Statics and Dynamics
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Lecture - 03

Hello. Let us talk a little bit about equilibrium. This is a word that occurred when we were defining Newton's laws of motion. And, we want to understand what equilibrium is and how do we quantify equilibrium in statistics systems especially. So, when we discussed Newton laws especially the first law, we said everybody remains in a state of rest or uniform motion when no external force acts upon it. So, that in a sense is our definition of equilibrium that, a body at rest or in a state of uniform motion remains unchanged unless an external force acts upon it; which also means that, you require an external force to change the state from that point forward. Therefore, we are going to define a body at rest or in a state of uniform velocity as being exactly the same; and, that is what we will call equilibrium. So, let us define it.

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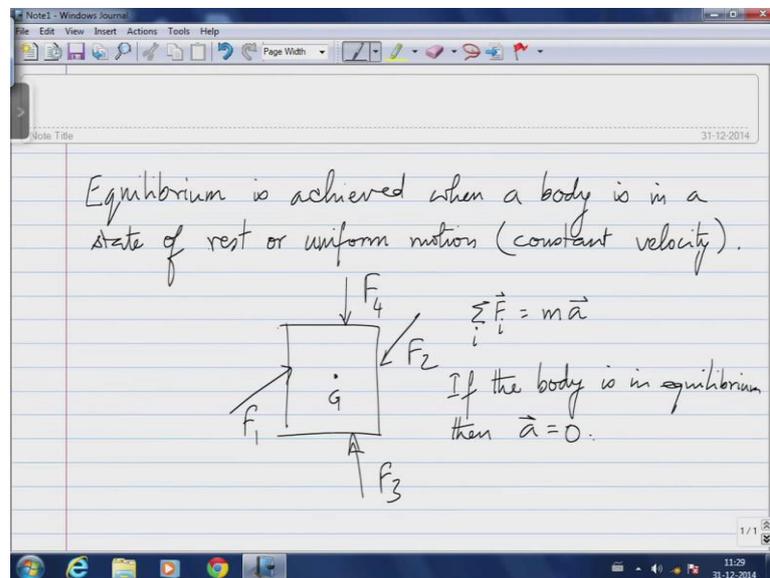
Equilibrium is achieved when a body is in a state of rest. Now, this I am sure you would not complain; neither would the people before Isaac Newton. His contribution like we discussed in the previous set of lectures is adding this second part that, a body in a state of uniform motion defined as one where we have constant velocity is also analogous. So, this is basically, if you think of it as primary contribution to the field of dynamics. To say

that, if force, which is a push or a pull causes acceleration and not velocity; when I push something, it acquires velocity and it changes position. So, there are if you think of it, three ways of – three levels of rationalizing this experimental observation. If I push, an object changes position that is true. If I push, an object acquires velocity. If I push, an object accelerates. These three are all true experimental observation, but what he showed is that, mathematically, the force exerted is professional to acceleration and not the first two.

So, now, that I have defined equilibrium as a state of rest or uniform motion, let us quantify what it would look like from Newton's second law. So, let us say I take a body; I exert forces on the body. So, let us say I have several of these forces. Now, all these forces acting on a body would create an acceleration of this body. So, if I add all the vectors sum; if I add the forces as a vector sum; that creates an acceleration of this body. Now, we are still dealing with small bodies. So, we will come back and define what a finite-sized body would look like and behave like. But, let us assume the body is quite small in that; all these forces are passing through some known point G. If I take the vector sum of all the forces acting on this body; that vector sum is equal to the mass times the acceleration of this body. If a body has to be in a state of equilibrium, then the acceleration is 0.

