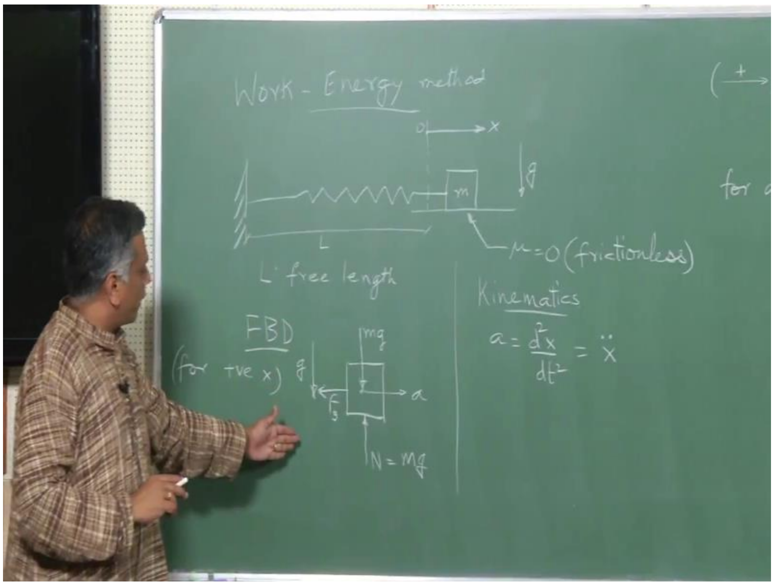
Statics and Dynamics

Dr. Mahesh V. Panchagnula
Department of Applied Mechanics
Indian Institute of Technology, Madras

Lecture – 35

[FL] We are going to continue our discussion of work energy methods and we are going to look at a different application today, where you have spring potential energy coming into play along with work done in a body as well as kinetic energy. So, we will start with the simple system and grow into a slightly more complicated system.

(Refer Slide Time: 00:24)

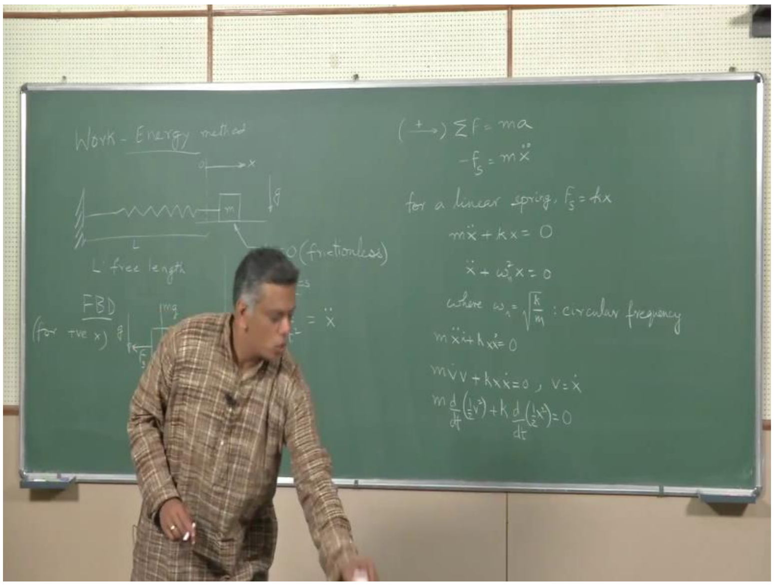


So, we are going to look at a work energy method with a spring, so let us take a very simple example, some mass m and this happens to be slight on a surface with no friction. I am going to place my origin at some point here, say that is my positive x axis and I am going to denote this length of the spring L as the free length. So, a spring typically has something called a free length and that is a length at which the force becomes 0.

So, the force on this body is 0 when the spring is of that length, but in this particular instance I am going to place my origin there and look at the position with respect to that free length. So, the mass is likely to be both to the right as well as to the left of this origin o and we will see what the forces look like on either side. So, I am going to draw a free body diagram of this body. So, there is a mass m center of mass, there is a weight of the body, because it is in a gravitational field, there is an equal and opposite normal reaction.

