not equal to 2, you can check that.

So, I would like you to just sort of think that, if this is the solution what is this even mean what happens. So, what happens, I would like you to think I can discuss this in the next module, but I would you know ask you this. So, let me know what happens this beta is equal to you know think. So, what we will do is in the next module is now that you we got an expression for this delta star. So, basically what we will get is you know a function of eta the like we did previous case for the flat plate right. So, we will get an expression for eta for y and x and then that should give us that should basically give us a solution. So, you will see where we end up doing that. So, we shall continue with this in the next couple of modules and we look into that.

So, let me kind of summarize. So, what essentially we did here is we looking at solutions to other than two bodies other than flat plate. And what we been able to do is, right now we kind of classifying problems. We do get you know an equation like four here. So, we do get an equation like four and it is an ODE. Now, what we do here is that you know it is a complicated equation. So, we kind of classifying it. So, one of the thing, there are two cases, one is when the boundary layer is with outer flow and the other is without outer flow.

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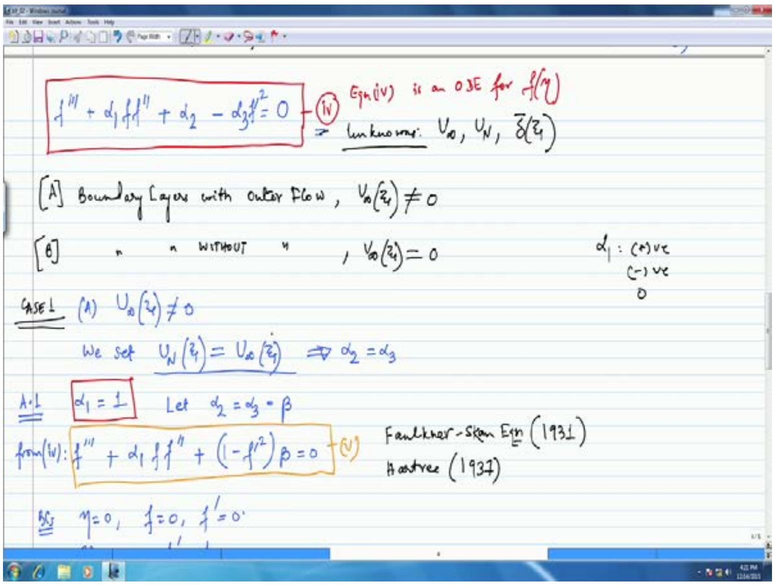

So, now, when I do that, so the first case is we know is a boundary layer is without outer flow and we set U N equal to U infinity here, and then we said alpha 1 is equal to 1. So, basically we will looking at solutions for alpha 1 to be positive, alpha 1 to be you know positive, negative or 0. So, the case we consider here is basically alpha 1 is positive and we say that equal to 1 without loss of any generality and then we continue so far.

So this is the equation, so what we land up with when we do that, so we get alpha 2 is equal to alpha 3 we will land up with a Falkner-Skan equation. And we have finally, come up with and we saw that and Blasius saw that if the outer flow base a power law, we could get similar solutions. So, what we landed up with an expression for the delta bar. So, now, we going to try and get a solution for this you know using getting into eta. So, we shall do that in the next couple of modules and I will stop for now.

Thank you.