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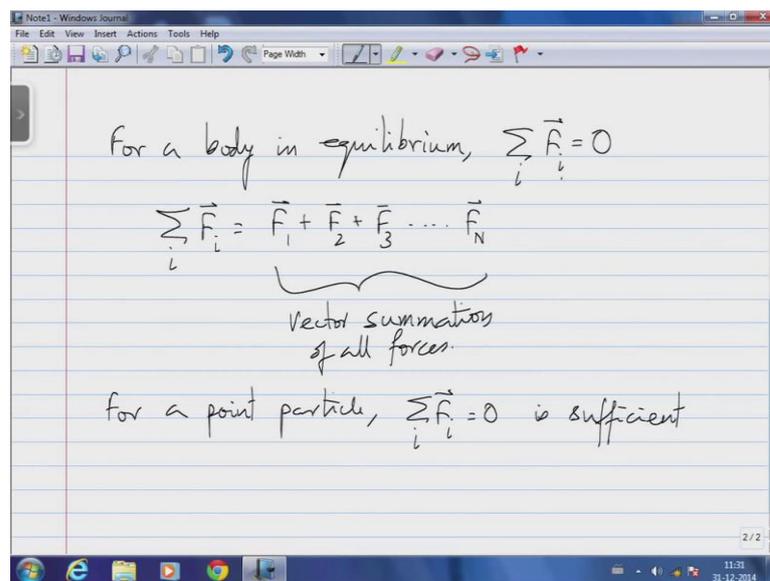
This automatically means that, for a body in equilibrium, summation of all the forces is 0. So, let us just make sure we are all on the same page with our notation. Summation over i F i is basically F 1 plus F 2 plus F 3 all the way up to F n if there are N forces and, this summation is a vector summation. So, this is our mathematical condition that we can use to identify whether a body is in equilibrium. So, for a point particle… is sigma F i equal to 0 is sufficient; that is, if the forces on a particle – all add up in a vector sum to 0; then, the particle is going to remain in a state of rest or uniform motion.

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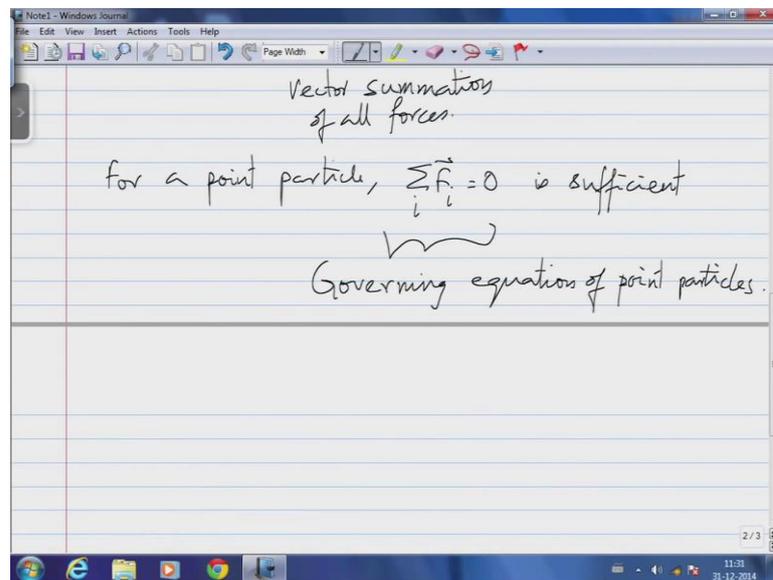
So, this… If we look at this; so, this is going to be our governing equation for point particles in static equilibrium. So, as we saw, this follows from Newton's second law of motion.
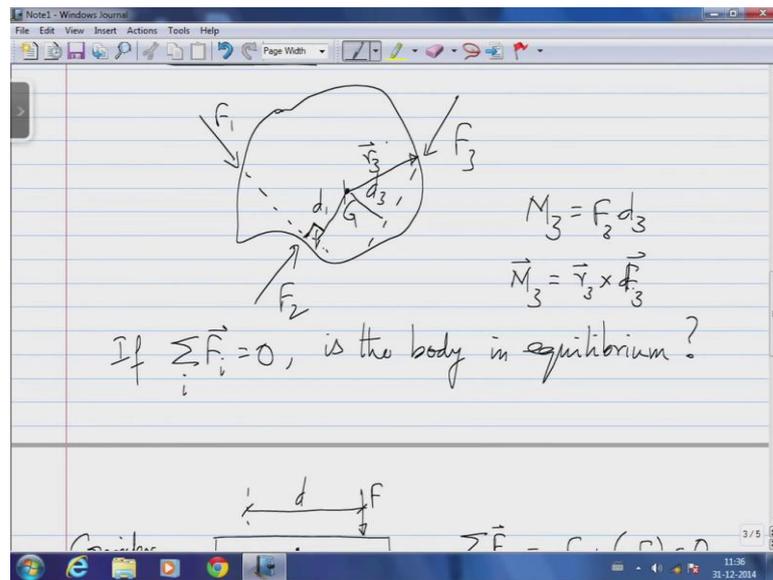
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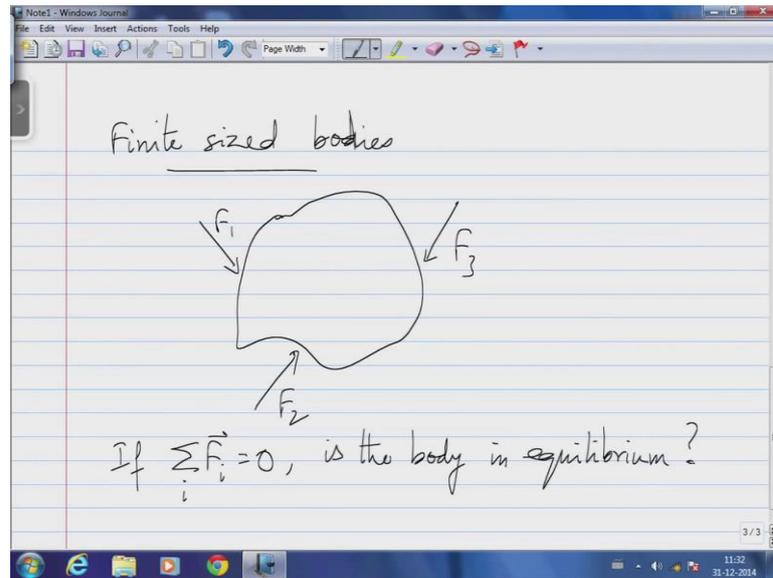
Now, let us move on and define what finite-sized bodies would behave like. So, almost none of our engineering systems are point particles. Although we will see where point particle is a good model of a real engineering system. We will come to that a little later. But, for now, let us see how to extend Newton's laws of motion to finite-sized bodies. So, let us say I have a body kind of arbitrarily shaped; and, there is a whole set of forces acting on this body. Now, if I have a finite-sized body under the action of multiple forces, what would the body do? So, if… So, let us take the case that we are familiar with from static equilibrium, which is saying, if sigma i in a vector sum is equal to 0, is the body in equilibrium?