Now, where the spring is tagged, there is a force F and this body has an acceleration a. So, this is our free body diagram, so now, let us start with the kinematics part. The kinematics in this particular instance tells us that the acceleration is the second derivative of position which for compact a notation using compact notation I will use the idea of double dot., so a equals x double dot.

(Refer Slide Time: 03:14)



So, now, if I go through and complete the calculation, taking all forces to my right positive, sum of all forces equals mass times acceleration. The only horizontal force is the one due to the spring and I will call this F sub s, the only horizontal force is F sub s all vertical forces have already cancelled out. So, minus F sub s equals m x double dot, now for a linear spring F sub s equals k times x, the magnitude of F sub s is equal to k times x.

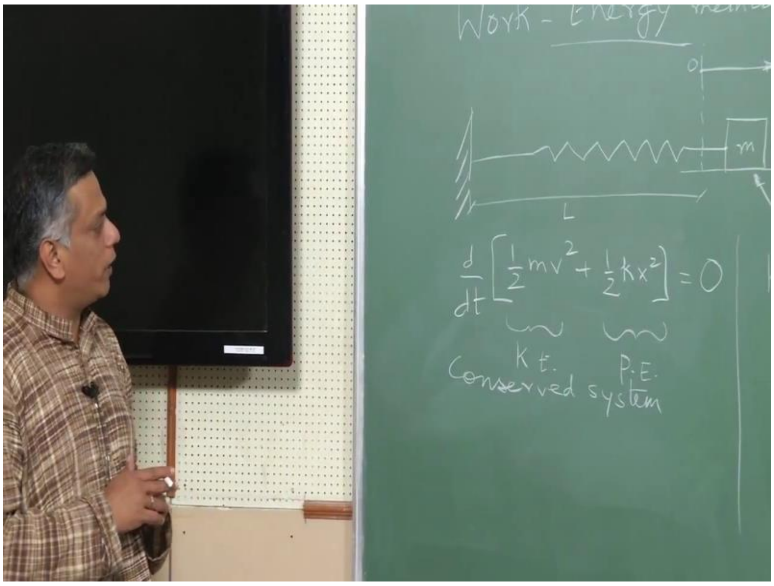
That means I have already taken care of the fact that is in the negative x direction, because the extension is in the positive x direction. So, I am going to place this for positive x, F sub s is minus k x and that is how I end up with F s be k x. So, if I make that substitution I will end up with the equation that says or I can write this is in the form of x double dot plus omega sub n square x equal to 0, where omega n is square root of k over m also called the circular frequency.

So, what… We started with force balance and then ended up with a situation, where I have this actual force acting on the body is k x is mass time acceleration. And if the body is to the right of the origin in the positive x space, the force is to the left. If the body is to the left of the origin, the force is to the right and that accounts for this kind of an oscillatory motion of this body on this friction less plane.

So, this is what we call natural oscillations of this body. So, in the instance where I let say the body is at the origin, there is no perturbation; there is no spring force, the body would remain there. If I pull the spring slightly and let go, it would undergo a cyclic oscillation and the frequency of that cyclic oscillation is given by this omega n equals square root k over m. So, we got this information from prime Newton second law. So, I am going to do, look at this in a slightly different way I am going to take at m x double dot plus k x equal to 0, I am going to multiply this by x dot.

So, if I multiply this by x dot what we end up showing is that m times v dot times v plus k x times x dot equal to 0, where v equal to x dot. So, I am going to simply make a substitution of variables here, I am going to call x dot as some v, I am going to only denote that in a first time and you will see quickly why I did that v times v dot is the… I can write it more compactly as the derivative of half v square, the time derivative of half v square; likewise, this becomes the derivative of half x square equal to 0.