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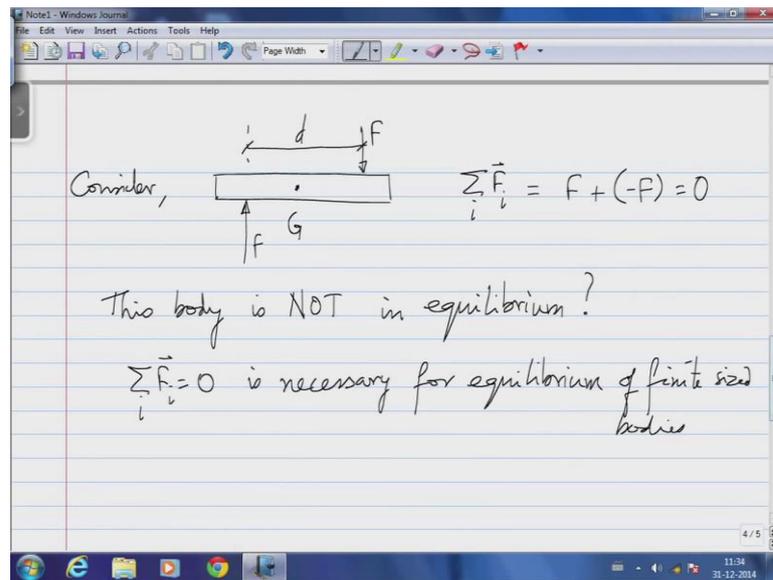
If we ask this question, the simplest way to answer this is to do a simple thought experiment. So, let us say I take a bar; I exert a force F and another force F separated at distance d apart. So, if I consider this example, what do I find? I only have two forces. So, sigma i F i is equal to F plus minus F. So, if I call this, both forces have the same magnitude F, but they are opposite in direction. So, if I choose one as being F, the other has a negative sign to it. So, therefore, summation of the forces is 0. But, would this body remain at rest or in a state of uniform motion? The answer is no. So, this body is not in equilibrium. Correct? Which also means that, while sigma F i equal to 0 is necessary for equilibrium of finite-sized bodies, it is not sufficient. What is sufficient is if I take the center of mass of this body is G.