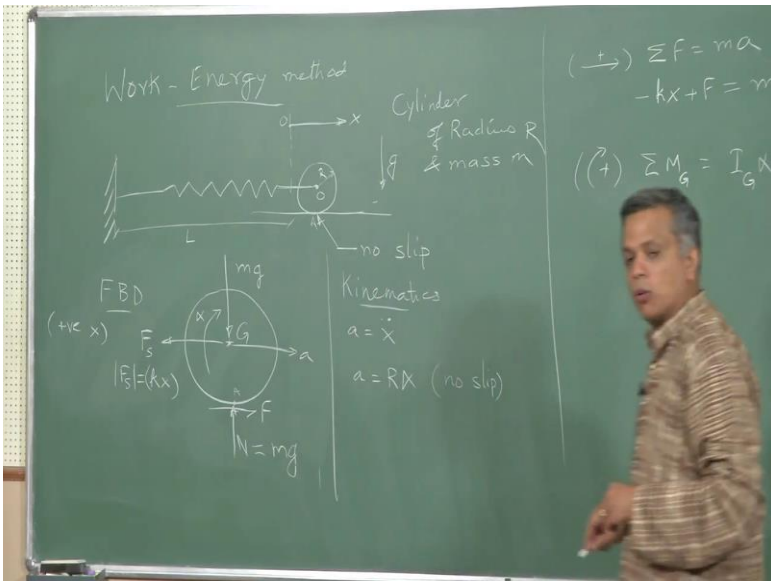
(Refer Slide Time: 08:01)



So, if I now come back and use that information here I can write that equation more compactly as the time derivative of half m v square plus half k x square is equal to 0. And what this mean is that the sum total kinetic energy plus the spring potential energy is constant. So, at the sum total energy kinetics plus spring potential does not change with time. So, this kind of a system is called a conserved system, so because it conserves energy, this is simplest case of an oscillated and undamped.

Because, there is no damping of energy, there is no convention of mechanical energy to any other form, this is basically a simplest example of energy stored in the mass in the form of half m v square and stored in the spring in the form half k x square interchanging the magnitudes. So, we are going to add a next level of complexity to this and let see what we learn.

(Refer Slide Time: 09:49)



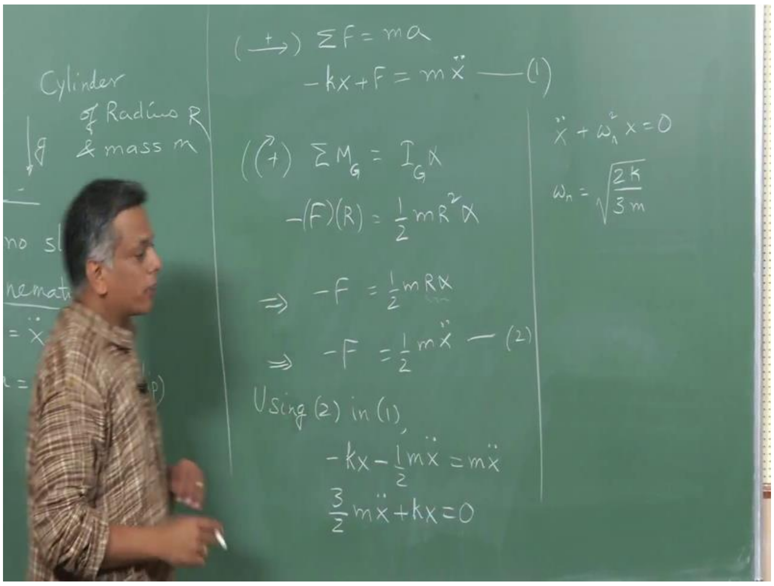
I am going to keep the rest of the system the same, except instead of a block I am going to use a cylinder that is on a surface with no slip. So, the spring is now connected to the center of a cylinder of some radius R. So, this is some radius capital R and this is sitting on a plane where the contact point between the plane in the cylinder shows no slip, we go and understand the dynamic of this, rest of it remains the same.

So, we will start by drawing a free body diagram like we will always done, this is my cylinder is the weight of the cylinder m g, there is a normal reaction N. I am going to draw this likewise said in the positive x although it does not matter. When I choose that for positive x, the only difference is I am able to give a direction to the spring force. Now, this one additional force here, because I have no slip at this point.

Because, I have no slip at this point I have friction and I am going to know which way the friction acts, I am going to assume it is in the positive direction and it is magnitude is some F. I have a linear acceleration of this point a, as well as an angular acceleration alpha. So, this object has an angular acceleration alpha as well as the linear acceleration a.