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Let us say in here I define the center of mass G.

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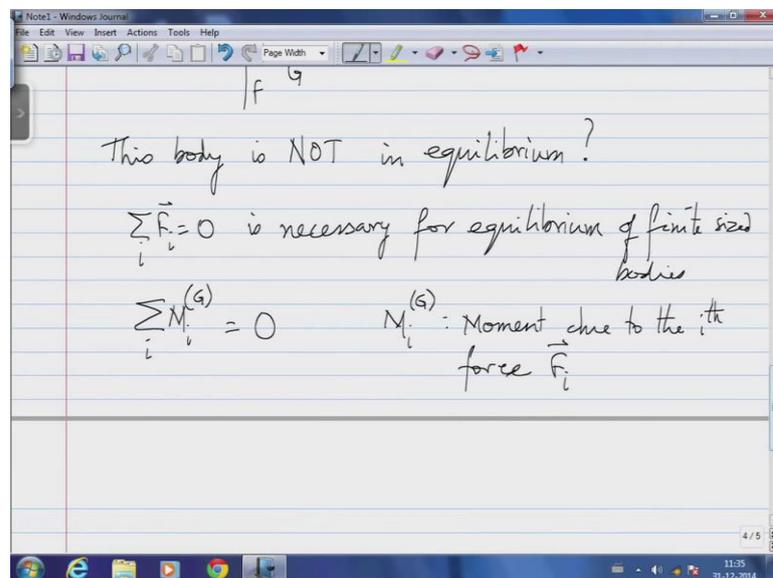
The summation of all the moments about the center of mass have to add up to 0. So, M i super script G is the moment due to the i-th force F i. So, how do I calculate a moment of a force about a point G? There are some simple ways. If I take this distance d 1, which is basically the perpendicular distance of the line of action F to the point about which I am interested in computing the moments. So, this here is d 3. M 3 is F 3 times d 3. So, this is in a planar sense, if the forces are all coplanar; then, the moment due to a force F about a

point G is simply the product of F 3 – the magnitude and the perpendicular – the distance between the point of interest and the line of action of that force. In a more generalized sense, I can define this as a vector as r 3 cross F 3. r 3 being the vector pointing from the point of interest to the point, where the line of action intersects the body. So, these are all ways of calculating the moments. So, now, essentially for equilibrium, apart from the summation of the force being equal to 0, we also have one more condition that, the summation of all the moments has to be 0. And, only when both these conditions are satisfied; that the body is neither translating nor rotating. So, it remains in a state of rest or uniform motion when both these conditions are satisfied together. So, this is a simple understanding.

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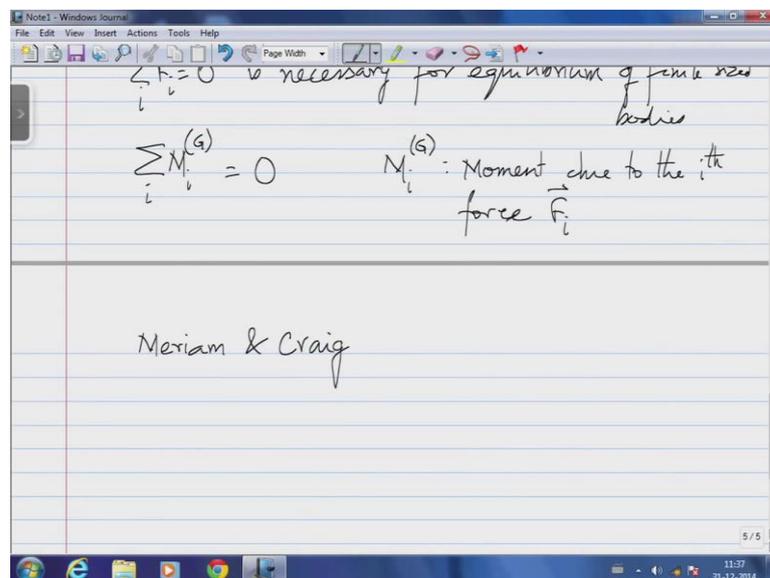
For more detailed discussions, you can refer to any standard text books such as Meriam and Craig; the first two chapters would discuss the derivation – a more rigorous derivation of these laws of equilibrium for static systems. But, this will suffice for the purpose of this course.