So, let us start by applying the loss of kinematics, first like we always done. Kinematics says two things, one let just like we had in the first instance a equals x double dot that does not change. In addition we also have another condition that a equals R times alpha that comes from no slip, so now, let us apply the laws of kinetics.

(Refer Slide Time: 12:42)



So, the first thing taking all forces to my right positive sum of forces is mass times acceleration. So, minus k times x plus F, this F is due to the friction force ((Refer Time: 15:56)) F sub s has the magnitude k x. So, minus k x plus F equals m x double dot, so this is what our force balance applying Newton second law in the positive x direction tells us. In the vertical direction all it tells me is that the normal reaction is exactly equal and opposite to the weight of the cylinder itself.

Now, I have a one more additional law which is our Euler's law that is sum of the moments about the center of mass G equals I G times alpha. Now, I must repeat this one last time once more I think it is worth it, if I choose moments about the point of contact I call this A I will have to be very careful. Because, that point a in this instance is addressed, but in general if it is not addressed or if it is in a non constant velocity mode I will get erroneous answers.

So, like I have always advocated, strict to this center of mass being your point of reference for applying moments, for applying the Euler’s second law and you will always be. So, let see what are all the forces that are responsible for a moment, the normal reaction, the mass of the body as well as the spring force all act through G. So, they do not causes a movement, the only forces that causes a movement is the friction force and that friction force has a moment arm of value R, I G in this particular instance is half m R square alpha.

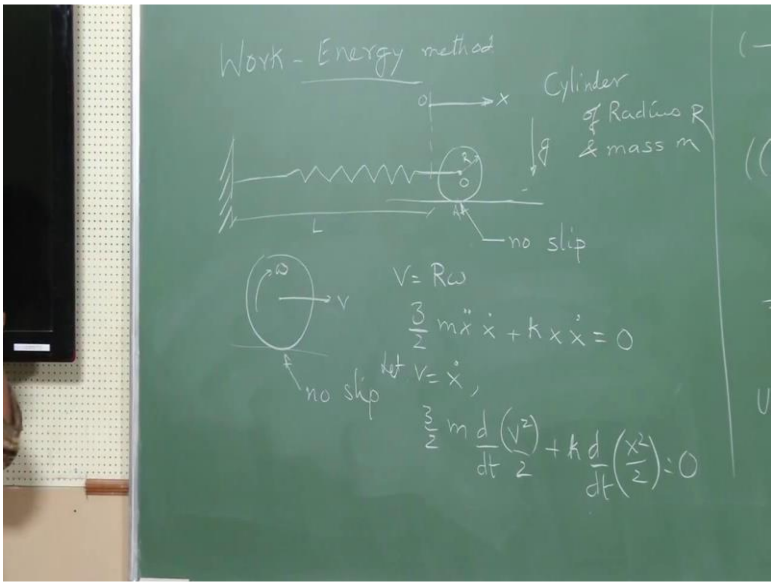
So, let us go to simplify this minus F a equals half m R alpha minus which implies minus F this half m x double dot. I am going to replace R alpha with x double dot, because a equals x double dot as well as R alpha. So, if this is my question 1 and this is equation 2, using equation 2 in 1 what do I get minus k x minus half m x double dot equals m x double dot and if I simplify this, I have 3 half m x double dot plus k x equal to 0.

So, when this object was a square was a just rigid object sliding on a frictionless in claim plane, I had this equation come out to be m x double dot plus k x equal to 0. Whereas, now I have this additional factor 3 half's and that is come, because this cylinder has a rotation to go with it now. So, this cylinder is translating as well as rotating about it is center of mass. So, the previous example have just the block sliding on a friction less plane was a single degree of freedom block that is it had only 1 degree of freedom to store kinetic energy.

Whereas, this mass which is the cylinder of radius R and mass m has 2 degrees of freedom to store kinetic energy. One it could translate and other it could rotate, both of those are ways of storing energy as far as the object is concerned. So, let us complete this calculation, so the first thing if I write this in the standard form of an undamped oscillator, we find that omega n now is 2 3rds has a factor 2 3rds under the radical.

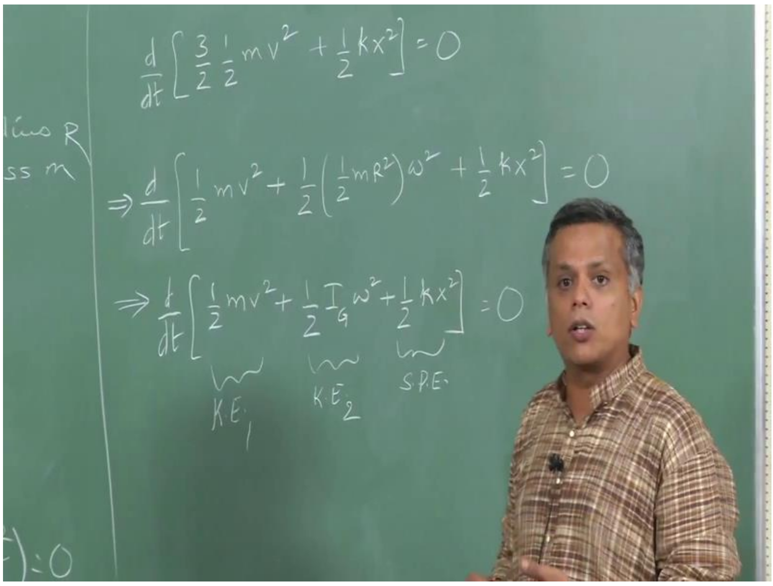
So, that is lower natural frequency for the same mass. Now, if I look at the instantaneous velocity of this body, so if the translational velocity of this bodies. So, let see we can do this same kind of work energy calculation for this as well.

(Refer Slide Time: 18:53)



If I have the cylinder moving with some velocity v and an angular velocity omega, because of no slip v equals R omega. So, the only way this body can move at any point is if it rotates correspondingly. So, if I do the same calculation as what I have, what I did before same kind of a argument we will find from multiply by x dot and making a substitution at v equal to x dot.

(Refer Slide Time: 20:29)



Now, this is still not in the form we wrote towards the end of the calculation the last time 3 half’s into half m v square plus half k x square equal to 0, I want to rationalize the origin of the number 3 half. So, what that tells me is that I can write this as 1 half m v square plus 1 half, half 1 half m R square omega squared knowing that r omega is v from here plus half k x square, the time derivative of this whole quantity in the square parentheses is 0.

So, now, completing this calculation we find half I sub G omega square plus half k x square equal to 0. So, here is an explicit to it to see that there is kinetic energy store in two forms, this is kinetic energy 1, this is kinetic energy 2 and this is spring potential energy. So, this shows of two things, one you can use Newton's law as well as Euler's laws to understand problems involved in vibration and you can also rationalize the energy content in each degree of freedom by converting the problem to work energy kind of energy for formulation.

Now, you will notice here as well that the time derivative of all the three forms of energy in this particular problem, the sum total energy does not change with time. So, this is also an example of a conserved system, now what do you understand one thing I do have friction at the point of contact. But, it is still conserved system primarily because that friction does no work, if there is no relative motion at this point and because there is no relative motion at that point, there is net destruction of mechanical energy.

So, essentially even though you have a friction, this is an example of a conserved system. So, the moral of that part of the story is that existence of friction force alone is not a bad thing, it is relative motion along with frictional force that is responsible for dissipation of mechanical energy. I hope this brat a flavor of the various kinds of dynamic problems that occur to you, I hope you enjoyed this class, I mean spin couple of months of fairly instance discussion both in statics and in dynamics.

And I hope you continue to learn from NPTEL and from all the other socials of information that are available to you, full free to write to us if you have any questions. And we look forward to register for the certification course, the last date is will extended it is now up to march first, you can register for the course and we believe it will add value to yourself, if you test for yourself your own abilities in a popular example situation. So, we invite you to register for the course and we invite you to learn along with us.

Thank you very much.